Optimum Production Lot Size Model for a System With Deteriorating Inventory

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Optimum production lot size model for a system with deteriorating inventory

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A production lot size model has been developed for an inventory system with deteriorating items. Both the varying and constant rate of deterioration have been included in the analysis. For the case of a varying rate, it seems impossible to obtain a simple expression for the production lot size, so a numerical method has been suggested. For the constant rate of deterioration case, an approximate expression has been derived for the production lot size. Finally, a numerical example is solved to show the impact of deterioration.

Introduction

An inventory system in a somewhat narrow way may be thought of as a system in which certain items are stocked. The demands are met and the new orders are placed to get the supply. The basic question here is when to order and how much to order at a time. The answer to this question is dependent on a large number of factors, for example, the nature of demand, circumstances governing replenishment, various costs such as inventory carrying cost, shortage cost and replenishment cost and characteristics of the item being stocked. It is the last of these factors, the characteristics of the item, that will be addressed in this paper. The item may be perishable, hence its price might go down depending on its age or the item may improve its quality as time passes and as a result, its price may have an increasing trend. The item may become obsolete depending upon change in style or technological development. A number of researchers have attempted to solve these cases in some way or other. Brown *et al.* (1967) introduced a Bayesian procedure to solve the problem of obsolescence. Pierskall (1969) considered a finite-period one item system having known demand distribution without any backlogging. He assumed a sequence of probabilities that the item becomes obsolete in a certain period and applied dynamic programming to obtain a solution. Whitin (1957) studied the case of deterioration of fashion goods at the end of the storage period. Ghare and Schrader (1963) developed a simple EOQ model for an inventory with a constant rate of deterioration. Recently, Covert and Philip (1973) developed an EOQ model for items with a variable rate of deterioration. Both of the above models assume an infinite production rate. In this paper this condition has been relaxed and an attempt is made to present a more general model where the inventory is deteriorating at an increasing, a decreasing or a constant rate. Standard terminology for increasing and decreasing rates is a Weibull rate and an exponential rate is used for a constant deterioration rate. A two parameter Weibull rate will be used here, denoted by $D(t)=\alpha\beta t^{\beta-1}$. The implication of the two parameter Weibull rate is that the items in inventory

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2 L 2

start deteriorating the instant they are received into inventory. A more general case would be the use of a three parameter Weibull rate which will permit already deteriorated items to be received by the inventory system and also those items which may start deteriorating some time in the future. For mathematical simplicity, the two parameter rate has been used in this paper. However, the case of a three parameter rate can successfully be handled as has been shown for a simple EOQ model by Philip (1974).

It is shown here that an expression for the lot size Q can be derived in terms of the lot size for items without any deterioration. This is possible after making some simplifying assumptions. Finally, the numerical results showing the impact of deterioration are included.

Notation

The notation used in this paper is as follows :

ψ = production rate given in number of units/year ;
λ = demand rate given in number of units/year ;
c_3 = cost of placing an order ;
I = the inventory level at time t ;
I_0 = maximum inventory level within a cycle ;
c = cost of a deteriorated unit ;
c_1 = inventory carrying cost/unit/unit time ;
Q = production lot size ;
T = cycle time ;
T_1 = time required to produce Q units ;
T_2 = time during which there is no production in a cycle, i.e. $T_2 = T - T_1$;
$D(t)$ = the deterioration rate, given by $\alpha\beta t^{\beta-1}$ where $\alpha, \beta, t > 0$. When $\beta = 1$, $D(t)$ becomes a constant which is the case of an exponential decay. When $\beta < 1$, the rate of deterioration is decreasing with t and when $\beta > 1$, it is increasing with t ;
K = total cost/unit time ;
$T_1{}^*$ = optimum value of T_1 ;
$T_2{}^*$ = optimum value of T_2 ;
$T_{1_c}{}^*$ = optimum value of T_1 for conventional production rate model ;
$T_{2_c}{}^*$ = optimum value of T_2 for conventional production rate model ;
$Q_c{}^*$ = optimum value of Q for conventional production rate model.

Development of the model

Assumptions

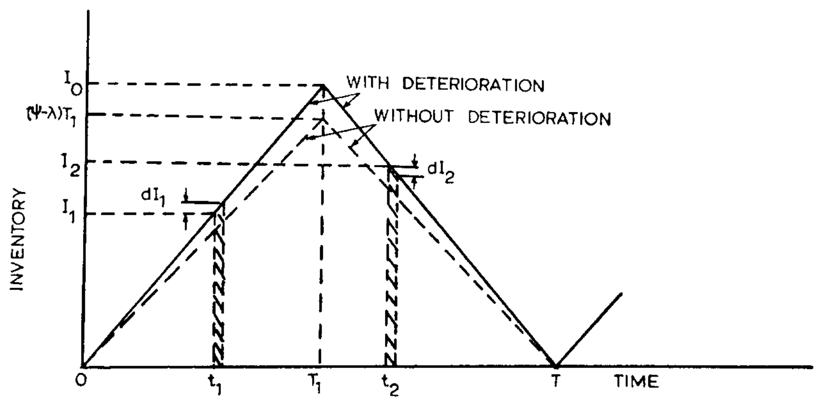
The model will be developed using the following assumptions :

(1) Demand is known and has a constant rate.
(2) Shortages are not allowed.
(3) Production rate governing supply is finite.
(4) Units are available for satisfying demand immediately after their production.
(5) A deteriorated unit is not repaired or replaced by a good unit.
(6) The cost of a deteriorated unit is constant and equal to c. This will account for the salvage value, if any.

(7) The units start deteriorating only when they are received into inventory. This assumption allows us to use a two parameter rate as discussed earlier.
(8) The production rate is independent of the size of the production lot.
(9) The system is in steady state, i.e. the production rate is greater than the demand rate.
(10) The number of units will be treated as a continuous variable.
(11) There are no constraints on space, production lot size, number of production set-ups, etc.
(12) The production lot size, though unknown, is fixed ; i.e. it will not vary from one cycle to another.

Mathematical development

An inventory cycle for a finite production rate model is shown in the figure. The inventory level at the beginning and end of the cycle is zero. The cycle length is equal to Q'/λ where Q' is the number of good units out of a batch of Q units. The production will take place for a duration of T_1 time units and at the end of this period enough units should be left that will take care of the demand in the period $T-T_1$ and the deteriorated units. Let $D(t)$ represent the instantaneous deterioration rate function for the items stocked.



A finite production rate model with deterioration of inventory.

The change in the inventory level, dI during a small interval of time dt is a function of the deterioration, the demand rate λ, production rate ψ and the remaining inventory. Thus

$$-dI_1 = ID(t)\,dt + \lambda\,dt - \psi\,dt \quad \text{for } 0 \leqslant t_1 \leqslant T_1, \tag{1}$$

and

$$-dI_2 = ID(t)\,dt + \lambda\,dt \quad \text{for } T_1 \leqslant t_2 \leqslant T. \tag{2}$$

Equations (1) and (2) can be rewritten as

$$\frac{dI_1}{dt} + ID(t) = (\psi - \lambda), \qquad 0 \leqslant t \leqslant T_1, \tag{3}$$

and

$$\frac{dI_2}{dt}+ID(t)=-\lambda, \qquad T_1\leqslant t\leqslant T. \tag{4}$$

The solutions of these differential equations are given in Spiegel (1960). These are

$$I_1=\frac{\int_0^{t_1}(\psi-\lambda)\exp\left(\int D(t)\,dt\right)dt+B_1}{\exp\left(\int_0^{t_1}D(t)\,dt\right)}, \tag{5}$$

and

$$I_2=\frac{\int_{T_1}^{t_2}(-\lambda)\exp\left(\int D(t)\,dt\right)dt_2+B_2}{\exp\left(\int_{T_1}^{t_2}D(t)\,dt\right)}. \tag{6}$$

The values of the constants of integration B_1, B_2 can be found by using the boundary conditions. That is, at $t_1=0$, $I_1=0$, the initial inventory, and at $t_2=T_1$, $I_2=I_0$. Applying these boundary conditions yields $B_1=0$, $B_2=I_0$. This gives

$$I_1=\frac{\int_0^{t_1}(\psi-\lambda)\exp\left(\int D(t)\,dt\right)dt}{\exp\left(\int_0^{t_1}D(t)\,dt\right)}, \tag{7}$$

and

$$I_2=\frac{\int_{T_1}^{t_2}(-\lambda)\exp\left(\int D(t)\,dt\right)dt+I_0}{\exp\left(\int_{T_1}^{t_2}D(t)\,dt\right)}. \tag{8}$$

In order to simplify the expressions of I_1, I_2 further, it is imperative that the deterioration rate function $D(t)$ is known.

Two types of deterioration rate can be encountered in reality. The rate can vary with time or remain constant. Both of these cases will be taken in this paper.

Case 1. Varying rate of deterioration

The function $D(t)$ can be written for this case as $D(t)=\alpha\beta t^{\beta-1}$ where α, β are some constants determined by the deterioration process. Substituting this value of $D(t)$ in eqns. (7) and (8) yields

$$I_1=\frac{\int_0^{t_1}(\psi-\lambda)\exp(\alpha t^\beta)\,dt}{\exp(\alpha t_1{}^\beta)}, \tag{9}$$

and

$$I_2=\frac{\int_0^{t_2}(-\lambda)\exp(\alpha t^\beta)\,dt+I_0}{\exp(\alpha t_2{}^\beta)}. \tag{10}$$

Now at $t_2 = T - T_1 = T_2$, $I_2 = 0$ hence

$$I_0 = \int_0^{T_2} \lambda \exp(\alpha t^{\beta})\, dt.$$

Substituting this in eqn. (10) yields

$$I_2 = \frac{\int_0^{t_2} (-\lambda) \exp(\alpha t^{\beta})\, dt + \int_0^{t_2} \lambda \exp(\alpha t^{\beta})\, dt}{\exp(\alpha t_2{}^{\beta})}. \tag{11}$$

There is another condition that must be satisfied by this system. This condition gives a relationship between T_1 and T_2. That is at $t_1 = T_1$ and $t_2 = 0$, I_1 is equal to I_2. This gives

$$I_0 = \frac{\int_0^{T_1} (\psi - \lambda) \exp(\alpha t^{\beta})\, dt}{\exp(\alpha T_1{}^{\beta})} = \int_0^{T_2} \lambda \exp(\alpha t^{\beta})\, dt. \tag{12}$$

This equation is not easy to simplify because of the difficulty in integration. The average carrying cost/unit time can be written as follows :

$$c_1 \frac{1}{T_1 + T_2} \left[\int_0^{T_1} I_1\, dt + \int_0^{T_2} I_2\, dt \right],$$

where I_1, I_2 are as given in eqns. (9) and (11), and ordering cost/unit time

$$= \frac{c\psi T_1}{T_1 + T_2} + \frac{c_3}{T_1 + T_2}.$$

Thus the total cost equation is

$$K = \frac{c_1}{T_1 + T_2} \left[\int_0^{T_1} I_1\, dt + \int_0^{T_2} I_2\, dt \right] + \frac{c\psi T_1}{T_1 + T_2} + \frac{c_3}{T_1 + T_2}. \tag{13}$$

There are two variables T_1 and T_2 in eqn. (13). However, they are not independent and are related by eqn. (12). If we can solve T_1 in terms of T_2 or vice versa, eqn. (13) can be written as a function of only one variable, T_1 or T_2. Then the cost can be minimized by differentiating it with respect to that variable (T_1 or T_2), equating it to zero and solving for that variable (T_1 or T_2). Unfortunately, this is satisfactory theoretically, but practically it is almost impossible since the integrals in eqn. (13) are not integrable and eqn. (12) cannot be solved explicitly for T_1 and T_2. Thus, other ways to solve the problem have to be found. It is proposed that eqn. (12) be solved by a trial and error method. Say, as a result of this we find $T_1 = kT_2$ where k is a constant obtained by trial and error. Then eqn. (13) can be written only in terms of one variable (say T_1). However, this still is not easy to differentiate because of the integrals. The simplest way to tackle this problem is the tedious and long method of expanding the exponential terms in a series form and integrating term by term. Since any such series contains an infinite number of terms, an assumption may have to be made so that the higher order terms could be ignored. This will result in a simple expression which can be differentiated with respect to T_1 and equated to zero in order to find the optimum value of

T_1, $T_1{}^*$. Again, it may not be easy to solve for $T_1{}^*$, so some numerical techniques may have to be used in order to find $T_1{}^*$. Covert and Philip (1973) have applied the Correction Method of Newton (Stiefl 1963) for a simple lot size model with a changing rate of deterioration. This technique can also be successfully used for the finite production rate model.

From the above discussion, at least one thing is clear, that is obtaining a solution for this model is not easy. The problem as a whole can be considerably simplified if we make the following approximation.

Approximation

We treat the inventory depletion curve as a straight line, even though it is not linear. This is the same as in simple models without deterioration. Thus, the total cost equation can be written as

$$K \simeq \frac{c((\psi-\lambda)T_1 - I_0)}{T} + \frac{c(I_0 - \lambda T_2)}{T} + c_1 I_0 \left(1 - \frac{\lambda}{\psi}\right) + \frac{c_3}{T}. \tag{14}$$

The first two terms in eqn. (14) represent the cost due to spoiled units, the third term represents the average inventory cost and finally the fourth term is the ordering cost. The eqn. (14) can be simplified to yield

$$K \simeq \frac{c\psi T_1}{T} - \lambda c + c_1 I_0 \left(1 - \frac{\lambda}{\psi}\right) + \frac{c_3}{T}. \tag{15}$$

It can be seen that eqn. (15) is a lot simpler than eqn. (13). Substituting from eqn. (12) the expression for I_0 in terms of T_2 yields

$$K \simeq \frac{c\psi T_1}{T_1 + T_2} - \lambda c + c_1 \left(1 - \frac{\lambda}{\psi}\right) \int_0^{T_2} \lambda \exp(\alpha t^\beta)\, dt + \frac{c_3}{T_1 + T_2}. \tag{16}$$

We have already discussed how to find a relationship between T_1 and T_2 by trial and error from eqn. (12). As proposed earlier, let $T_1 = kT_2$ where k is a constant determined by trial and error. Substituting the value of T_2 in eqn. (16), differentiating it with respect to T_1 and equating to zero yields

$$c_1 \left(1 - \frac{\lambda}{\psi}\right) \exp(\alpha T_2{}^\beta) - \frac{c_3}{(1+k)T_2{}^2} = 0. \tag{17}$$

Equation (17) can be solved by the numerical method proposed by Covert and Philip (1973). Once $T_2{}^*$ is known, $T_1{}^*$ can be found from $T_1{}^* = kT_2{}^*$. This is the time duration for which the production should take place. Thus, the production lot size Q will be

$$Q^* = \psi T_1{}^*. \tag{18}$$

In absence of any specific relationship for $T_1{}^*$ in terms of the system's variables c, c_1, c_3, λ and ψ, it is not possible here to see their relative impact on $T_1{}^*$ and thus on the production lot size Q^*. However, such a study can be performed with the help of a computer.

Case 2. Constant rate of deterioration

Most of the difficulties encountered in the previous case of changing rate of deterioration occur because the rate is not constant. But if the deterioration rate is constant, the function $D(t)$ can be written as $D(t)=\alpha$. For this case, the expression for I_1, I_0, I_2 can be written by simply substituting $\beta=1$ in eqns. (9), (10) and (11). This gives

$$I_1=\frac{\int_0^{t_1}(\psi-\lambda)\exp(\alpha t)\,dt}{\exp(\alpha t_1)}=\frac{(\psi-\lambda)}{\alpha}[1-\exp(-\alpha t_1)], \tag{19}$$

$$I_0=\int_0^{T_2}\lambda\exp(\alpha t)\,dt=\frac{\lambda}{\alpha}[\exp(\alpha T_2)-1], \tag{20}$$

$$I_2=\frac{\int_0^{t_2}(-\lambda)\exp(\alpha t)\,dt+\int_0^{T_2}\lambda\exp(\alpha t)\,dt}{\exp(\alpha t_2)}$$

$$=\frac{\lambda}{\alpha}\left(\frac{\exp(\alpha T_2)-\exp(\alpha t_2)}{\exp(\alpha t_2)}\right). \tag{21}$$

The carrying cost/unit time is

$$c_1\left[\frac{1}{T_1+T_2}\int_0^{T_1}\frac{(\psi-\lambda)}{\alpha}[1-\exp(-\alpha t_1)]\,dt_1 + \int_0^{T_2}\frac{\lambda}{\alpha}\left(\frac{\exp(\alpha T_2)-\exp(\alpha t_2)}{\exp(\alpha t_2)}\right)dt_2\right],$$

or, after simplification $c_1/(T_1+T_2)\,[(\psi-\lambda/2)T_1^2+\lambda T_2^2/2]$.

The cost of deterioration/unit time is $(c\psi T_1)/(T_1+T_2)-\lambda c$ (this was obtained in the earlier case). The order cost/unit time is $c_3/(T_1+T_2)$. Summing all these three costs gives the total cost

$$K\simeq\frac{c\psi T_1}{T_1+T_2}-\lambda c+\frac{c_1}{2}\frac{((\psi-\lambda)T_1^2+\lambda T_2^2)}{T_1+T_2}+\frac{c_3}{T_1+T_2}. \tag{22}$$

The next step before optimizing K is to reduce K to a function of either T_1 or T_2. This is done by rewriting the condition given by eqn. (12) for this case, and simplifying it. This yields

$$(\psi-\lambda)[1-\exp(-\alpha T_1)]=\lambda[\exp(\alpha T_2)-1].$$

This equation can be further simplified to give a simple relationship by making an assumption that αT is a relatively small quantity so that the higher power terms can be neglected. Keeping this in view and expanding the exponential terms in a series form yields

$$\frac{\alpha T_2^2}{2}+T_2-m\left(T_1-\frac{\alpha T_1^2}{2}\right)\simeq 0 \quad \text{where} \quad m=\frac{\psi-\lambda}{\lambda}. \tag{23}$$

This is a quadratic equation. Solving it for T_2 gives approximately

$$T_2\simeq m\left(T_1-\frac{\alpha T_1^2}{2}\right). \tag{24}$$

Equation (23) can also be solved for T_1. This gives

$$T_1 \simeq \frac{T_2}{m}(1+\alpha/2T_2). \tag{25}$$

From eqns. (24) and (25) a useful relationship can be established.

$$T_1+T_2=\frac{\psi}{\lambda}T_1-\left(\frac{\psi-\lambda}{\lambda}\right)\frac{\alpha T_1^2}{2}. \tag{26}$$

To optimize one has to differentiate eqn. (22) with respect to T_1 using eqns. (24) and (26) and equate the result to zero. The number of terms obtained as a result of differentiation are too many to write all at once. However, after simplification and neglecting the terms involving higher order α terms (α^2 and more), this reduces to

$$\frac{c\psi\alpha}{2}T_1^2+\frac{c_1}{2}\frac{\psi^2}{\lambda}T_1^2-c_3\left[\frac{\psi}{\psi-\lambda}-\alpha T_1\right]=0.$$

If $\alpha T_1 \ll 1$, the above equation can be solved for the optimum T_1^*. This gives

$$T_1^*=\sqrt{\left(\frac{2c_3}{c_1\left(\frac{\psi-\lambda}{\lambda}\right)\cdot\psi\left[1+\frac{c}{c_1}\alpha\frac{\lambda}{\psi}\right]}\right)}. \tag{27}$$

Recalling that T_1^* for items without deterioration is

$$T_{1_c}^*=\sqrt{\left(\frac{2c_3\lambda}{c_1(\psi-\lambda)\psi}\right)},$$

eqn. (27) can be rewritten as

$$T_1^*=\sqrt{\left(\frac{1}{1+\frac{c}{c_1}\frac{\lambda}{\psi}\alpha}\right)}\cdot T_{1_c}^*. \tag{28}$$

The expression for T_2^* can be found in a similar manner by using eqns. (22), (25) and (26). This gives

$$T_2^*=\sqrt{\left(\frac{1}{1+\left[\alpha\cdot\frac{c}{c_1}\frac{(\psi-\lambda)}{\psi}\right]}\right)}\cdot T_{2_c}^*, \tag{29}$$

where $T_{2_c}^*=\sqrt{[(2c_3)/(c_1\lambda\psi)(\psi-\lambda)]}$ for items without deterioration.

Thus the scheduling period T^* is the sum $T_1^*+T_2^*$, and the optimum production lot size is

$$Q^*=\psi T_1^*=\sqrt{\left(\frac{1}{1+\frac{c}{c_1}\alpha\frac{\lambda}{\psi}}\right)}\cdot Q_c^*, \tag{30}$$

where $Q_c^*=\sqrt{[(2c_3\lambda)/(c_1)(\psi)/(\psi-\lambda)]}$ is the lot size for items without deterioration.

It can be observed from eqn. (30) that the effect of deterioration is one of decreasing the production lot size. That is, it will be more desirable to produce less at a time but more frequently. Also, it can be observed from eqn. (30) that if $\alpha=0$, i.e. there is no deterioration, the lot size is equal to the conventional lot size. On the other hand, this general case should give the standard results if the production rate is infinite. This is the case solved by Ghare and Schrader (1963). They obtained the following condition for optimal cycle time T:

$$\frac{c\lambda\alpha}{2}+\frac{c_1\lambda}{2}+\frac{c_1\lambda\alpha T}{2}-\frac{c_3}{T^2}=0. \tag{31}$$

It should be recalled that in the derivation of eqn. (30), αT was assumed to be quite small. Applying this to eqn. (31) gives

$$\frac{c\lambda\alpha}{2}+\frac{c_1\lambda}{2}-\frac{c_3}{T^2}=0.$$

This equation when solved for T gives

$$T^*=\sqrt{\left(\frac{2c_3/c_1\lambda}{1+\alpha\dfrac{c}{c_1}}\right)},$$

which is the expression for T_2^*, given by eqn. (29) after substituting $\psi=\infty$. Thus, the relationships established in this paper are consistent.

A numerical example

A numerical problem is solved here. The values of various variables are as follows:

$$\begin{aligned}\lambda &= 2500 \text{ unit/year},\\ \psi &= 7500 \text{ unit/year},\\ c_1 &= \$0{\cdot}60\text{/unit/year},\\ c &= \$3{\cdot}00\text{/unit},\\ c_3 &= \$50{\cdot}00\text{/order},\\ D(t) &= \alpha=\tfrac{1}{50} \text{ (exponential decay)}.\end{aligned}$$

Solution:

$$Q_c^*=\sqrt{\left(\frac{2\times 50\times 2500}{0{\cdot}60}\right)}\cdot\frac{7500}{(7500-2500)}=791,$$

$$Q^*=\sqrt{\left(\frac{1}{1+\dfrac{3}{0{\cdot}6}\times\dfrac{1}{50}\times\dfrac{2500}{7500}}\right)}\cdot 791=685,$$

$$T_1^*=\frac{685 \text{ years}}{7500}=33{\cdot}3 \text{ days},$$

$$\begin{aligned}T_2^*&=\frac{7500-2500}{2500}\left(33{\cdot}3-\frac{33{\cdot}3^2}{2}\frac{1}{50}\right) \text{ years (from eqn. (24))}\\ &=44{\cdot}4 \text{ days.}\end{aligned}$$

$$\text{Cycle length, } T = 33{\cdot}3 + 44{\cdot}4 = 77{\cdot}7 \text{ days.}$$

$$\text{Actual demand during } T = \frac{77{\cdot}7 \times 2500}{365} = 681 \text{ units.}$$

$$\text{Total deteriorated units in a cycle time} = 685 - 525 = 160 \text{ units.}$$

For example, if the conventional lot size $Q^* = 791$ was used then

$$T_1^* = 38{\cdot}5 \text{ days,}$$

$$T_2^* = 47{\cdot}2 \text{ days,}$$

$$T^* = 85{\cdot}7 \text{ days,}$$

$$\text{Actual demand} = 587 \text{ units.}$$

$$\text{Total deteriorated units in a cycle time} = 791 - 587 = 204 \text{ units.}$$

Thus, by ordering the optimum amount, the number of deteriorated units is reduced.

Conclusion

A production lot size model has been developed for an inventory system with deteriorating items. Both the varying and constant rate of deterioration have been included in the analysis. For the case of varying deterioration rate, it seems impossible to obtain a moderately simple expression for the production lot size so some numerical method has to be used. Covert and Philip (1973) have illustrated one such method which also can be used for finding the production lot size. If the rate is constant (exponential case), it is possible to obtain a relatively simple expression as has been shown in this paper. A numerical example has been solved to see the impact of a constant deterioration rate. It reduces the optimum production lot size. The new production lot size effectively balances these three costs—the cost of carrying inventory, the cost of deteriorated units and the ordering cost.

Un modèle de production en lots a été développé à usage d'un système à articles de détérioration. Aussi bien le régime constant que le régime variable de détérioration ont été incorporés à l'étude. Pour le cas du régime variable, il semble impossible d'obtenir une expression simple pour la production en lots, de sorte qu'une méthode numérique a été proposée. En ce qui concerne le régime de détérioration constant, une expression approximative de la production en lots a été réalisée par dérivation. Finalement un example numérique est résolu pour illustrer l'impact de la détérioration.

Für ein Lagerbestandssystem mit Artikeln, die eine Wertminderung erfahren, wurde ein Modell mit Produktionslosen/Größe entwickelt. In der Analyse sind veränderliche und konstante Wertminderungen enthalten. Bei der veränderlichen Wertminderung scheint es unmöglich zu sein, die Produktionslose/Größe einfach auszudrücken, daher wurde ein numerisches Verfahren vorgeschlagen. Bei der konstanten Wertminderung wurde für die Produktionslose/Größe ein ungefährer Ausdruck abgeleitet. Abschließend wird ein numerisches Beispiel gelöst, um die Bedeutung der Wertminderung zu verdeutlichen.

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