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Note on Optimal Inventory Management Under Inflation

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ABSTRACT

This paper develops a discounted-cost model that is similar to the classical economic order quantity model but includes inflation rates as parameters of the inventory system. A numerical problem is solved to illustrate the effects.

Most of the literature in the field of inventory management has not included inflation as a parameter of the system. This has happened mostly because of the belief that inflation (which was quite low in the United States prior to the 1970's) would not influence the policy variables to any significant degree. In 1975, Misra [5] and Buzacott [2] developed economic-order-quantity (EOQ) models which incorporated inflationary effects into the model. The models assume a uniform inflation rate for all the costs and minimize the average annual cost to derive an expression for the EOQ. It was also shown that if the unit selling price is changed only at the beginning of each cycle (as practiced by many grocery stores, i.e., charge more if you pay more), the objective function should be maximization of profit instead of minimization of cost. In the situation in which the selling price is increased continuously at the inflation rate, minimizing cost also maximizes profit. In a recent paper, Bierman and Thomas [1] have proposed an inflation model for the EOQ which also considers the time value of money. They too have assumed a single inflation rate for all cost factors. The cost equation in the model does not lend itself to the derivation of an expression for the EOQ, therefore the authors have suggested the use of a search method. Misra and Wortham [6] encountered a similar cost equation and suggested an approximation to derive an expression for the EOQ. For various problems the approximate EOQ was found to be within 1% of the exact EOQ. The purpose of this paper is to present a model which considers the time value of money and different inflation rates for various costs associated with an inventory system.

THE PROPOSED GENERAL EOQ MODEL

In the analysis of an inventory system, normally three types of costs are considered. These are replenishment cost, inventory carrying cost, and shortage cost. In the basic model shortages are not allowed, so only the first two costs are included in the analysis. Purchasing cost is not included, because it is constant. This is not so if we consider inflation, hence this cost will be included in the analysis. The most general and realistic model will be the one which considers a separate inflation rate for each of its cost components [3,4]. Writing a cost expression for such a model is straightforward, but its optimization is very difficult, and will require the use of search procedures [7]. However, one can put these costs into two categories;

category 1 consists of all those costs which increase at the inflation rate that prevails in the company, and category 2 consists of those that increase at the inflation rate of the general economy or of the supplier company. These will be called the internal (company) and external inflation rates respectively. Their values can be arrived at by some form of averaging (simple or weighted) of the individual inflation rates of costs in each category.

In general, replenishment cost will increase at the internal inflation rate and the unit purchasing cost at the external inflation rate. The cost of carrying inventory consists of the opportunity cost and the real out-of-pocket costs such as costs of insurance, taxes, and costs of storage. The amount of capital tied up in inventory changes with the unit cost, which increases with the external inflation rate. The cost of storage can be in either category or in both, depending on whether the company owns the storage space, or rents it, or both. Van Hees and Monhemius [4, pp. 81-101] have given an excellent breakdown of the various costs, which can be used as a guide in categorizing them along the lines suggested here. The classification would also vary depending on whether the goods are ordered from outside or are manufactured within the company. For instance, if goods are manufactured within the company, the unit cost is governed by both the internal and external inflation rates. This is so because part of the unit cost (material cost, for instance) increases with the external rate and part (setup cost + direct costs incurred in production) with the internal rate. Thus, while a clear-cut categorization of these costs is generally not possible, for a given inventory system it can easily be done. In the formulation that follows, it is assumed that this has been done and the corresponding costs determined. Also, it will be assumed that the costs vary with instantaneous inventory level. One can include additional terms if some costs depend on the maximum inventory.

Formulation

The present worth of the total cost for the first cycle is

$$P_1 = Qc + A + c_1\int_0^{Q/\lambda} (Q-\lambda t)e^{i_1 t} e^{-rt}\, dt + c_2 \int_0^{Q/\lambda} (Q-\lambda t)e^{i_2 t} e^{-rt} dt\ , \tag{1}$$

where Q = reorder quantity, λ = demand per unit time, A = ordering cost, c = unit cost, c_1 = internal inventory cost per unit per unit time, i_1 = internal inflation rate, r = discount rate or cost of capital, c_2 = external inventory cost per unit per unit time, and i_2 = external inflation rate.

Equation (1) simplifies to

$$P_1 = Qc + A + \frac{c_1 Q}{R_1} - \frac{c_1\lambda}{R_1^2}(1-e^{-R_1 Q/\lambda}) + \frac{Qc_2}{R_2} - \frac{\lambda c_2}{R_2^2}(1-e^{-R_2 Q/\lambda})\ , \tag{2}$$

where $R_1 = r - i_1$ and $R_2 = r - i_2$.

For convenience let us define E_1 and E_2 such that

$$E_1 = A + \frac{c_1 Q}{R_1} - \frac{c_1\lambda}{R_1^2}(1-e^{-R_1 Q/\lambda}), \quad E_2 = Qc + \frac{Qc_2}{R_2} - \frac{\lambda c_2}{R_2^2}(1-e^{-R_2 Q/\lambda})\ ,$$

The cost diagram for N cycles is

	1	2	N
E_1	$E_1e^{i_1t}$	$E_1e^{2i_1t}$	$E_1e^{(N-1)i_1t}$
$+$	$+$	$+$	$+$
E_2	$E_2e^{i_2t}$	$E_2e^{2i_2t}$	$E_2e^{(N-1)i_2t}$

The present worth of the total cost for N cycles is

$$(3)\qquad P_T = E_1\left(1+e^{-R_1t}+e^{-2R_1t}+\dots\right)+E_2\left(1+e^{-R_2t}+e^{-2R_2t}+\dots\right)$$

$$=\left(\frac{A+c_1Q/R_1}{1-e^{-R_1Q/\lambda}}-\frac{c_1\lambda}{R_1^2}\right)\left(1-e^{-R_1\frac{Q}{\lambda}N}\right)+\left(\frac{Qc+Qc_2/R_2}{1-e^{-R_2Q/\lambda}}-\frac{c_2\lambda}{R_2^2}\right)\left(1-e^{-R_2\frac{Q}{\lambda}N}\right)$$

The total cost in equation (3) will converge if R_1 and R_2 are positive, i.e., the inflation rates are smaller than the discount rate, even for the infinite planning horizon, $N \to \infty$.

If the inflation rates are higher than the discount rate, the total-cost equation (3) is unbounded as $N \to \infty$. Thus, a finite horizon must be used for optimization, which we accomplish by differentiating equation (6) with respect to Q, equating it to zero, and solving for Q. This yields a complicated expression which cannot be solved for Q directly and requires the use of search techniques. In this situation, if the costs c_1 and c_2 are zero, it is optimal to have Q as large as possible. This is not a stable situation. To have finite Q, either the inflation rates should be less than the discount rate, or c_1 and c_2 should be very high. The length of planning horizon N will be determined by the forecast of the period before which the inflation rates will become less than the discount rate. In a planning horizon of unit time there are λ/Q cycles, i.e., $N = \lambda/Q$. For this case equation (3) yields

$$(4)\qquad P_T=\left(\frac{A+c_1Q/R_1}{1-e^{-R_1Q/\lambda}}-\frac{c_1\lambda}{R_1^2}\right)(1-e^{-R_1})+\left(\frac{Qc'}{1-e^{-R_2Q/\lambda}}-\frac{c_2\lambda}{R_2^2}\right)(1-e^{-R_2}),$$

where $c' = c + \dfrac{c_2}{R_2}$.

Case when (i_1 and i_2) $< r$

In this case, the cost equation (4) will be differentiated with respect to Q and equated to zero. This yields

$$(5)\qquad K\cdot\frac{\frac{c_1}{R_1}(1-e^{-R_1Q/\lambda})-(A+c_1Q/R_1)\frac{R_1}{\lambda}\,e^{-R_1Q/\lambda}}{(1-e^{-R_1Q/\lambda})^2}+\frac{c'(1-e^{-R_2Q/\lambda})-Qc'\frac{R_2}{\lambda}\cdot e^{-R_2Q/\lambda}}{(1-e^{-R_2Q/\lambda})^2}=0,$$

where $K = (1-e^{-R_1})/(1-e^{-R_2})$.

Equation (5) can be solved by the use of search techniques. However, an approximate analytical solution can be obtained if we expand the exponential terms up to the first three terms and neglect the higher-order terms. This approximation has been found to yield good results in other situations with similar expressions [6]. After considerable simplification equation (5) reduces to

$$\frac{c'}{2\lambda} - \frac{AK}{R_1 Q^2} + \frac{c_1 K}{2R_1\lambda} \approx 0 . \tag{6}$$

Equation (6) yields

$$Q^* = \sqrt{\frac{2\lambda A}{I'c}}, \text{ where } I' = R_1\left(1+\frac{c_2}{cR_2} + \frac{c_1 K}{cR_1}\right)/K . \tag{7}$$

I' can be called an adjusted inventory carrying cost, following the terminology of Hadley and Whitin [2]. Thus, in practice all that is needed is to calculate I' and use it in place of I in the Harris-Wilson-Camp formula [2,3]. This does not give the optimum Q, but the approximation is quite good as will be seen later in an example. To find the optimum Q by search techniques, we can use this approximate value as a starting point.

EXAMPLE

Given λ = 10,000 units/year, A = \$40, c = \$4.00, r = 0.20, i_1 = 0.08, i_2 = 0.14, c_1 = \$0.20 per unit time, and c_2 = \$0.16 per unit per unit time, then I' = 0.153, and

$$Q^* \approx \sqrt{\frac{2\times 10,000\times 40}{0.153\times 4}} = 1148 .$$

To check the accuracy of the approximation, the exact value of Q was calculated from equation (5) by trial and error. The exact value of Q obtained was 1160, thus the approximation is quite good.

The corresponding Q from the Harris-Wilson-Camp formula is

$$Q^*_w = \sqrt{\frac{2\times 10,000\times 40}{4\times 0.2+0.20+0.16}} = 831 .$$

Thus, as a result of inflation the optimum order quantity has increased. The corresponding costs are

$$P_T = \$33,106 \text{ for } Q = 831$$

and

$$P_T = \$32,822 \text{ for } Q = 1148 .$$

In summary, the optimum order quantity is changed significantly when inflation is included in the analysis. However, the reduction in costs is slight. The cost function in the EOQ model is known to be insensitive in the neighborhood of the optimum Q. It is even less sensitive when given in present-worth terms. A further extension of this research is the interesting case in which lead time is significant. Since the time value of money is considered in the model, the payment policy, i.e., whether the payments are made in advance or at the time of delivery, will influence the model.

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