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“It’s Just Lines”: A Qualitative Analysis of Emergent Structures and Experiences within STEAM Education Initiatives for Secondary-Level Students

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Cover Page Footnote
I would like to thank all members involved in LASER, particularly Drs. Ashwin Vaidya and Bogdan Nita, as well as my fellow student ambassadors from the mathematics department that assisted in the logistics and planning of the event. I would also like to extend immense thanks and gratitude to Dr. Steven Greenstein, whose time, dedication, and feedback vastly enhanced the quality of this manuscript. Additionally, thanks to all of the reviewers, whose time and feedback contributed greatly to synthesizing the analysis of the data. Additionally, thank you to all participating students whose invaluable efforts and commitment to engaging in the workshop cannot be understated.

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Abstract

This qualitative analysis is oriented around the experiences and discourse of high school students participating in LASER (Linking Art and Science through Education and Research), an initiative at Montclair State University in Montclair, NJ that seeks mathematics and physics education reform through an immersive and innovative approach to STEAM (Science, Technology, Engineering, Art, and Mathematics) experiences. This work focuses on the concluding task of a one-day workshop wherein students are led on a campus walk to observe the local campus architecture, and then must recreate one of the observed structures within the Desmos graphing utility. Student approaches and narratives reveal a willingness to explore the complexity of the modeling task that might not otherwise be encouraged in their traditional learning environments. The analytical framework of the three worlds of mathematics proves to be a particularly useful tool in making sense of the complexity of student engagement. Implications for the future of STEAM education and interdisciplinary curricula are explored, with emphasis on the roles of creativity and a non-traditional learning environment.

Keywords: three worlds of mathematics, creativity, constructivism, STEAM, Desmos

1 Introduction

In many United States secondary school districts, an increase in STEM (Science, Technology, Engineering, and Math) education initiatives has contributed to a significant reduction, both with respect to funding and time allocation, in arts and humanities programs [10]. Often, maintenance of art programs at the secondary and tertiary level are dependent on both the size of the school and the magnitude of funding available to the district [7]. The false dichotomy between art and STEM is a phenomenon recognized across the world in varying sectors of society [4], and appears to be agnostic...
with respect to both eastern and western cultural ideals [8]. However, since at least the end of the twentieth century, there has been a call for an interdisciplinary approach to art education wherein content is filtered through its connections to other academic disciplines and programs of study [20]. This has led many to replace the STEM moniker with STEAM, which adds “Art” to the acronym. Educational programs that advocate for STEAM instead of STEM have produced significant outcomes with respect to enhanced student creativity and problem-solving skills (e.g. [5]), proving constructive in the rejection of narratives that consider the arts and humanities to have little to no impact on one’s ability to engage in scientific and/or mathematical thinking.

With respect to the STEAM movement, Allina [2] argues that education which promotes creativity “recognizes student learning diversity, increases student engagement and can potentially enhance Science, Technology, Engineering, and Mathematics (STEM) learning by embracing cross-cutting translational skills common to STEM and arts and design disciplines” (p. 77). Creativity, historically considered to be a driving quality leading to success within the arts [19], is observed to be an emergent and integral feature that coexists in the space of scientific inquiry and modes of thinking conducive to engaging in research and posing novel questions within the fields of mathematics and science [15].

While some have argued for the use of the arts as a means toward engaging students in mathematical exploration [6], this article pushes for the equitable coexistence of the disciplines that comprise STEAM without any one discipline taking priority over the other. Instead of using one as an access point for the other, I claim that weaving between and amongst the disciplines within STEAM without rigid borders allows for one to engage in exploration that is vastly greater than the sum of its parts. The constructivist theory of learning [21] asserts that perceived meaning is constructed by the learner and is inherently a subjective and richly unique experience. This perspective is highly enabling for one to conceive the inherent connections that exist among and between science and the arts, and lies at the foundation of the outcomes that become illuminated within this particular student experience with which I engaged.

2 Background and Context

The LASER (Linking Art and Science through Education and Research) initiative at Montclair State University “aims to reform the state of mathematics and physics education at Montclair State University through a STEAM experience which involves: innovative teaching practices in our physics and mathematics courses, involvement of students in research and development of new courses, all at the interface of science and art” [13]. In April 2023, this initiative hosted a “Math & Art Day”, an on-campus full-day workshop whose primary focus was to engage high school students in an abstract and holistic STEAM-based approach to mathematical thinking. Participating students came from a wide range of school districts across the state. The majority of these students were either freshmen or juniors in high school.

Out of forty initial responses indicating an interest in attending the workshop, a
total of twenty-three students attended and engaged in all of the provided activities. These involved students exhibited a wide variety of investment in the workshop, including but not limited to: leveraging mathematics and art in pursuit of social justice, and deepening their knowledge of how to apply mathematics to solve real-world problems. Many students identified mathematics as their favorite subject in school, with geometry emerging as a particularly favored subject. While students participated in a variety of engaging experiences, the final activity of the day was made up of three parts: first, (i) a 30-minute campus walk/scavenger hunt led by members of the mathematics department in which students were given images of various architectural structures to locate throughout the campus (see Figure 1):

![Figure 1](image1.png)

Figure 1: LASER Math & Art Day cohort embarking on campus walk (left); group of students engaging in scavenger hunt for architectural structures (right). Students consistently shifted their physical perspectives in order to uncover new information from these structures.

Second, (ii) a 30-minute discussion and demonstration of how one might use Desmos’s graphing utility in order to recreate architectural structures using functions, domain restrictions, and various plotted curves (see Figure 2):
And finally, (iii) a 1-hour workshop in which students were invited to select one of the provided photos and recreate the image in Desmos on their personal computers.

Students were encouraged to make use of any external tools they wish, such as calculators, paper and pencil, rulers, and anything else they might need. This final activity, according to the organizers of the workshop, was designed to enable students to find tangible examples of mathematics in architecture throughout the environment in which they are having these mathematical experiences to support an embodied perspective of STEAM engagement.

The qualitative analysis and methods that follow are applied particularly to (iii), with a central focus on student thinking processes, discussions, and the various stages of their Desmos output. While many mathematical visualization softwares would have served as excellent choices, Desmos is one that has well-documented support regarding constructivist mathematical thinking [14], creativity in modeling [17], and support of positive attitudes towards mathematics and learning [12].

3 Methods

I am a Mathematics Education Ph.D. student here at Montclair State University. Prior to enrolling in this doctoral program, I earned my M.S. in Mathematics (also at Montclair State University) with a concentration in Mathematics Education. I was initially invited to attend this workshop as a student ambassador for the mathematics department. My main responsibilities were logistical concerns, such as helping students and their parents find the location of the workshop. However, with the support of the two mathematics professors that oversaw the workshop, I served a secondary role of ‘interloping observer’. I spoke directly with the students throughout the entire day to gain insight into their personal academic experiences up to this point. Because I aimed
to have meaningful conversations with the students in order to establish rapport, this secondary role extends beyond that of an observer. Throughout this final activity of the day, I was amazed at the complexity of student considerations in their construction of a Desmos model. Despite having started the day without any specific protocols or observations, my goal was to think through the many possibilities of preserving the integrity of the students’ work while also uncovering the mathematical mindsets of students who presented themselves through narratives that served to explain their work. The two data sources analyzed were student-generated notes and Desmos outputs, and my own field notes collected through discussions with students and my own observations of their notes and Desmos outputs.

Sociocultural factors, among others, are implicit in the ways that students interact with academic content. One’s process of accumulating knowledge can be conceived as an inherently complex system that is multifaceted and influenced by a myriad of factors [9][16]. Such a perspective is empowering to the notion of the student as a unique individual capable of complex thought processes, which is a prominent tenet of modern inclusive educational practices [3]. In my search to find methods of categorizing student work and discourse, I came across the analytic framework of “three worlds of mathematics” [18], a protocol for analyzing the richness of demonstrative mathematical thinking. These three worlds of mathematics are:

- “Conceptual-embodied”: based on perception, action, and thought experiment.
- “Proceptual-symbolic”: compressing processes such as counting into concepts such as number.
- “Axiomatic-formal”: implementation of concept definitions and/or mathematical proof.

The premise of utilizing this framework is that successful STEAM engagement is determined based on the qualitative analysis of student actions, considerations, and response to prompts. Tall [18] identifies mathematical work to fall in the category of “Conceptual-embodied” if the work incorporates the student’s own body or first-person perspective as tied directly to a mathematical concept. “Proceptual-symbolic” work is achieved if students demonstrate the ability to come up with a system of efficiency that allows them to compress some process into a more efficient mathematical method. Finally, “Axiomatic-formal” emerges when students explicitly incorporate mathematical constructs, such as proof or definitions, into their work. This framework is helpful in that it provides a structured approach to analyzing student data in order to uncover the deeper mathematical processes that underlie the student experience. It is a tenet of this choice that such an activity affords the opportunity to find much more than meets the eye at first glance.

The intent is not to place labels on these students’ ways of thinking or experiencing. Instead, the intention is to use the “three worlds” as an analytical framework to interpret the work each student completes throughout the provided prompt. This perspective aims to provide not only support for the benefits of STEAM education initiatives, but also to advocate for an increased emphasis on the unique ways in which
learners construct meaning and contribute to their existing knowledge base through the variety of experiences and perspectives in this workshop’s common task.

4 Research Questions

The research questions that underlie the analysis of this student experience are:

1. How does the “three worlds of mathematics” analytical framework [18] illuminate the mathematical considerations exhibited by participating high school students in the concluding task of LASER Day?

2. In what ways are these student experiences indicative of the benefits of STEAM-based learning opportunities versus STEM-based learning opportunities?

5 Findings

STEAM Engagement via Three Worlds of Mathematics

Conceptual-embodied. This dimension of the three worlds affords a lens rooted in student perception, action, and experimentation of thought. While there exist philosophical claims that all thinking is embodied [11], many students in this workshop gravitated towards leveraging either familiar devices or their own bodies in order to make sense of the modeling task at hand. This manifested itself in the form of two primary qualities of the work: scaling (see Figure 3, left) and off-centered perspective (see Figure 3, right).

There was also an abundance of alternating between student work that was computer-based and working with pencil and paper. Many students reasoned that it “felt comfortable” to start with pencil and paper before translating these ideas to a digital environment, and that they “got a good sense” of the shapes in the architectural structures.

Figure 3: Exhibitance of conceptual-embodiment in the form of choice of scaling (left) and accurate modeling of angled perspective (right) from two individual students.

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in this mode. With respect to scaling, many students made sense of their global scaling by choosing a trial-and-error approach with the inner scaling of smaller objects in their chosen structure, such as windows or archways. Once these were designed to a satisfactory resolution, they were able to work their way outwards and reason what the necessary scaling of external outlines and larger objects might need to be.

With respect to accounting for angled perspectives, one student commented that they considered what the structure might look like if the photo were taken perpendicular to the structure, and reasoned that they might account for properties such as depth and angled images by altering the slopes of their constructed lines in either the positive or negative direction. One student, who chose to attempt to create a clock with moving hour and minute hands (see Figure 4), attempted to time the movement of these hands themselves in order to attempt to determine at what rate Desmos refreshes input values. While stemming from an embodied disposition, this also demonstrates the ability to translate precursory processes, or “phenomenal grounds” [1] into mathematical concepts, revealing the potential for student thinking to occur in multiple worlds at once. This is a feature that Tall [18] explicitly allows for, as it is the “interrelationship of worlds working together” (p. 9) that affords meaning-making in the presence of complex student work.

Figure 4: Student construction of a functional clock in the Desmos graphing utility environment.

**Proceptual-symbolic.** This dimension of the three worlds seeks to identify student behavior that compresses processes into concepts. In many cases, novel methods and ideas for constructing their chosen campus image to reconstruct in Desmos were abundant throughout the students’ dialogue. One student began their work with an archway as their reference point, whose outline was made up of two parallel lines and a curve connecting the two. Their initial struggle arose when they correctly deduced that the two parallel lines could be written in the form \( x = d \) and \( x = -d \) (for some real number \( d \) spaced evenly from the origin), but were unsure how to shorten the lines
into segments. They chose to come back to this and move on to building the curve that connects to the two lines at two evenly spaced points with the same y-value.

Despite the emphasis on constructing circles to represent curves (see Figure 2), the student felt that a parabola would be more fitting for an archway. They immediately wrote down the standard form of a quadratic equation with parameters a, b, and c, and knew that it needed to go through the points (0, 10), (-6, 8), and (6,8) based on their chosen scaling, but were unsure how to find the correct equation. When given some time to continue working through this problem, it was observed that the student decided to guess-and-check their parameters until the parabola appeared to fit the indicated points. When asked why they chose to persevere through trial-and-error, they responded that:

“I know how to do it [meaning solve a system of equations to obtain the parameters], but I didn’t feel like going through all of it … if it was a math test, I would.”

While the specific choice of curve deviated from the demonstration in (ii), this line of thinking demonstrates the students’ ability to bridge the observation of curves and straight lines to the mathematical concept of plotting vertical lines with an undefined slope, associating of parabolas with quadratic equations, and the need to construct piecewise functions at common points to maintain the continuity that appears in structures that surround us in the observable world. The student also displayed their ability to apply familiar content knowledge to a novel modeling problem, again scaffolding the link from observable native processes into mathematical concepts.

Another student’s work was initially centered on the construction of a clock that appeared on the front of a tower in their chosen image. Initial scaling of their model was oriented around reproducing this clock as accurately as possible. The two main areas of focus that the student spent considerable time constructing were the roman numerals on the face of the clock, and the moving hour and minute hands on the clock. When asked how they might start, the student made the simple statement, “it’s just lines,” in reference to both the numerals and the hands on the clock.

From here, they commented on how they can construct the numerals from straight line segments, and adjust their orientation around the clock by ensuring that their coordinates are based on evenly spaced positioning in a circular shape. With this line of thinking, they were able to construct relationships between point-slope forms of linear equations, the relationship between the slope of a line and the angle at which a segment rests, and the relationship that a circular curve shares with concepts in trigonometry, such as the tangent function (see Figure 4).

Moving to the construction of movable hour and minute hands, the student recognized that having the hands move in real time would be highly impractical, and likely imperceptible to anyone who looked at their model. Recognizing that the hour hand ought to move at 1/12 the rate of the minute hand, as well as the need to obey circular motion, the student constructed four total segments that incorporate the radian measure of a circle, related rates between the hands of the clock, and considering how
frequently Desmos refreshes parametric values to obtain a seemingly continuous output. The last of these points demonstrates the student’s understanding of discrete and continuous processes, and how computer precision might take discrete processes and apply them at a particular rate to simulate a seemingly continuous output.

**Axiomatic-formal.** This dimension of the three worlds allows for the identification of the role of definitions and/or proof in student engagement. While many students initially favored structures that made minimal use of curves, their perseverance in modeling revealed the affordances of curves and their mathematical properties. A handful of students considered how one might find a closed-form equation that plots a square, similar to the standard equation of a circle. This pursuit took a couple of different forms. One such output, obtained through trial-and-error, revealed that taking the framework of the standard equation of a circle but raising $x$ and $y$ to higher and higher even powers generated a shape that appeared to asymptotically approach that of a square. When asked why they think that might be, they responded that they felt it was related to the fact that a number in the interval $(0, 1)$ raised to a large power yielded an even smaller number. This construction, despite lacking some details, reveals an understanding of both limiting behavior and an insight into early Archimedean approximations of circles, and thus the value of pi itself. Another approach taken to constructing a square was the parametric construction of four line segments of the same length that join at the corners, forming an enclosed circle. The student commented that, “circles might be easier because I only need one equation”. This student then chose to start over with a new image that made more use of curves, and constructed these various curves by joining restricted portions of circular boundaries together to create the outline of the building (see Figure 5). Although they expressed concern about the lack of time remaining, they commented that they would “rather be correct with less than wrong with more.”

Figure 5: Initial stages of a student’s reorientation to the modeling task in favor of curve-based architectural structures. Here, the student has begun replacing a large set of line segments with a smaller set of curves to achieve a more precise image with a lesser amount of data.
6 Discussion: Emergent Properties of Student Engagement

Despite starting with volumes of data and being unsure of how to make sense of it, Tall’s three worlds of mathematics [18] framework has proven particularly useful in categorizing and evaluating higher-order mathematical thinking. This analysis is of particular interest in an interdisciplinary setting, as one might consider how conducive other disciplines might be in their pairing with mathematics/STEM in general, and how inspiration from an artistic lens might lend itself to the ability to be a mathematical thinker across multiple levels.

Students generally chose their architectural structure to model based on their own perceived convenience. This includes factors such as the ease of plotting rectangles versus curves, as well as the ease of plotting images that were taken perpendicular to the face of the structure versus at an off-centered angle. The three characteristics that appeared to have the largest modality across all involved students were: willingness to take risks, connection of modeling tasks to previous mathematical knowledge, and leveraging of complexity in varying forms to maintain accuracy in the translation from the provided images to one’s constructed Desmos model. This complexity can be exemplified by the work from two particular students. The first is that of implementing dynamic features into their Desmos model, such as a functional clock with moving hour and minute hands (see Figure 6, top). The second is the implementation of high-resolution granular detail to depict as much nuance in the image as possible, in total containing 227 distinct equations and/or restrictions on either the domain or range (see Figure 6, bottom). While both taking drastically different approaches to constructing two different structures, both exhibit layers of connections between different varieties of functions’ graphical representations in order to promote creativity and realism in their modeled reconstruction of local and easily observed architecture.

My impressions of students' initial approaches are sorted into three overarching themes. The first, emerging early on in their work, was the myriad of methods taken on by students who felt unsure of either how to start or how to proceed from an early state. Students who self-identified as struggling often reverted back to initial cues taken from (ii). They also requested printed versions of the Desmos code and plots from the demonstration in order to draw parallels between what had been constructed in (ii) and what they aimed to construct. These students displayed a willingness to initiate discussions with their surrounding neighbors, although they seemed unwilling to stray outside of the sphere of those that they knew prior to attending the workshop. Many students made use of online resources, such as MathStackExchange posts, other Desmos projects publicly available online, as well as videos published by educational content creators who discussed topics such as the relationship between line segments and curves, and the scaffolding from initial observations to rigorous mathematical modeling. The willingness to utilize external resources is conducive to an embodied perspective where students draw from their own and other’s perceptions in order to strengthen their resultant outputs.
Many students appeared to have an exceeding reliance on the precision of Desmos. In the graphing utility, coordinates are often truncated at three or four decimal places, even if those values are irrational or repeated decimals. This was interpreted by many students to be the exact value, and domain restrictions were often dependent on these approximations instead of utilizing the true value at these coordinates. For some, this led to errors in further calculations that were dependent on this interpretation, but there was little to no discussion on why this might be the case. Students seemed convinced that something was wrong, but were unsure of what specifically might be causing the issue due to their unwillingness to challenge the accuracy of these approximations. Future work in this area may anticipate such a perspective from students and generate follow-up questions that further probes reasoning that leads to a strengthened utility of function outputs or other mathematical expressions that preserves accuracy through multiple levels.

A final theme was the connections formed between a formal classroom environment and the informal environment of the LASER initiative. Students appeared to be aware of the need to leverage formal knowledge that had previously been acquired in their education, but this awareness manifested itself in multiple forms. One such form was the preference to stray from rigorous construction, such as solving systems of equations to obtain exact forms of models, justified by the reasoning that such expectations can be time-consuming and are only useful on graded assessments. Despite very different takes, most students agreed that the informal environment afforded the
opportunity for flexible thinking and experimentation in their outputs, and that taking multiple approaches to the same problem can reveal unexpected outputs. While traditional learning environments hope to instill this, it is of interest to the educational community as a whole how this outcome can be further instilled in learners in a more formal setting. Future studies hope to continue to investigate this dichotomy, as well as the surrounding reasons why a student might leverage one perspective versus another based on the perceived utility of a formal academic environment.

7 Conclusion

Tall’s “three worlds of mathematics” analytical framework proved to be helpful in making sense of the mathematical engagement displayed across all participating students in LASER’s “Math & Art Day”. Many of these students coexisted within the three worlds simultaneously, yielding rich data and high-level engagement in the given task as well as highly meaningful reflection. Students’ observations of local architecture lends itself to a novel experience in which participants were able to observe and how they might model such observations in a mathematical way. This particular STEAM educational initiative supported positive student perceptions of mathematics as a discipline and a framework for analytical thinking. Student discourse appears to be representative of complex thought processes and a favoring of interacting with mathematics through an interdisciplinary approach. This experience aims to lay the foundation for future work that continues to examine the complex relationship between mathematics and the arts, and how each might be leveraged as a lens into and within the other. While the “three worlds of mathematics” is one of many helpful tools to analyze student discourse and work, this choice showcases the underlying complexity of high school student engagement in mathematical modeling. This outcome is deserving of significant attention, and could serve to address the disconnect many students perceive to exist between their traditional schoolwork and this engaging STEAM experience.

8 Declaration of Potential Conflicts of Interest

I am both a doctoral student and adjunct instructor in the mathematics department at Montclair State University. Participation in this research effort was prefaced by strict adherence to university ethics policies, and in no way did my affiliation with the university have an evident adverse impact on the experience of the student participants, the observed results, nor the conduct of the event itself.

9 Acknowledgements

I would like to thank all members involved in LASER, particularly Drs. Ashwin Vaidya and Bogdan Nita, as well as my fellow student ambassadors from the mathematics department that assisted in the logistics and planning of the event. I would also like
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