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Exploring connections between advanced and secondary mathematics

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EXPLORING CONNECTIONS BETWEEN ADVANCED AND SECONDARY MATHEMATICS

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The second meeting of this Working Group continues to explore questions about the connections between abstract algebra and school mathematics. Our goal is to focus in on questions around the way in which teachers' practice might be influenced based on their understanding of such connections. In particular, we will gather interested individuals in an effort to deepen our understanding of existing connections between abstract algebra and secondary mathematics and which of these connections are important for secondary teachers to know and understand. Moreover, we aim to further research in this area by first considering connections between abstract algebra and school mathematics present in tasks and curricular materials. Second, we will discuss efforts to design and implement professional development focused on these connections. Through our working group meetings, we plan to develop on-going collaborations to further research around the mathematical content preparation of teachers.

Keywords: Advanced Mathematical Thinking, Teacher Education-Preservice, Teacher Education-Inservice/Professional Development, Teacher Knowledge

Brief History of the Working Group

The first working group on connections between advanced and secondary mathematics was held during PME-NA 37 (Bartell, Bieda, Putnam, Bradfield, & Dominguez, 2015). During this working group, facilitators shared definitions of connections, and presented current work about important connections between abstract algebra and secondary mathematics. Through facilitator presentations and whole and small group discussions, the working group addressed the following questions: (1) What are the important connections between abstract algebra and secondary mathematics? (2) How does knowledge of connections between advanced and secondary content impact instruction in secondary classrooms? (3) How can we better support teachers to understand connections between advanced and secondary content and to use pedagogy that employs these connections? (4) How do we determine the depth of teacher knowledge of the connections between abstract algebra and secondary mathematics? (5) What are indicators that teachers have gained particular understandings? (6) What do we want teachers to be able to do with this knowledge?

After considering these questions, the group identified concrete next directions for collaboration. These include both examining tasks or curricular materials and designing and implementing professional development with secondary teachers around connections between abstract algebra and secondary mathematics. These projects were born from participants' interests in next steps as well as the group's desire to better understand teachers' knowledge of connections, how this knowledge could be assessed, and how mathematics teacher educators and teacher leaders might be able to support secondary teachers' knowledge of connections.

To ensure that our work was immediately beneficial to the community as well as of sustained duration, our initial working group proposal contained three primary follow-up activities: (1) submitting an article to the Notices of the American Mathematical Society, the most widely-read journal by professional mathematicians; (2) preparing an entry for the American Mathematical

Society's blog on the Teaching and Learning of Mathematics; and (3) longer-term work via continued meetings at other conferences and establishing a collective research agenda. We are actively working to write an article for the Notices, and we anticipate its completion by the end of the Spring 2016 semester. We completed our blog entry, which focused on providing several examples of connections between abstract algebra and high school mathematics, and it appeared in December 2015 (Baldinger, Broderick, Murray, Wasserman, & White, 2015). Due to its ongoing nature, we are continually working on the third item. Thus far various subgroups have met virtually, via regular email correspondence, and in person at the Joint Math Meetings and the Association of Mathematics Teacher Educators Annual Meeting.

Background and Theoretical Perspective

Connecting Abstract Algebra and High School Algebra

This working group explores connections between advanced mathematics content (in particular, abstract algebra) and secondary mathematics content. We are particularly interested in considering those connections that influence in some way a teacher's instructional practice. Recent policy documents (e.g., Conference Board of the Mathematical Sciences, 2012) advocate for including abstract algebra as a fundamental part of secondary teacher preparation. This provides motivation for better unpacking the connections between this advanced mathematics and school mathematics. Research focused on these connections has begun to gain traction in the mathematics education community. Such research addresses (a) the connections that exist between particular advanced content courses and secondary mathematics (e.g., Baldinger, 2014; Cofer & Findell, 2007; Suominen, 2015; Usiskin, 2001; Wasserman, 2014) and (b) the impact learning connections between advanced and secondary content has on teachers and their instruction (e.g., Baldinger, 2013; Wasserman, 2014).

Researchers have identified and listed connections between abstract algebra and school mathematics using a variety of approaches. For example, using the Common Core State Standards to represent the school mathematics curriculum, Wasserman (2015) considers teachers' potential practices for teaching specific elementary, middle, and secondary topics, before and after learning about introductory concepts in abstract algebra. He developed a framework that considered the K-12 content areas for which teaching might be influenced by teachers' knowledge of abstract algebra. This provides a different perspective than the traditional listing of connections between abstract algebra and secondary mathematics (e.g., (Z, +) is a group). Suominen (2015), in contrast, considered connections by analyzing commonly used abstract algebra textbooks and interviewing abstract algebra professors. She found that the connections made in textbooks could be categorized in different types of connections (e.g., generalizations, real-world applications). After developing a list of connections, she found that the mathematicians interviewed prioritized different connections. Baldinger (2015) took yet another approach, and explored connections as they related to pre-service teacher engagement in mathematical practices. She found that in an abstract algebra course designed for pre-service secondary teachers, participants had opportunities to learn about and engage in mathematical practices such as justification, attending to precision, and communicating mathematical ideas. Following the course, participants were able to better engage in these practices while solving high school algebra tasks.

Influencing Instructional Practice

The connections identified through previous research are particularly important to the extent that they influence teachers' instructional practice. For example, Hill (2003) describes how a secondary teacher was able to build on the axiomatic approach to abstract algebra in a unit on complex numbers for her secondary students. Wasserman (2014) investigated teachers who studied abstract algebra topics from a perspective emphasizing algebraic structures and their connections to school mathematics. He found that they began to change their beliefs and practices around teaching content such as number operations and solving equations. Cofer (2015) explored how pre-service secondary teachers were able to incorporate ideas from abstract algebra when explaining topics in school mathematics. She found that some participants were able to give explanations that incorporated an "advanced mathematical argument" and clearly drew on their abstract algebra knowledge. In other cases, participants relied on analogy or rules in their explanations, showing a lack of integration of abstract algebra knowledge. Cofer's framework provides a lens for investigating how knowledge of connections might be manifested during instructional practice.

Supporting Teachers to Learn about Connections

Building on the importance of connections between school algebra and abstract algebra, the question comes up about how we can support teachers to learn about such connections. Research suggests that pre-service teachers struggle in traditional abstract algebra courses, and these courses tend to feel disconnected from school algebra (Clark, Hemenway, St. John, Tolias, & Vakil, 1999; Zazkis & Leikin, 2010). Additionally, many do not understand the purpose of taking advanced mathematics courses (e.g., Cuoco, 2001), especially the relevance to teaching secondary mathematics (e.g., Cuoco & Rotman, 2013). For example, Broderick (2013) interviewed prospective secondary teachers about the usefulness of their college math courses. He found their comments were consistent with the literature (e.g., Cuoco, 2001; Hill, 2003), with one caveat. One participant had not passed abstract algebra the first time and went through it again. She found more relevance the second time through and was more satisfied with taking the course. Such findings have led to several efforts to make abstract algebra more accessible and applicable.

At the pre-service level, researchers have documented different strategies for better supporting teachers to learn about these connections (Murray & Star, 2013). Some courses, for example, intentionally utilize cooperative learning environments (Barbut, 1987; Cnop & Grandsard, 1998; Grassl & Mingus, 2007) or include a component of technology (Clark et al., 1999; Leron & Dubinsky, 1995). Through emphasizing non-traditional pedagogy, these courses provide pre-service teachers with a new model of teaching mathematics. Such courses support learning connections by helping pre-service teachers more deeply understand the abstract algebra content, and their non-traditional pedagogical approach may support changes in practice. However, the connections across abstract algebra content and school mathematics are largely implicit. Other courses intentionally address connections with school mathematics (e.g., Baldinger, 2013; Cuoco & Rotman, 2013; Wasserman, 2014). In these cases, the connections are an intentional part of the course design, and pre-service teachers are supported to learn about connections alongside learning about abstract algebra content.

For in-service teachers, research suggests that professional development driven by a focus on mathematics content tends to have a stronger impact on practice than professional development with, for example, an activity focus (Garet, Porter, Desimone, Birman, & Yoon, 2001; Marra et al., 2011). Furthermore, Sowder and colleagues (1998) found that increasing teachers' content knowledge could have impact on teachers' practice. This helps motivate a need to focus explicitly on connections between abstract algebra and school mathematics within professional development, as doing so helps make the content more relevant to classroom practice. This can help mitigate the tension teachers feel when professional development is disconnected from their work with students (Nipper et al., 2011).

Taken together, this research suggests important starting points in considering how to support teacher learning around connections between school algebra and abstract algebra. In this working group, we plan to build on this research to discuss strategies to support teacher learning of these connections in ways that help teachers to make positive changes in their practice based on their deepened understanding.

Theoretical Perspectives

We view teacher knowledge of the connections between school algebra and high school algebra as part of their mathematical knowledge for teaching. We build on the work of other scholars (Ball, Thames, & Phelps, 2008; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) in taking a practice-based approach to conceptualizing teacher knowledge, since we are interested in connections that have an impact on instructional practice. Though there is debate about whether some of the categories of mathematical knowledge for teaching identified by Ball and colleagues (specialized content knowledge, in particular) can be appropriately transferred to secondary level mathematical knowledge for teaching (Speer, King, & Howell, 2015), we draw on the approach of considering knowledge in practice. In thinking about connections, we are particularly interested in what Ball and colleagues call "horizon content knowledge". Wasserman and Stockton (2013) unpack this category of knowledge, and explore how it can be applied to the work of teaching.

In considering how teachers develop mathematical knowledge for teaching, and horizon content knowledge around connections in particular, we are guided by the construct of "key developmental understandings (KDUs)" (Simon, 2006). A KDU is a "conceptual advance that is important to the development of a concept" (Simon, 2006, p. 365). Silverman and Thompson (2008) use this construct in their framing of teachers' mathematical knowledge, by proposing that teachers develop mathematical knowledge for teaching of a topic if: (a) they have achieved a KDU that encompasses mathematical understanding of the topic; and (b) they have an understanding of how the topic may evolve instructionally in support of students' reasoning in the K-12 classroom. We apply this notion to understanding of connections, by investigating in particular how understanding those connections contributes to teacher practice.

Connections to PME-NA

For many teachers, the border between high school algebra and abstract algebra is significant. However, the connections between these two content areas that we focus on here serve as key bridges across that border. Our working group this year seeks to address questions that support teachers in questioning this border themselves—beyond just identifying connections between the two areas, we want to support teachers to apply knowledge of these borders to their classroom practice.

Working Group Goals

A long-term collective goal is to fundamentally reconsider the teaching of more advanced content courses for teachers in ways that are useful both generally for the teaching of all mathematics majors and also specifically for the unique considerations and professional practices of secondary teachers.

The goals for the second meeting of the working group are to share the work done for the two projects begun at PME-NA 37, to discuss next steps for these projects, and to consider new research that could advance our collective understanding of connections between advanced and secondary content and the role of connections in secondary mathematics teaching and learning.

The overall goal for this working group remains to strengthen our collective understanding of connections between advanced and secondary content and their role in secondary mathematics teaching and learning. We focus on abstract algebra as an entry into this domain, particularly for its rich connection to secondary mathematics topics such as algebra and functions, while also anticipating that the impact of these conversations will prompt research into other content areas (e.g., calculus, linear algebra, and analysis).

Plan for Working Group

Across the three sessions, participants will have the opportunity to interrogate the current state of the research through brief presentations and guided discussions.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

Session 1: Examination of Tasks and Curricula

In the first session, group members will present work about the analysis of tasks and curriculum around the connection between abstract algebra and secondary mathematics. Leading up to the 2016 working group, a subset of participants from the previous year have been exploring this idea from two perspectives. The first perspective is to interrogate mathematics courses taught for prospective secondary teachers (e.g., capstone courses; content courses; content-focused methods courses) for connections, including where the connections are located, which ones are addressed, and how they are addressed. The intent of this analysis is to inform what is going on currently in mathematics teacher preparation and compare the nature of these connections to those that are highlighted in mathematics courses.

The second perspective is to merge the frameworks developed by Wasserman (2015) and Suominen (2015) discussed above. The goal of this work is to see if there are commonalities in the connections noted in both the Common Core State Standards and higher education textbooks. Moreover, the group is interested in determining how meaningful the connections are to instruction in secondary school, particularly those connections that are less commonly noted in the literature. By merging these two approaches, the group is attempting to narrow down what to look for in tasks and curricular materials for secondary mathematics. This attempt will further the group's understanding of what sections, courses, and topics in secondary mathematics could be further explored for connections.

After the presentation, facilitators will lead small group discussions so that participants can provide their own input on connections embedded in current tasks for secondary mathematics students, regardless of whether these tasks are being used for high school students or prospective teachers. At the end of this session, the facilitators will create a summary of important connections and tasks discussed to help organize and motivate the final session.

Session 2: Designing Professional Development

In the second session, group members will present work regarding the design and implementation of professional development with secondary teachers around connections between abstract algebra and secondary mathematics. Based on ongoing discussions among participants of the original working group and others at various professional meetings, the professional development is being planned to have a strong pedagogical component, which involves using classroom-based scenarios or vignettes that highlighted some aspect of high school mathematics connected to abstract algebra. As we design this professional development, we consider the following questions: (1) What are the objectives, goals, and outcomes of professional development focused on the connections between abstract algebra and secondary mathematics? (2) How we can use the professional development to examine curricula and tasks and to solicit teachers' ideas about the connections?

In the second part of this session, we will engage in small group discussions of this particular professional development design. Through this conversation, we will collect ideas and feedback for further professional development workshops and consider the following questions: What are the research questions aligned with this professional development? How do we answer these questions, or measure what we are interested in? How might the internalization of connections between abstract algebra and secondary mathematics changes teachers? At the end of this session, the facilitators will create a summary of ideas generated during the discussions to help organize and motivate the final session.

Session 3: Determining Concrete Next Steps

In the final session of this working group, we will begin by sharing our summaries from the previous two sessions. We will engage in a whole group discussion of possible research questions that were generated in previous conversations. We will conclude the session by splitting up into groups, based on interest, to begin planning next steps.

Anticipated Follow-up Activities

We will actively continue our work together beyond the working group meetings at PME-NA 38. The formation of new connections among scholars, as well the deepening of the current connections, will positively impact post-conference activities. Our two subgroups (the tasks and curriculum subgroup and the professional development subgroup), along with any new subgroups arising out of the meetings at PME-NA 38 will work on specific research questions generated through our meetings at PME-NA. We also plan to collaborate on grant and journal submissions; an NSF DRK-12 grant is a likely starting point. In addition, the subgroup focused on professional development will take direct action-steps toward producing, testing, and investigating professional development focused on connections between abstract algebra and school mathematics. Through participation in the working group, several members are already sharing and providing feedback on the work of other members, and we will expand and deepen these efforts. We anticipate that these connections among scholars can be maintained and potentially lead to collaboration on related research.

We anticipate that members of this working group will continue to meet virtually, correspond regularly about the work via email, and meet in person at various professional meetings.

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