A New Method of Measuring Financial Risk Aversion Using Hypothetical Investment Preferences: What Does It Say in the Case of Gender Differences?

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A New Method of Measuring Financial Risk Aversion Using Hypothetical Investment Preferences: What Does It Say in the Case of Gender Differences?

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ABSTRACT
Aversion to risk is one of the main factors driving investment decisions. Studies have been based on either simple decisions in a laboratory setting or real-life decisions viewed in retrospect. The study's main contribution to the literature consists of a new and elaborate method of measuring risk combined with a real-world investment task brought into a laboratory setting and show that in this controlled environment on average women are more risk averse than men. Unlike previous studies, the authors measure risk tolerance in units that naturally map into the risk-return space used by investors, giving them the missing tool to identify the optimal portfolio among the set of investment options that comprise the efficient frontier.

KEYWORDS
Risk aversion; Portfolio optimization; Mean variance analysis; Utility theory; Gender differences

Introduction

The standard approach to portfolio optimization describes in detail how to determine the portfolios along the efficient frontier and capital market line. However, this approach leaves it to the investor to choose the optimal portfolio from this set of possible portfolios. To do so, one needs to know her level of risk aversion. Once the level of risk aversion is known, one can solve the min-max problem, as one knows both the set of optimal portfolios and the set of indifference curves based on one's level of risk aversion.

Aware that investors should be offered more guidance than what is offered in the standard portfolio theory, in this paper we develop a method for measuring the investor's level of risk aversion. In essence, this gives the investor the missing tools to identify his optimal investment portfolio on the efficient frontier. This method requires minimal instructions to investors and has an analysis method that converts the investor responses into a measure of risk aversion. As a demonstration of the power of this method, we illustrate this tool by comparing the average levels of risk aversion of men and women.

Our approach is based on the evidence presented by Markowitz ([2014], see also Simaan [2014]) arguing that the mean variance framework is a robust model of utility maximization. For this reason we use the mean and variance to describe the investment space and use the mean and variance as the model on which we assess the risk aversion of each subject.

With due respect for the many experimental studies (see an overview by Smith [1991]) evaluating assumptions of the expected utility model, we believe that constant absolute risk aversion (CARA) is the only viable model that can be applied in the investment advisory world as portfolio decisions are often made without considering the full value of the investor's worth including real, physical assets and intangible assets such as education level and professional accomplishments. Only after valuing these attributes can one determine the investor's baseline asset level which is needed to assess their relative risk aversion or any decreases or increases in absolute risk aversion for any given dollar gain or loss in the investment value.

Subsequently we describe the literature on the underlying theory and its application to portfolio optimization, as well as how our study relates to other studies on variations in preferences in risk taking due to socioeconomic, personality and gender differences. Next, we describe our data collection procedure and analysis methodology to determine the risk aversion score. We use the method to analyze subject responses to measure how women and men differ in their level of risk aversion. We then describe extensions and modifications of the data analysis to incorporate models beyond the expected utility model.

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Color versions of one or more of the figures in the article can be found online at http://tandfonline.com/hbhf.

Literature review

Utility theory

The assessment of human decision-making has a long history, dating back to the expected utility (EU) model proposed by Daniel Bernoulli in 1738. In his work, Bernoulli assumes that the value of items we consume is determined by the utility they yield and the risks surrounding these choices. In sum, people faced with choices will be risk averse. Following Bernoulli's pioneering work and that based on marginal utility almost one century later, the latter being widely credited to the Austrian economist Friedrich von Wieser [1891], economists have come to assume that preferences are exogenous and stable over time.

The expected utility model was formalized by von Neumann and Morgenstern [1944]. Arrow [1963] and Pratt [1964] linked certain families of utility curves to the concept of risk aversion. Markowitz [1952, 1959] integrated the concepts of risk aversion into the process of deciding an investor's more preferred portfolio on the efficient frontier. Together they form an axiomatic system of preferences based on assumptions about the consistency of judgments across different levels of value. These are in turn extended to a normative model of investment decision-making based solely on the mean and covariances across a set of investment options.

Since then, decision science has often concentrated on finding fault with the expected utility model showing that individuals do not always act according to the paradigms of expected utility theory, which basically imply that individuals are rational and risk averse (Allais [1953], Ellsberg [1961], Kahneman and Tversky [1979], March and Shapira [1987]). According to these newer models, risk as perceived at the individual level of decision making cannot be neatly framed in the broad normative models of choice described by expected utility theory.

Instead of viewing these departures from the expected utility model as a debate between behavioral psychologists and experimental economists, we view it as seen by Smith [1991] where "most standard theory provides a correct first approximation in predicting motivated behavior in laboratory experimental markets, but the theory is incomplete, particularly in articulating convergence processes in time and ignoring decision cost."

With the evidence on mean variance approximations to expected utility in mind, we base our model on the arguments put forth by Markowitz [2014] in support of the mean variance model of investment and the CARA model outlined by Arrow [1963] in which he purported that risk aversion is constant as wealth increases.

We also view the simple binary decision tasks that have dominated laboratory research as more suitable to the short-term tasks undertaken by traders who need to make quick decisions in a timely fashion. However, this is only one side of money management. Another type of task is that faced by portfolio managers and sophisticated individual investors who needs to combine a multitude of small decisions into an overall assessment of investment opportunities. Their more complex decisions are also by nature less subject to the same time constraints faced by traders aiming to take profits on small price changes.

With this in mind, we designed a method that follows Vershoor et al. [2016], who showed that the closer a laboratory experiment is to a real-life situation, the higher the likelihood that a subject's laboratory performance becomes predictive of real-life decision making. Our approach also heeds the advice of Rabin [2000], who argued that not all laboratory results necessarily scale up to real-life decisions. Based on these 2 recommendations, we became cognizant that the best predictors of complex real-world decisions are those experiments that closely mimic real-life decision making such as those faced by a portfolio manager.

Last, we also wanted a method that could be used in both a laboratory setting as in the work of Allais [1953] and in field studies such as done by Gneezy and Potters [1997]. These views became an important aspect of the questionnaire we developed.

Risk taking is a function of socioeconomic and other personal factors

Individuals have different risk appetites. Risk taking has been measured as a function of socioeconomic factors such as age (Jaggia and Thosar [2000]), marital status (Sundén and Surette [1998]), level of wealth, couples with one income versus couples with dual incomes (Säve-Söderbergh [2012]), the holding of risky assets such as equities (Paas et al. [2007], Wang and Hanna [2007]), engaging in entrepreneurial activities that generally involve risky decision-making (Polkovnichenko [2005], Yao et al. [2004]), and the level of education (Kristjanpoller and Olson [2014]) among other self-reported factors.

For the most part, these personal attributes are found to be determinants of an individual's level of financial risk aversion or tolerance. For instance, all existing studies find that individuals tolerate higher levels of risk as their wealth, income, or level of education becomes more substantial. Risk taking, on the other hand, tends to decrease with age, especially with the onset of income uncertainty and increased health expenses during retirement. Marital status was also found to generally make both men and women more averse to financial risk, especially if such status results in children, due to perceived responsibilities toward offspring. But, studies that further dwelled into the significance of marital status found it less important, particularly for married couples with dual incomes. Apparently, they perceive a second income as insurance against the loss of their own. All in all, during the process of determining the impact of these socioeconomic factors on financial risk aversion, these studies have generally found that women generally choose more conservative investments than their male counterparts (for a contrary point of view, see Nelson [2012]).

Risk taking is a function of gender and testosterone levels

Proposing an alternative description of choice behavior toward risk, other gender studies explore how men's hormonal drive can interfere with good judgment and lead them into risky decision making (Wingfield et al. [1990], Dixson [1998], Mazur and Booth [1998], Nelson [2005], Coates and Herbert [2008]). This body of literature on this tendency, known as "testosterone investing," and sometimes more pointedly as "testicles investing," has explored gender differences toward financial risk in both laboratory and field studies and generally showed that women are more risk averse than men.

In sum, studies of both personal and testosterone factors have explored the relationship between gender and financial risk taking in a large body of literature. The following general conclusions can be drawn from this literature: (a) women generally take less risk than do men, as they tend to exhibit a greater perception of risk and consequently react with a more conservative response to risky financial situations; (b) risk-taking differences between men and women can also be context specific, especially in situations where these differences are dictated by socioeconomic factors; (c) risk-taking differences can also be shaped not by gender, but rather by a better understanding of the financial risks involved, a mental capacity that the literature associates with education, experience, and level of expertise; and (d) risk taking could be related to the amounts of testosterone versus estrogen in men versus women.

We explore this area using a methodology we believe to be more in tune with what women and men realistically face when making investment decisions. The methodology is described in the next section.

Methodology

Our experimental design is built to maximize its diagnostic power while testing for decision consistency in a setting that is familiar to portfolio managers. In this way we standardize the task to facilitate comparisons across individuals and measure their preferences on a standardized metric. We also present subjects with a task that would be familiar to investment professionals such as portfolio managers.

Potential investment set

The potential investments presented to subjects were selected to have returns and risks that would not be out of place for an investor in the U.S. markets. The potential investments also had the stimuli needed to cover a range of both returns and risk and be of sufficient number to be diagnostic across a wide range of risk aversion scores. For this reason, investment stimuli with annual returns between 0% and 15% and annualized volatilities of up to 20% were presented to the respondents. Ten investments were presented to each subject, as that number was deemed a good trade-off between sufficient diagnostic granularity and the time needed to complete the task. Subjects were asked to order the investments from most preferred to least preferred. Their ensuing rankings were used to measure subject preferences covering the range of risk aversion: from the least volatile investment regardless of returns to the highest-returning investment regardless of the level of volatility. Figure 1 shows an example of 1 of the 10 investment stimuli.

Each ranking of the 10 investment alternatives can be decomposed into 45 pairwise comparisons (10 investments picked 2 at the time). In each pair, the subject's preference for one investment over the other measures the subject's upper or lower bound of risk aversion. Each of these implied 45 investment decisions constitutes a cutoff point for how much the respondent values a return in relation to the risk associated with it. This is according to the definition and measure of risk aversion as derived from a standard expected utility model, which predicts that an investor's preference is based on a linear combination of risk and return. The model is outlined in the Appendix. Each of the 45 cutpoints determines the upper and lower bounds of the subject's risk aversion score, the space is therefore divided into a maximum of 46 regions for risk scores along the real time.

The pattern of investments presented to subjects is an important part of the experimental design. If the 10 investment alternatives are equally spaced along an arc consisting of portfolios on an efficient frontier as conceived by Markowitz, the most preferred investment is highly diagnostic of the subject's risk aversion. In some cases, this information is sufficient to predict the ordering of the other 9 investments.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
2111	−4.08	1.06	−4.23	5.68	0.34	7.41	−0.20	3.69	0.21	−4.46	8.51	−8.49	4.03
2110	9.20	3.40	−3.87	−2.03	8.20	6.19	−5.09	−4.70	−7.38	3.59	7.05	1.23	14.91
2109	−1.59	10.82	−3.45	−1.85	4.99	−1.92	−0.28	2.21	7.20	−0.91	9.25	1.60	27.88
2108	0.35	−1.26	3.51	−0.67	1.69	3.90	6.25	−3.01	3.01	−5.15	6.89	−3.51	11.78
2107	4.43	−2.16	2.74	1.34	−0.56	6.79	−4.10	4.94	−0.31	−5.23	4.89	1.30	14.14

mean	1.22
sd	4.62
skewness	0.0739
kurtosis	−0.9138
max DD	16.23
cum return	95.04

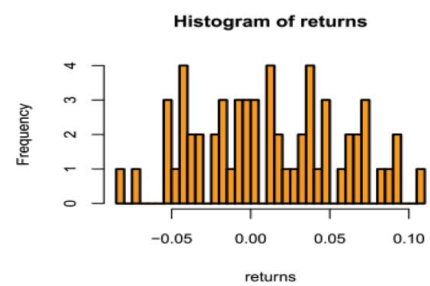

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Figure 1. Investment 1.

However, the efficiency of this design also makes it possible for subjects to order all the stimuli by either increasing (decreasing) risk or increasing (decreasing) return. Both schemes order the stimuli in the same way, and subjects would have reduced the problem from a 2-dimensional to a 1-dimensional decision task.

To challenge the subjects, the stimulus set shown in Figure 2 was designed to force them to make decisions considering both risk and return for all pairs by including investments to the right of the efficient frontier. Now subjects were pushed out of their comfort zone and needed to consider risk, return and the possibility that one investment stochastically dominates another.

Accordingly, 10 potential investments were presented to each subject. The layout for the first of the 10 investments is shown in Figure 2. Each potential investment was described by hypothetical monthly returns, supplemented by plots and statistics based on those returns. Information on each of the 10 hypothetical investments was presented in a form used by many portfolio managers, which includes summary statistics over a 5-year period, monthly returns, a graph of cumulative returns, and a histogram of monthly returns.

Presentation method

Each subject was presented with an answer sheet containing the numbers 1 to 10. To make it harder for the subjects to confuse investment 1 with their most preferred choice, they were asked to place an *M* next to the

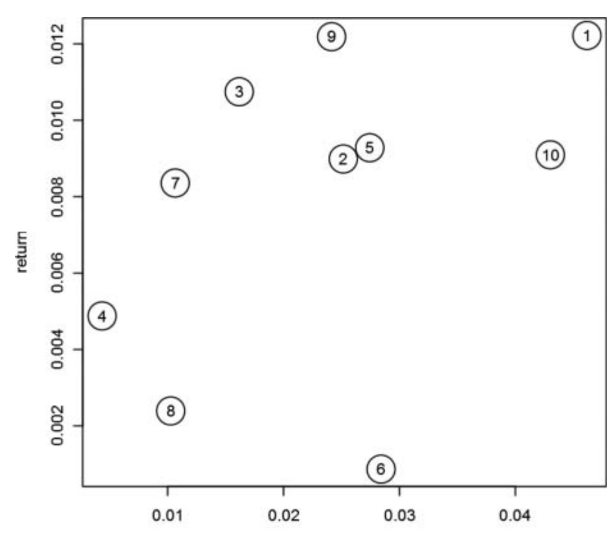

Figure 2. Risk-return space of investment choices.

investment they most preferred. They were asked to place a 2 next to the investment that was their second choice. And they were asked to assign unique numbers from 3 to 10 to each of the remaining investments, with a 10 indicating their least-preferred investment.

Each subject's level of risk aversion was measured by their ranking of the investments from most to least preferred. If the subject was concerned only with getting the highest possible returns without regard for the investment volatility, then that subject would most prefer investment 1, as indicated in Figure 2, which was the investment with the highest return, followed by 9, then 3 and so forth. The full ordering using this method was [1 9 3 5 10 2 7 4 8 6].

By contrast, another subject who was concerned only with minimizing risk would most prefer investment 4, as indicated in Figure 2, which was the investment with the lowest volatility, followed by 8, then 7, and so forth. The full ordering for this subject was [4 8 7 3 9 2 5 6 10 1]. Whereas the latter's investment choices were consistent with the greatest of risk aversion, the former can be considered risk neutral. Other preference orderings indicated that the subject evaluated a mixture of returns and risks to assess the attractiveness of the investment.

Data analysis methods

The subjects' responses were analyzed using the following 3 methods.

First analysis method: Guttman Scale

The first analysis method counts the number of times risky investments are preferred to less risky investments (the classification of each comparison is described in the Appendix). Using a Guttman Scale (Guttman [1944, 1950]) allows us to take advantage of the unipolar ranking of risk: herein, to capture the full spectrum of attitudes toward risk from risk seeking to risk averse and everything in between. The first method does not assign a risk tolerance coefficient to each subject, but merely counts the number of times the riskier investment is chosen or the less risky investment. In the face of such a challenging task, herein one would expect a certain number of inconsistencies in our respondents' answer choices, and this counting method does not require additional modeling.

Second analysis method: Closest match to a consistent risk ranking

The second analysis considers that there are 3.6 million ways to order the 10 investment choices that the respondents see, but only 46 of these orderings are consistent with the utility model presented in the Appendix. We assess the level of risk aversion by determining the best match of the subject's investment ordering to 1 of the 46 risk-consistent rankings. Each of these 46 orderings is consistent with the utility model as defined in the Appendix and corresponds to a specific level of risk aversion. To do this, we compute Kendall's tau correlation [1938][1,2] between the subject's ordering and each of the 46 possible risk rankings. Furthermore, we pick the ordering with the highest tau as the best fit. The risk aversion of this ordering is the subject's risk aversion level.

In the second method, deviations from the 46 risk-consistent rankings are assumed to be equally probable, so minimizing the number of deviations from the risk-consistent rankings is the best mapping of subject responses to their measured risk tolerance level. The additional modeling requirements are balanced by the ability to quantify each subject's risk aversion. Because this method assesses only 46 levels of risk tolerance, each subject's risk aversion is the mean value of all values within that level.

Third analysis method: Ordinal regression to determine the risk aversion

A third method utilizes a stricter model of the decision process. Here, each subject's evaluation of the investment choices is driven by a latent scale on which each investment is placed. The scale is derived from the quadratic expected utility that arises from a constant level of absolute risk aversion (Campolieti and Makarov [2014]:

$$u = \mu - \beta \frac{1}{2} \sigma^2 \tag{1}$$

Deviations from the 46 risk-consistent rankings arise due to noise in the decision process. This noise is modeled as arising from a logistic distribution around the latent assessment of the investments. A proportional odds, cumulative logit model (McCullagh [1980], Agresti [2002]) is fit to the data to estimate the values on the latent scale by minimizing the likelihood ratio that the subject ordering is consistent with the latent scale.

The third method utilizes the most model assumptions but also estimates each subject's risk aversion based on a statistical model of the errors. Deviations from any of the 46 risk-consistent rankings are incorporated into the estimate of risk aversion that allows the method to differentiate among the types of deviations and adjust the estimated risk aversion level accordingly.

Results

Using a classroom setting, 217 finance students, mostly undergraduates attending the school of business administration at Montclair State University, successfully completed the preference ordering. Students attending Montclair State University mostly come from within the state of New Jersey. They are diversified racially, ethnically and socioeconomically. Many of them are the first generation to be born in the United States and also the first in their family to pursue higher education. They also often hold several small jobs to finance their education while balancing academics.

Students were presented with an 11-page survey including the 10 investments on individual pages as illustrated in Figure 1 and given 15 min to complete the survey.

Approximately 250 students took part in the survey, but some responses were eliminated due to errors such as not indicating their gender or not ranking all the investments. Data were collected from 122 men and 95 women. We analyzed these responses by taking the ordering of the 10 investment possibilities and converting each ordering into 45 paired comparisons.

Each of these 45 pairwise comparisons indicates whether they chose a more-risky investment over a less risky investment. In essence, each pair is diagnostic of a level of risk preference indicating that the subject's risk aversion is either above or below the risk aversion coefficient referred to in the Appendix. These 45 cutpoints divide the risk aversion scores into 46 regions. Because these choices can be organized into a Guttman Scale (Guttman [1944, 1950]), each respondent's level of risk aversion is the number of times she chose a less risky investment over a riskier investment. The resulting risk score is the number of risky items from each pair chosen.

Study outcomes

We employed these 3 approaches to compare the level of risk aversion of the 95 women and 122 men.

In the first method, we counted the number of times risky investments were preferred to less risky investments. Of the 45 pairs, women on average chose 28.88 less risky investments with a standard deviation of 8.90. Men chose on average 26.47 less risky investments, with a standard deviation of 6.80, as shown in Table 1. An independent 2-sample t test results in t equal to 2.184 with 171.37 degrees of freedom and a 1-tailed p value equal to 0.015. This indicates that women were significantly more risk averse than men in terms of choosing less risky investments.

In the second method, we calculated a risk aversion score for all subjects by determining each subject's best fit to 1 of the 46 risk-consistent orderings. The subject's risk aversion score was that of the best-fitting risk-consistent ordering. For the 217 subjects, women had an average risk aversion score of 158.65 with a standard deviation of 424.94, while men had an average risk aversion score of 57.92 with a standard deviation of 275.12. Results of an independent 2-sample t test, t(152.74) = 2.006, 1-tailed p value = 0.023, indicated that women were significantly more risk averse than men in terms of their risk aversion score.

In the third method, we ran an ordinal regression to determine the risk aversion score. Women had an average risk aversion score of 40.164 with a standard deviation of 200.093, while men had an average risk aversion score of –0.526 with a standard deviation of 59.993.

Results of an independent 2 sample t test, $t(107.21) = 1.916$, 1-tailed p value = 0.029, indicated that women were significantly more risk averse than men in terms of their risk aversion score.

The results are summarized in Table 1.

The means and standard errors by gender for each of the 3 analyses are also summarized in Figure 3.

Table 1. Survey results by method of analysis.

	Method 1	Method 2	Method 3
Female	28.874	158.652	40.165
Female SD	8.903	424.943	200.093
Male	26.467	57.925	−0.526
Male SD	6.802	275.121	59.994
t	2.184	2.006	1.916
df	171.371	152.736	107.213
p	0.015	0.023	0.029

Discussion

We have shown that the 10-investment ranking task can be used to measure a subject's risk aversion using 3 different approaches to analyzing the results. We have also

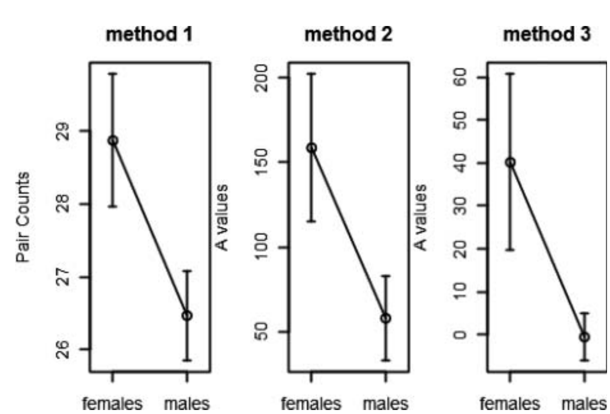

Figure 3. Means and standard errors by gender and method of analysis.

shown that the risk measures are sensitive to the differences in perspective between women and men.

Advantages of the data collection method used

The method we used to collect the data has several advantages over other data collection methods. First, the method calculates a measure of risk aversion derived from the Arrow-Pratt definition of risk that is consistent with the measure needed to calculate the most-preferred portfolio on the efficient frontier. Second, the task's demands match those faced by financial professionals such as portfolio managers. This means this questionnaire can be taken by subjects with some financial experience as well as experienced investment managers. Third, this questionnaire does not require that the respondent have a track record in investing or information on prior decisions.

Unlike many of the studies cited by Nelson [2012], our study shows how the average preferences for women and men differ when evaluating specific investment decisions. For instance, the number of times the riskier investment was chosen for each of the 45 pairs can be tabulated for men and for women and are displayed as points in Figure 4. If the percentage of women preferring one specific investment over another equals the percentage of men preferring the one over the other, then the pair is represented as a dot on the 45 degree line. In the figure, the majority of dots fall below the 45° line, indicating that men more frequently chose the riskier asset over the less risky asset in the pairs. The degree of difference in male and female percentages is indicated by the dashed lines showing the 1% 2-tailed confidence intervals around the 45° line. In the most extreme case, 61% of men preferred investment 10 over investment 8. By contrast, only 34% of women preferred the riskier investment 10 over the less risky investment 8.

Despite its successful application in our approach, expected utility theory has its critics. The analysis of the ranking data can be augmented to incorporate these concerns as the richness of the subject responses can be used to assess the robustness of the expected utility model.

Expanding on EU Theory

One way to extend the model is by adding terms to the linear model we refer to in the following equation as "other risk factors."

$$u = \mu - \beta \frac{1}{2} \sigma^2 - \textit{other risk factors} \tag{2}$$

More specifically, Machina [1982], Yaari [1987], and Quiggin [1991, 1993] incorporated a subjective transformation to the tail probability distribution as well as the utility function in the expected utility model. In terms of Quiggin's presentation, the cumulative probability mapping is a function f: $R \Rightarrow R$ where f(0) = 0 and f(1) = 1. However, a remapping of objective probabilities to subjective probabilities using an inverse normal transformation, such that

$$f(p) \; = \; G^{-1}(F(p)) \tag{3}$$

where $F \sim N(0,\sigma^2)$ and $G \sim N(0,\beta^2\sigma^2)$ as in Figure 5 leads to the same 1-factor linear model as that derived from the expected utility model. In that model, the interpretation of the beta coefficient becomes the standard

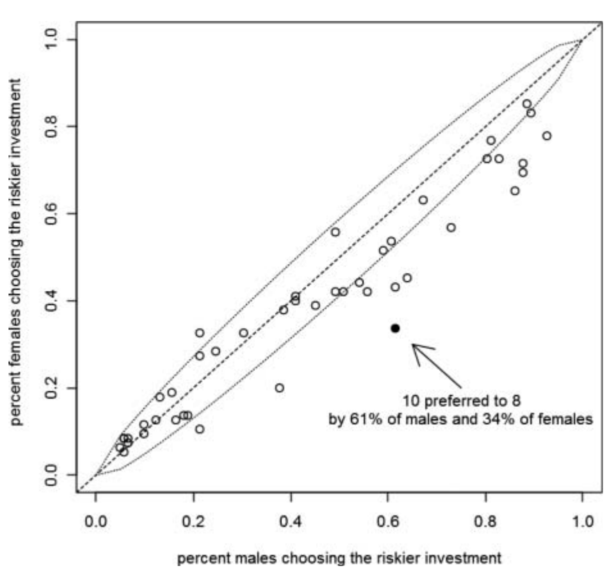

Figure 4. Percent choosing the riskier investment.

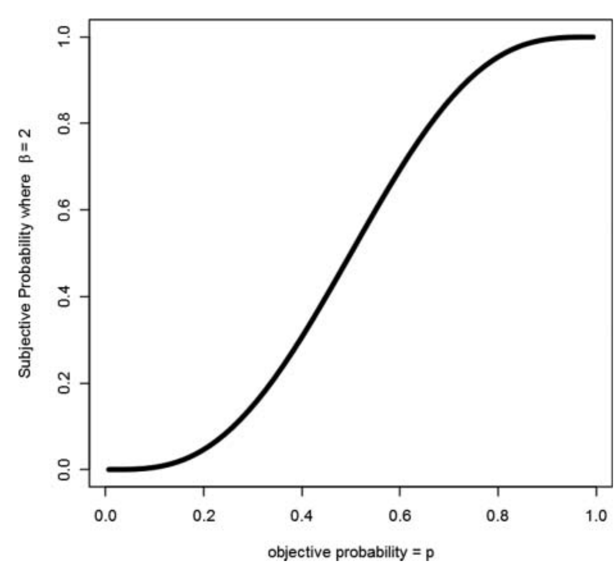

Figure 5. Probability map from objective to subjective probability.

deviation of the inverse normal transformation of the probabilities. In this interpretation, β, is the shrinkage factor for the subjective variance of the distribution as opposed to the curvature of the utility function.

Evaluating the robustness of the mean variance model

One of the foundational assumptions of the mean variance model is that investors will prefer an investment with a higher return and a lower volatility to one with a lower return and higher volatility. The higher returning investment with the lower volatility is called dominant and the lower returning investment with the higher volatility is called nondominant. The experimental design (Figure 2) includes some investment pairs that were dominated in the mean variance space. These included investment 6, which was dominated by all investments except for investments 1 and 10. Figure 6 shows the dominance relationships when one investment had a higher expected return and lower standard deviation than another investment.

The mean variance models assumes that subjects will choose a dominant investment over nondominant (dominated) investment. To see whether this pattern holds true for the 217 subject rankings, we counted the number of times subjects chose the dominant investment and the number of times they chose the nondominant investment.

If the subjects are using the means and volatilities of each pair of investment while creating their preference orderings, then the dominant investment should generally appear higher in the ranking than the nondominant investment. This is supported by the number of times in which the row investment is chosen over the column investment in Table 2. White cells represent the number of times the dominant investment is preferred to the nondominant investment and the cells where the nondominant investment is preferred to the dominant

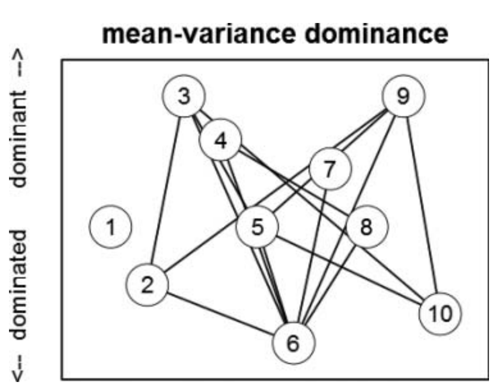

Figure 6. Mean variance dominance.

Table 2. Mean variance dominance.

	1	2	3	4	5	6	7	8	9	10
1		125	68	102	121	187	92	167	52	172
2	92		***35***	108	104	202	83	170	***57***	181
3	149	182		143	180	205	142	188	129	196
4	115	109	74		117	202	96	194	101	152
5	96	113	***37***	100		205	89	167	***57***	181
6	30	***15***	***12***	***15***	***12***		***15***	***33***	***16***	44
7	125	134	75	121	128	202		189	109	185
8	50	47	29	***23***	50	184	28		42	110
9	165	160	88	116	160	201	108	175		190
10	45	36	***21***	65	***36***	173	32	107	***27***	

investment are in light gray cells with bold and italicized numbers. Dark gray cells represent comparisons in which neither investment dominates the other, and the ordering given by any subject is based on the subject level of risk preference.

The distribution of the cell counts shown in Table 2 are represented by the 3 histograms in Figure 7. Decomposing the 217 rankings, subjects chose the nondominant investment over the dominant investment (as indicated by the italicized numbers in Table 2 and the bars in the middle graph in Figure 7) 411 times. As there are 15 dominated pairs, the expected count is 1627.5 [15*217/2] with a standard deviation of 44.9. So the observed numbers are significantly below the number expected if subjects were randomly reporting their investment rankings.

In an alternative analysis, we constructed a bootstrap sample in which we randomly renumbered the

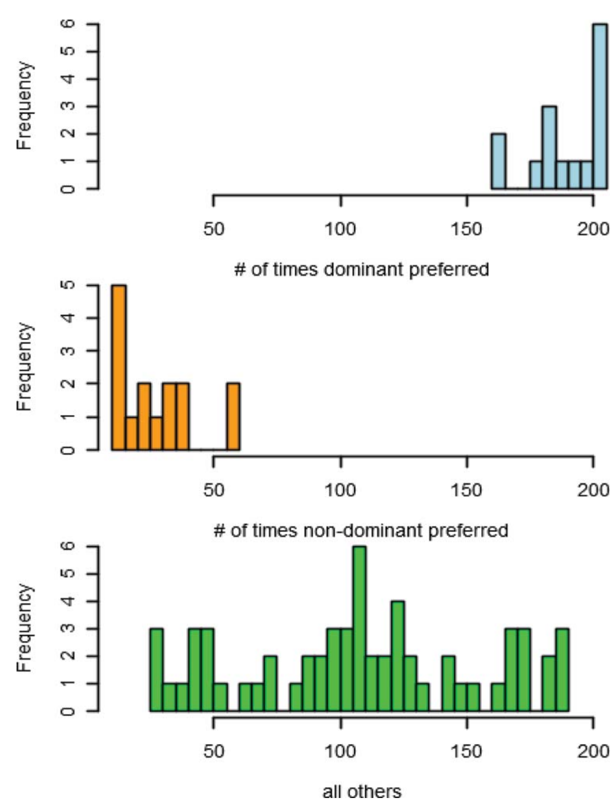

Figure 7. Investment rankings decomposition.

investment choices and sampled 217 subjects with replacement to create a sample in which the 217 subjects had consistently mislabeled their investment rankings. We counted the number of violations of dominance from this set of rankings. We repeated this procedure 100,000 times to estimate the mean and standard deviation of the distribution of nondominant counts selected. The mean number of counts was 1840.9 with a standard deviation of 28.7. Once again, the 411 observed violation of mean variance dominance was far fewer than the expected value of 1840.9.

From these 2 tests of the mean variance judgment space, we know that subjects are making judgments consistent with the predictions of the mean variance model.

Capping the capital asset pricing model: Closing the loop

An investment-based approach to measuring risk aversion also fits well with the portfolio construction methods for the capital asset pricing model (CAPM) model proposed by Markowitz, in which a rational investor's goal in creating a portfolio of assets is to generate the maximum return for the level of risk with which this investor is comfortable. The efficient frontier is the set of optimal portfolios that offers the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. For any investor, the optimal point on the efficient frontier is dependent on one's appetite for risk.

Because risk is in the eyes of the beholder, the optimal portfolio is the min-max set in which the investor maximizes utility while minimizing risk for the investments on the efficient frontier. Hence, the individual investor's preferred portfolio is only specified once the investor's utility function is known. Here, the investor's utility function is determined by our method of ranking investments. We translate the ranking into the risk aversion score that can be applied to the CAPM model.

The standard approach to portfolio optimization describes in detail how to determine the portfolios along the efficient frontier and capital market line. However, this approach leaves it to the investor to choose the optimal portfolio from this set of possible portfolios. To do so, one needs to know one's level of risk aversion as expressed by beta. Once beta is known, one can solve the min-max problem, as one knows both the set of optimal portfolios and the set of indifference curves based on one's level of risk aversion.

Our data collection process and analysis method determine a risk aversion score that fills this gap by unambiguously telling us what the individual's optimal portfolio is based on one's risk-return preferences. This optimal portfolio is the min-max solution of the intersection between a minimization of the risk for a given return and the maximization of one's utility. Indeed, by asking our respondents to rank their investment decisions, we can determine the parameters of the investor's utility function from their preferences without needing to refer to noninvestment variables such as their lifestyle, education, level of wealth, goal horizon, etc. This is unlike anthropological and other experimental studies referred to in the review of the literature, which use these socioeconomic variables as important screening tools for determining who should be a participant in a study.

Illustrating the usefulness of the study's aversion scores

As an illustration of the usefulness of aversion scores, we show how one particular subject's answers are translated into indifference curves for determining the optimal investment on the efficient frontier. This subject gave the top rating to investment 9, the second rating to investment 3, and so forth. The subject's ratings for the 10 investments can be summarized by the list [6 4 2 7 5 10 3 9 1 8]. Running an ordinal regression (see method 3) on this list with the variance and return as the independent variances yields a risk aversion score of 4.97. The risk aversion score determines the subject's preferences for each investment in the risk-return space, so the subject's pattern can be summarized by the preference contours in Figure 8. In that figure, the subject prefers investments that are to the left (low risk) and higher (high return) in the space.

Given the subject's aversion score of 4.97, we can determine the subject's optimal portfolio on any efficient frontier. In Figure 9, an efficient frontier appears in red

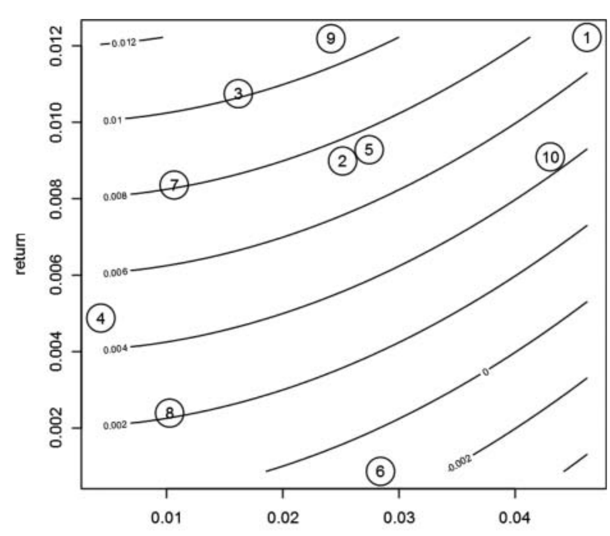

Figure 8. Utility curves for A = 4.97.

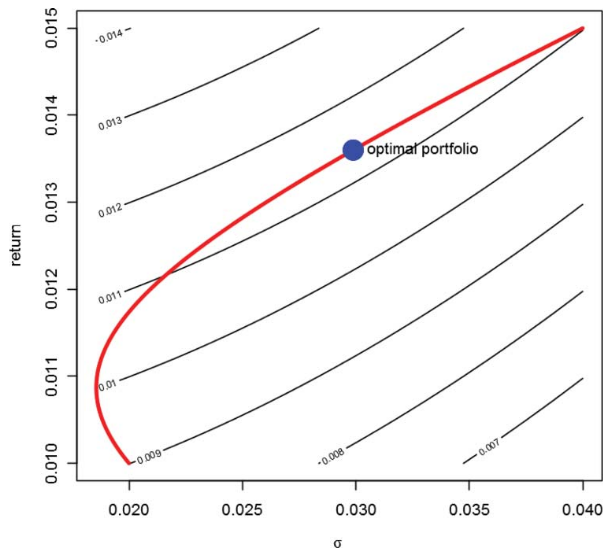

Figure 9. Utility curves for A = 97.

and the subject's optimal portfolio has a return of 0.0136 and risk of 0.0299. This is the point on the efficient frontier with the highest expected utility. With this score, we can now tell which of the portfolios on the efficient frontier represents the optimal portfolio for this particular subject.

Concluding remarks

This study provides a path to determining the optimal portfolio on the efficient frontier for any given investor depending on his or her appetite for risk. To obtain the greatest predictive power, we designed a laboratory task that was verisimilar to decisions made by a portfolio manager. The subject's rating from this task was translated into a risk aversion score via an ordinal regression. The risk aversion score can then be used to select the optimal portfolio on the efficient frontier calculated from a selection of investment options.

Our method of data collection can also be used to compare levels of risk aversion across subjects. We employed 3 different methods for comparing risk aversion level: counting the number of risk-averse decisions made within each subject ratings, matching each subject's ratings to the rating for levels of risk aversion, and regressing (using an ordinal regression) the ratings on the risk and returns for each investment option. Using these 3 risk aversion comparison methods, we showed that within our subject population, women were more risk averse than men.

We compared differences in risk aversion by women and men, but the method can also be expanded in future research to the comparison of groups beyond gender. It does not need to be calibrated by country, language, lifestyle, or other variables. Also, anyone from a novice investor to a professional portfolio manager can use our method to create a ranking of investments according to their preferences while selecting the information that they feel is most relevant for their investment decision.

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Notes

1. We use a rank-order correlation to measure the similarity between the subject's ranking and the 46 risk-consistent rankings because the subjects' responses are on an ordinal scale.
2. Kendall's tau (Kendall [1938]) is a rank correlation based on counts of the number of concordant and discordant pairs across the 2 rankings. Each tau statistic represents a goodness of fit between the subject's investment ordering and 1 of the 46 risk-consistent rankings.

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Appendix

Analysis of the subject preferences is based on a model of how investors value an investment. The model translates each pairwise preference into either an upper or lower bound of a coefficient of risk aversions. By breaking the rank ordering into pairwise preferences, we can determine the range of risk aversion values that are consistent with the full rank ordering of the investments.

The model assumes that each investor has a utility U_x for each investment x. If $U_x > U_y$, then the investor prefers investment x to investment y. Assuming investors have a constant level of Arrow-Pratt absolute risk aversion (Campolieti and Makarov [2014]),

$$r(x) = \frac{u''(x)}{u'(x)} \tag{4}$$

the utility for each investor is determined by a weighted sum of the investment return and the risk with investors preferring higher returns ($\beta_r > 0$) and usually preferring less risk ($\beta_\sigma < 0$):

$$u = \beta_r r + \beta_\sigma \frac{\sigma^2}{2} \tag{5}$$

For each investor, their risk aversion is ratio of the betas:

$$A = -\frac{\beta_\sigma}{\beta_r} \tag{6}$$

If the investor chooses investment x over investment y, this indicates that:

$$\beta_r r_x + \beta_\sigma \frac{\sigma_x^2}{2} > \beta_r r_y + \beta_\sigma \frac{\sigma_y^2}{2} \tag{7}$$

If we assume:

$$\sigma_x^2 > \sigma_y^2 \tag{8}$$

Choosing x over y indicates the investor's risk aversion coefficient, A, is bounded above by:

$$A = -\frac{\beta_\sigma}{\beta_r} < 2\frac{r_y - r_x}{\sigma_y^2 - \sigma_x^2} \tag{9}$$

The choice on each pair creates upper or lower bounds on A, with the more risky choices made by an investor, the lower the investor's aversion to taking risk.