Understanding the Flow Structure of Low Reynolds Number Flows

Albert Jarvis
Montclair State University

Follow this and additional works at: https://digitalcommons.montclair.edu/etd

Part of the Applied Mathematics Commons

Recommended Citation
https://digitalcommons.montclair.edu/etd/184

This Thesis is brought to you for free and open access by Montclair State University Digital Commons. It has been accepted for inclusion in Theses, Dissertations and Culminating Projects by an authorized administrator of Montclair State University Digital Commons. For more information, please contact digitalcommons@montclair.edu.
Abstract

Ocean flows and the mechanisms by which their contents are organized has been a longstanding area of interest in applied mathematics. In recent years, a new theory has been developed to identify the structures responsible for the organization of fluid particles within complex geophysical flows. This theory is known as the theory of Lagrangian Coherent Structures (LCS) and details which structures are responsible for the organization of the flow and how to identify them. Being able to identify these LCS in real time has far reaching implications ranging from developing strategies for search and rescue missions to identifying the best intervention strategy to clean up an environmental disaster. A strategy has been developed to identify these structures in real time using autonomous ocean robots. Although there is a strong understanding of how LCS affect fluid particles, the study of how LCS affect inertial particles is an area wide open for exploration. The robotic strategy depends on understanding the affects the structures will have on the motion of these robots. We focus on gaining a fundamental understanding of how LCS affect inertial particle motion by performing experiments of inertial particles in a variety of flows. We use numerical simulations and theory to guide our experimental work. We lay a strong framework for future experiments and make some novel observations along the way.
MONTCLAIR STATE UNIVERSITY

Understanding the Flow Structure of Low Reynolds Number Flows
by
Albert Jarvis
A Master’s Thesis Submitted to the Faculty of
Montclair State University
In Partial Fulfillment of the Requirements
For the Degree of
Master of Science
August 2018

College of Science and Mathematics
Department of Mathematical Sciences

Thesis Committee:

Dr. Eric Forgoston
Thesis Sponsor

Dr. Lora Billings
Committee Member

Dr. Aslawn Vaidya
Committee Member
Acknowledgments

AJ was supported by the National Science Foundation (Award Numbers CMMI-1462884 and DMS 1418956). The content is solely the responsibility of the author and does not necessarily represent the official views of the National Science Foundation.
## Contents

1 Introduction 8

2 Theory and Methods 10
   2.1 Lagrangian Coherent Structures .......................... 10
   2.2 Finite Time Lyapunov Exponents .......................... 12
   2.3 Robotic Strategy for Tracking LCS ........................ 14
   2.4 Maxey-Riley and Inertial Particle Aggregation .......... 15

3 Model and Numerical Results 16
   3.1 Double-Gyre model ....................................... 16
   3.2 Numerical Results ....................................... 16

4 Experimental Setup/Methods 17
   4.1 Setup ................................................. 17
   4.2 Methods .............................................. 18
      4.2.1 Particle Image Velocimetry (PIV) .................... 18
      4.2.2 PIV Experiments .................................. 18
      4.2.3 Particle Tracking Velocimetry (PTV) ............... 19
      4.2.4 PTV Experiments .................................. 20

5 Experimental Results 20
   5.1 Tracer Particle Experiments ............................... 20
      5.1.1 Time-Independent PIV ............................... 20
      5.1.2 Time-Dependent PIV ............................... 22
   5.2 Inertial Experiments .................................... 23
      5.2.1 Time-Independent flow with Neutrally Buoyant Particles 23
      5.2.2 Comparing shapes in Time-Independent DG ........ 24
      5.2.3 Periodically Switching DG Flow ................... 28
      5.2.4 Comparing Shapes in Periodically Switching DG Flow 30

6 Summary and Conclusion 34

A Experimental Procedure 37
   A.1 Tracer Experiments ...................................... 37
      A.1.1 Setup ............................................ 37
      A.1.2 Procedure ........................................ 38
      A.1.3 Processing ....................................... 39
      A.1.4 Cleanup .......................................... 40
   A.2 Inertial Experiments ..................................... 40
      A.2.1 Setup ............................................ 40
      A.2.2 Procedure ........................................ 40
      A.2.3 Processing ....................................... 41
      A.2.4 Cleanup .......................................... 41

B Tank Maintenance 41
   B.1 Basic Upkeep/Cleaning .................................. 41
   B.2 Gears/Shaft Collars .................................... 42
   B.3 Miscellaneous Problems ................................ 43
List of Figures

1. (Left) Deepwater Horizon Explosion. Credit: NOAA. (Right) Boat attempting to skim up oil after Deepwater disaster. Credit: NOAA.

2. (Left) LCS obtained from numerical simulation [2]. (Right) Tiger tail formed from BP oil spill. Credit: NASA Earth Observatory.

3. (Left) Location of Great Pacific Garbage Patch. Credit: NOAA. (Right) Garbage aggregate in the ocean.

4. Attracting (top) and repelling (bottom) LCS deforming fluid parcel [2].

5. (Left) Initial particle position would have a low FTLE value. (Right) Initial particle position would have a high FTLE value.

6. Three theoretical (massless) autonomous robots cooperatively tracking a LCS in the Santa Barbara channel [4].

7. (Left) FTLE field of time-independent DG flow. (Right) FTLE field of time-dependent DG flow obtained at $t = 15$. Finite time $T = 15$, $\omega = \frac{6\pi}{10}$, $\epsilon = 0.25$, $A = 0.1$.


10. (Top Left) Raw image from experiment. (Top Right) Velocity field from experimental run. (Bottom) FTLE field calculated from velocity field.

11. (Top Left) Filtered image from experiment (Top Right) Velocity field when right rod is on. (Bottom Left) FTLE field at $t = 12.5s$ (Bottom Right) FTLE field at $t = 15s$.

12. (Top Left) FTLE field of experimental run. (Bottom) FTLE computed from velocity field.

13. (Top Left) Raw image from experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 80s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

14. (Top Left) Raw image from sphere experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 100s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

15. (Top Left) Raw image from rod experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 100s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

16. (Top Left) Raw image from triangle experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 100s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

17. (Top Left) Raw image from switching experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.
18 (Top Left) Raw image from switching experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code. 

19 (Top Left) Raw image from switching sphere experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code. 

20 (Top Left) Raw image from switching rod experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code. 

21 (Top Left) Raw image from switching triangle experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.
1 Introduction

The theory of Lagrangian Coherent Structures (LCS) is invaluable for understanding material transport in geophysical flows. LCS are robust skeletal structures that govern fluid transport and trace out boundaries for dynamically distinct regions in a flow. Understanding a flow’s LCS structure and being able to identify them in real time has far reaching implications. In April 2010, we experienced the worst oil spill in the history of U.S. waterways when BP’s Deepwater Horizon drilling rig exploded, releasing over 200 million gallons of oil into the Gulf of Mexico. The effect these spills can have on the ecosystem is massive. The oil coats the marine life and prevents some animals from staying warm. The oil negates the insulating effects of fur on mammals and prevents birds’ feathers from repelling water as they are meant to. Both of these byproducts of the oil spill can induce hypothermia and kill the animals [13]. The theory of LCS aims to identify the underlying structures which herd these contaminants in hopes of developing optimal cleanup strategies. The usefulness of applying this theory to contaminant spills was shown by Haller & Peacock [2] when they applied their method (after the incident had occurred) to identify the underlying LCS for the BP oil spill and showed how it shaped the evolving form of the spill. We can see in Figure 2 that they have identified an attracting LCS using Haller’s strainline approach in backwards time which shaped what is known as the “tiger tail” of the spill. If this insight was available at the time of the disaster, it could have assisted in the intervention strategies being implemented at the time. Current strategies for identifying LCS in real time rely on having accurate global velocity field data which is often not available. A new strategy involving the use of autonomous underwater and surface vehicles (AUVs and ASVs) is being developed with the aim of solving this problem.

Figure 1: (Left) Deepwater Horizon Explosion. Credit: NOAA. (Right) Boat attempting to skim up oil after Deepwater disaster. Credit: NOAA.

AUVs and ASVs are being deployed into our oceans to perform a variety of different sensing and monitoring tasks. These tasks range from procuring data to improving weather forecasting and providing insight into contaminant transport, to monitoring biological phenomena such as plankton assembly and algae bloom. In addition to these tasks, Michini, et al. [4] have developed a strategy (Section 2.4) to use these AUVs to identify and track LCS in the ocean in real time using only local velocity fields. Since the motion of these sensors will be determined by the underlying dynamics of the oceans they reside in, the need to better understand how the LCS affect these AUVs becomes ever more important. While there is a solid understanding of how LCS affects massless point particles in steady, periodic and quasi-periodic flows, there is still much to be
learned about how LCS affects bodies of a finite mass and size in a variety of steady and unsteady flows. Though not the main aim of this work, as the theory progresses it can be applied to environmental problems having to do with contaminant transport within our oceans.

Giant garbage patches have formed in most of our planet’s oceans. The largest of these is the Great Pacific Garbage Patch (GPGP). It is estimated that this garbage patch contains 1.8 trillion pieces of plastic, weighing 79,000 tonnes and spanning a roughly 1.6 million km\(^2\) area [14] (twice the size of Texas). These garbage patches (see Figure 3) are made up almost entirely of plastics ranging in size from 0.05cm to over 50cm. The most prevalent of these is the smallest, known as microplastics which is plastic debris that has broken down to an extremely small size (0.05 - 0.5cm) over time. These microplastics are easily ingested by marine life. Not only can this be toxic to the marine life but these plastics make their way up the food chain and can eventually reach humans. Needless to say, it is essential to gain a deeper understanding of the mechanisms by which both passive and inertial contaminant transport is governed in hopes of developing intervention strategies to attempt to mitigate the damaging effects of these disasters.

![Figure 2: (Left) LCS obtained from numerical simulation [2]. (Right) Tiger tail formed from BP oil spill. Credit: NASA Earth Observatory.](image-url)

![Figure 3: (Left) Location of Great Pacific Garbage Patch. Credit: NOAA. (Right) Garbage aggregate in the ocean.](image-url)
2 Theory and Methods

2.1 Lagrangian Coherent Structures

Previous approaches to dealing with material transport in complex geophysical flows relied on running large numerical simulations of ocean flows and obtaining velocity data from these simulations. This data would then be used to advect particles through these velocity fields to predict their future locations [2]. The problem is that these simulations are extremely sensitive to initial conditions and therefore unreliable in most practical applications. The LCS approach is a relatively new way to make sense of complex, time-dependent, nonlinear dynamics that looks beyond single particle trajectories and identifies the key, longstanding organizing structures hidden within the flows. Simpler steady, periodic and quasiperiodic systems give rise to stationary fixed points and corresponding stable and unstable manifolds. These fixed points and stable/unstable manifolds are typically simple to compute and it’s usually just as straightforward to understand their implications on the dynamics of the flow. Geophysical flows arising in nature, however, are generally more complicated, time-dependent, aperiodic flows. In these flows, the notion of stable/unstable manifolds become ill-defined mathematically. In addition, dealing with material transport in geophysical flows inherently necessitates an objective, frame invariant approach. Whether data is being processed at different radar systems across the earth or at a satellite orbiting above our planet, the results must be objective regardless of where the data was obtained. Previous methods which depend on Eulerian velocity fields often fail to be objective under time-dependent rotations and translations of the reference frame. This brings about a necessity for a new approach that can deal with unsteady systems and identify these structures objectively. Out of these needs, the theory of LCS was born [1]. One of the first steps towards the study of Lagrangian coherent structures came from Pierrehumbert and Yang [10] and involved the use of Finite Time Lyapunov Exponent fields in atmospheric flows. In the years that followed, Haller and collaborators wrote a series of articles [3], [5], [6], [11] that laid the groundwork for LCS. More recently Haller has been developing a rigorous theory for LCS as well as creating and improving techniques to compute LCS [1], [7].

The LCS approach is often applied to geophysical flows to obtain a framework for material transport in the given flow. The LCS separate dynamically distinct regions within a flow and can be powerful tools for understanding how a flow and its contents will be organized over time. It is based on identifying material surfaces that are most prominent in attracting or repelling nearby fluid elements over some finite time span. The LCS are the material surfaces which attract and repel nearby trajectories at the greatest local rate relative to other nearby material surfaces [1]. In 2-D these material surfaces are material lines \((n - 1)\) dimensional). To identify them (via Haller’s most recent approach [1]), consider a 2-D velocity field on \(U \subset \mathbb{R}^2\) given by

\[
\dot{x} = u(t, x); \quad x \in U, \quad t \in [t_0, t_1].
\]

(1)

A particle’s position at time \(t\) which was initialized at time \(t = t_0\) and position \(x = x_0\) is the solution to the following differential equation,

\[
\begin{align*}
\dot{x}(t; t_0, x_0) &= u(t, x(t; t_0, x_0)), \\
x(t_0; t_0, x_0) &= x_0 = (x_0, y_0).
\end{align*}
\]

(2)

The solution of this dynamical system can be thought of as a map which takes a particle at initial position \(x_0\) and initial time \(t_0\) to its position at time \(t\). This map is known as
the flow map, denoted by \( \phi_{t_0}^t(x) \) which satisfies,

\[
\phi_{t_0}^t(x) : U \rightarrow U : x \mapsto \phi_{t_0}^t(x) = x(t; t_0, x_0).
\]

We can then compute derivatives of the flow map with respect to variations of the initial positions, giving us the deformation-gradient tensor

\[
\nabla \phi_{t_0}^{t_1}(x_0) = \begin{bmatrix}
\frac{\partial x_1}{\partial x_0} & \frac{\partial x_1}{\partial y_0} \\
\frac{\partial y_1}{\partial x_0} & \frac{\partial y_1}{\partial y_0}
\end{bmatrix}.
\]

The right Cauchy-Green strain tensor is defined as

\[
\Delta_{t_0}^{t_1} = [\phi_{t_0}^{t_1}(x_0)]^T[\phi_{t_0}^{t_1}(x_0)].
\]

Haller says, “Strainlines are tangent to the eigenvector field of the right Cauchy-Green tensor’s smallest eigenvalue. It can then be shown that LCS positions at the initial time are given by the strainlines with the locally highest averaged values of the right Cauchy-Green strain tensor’s largest eigenvalue” [2]. These LCS in aperiodic systems play the role that stable and unstable manifolds play in steady, periodic and quasiperiodic dynamical systems.

To see how these structures affect fluid parcels in 2-D we can look to Figure 4. In the top figure we can see a circular fluid parcel straddling a few material lines with the blue denoting the material line with the highest local rate of attraction relative to nearby material lines. After some time \( t_1 \), we can see that the originally circular fluid parcel has now stretched out along or parallel to the dominant material line. As its name suggest, an attracting material line will deform a fluid parcel by pulling in towards the line. Conversely, if we look at the bottom figure in Figure 4, we again see a circular fluid parcel straddling a few material lines. In this case, the red line denotes the material line with the greatest local rate of repulsion relative to nearby material lines. Again, after some time, we can see that the once circular fluid parcel has now been pushed away from the dominant material line in a perpendicular fashion. This repelling material line will deform a fluid parcel by deforming and repelling it in the perpendicular direction.

As mentioned in the introduction, the theory has been applied to data and its merit has been confirmed [2]. One of the main goals moving forward is to be able to forecast the locations of these LCS in geophysical flows. While that is not yet attainable, what Haller calls ”nowcasting” [2] is the next step. Knowing the locations of LCS in real time allows for the identification of key transport barriers in a given flow. That knowledge can be used for numerous tasks including search and rescue strategies, directing a contaminant to a more desirable region or optimizing an object’s energy output as it monitors a region or traverses from one region to another in some geophysical flow.
2.2 Finite Time Lyapunov Exponents

There have been a number of different methods, each possessing strengths and weaknesses, proposed to find these coherent structures numerically. One of the most popular methods connects Finite Time Lyapunov Exponent (FTLE) fields and LCS [11]. Lyapunov exponents are a measure of particles’ asymptotic behavior based on a sensitivity to initial conditions. In essence, they are a measure of stretching or contracting of the separation of initially nearby particles. Since we are interested in the dynamics of flows over finite times, it is necessary that we instead use Finite Time Lyapunov Exponents which are Lyapunov exponents computed over some finite time window $[t_0, t_0 + T]$. In Figure 5 we can see the two opposing values possible for the FTLE calculation. In the figure on the left, we see two particles that are initially nearby remaining nearby after some finite time. We would say that the point at which these particles are initialized has a low FTLE value. In the figure on the right, we see that again, two particles are initially nearby though this time they straddle a heteroclinic trajectory. After some finite time these particles end up far apart and their initial starting position is said to have a high FTLE value. Since we are dealing with 2-D flows I will demonstrate how one would compute FTLEs in a 2-D flow but note that this method could easily be extended to 3-D.

Consider a 2-D velocity field on $U \subset \mathbb{R}^2$ given by

$$\dot{x} = u(t, x); \quad x \in U, \quad t \in [t_0, t_0 + T].$$

A particle’s position at time $t$ which was initialized at time $t = t_0$ and position $x = x_0$ is the solution to the following differential equation,

$$\begin{cases}
\dot{x}(t; t_0, x_0) = u(t, x(t; t_0, x_0)), \\
x(t_0; t_0, x_0) = x_0 = (x_0^1, x_0^2).
\end{cases}$$

The solution of this dynamical system can be thought of as a map which takes a particle at initial position $x_0$ and initial time $t_0$ to its position at time $t$. This map is known as the flow map, denoted by $\phi_{t_0}^t(x)$ which satisfies

$$\phi_{t_0}^t(x) : U \to U : x \mapsto \phi_{t_0}^t(x) = x(t; t_0, x_0).$$

FTLEs are a measure of the average (over the finite time) of the maximum separation rate of initially nearby particles. To compute that separation rate, we must introduce an initially, infinitesimally nearby particle $y = x + \delta x_0$ where $\delta x_0$ is an infinitesimal quantity oriented in an arbitrary direction. After some finite time $T$ the perturbation
has evolved into
\[
\delta x(t_0 + T) = \phi^{t_0+T}(y) - \phi^{t_0+T}(x) = \frac{d\phi^{t_0+T}(x)}{dx} \delta x_0 + O(\|\delta x_0\|^2). \tag{9}
\]
Since \(\delta x_0\) is infinitesimal, \(\|\delta x_0\|^2 << 1 \Rightarrow O(\|\delta x_0\|^2)\) is negligible.

The magnitude of the perturbation (under the standard Euclidean norm) is
\[
\|\delta x(t_0 + T)\| = \sqrt{\langle \frac{d\phi^{t_0+T}(x)}{dx}, \frac{d\phi^{t_0+T}(x)}{dx} \delta x_0 \rangle} = \sqrt{\langle \delta x_0, \Delta \delta x_0 \rangle}, \tag{10}
\]
where \((\cdot)^*\) denotes the adjoint, and
\[
\Delta = \left( \frac{d\phi^{t_0+T}(x)}{dx} \right)^* \frac{d\phi^{t_0+T}(x)}{dx} \tag{11}
\]
denotes the finite time version of the right Cauchy Green deformation tensor which is a symmetric matrix.

To find the maximum stretching between initially nearby particles \(x_0, y_0\) note that this will occur when \(\delta x_0\) is chosen to be in the direction of the eigenvector corresponding to the maximum eigenvalue of \(\Delta\). If we let \(\lambda_{max}(\Delta)\) denote the maximum eigenvalue of \(\Delta\) corresponding to eigenvector \(\xi_{max}\), then
\[
\max_{\delta x_0} \|\delta x(t_0 + T)\| = \sqrt{\langle \delta x_0, \lambda_{max}(\Delta) \delta x_0 \rangle} = \sqrt{\lambda_{max}(\Delta)\|\delta x_0\|}, \tag{12}
\]
where \(\delta x_0\) is an initial separation in the direction of \(\xi_{max}\).

If we define
\[
\sigma^{t_0+T}_{t_0}(x) = \frac{1}{|T|} \ln \sqrt{\lambda_{max}(\Delta)}, \tag{13}
\]
then the finite time average of the maximum stretching is given by
\[
\max_{\delta x_0} \|\delta x(t_0 + T)\| = e^{\sigma^{t_0+T}_{t_0}(x)|T|\|\delta x_0\|}. \tag{14}
\]

Then \(\sigma^{t_0+T}_{t_0}(x)\) denotes the largest FTLE for the point in the domain \(x_0\) over a finite time of \(T\). This computation is performed over the entire domain to obtain a FTLE field for a given flow over some finite time \(T\) [3].

In 2001, Haller observed that maximal ridges of the FTLE field are indicators of repelling LCS in forward time and attracting LCS in backwards time [11]. Where there is a high magnitude of separation, high FTLE ridges reside. This seems like an alluringly simple method to find LCS but it does come with limitations. In certain circumstances FTLE ridges can lead to both false positives and false negatives when trying to find LCS. Also, FTLE ridges are often far from Lagrangian. Haller and Farazmand have recently devolved a geodesic theory which gives no false positives or negatives and produces truly Lagrangian structures. They have shown that the most locally repelling strainlines, which are curves that are everywhere tangent to the eigenvector field of the Cauchy-Green Strain tensor, are the repelling LCS [7]. For the flows we are interested in, the FTLE method is sufficient.
2.3 Robotic Strategy for Tracking LCS

An approach developed by Michini, et al. [4] proposes a method for tracking LCS in real time using a collaborative team of mobile networked robots. Unlike other methods including FTLE, this approach does not require global velocity field information of the flow and relies only on local sensing and predictor corrector methods. The robots utilize spatiotemporal sensors along with the ability to communicate with each other to interpolate local velocity fields and guide each other in the search for LCS. In Figure 6 we can see a team of three robots straddling and identifying a strong repelling LCS running down the Santa Barbara channel. For a more detailed explanation on how the strategy works, refer to Ref [4]. An important aspect of the collaborative guiding of these robots is that we understand the dynamics they undergo while moving through the flow. These robots will have a finite energy source and utilizing knowledge of the underlying ocean dynamics they are subjected to will be necessary for optimizing both energy usage and time of traveling as these robots move through the ocean. While there is a strong understanding on how massless point particles will behave in such a situation, work needs to be done to understand how these dynamics differ for inertial objects of a finite size and mass.

Figure 6: Three theoretical (massless) autonomous robots cooperatively tracking a LCS in the Santa Barbara channel [4].
2.4 Maxey-Riley and Inertial Particle Aggregation

As stated previously, there is room for improvement in understanding how particles of a finite mass and size behave and aggregate in a variety of unsteady flows. Maxey and Riley [8] developed the Maxey-Riley equation which is a second-order differential equation describing the motion of an inertial spherical particle in fluid flow. The equation is given by

\[
m_p \ddot{v} = m_f \frac{Du}{Dt} + \frac{1}{2} m_f \frac{d}{dt}[v - u(r(t), t)] - \frac{1}{10} a^2 \nabla^2 u(r(t), t)] - 6\pi a \mu X(t) + (m_p - m_f)g - 6\pi a \mu \int_0^1 \frac{dX(t)}{d\tau} \frac{\nabla u(r(t), t)}{\sqrt{\pi \nu (v(t - \tau)}}.
\]

(15)

where \(X(t) = v(t) - u(r(t), t) - \frac{1}{6} a^2 \nabla^2 u\).

The preceding equation is the dimensional mass formulation of the Maxey-Riley Equation. The parameters of this equation are as follows:

- \(m_p\): mass of the particle
- \(m_f\): mass of fluid displaced by the particle
- \(u(r(t), t)\): velocity of fluid at position \(r(t)\) at time \(t\)
- \(\mu\): viscosity of the fluid
- \(a\): radius of the particle
- \(\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \nabla u\) is the material derivative

The first term on the right \((m_f \frac{Du}{Dt} u(r(t), t))\) describes the forces exerted by undistributed fluid on the particle. The second term on the right accounts for the added mass effects. The third term is the Stokes drag term and the fourth is the buoyancy effect term. The integral term is known as the Basset history term. It is common place to ignore the Basset history term and the Faxen correction when dealing with particles of a sufficiently small particle radius. In our simulations we ignore the buoyancy term and force due to gravity as well. After making these assumptions and non-dimensionalizing the equation, we have

\[
\ddot{r}(t) = \frac{1}{St} (u(r(t), t) - \dot{r}(t)) + \frac{3}{2} R \frac{d}{dt}(u(r(t), t)),
\]

\[
St = \frac{(m_p + \frac{1}{2} m_f)U}{6\pi a \mu L}, \quad R = \frac{m_f}{m_p + \frac{1}{2} m_f},
\]

(16)

where \(St\) is the Stokes Number and is the characteristic relaxation time of the particle with respect to the carrier fluid with \(U\) and \(L\) being the characteristic velocity and length respectively. The density ratio of the fluid density to particle density is given by \(R\). There are three different density regimes that give rise to different dynamical behavior. When \(R < \frac{2}{3}\) \((\rho_f < \rho_p)\) we call these particles “aerosols”. When \(R > \frac{2}{3}\) \((\rho_f > \rho_p)\) we call these particles “bubbles” and when \(R = \frac{2}{3}\) \((\rho_f = \rho_p)\), we call these particles “neutrally buoyant”.

Given a flow field, one can use this equation to advect an initialized mesh of inertial particles through the flow and run LCS diagnostics on the data produced by the simulation. Sudharson, et al. did exactly this [9] and studied how inertial particles aggregate.
in a simulated, time-dependent, double-gyre model. What they found was that there is a certain threshold, depending on $R$ (which measures the density ratio between an inertial particle and fluid particle), that delineated between different dynamical regimes. In one regime of particle density (inertial particle density $>$ fluid particle density), the particles would be attracted to attractors in the system found by computing negative time FTLE (nFTLE). In the other regime (inertial particle density $<$ fluid particle density), these particles would be repelled from the same attracting ridges the more dense particles were drawn to. This was an interesting result with room to be expanded upon by performing similar numerical simulations on a variety of different unsteady flows and comparing with experimental results.

3 Model and Numerical Results

3.1 Double-Gyre model

A natural model to begin fundamental studies of ocean dynamics would be the double-gyre model (DG). The double-gyre model consists of two counter-rotating gyres. This model is governed by the differential equation

$$\begin{align*}
\dot{x} &= -\pi A \sin(\pi f(x,t)) \cos(\pi y) - \alpha x, \\
\dot{y} &= \pi A \cos(\pi f(x,t)) \sin(\pi y) \frac{df}{dx} - \alpha y, \\
f(x,t) &= \epsilon \sin(\omega t + \psi) x^2 + (1 - 2\epsilon \sin(\omega t + \psi)) x.
\end{align*}$$

When $\epsilon = 0$ there is no time-dependence and we get a steady flow. When $\epsilon \neq 0$ the separatrix between the gyres "sloshes" from side to side causing each gyre to expand and contract with a period of $\frac{2\pi}{\omega}$. The value of $A$ roughly tells us the amplitude of velocity vectors while the value of $\epsilon$ dictates the amplitude of left and right sloshing. The phase is $\psi$ and $\alpha$ is the dissipation. In Figure 7 we can see the FTLE fields of both a time-independent (left) and time-dependent (right) double-gyre flow. We look at the DG model in a domain of $[0, 2] \times [0, 1]$, with one gyre center located at $(0.5, 0.5)$ and the other gyre center at $(1.5, 0.5)$. If we look at the FTLE field of the steady DG we can see that the strong ridge running down the center is in fact a heteroclinic trajectory connecting two fixed points. This ridge separates dynamically distinct regions of the flow and, without any stochastic noise, will not allow fluid particles to traverse the LCS boundary. On the other hand, if we let $\epsilon \neq 0$, we obtain a time-dependent DG flow and our FTLE field looks much different. In the time-independent case, the FTLE field will look exactly the same for all time. In the time-dependent case, we are looking at a snapshot of an FTLE movie that shows how the FTLE field evolves over time. It is these cases involving a dependence on time that we expect to produce interesting dynamics.

3.2 Numerical Results

My colleague performed a slew of numerical simulations of both steady and unsteady flows in a double-gyre model [12]. While most are not possible to replicate with our current experimental setup, certain simulations such as steady double-gyre with both tracer and inertial particles can be compared. Although we cannot replicate most of the other simulations, they serve as a guide as we explore inertial particle dynamics in time-dependent flows. In the previous section we see the FTLE field of a steady double-gyre flow computed by Aucoin. We use this as the benchmark to confirm our experiment is working properly. In Figure 8, the left figure shows how neutrally buoyant particles in a steady DG flow are evenly spread throughout the flow. The right figure
shows how particles which are less dense than the carrier fluid (aerosols) aggregate over some finite time. We see that these particles tend to cluster towards the centers of the gyres themselves. With both, we aim to confirm numerical results with inertial particle experiments in the future.

Figure 8: (Left) Neutrally buoyant particles in steady DG flow. (Right) Aerosol particles in DG flow [12].

4 Experimental Setup/Methods

4.1 Setup

Our experimental setup consists of a flow tank (shown in Figure 9), a two Watt laser (for PIV experiments), a high speed camera and a program called DaVis which does all the pre-processing, computation and post-processing required by the PIV method and organizes all the data in a manner which can be exported easily. The tank consists of an 9 by 10 cm aluminum plate which was machined so there are sixteen equally spaced holes. This area is sectioned off by four Plexiglas pieces into an 8cm$^2$ domain. Sixteen metal rods go through the holes in the aluminum plate through the bottom so they are essentially flush with the top of the plate. Either metal discs or glass rods are attached to the metal rods and drive the flow. The sixteen metal rods are driven, in four quadrants, by four variable speed motors through gears attached to the metal rods. These variable speed motors can be operated at varying angular velocities and be turned
on at off at any time (could be done randomly) which allows our setup to produce a wide variety of both steady and unsteady flows. We use glycerin as our fluid and either essentially massless, micron-sized glass spheres (PIV experiments) or roughly 1mm in diameter polyethylene microspheres with density around 1.1g/cc (inertial experiments).

4.2 Methods

We perform two different types of experiments in our flow tank. In both, we are interested in computing FTLE and inertial FTLE (iFTLE) fields to compare with and hopefully confirm numerical work performed by my colleague [12] with the larger aim of providing insight into how coherent structures affect particles of different density ratios with respect to the carrier fluid.

4.2.1 Particle Image Velocimetry (PIV)

Particle Image Velocimetry (PIV) is a non-intrusive method to visualize fluid flows by adding and illuminating tracer particles. Because they are tracer particles, they follow the flow and can provide insight into underlying structures within the flow. We are interested in this method as a way to extract velocity fields from experimental data to compute FTLE fields and identify coherent structures within the flow. The basic method is as follows. First, one seeds their flow with tracer particles. Then these particles must be illuminated at least twice in quick succession, this is done with a laser. Light scattered by these particles must be recorded either in a double exposed single frame or a sequence of frames. Recorded images are then divided into small sub-areas called ”interrogation windows”. Local displacement vectors between the first and second capture is determined for each interrogation window by means of statistical methods (auto- or cross-correlation). Sophisticated post-processing is applied to deal with the immense amount of data. The projection of the velocity vector into the 2-D plane is calculated taking into account the time delay between successive images. This process of interrogation is repeated for all interrogation windows of the PIV recording. From this process, we can extract an Eulerian velocity field for each frame and then apply our own codes to compute FTLE fields on said data.

4.2.2 PIV Experiments

We do this type of experiment with the aim of identifying FTLE fields (and in turn coherent structures) corresponding to fluid particles by means of a PIV method. We first seed our glycerin with our micron-sized glass spheres which will act as tracer particles within the flow. We then add the the glycerin-particle mixture to the tank. The high speed camera is set up above the tank so its line of sight is perpendicular to the plane of...
the surface of the fluid. The laser is then setup, roughly a meter away, and is adjusted until we feel the illumination is sufficient to perform the experiment. For steady flow experiments, we turn on the tank and allow it to reach its steady state and then begin imaging. We have tried different capture rates for the camera ranging from 150 fps down to 50 fps. We have settled on the 50 fps for recent experiments as this appears to be sufficient and allows us to record for much longer times without creating an immense computational cost. The raw data is then processed in DaVis by means of built in operations. Typically we first apply a min max filter for intensity normalization. This filter is meant to even out the light intensity of our images as the illumination is often not perfect. This filter is applied in hopes of smoothing out the light intensity to produce cleaner velocity fields. We then apply a large batch operation called PIV time-series. The time series indicates that we opt to record a time-series sequence of frames instead of the double exposed single frame. This batch operation performs pre-processing, vector calculations and post-processing. The first operation in the PIV batch operation is the time series cross-correlation operation which sets the method we will be using. The next operation is Image Preprocessing. This operation filters out large intensity fluctuations in the background which is meant to further smooth out the illumination of the images with the aim of smoothing out our velocity fields. Next we apply a simple geometric mask so we are focusing only on the tank. The next operation is the Vector Calculation by means of a cross-correlation method. This operation is essential to the PIV method and determines the local displacement between vectors in successive frames which is necessary to produce velocity vectors for each point in the domain. We perform multiple passes of this method with decreasing sized interrogation windows. There are many parameters that can be set within this operation that can remove and replace vectors if they are a certain number of standard deviations away from neighboring vectors. We often employ these methods to get rid of spurious vectors. Following this we perform vector post-processing which applies further filters to the processed data again with the hopes of getting rid of and replacing any vectors that don’t belong. Once all this is complete, we are able to extract velocity fields for each frame. We then export this data and run it through our MATLAB code to compute the FTLE fields for the experimental flows. These FTLE fields are then compared with FTLE fields obtained from numerical simulations.

4.2.3 Particle Tracking Velocimetry (PTV)

Particle Tracking Velocimetry (PTV) is a derivation of PIV and therefore, the process is quite similar. Instead of producing Eulerian velocity fields, full Lagrangian particle trajectories can be obtained for each individual particle. The typical approach is to first identify the location of individual particles. Then a particle pairing algorithm is applied to match particles in successive frames. We use a collection of codes developed by Nicholas Ouellette and his team from the Civil and Environmental Engineering Department at Stanford University. These codes, given a series of images of particles in a flow, produce Lagrangian particle tracks using a predictive three-frame best-estimate algorithm. The codes take a series of images, and asks the user to stipulate thresholds (brightness/darkness relative to the background, minimum size of particle, if the particles are brighter or darker than the background and the maximum displacement from kinematic prediction of the particle). We use this code to deal with experiments run with inertial particles to identify particle trajectories and compute corresponding iFTLE fields.
4.2.4 PTV Experiments

We perform these experiments with the aim of identifying iFTLE fields corresponding to inertial particles by means of a PTV method. In this case, the glycerin is added to the tank without the suspended micron sized glass spheres. Instead, inertial particles are carefully placed on the surface of the fluid, usually arranged in an equally spaced grid. In this type of experiment, we do not employ a laser as the inertial particles we work with do not fluoresce. Instead we employ a clever use of over-exposed images coupled with a tedious mask to make sure the PTV code identifies the particles and only the particles when tracking. We start the tank and begin recording simultaneously. For the PTV experiments we only use DaVis to mask out the areas that would be problematic for the PTV code. After this mask is complete, we export the masked out, over-exposed raw images and run them through Oullette’s PTV code. The code returns the tracks of each identified particle and its corresponding velocity at each time step. This data is then put through our iFTLE code and iFTLE fields for the experimental flows are computed. These iFTLE fields are then compared with iFTLE fields obtained from numerical simulations of Aucoin [12].

5 Experimental Results

5.1 Tracer Particle Experiments

We perform experiments with micron-sized tracer particles to set a benchmark for comparison when investigating if and how LCS affect inertial particles differently.

5.1.1 Time-Independent PIV

We begin with a time-independent multi-gyre flow. We perform experiments following the procedure laid out in section 4.2.1. In our first set of figures, we can see the progression of the data from its initial stage of raw images, to extracted velocity fields utilizing a PIV method, to the final FTLE field of the flow. In Figure 10 we can notice that in the raw images, the lighting seems uneven. This was a challenge we have had throughout our time performing PIV experiments. With discs, we rarely were able to get a laser sheet we were happy with. When we switched to using glass rods this helped the problem some but still did not totally resolve the issue of uneven illumination. To assist in this, we applied the filter mentioned in 4.2.2 which was able to even out the lighting. We will see in the next set of figures to what degree this helped. The velocity field is also uneven and velocities vary from gyre to gyre. We attribute this to multiple things. One being the unevenness of the laser sheet, another being the unevenness of the objects creating the gyre flow. In the case of discs, they are often not perfectly flat and in the case of rods, they are often not perfectly straight. As time went on I was able to improve this but was never able to get them to be perfect. This brings us to the last issue which is the fact that we are attempting to image a 2-D flow, with no effect from the z-direction, when in reality we are imaging a 3-D flow. We expect that, in reality, we are getting some effects from the third dimension. The discs are not perfectly flat and sit beneath the surface of the flow (and beneath the plane which we are imaging) leading us to believe they are adding some 3-D effects which are not negligible. Without the rods being perfectly straight, they precess and we believe this not only adds 3-D effects but also accounts for the unevenness of the velocity field around the gyres. For time-independent flows we used a number of filters and methods (mentioned in section 4.2.2) to assist in cleaning up the data and producing smoother velocity fields. Before we had knowledge of how to use processing tools to clean things up, we can see all the
issues play out in the FTLE field as we notice the choppiness about it. Beyond that, it does not appear to be correct. We expect strong repelling ridges running in between the gyres, not ridges circling the gyres.

![Figure 10](image1.png)

Figure 10: (Top Left) Raw image from experiment. (Top Right) Velocity field from experimental run. (Bottom) FTLE field calculated from velocity field.

In Figure 11 we can see how our velocity field and FTLE field has improved as a result of filters and post-processing methods. In the first image we can see the result of applying min max normalization filter mentioned in the methods section. We can see that our velocity field is now much closer to uniform and we do not see large deviations between gyres. This results in a FTLE field that does indeed match expectations. We would like to continue to improve our imaging methods and processing methods to obtain higher resolution velocity fields and in turn, higher resolution FTLE fields.
5.1.2 Time-Dependent PIV

In addition, we looked at a time-dependent flow with periodic switching and different angular velocities driving the rods. Although we cannot emulate the sloshing from the DG model in our experimental tank, we attempt to create some similar time-dependent affects by producing a flow which will move the separatix to one side. We investigate a flow which has a period of 2.5 seconds and the rods have a velocity ratio of 2:1. One rod was spun at 37.5 rpm while the other was spun at 75 rpm. Once one rod turns off, the other turns on immediately. A finite time of 15 seconds was used to compute FTLEs. This experiment was done near the end of our work and could use some tweaking of the finite time or resolution used in processing to further improve the FTLE fields. It can be seen from the velocity fields in Figure 12 that the rod on the left is spinning faster than the one on the right. We can look at a snapshot of the FTLE field and observe that indeed the separatrix appears to be moving to one side (the right side) which makes sense as the left rod is spinning with a higher angular velocity and therefore we expect its "reach" to extend into the domain of the other gyre and occasionally pull some particles from the other’s region due to to its greater radial reach produced by its higher angular velocity. In the future we would like to increase the ratio of velocities to see if the shift becomes more pronounced.
5.2 Inertial Experiments

We perform experiments with inertial particles in an attempt to understand how coherent structures affect these particle’s trajectories. Our aim is to gather data and compute iFTLE fields from a variety of different flows. Obtaining iFTLE fields from experimental data has been trickier than previously expected and we have not yet perfected the method. We plan on honing this method in the near future to produce a clearer comparison. Considering this, we present a suite of preliminary results along with qualitative observations with the aim of laying the framework for future endeavors. We present the base case of a grid of neutrally buoyant particles in a time-independent Stokes four roll mill. From there we compare how particles of different shapes behave in a time-independent double-gyre flow. Following that we present two cases of a grid of neutrally buoyant particles in a time-dependent double-gyre flow with periodic switching. Lastly, we again compare particles of different shapes but this time, for the time-dependent flow mentioned above. In all cases we use glass rods to drive the flow and use neutrally buoyant particles which range in size from 1-1.08 mm in diameter and have a density of 1.1g/cc.

5.2.1 Time-Independent flow with Neutrally Buoyant Particles

The first experiment we will look at is a time-independent Stokes four roll mill with neutrally buoyant inertial particles. Glycerin was the carrier fluid and it was at a level of 2cm from the bottom of the tank. We used glass rods and they were spun at an angular velocity of 46.875 rpm. The particles are initially arranged in a grid, the tank is turned on and then we begin to image the experiment. The duration of the experiment
was 1 minute and 20 seconds. In Figure 13, we can again see the progression of data from the initial raw tank images, to the images post filtering and masking, to its final stage of particle tracks. In the top images we can see the first and final frames of the raw tank images. In the bottom images we can see the first and final frames of the masked image and in the last figure we can see the final image of the particle tracks. We see generally what we expect from neutrally buoyant particles in a steady gyre flow. They seem to circle around their respective rods and they tend not to leave this region.

Figure 13: (Top Left) Raw image from experiment at initial time \( t = 0 \). (Top Right) Raw image from experiment at final time \( t = 80 \text{s} \). (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

### 5.2.2 Comparing shapes in Time-Independent DG

Next we show a qualitative comparison between particle shapes. We compare spheres (the particles used in the previous experiment) with rods (two spheres glued together) and with triangles (three spheres glued together in the shape of a triangle). We compared these shapes in the time-independent double-gyre flow. The angular velocity of the rods is 65.625 rpm and the glycerin is again at a height of 2 cm. In all cases of comparison we placed the particles in a line below the gyres. The images in Figure 14 show spheres, Figure 15 shows rods, and Figure 16 shows triangles. In all cases we use seven particles.
In the set of images from Figure 14 we can see the initial and final stages of the spheres experiment as well and the final particle tracks. We notice that three particles circle the left gyre, three circle the right gyre and one appears to be missing but is really hidden under one of the masked areas. This was the particle that was initiated on the separatrix. This particle did not circle either gyre but was ejected up through the middle and out of the regions of attraction.

![Figure 14](image1.png)
the separatrix. Again, this particle did not circle either gyre but was ejected up through the middle and out of the regions of attraction. I attempted different parameters for the PTV code to obtain better tracks for the particles circling the left gyre but could not produce anything better than what is shown below. I suspect this either had to do with the lighting in this specific experiment or the PTV code not liking the shapes of the rods as they were problematic in later experiments as well.

Figure 15: (Top Left) Raw image from rod experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 100$ s. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

In the last set of time-independent comparison images from Figure 16 we can see the initial and final stages of the triangles experiment as well and the final particle tracks. We notice again that three particles are circling the left gyre, three circle the right gyre
and one is above the gyres taking a looping trajectory. The particle at the top left was the particle that was initiated on the separatrix. This time, the particle takes one large looping orbit around the left gyre in the duration of the experiment.

Figure 16: (Top Left) Raw image from triangle experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 100s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

Although it is not quite evident in the figures, when observing the movies of each experiment we can notice that the smaller the particle size (smallest being spheres, largest being triangles), the more likely the particle is to take a more looping trajectory and the more likely it is to escape a region of attraction. We saw that in both the sphere and rod case the particle that initially straddled the separatrix was pushed out of the regions of attraction while the triangle’s motion (that straddled the separatrix initially) was loosely governed by the left gyre.
5.2.3 Periodically Switching DG Flow

In addition to looking at time-independent flows, we observe particle dynamics in a time-dependent flow. We look at a double-gyre flow driven by rods which turn on and off periodically with different angular velocities. Although we cannot emulate the sloshing from the DG model, we attempt to produce some similar time-dependent affects by producing a flow which will move the separatrix to one side.

We first investigate a DG flow which has a period of 2.5 seconds and the rods have a velocity ratio of 2:1. One rod was spun at 28.125 rpm while the other was spun at 56.25 rpm. We use a grid of neutrally buoyant particles. All experiments here are run for 122 seconds. By observing the particle tracks in Figure 17, we can see something quite different from the time-independent case. In the time-independent case, we had a clear gap between the two gyres and trajectories never crossed. In this time-dependent case, though trajectories do not cross paths they do become quite close and the gap is much smaller. We also see that this gap is indeed shifted over to the right slightly, again do to the higher angular velocity of the left rod. We also notice jagged trajectories in the time-dependent case where we had essentially smooth trajectories in the time-independent case. This is due to the on-off effect of the gyres pulling the particles in between them closer to the rod that is on though in this case, no particles leave one basin of attraction and enter the other.
In the next set of images we are again looking at a DG flow with periodic switching happening every 2.5 seconds. We still use a velocity ratio of 2:1 but here, one rod is operated at 37.5 rpm and the other at 75 rpm. Again, we initialize a grid of neutrally buoyant particles. We see some similarities with the last experiment with some stronger effects and some new behavior to notice. If we look at the tracks in Figure 18, we again see some jaggedness to the trajectories due to the switching. A new thing to notice is that now the gap that was once prominent in the time-independent case and was noticeable in the first time-dependent case is completely gone. We have particle trajectories crossing each other somewhat often. We initially suspected that we did have particles from the weaker basin of attraction escaping to the stronger region of attraction but upon further inspection we realized that trajectories we crossing but no particles were escaping. We suspect with a higher velocity ratio we would see an escape from the weaker region of attraction to the stronger. We plan on investigating this supposition in the future.
5.2.4 Comparing Shapes in Periodically Switching DG Flow

In our last set of experiments we compare the shapes from 5.2.2 in the time-dependent DG flow from sections 4.1.2 & 5.2.3. In these experiments we again use a period of 2.5 seconds and use a 2:1 ratio of angular velocities, the velocities being 37.5 rpm and 75 rpm.

In the first set of images from Figure 19, we see spheres in a switching DG flow at the initial and final times along with their final particle tracks. We begin with seven spheres and we can observe some interesting behavior. This time, the particle that initially straddled where the separatrix would be in the time-independent case is pulled into the region of attraction of the stronger gyre (the one on the left) and the particle initially to the right of it acts as it is straddling the separatrix and is ejected out of the regions of attraction, never returning to either. The four particles on the left circle their rod and do not leave and the two left do the same until, near the end of the experiment another particle from the right region is ejected out the top and leaves its gyre region.
Figure 19: (Top Left) Raw image from switching sphere experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120$ s. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.

In the next set of images from Figure 20, we see rods in the same switching DG flow at the initial and final times along with their final particle tracks. We begin with seven rods and we observe much of the same thing. Again the particle that initially straddled where the separatrix would be in the time-independent case is pulled into the region of attraction of the stronger gyre (the one on the left) and the particle initially to the right of it acts as it is straddling the separatrix and is ejected out of the regions of attraction, never returning to either. This time the four particles on the left circle their rod and do not leave and the two left do the same although one of them is taking a much wider orbit.
In the final set of images from Figure 21, we see triangles in the same switching DG flow at the initial and final times along with their final particle tracks. We begin with seven triangles and see more similarities. Again the particle that initially straddled where the separatrix would be in the time-independent case is pulled into the region of attraction of the stronger gyre (the one on the left) and the particle initially to the right of it acts as it is straddling the separatrix and is ejected out of the regions of attraction. This time the four particles on the left circle their rod and do not leave and the two left do the same with tight orbits.

Figure 20: (Top Left) Raw image from switching rod experiment at initial time $t = 0$. (Top Right) Raw image from experiment at final time $t = 120s$. (Middle Left) Masked image of experiment at same initial time as image above. (Middle Right) Masked image of experiment at same final time as above. (Bottom) Final particle tracks produced by Ouellette PTV code.
Although these observations are tough to make by just looking at still images, if we look at the movies this behavior becomes clear. We see similar behavior as to the time-independent case but this time it is more pronounced. With spheres we see two particles leaving the weaker region of attraction. With the rods we see one leave but another taking a wide orbit and given enough time it may very well have left the region of attraction too. And in the case of triangles we see one particle leave the weaker region of attraction but the remaining particles keep tight orbits for the remainder of the experiment. This leads us to believe that less surface area equates to a higher probability of escape. In addition, we see more evidence that the separatrix has moved towards the weaker gyre as the particles that were initialized where the separatrix would be in the time-independent case are pulled into the stronger region of attraction and the particles immediately to the right of this point (where we expect the separatrix to be in the time-dependent case) are being ejected out of all regions of attraction, behaving as if they were straddling a separatrix.
6 Summary and Conclusion

We have presented a suite of experimental results involving both tracer particles and inertial particles in time-independent and time-dependent flows. We apply the FTLE method to data obtained from tracer particle experiments and use Ouellette’s PTV code to make qualitative observations about inertial particle behavior. Although we would like to hone these tools further and perfect the method of obtaining iFTLE fields for inertial particles, we were still able to produce some interesting results regarding inertial particle dynamics and have taken steps toward our main goal of understanding how LCS affect inertial particles.

In the early days of this work much time was spent refining and improving the experimental setup. This ranged from building a new housing for the tank to gaining a solid understanding of the many processing tools used to handle our data. Once we were satisfied with our setup we set our aim on performing a variety of experiments to investigate how coherent structures affect inertial particle dynamics. We began by developing a base case for comparison by obtaining experimental FTLE fields for a time-independent multi-gyre flow. While initially we were less than satisfied with the results, we improved on our imaging and processing methods to obtain smoother data and in turn, cleaner and more accurate FTLE fields. From there we investigated how neutrally buoyant inertial particles behaved in the same type of flow. Qualitatively, we saw agreement with both Sudharson et. al [9] and Aucoin [12] by noticing (through raw images and particle tracks) that these neutrally buoyant particles essentially follow the flow and circle around within their respective gyre in a time-independent double-gyre flow. In addition, we noticed that these particles never left the area of attraction they originally resided in.

Following these experiments we decided to investigate the role the shape of the particle plays in how it will behave in the time-independent double-gyre flow. We looked at three different particle shapes (spheres, rods and triangles). We were able to make some qualitative observations. In specific, we observed that the less surface area that a particle had the more likely it was to take wider orbits and in turn, the more likely it was to escape a region of attraction. We noticed that spheres had the greatest tendency to take these wide orbits, with rods being less likely to and triangles being the least likely.

From here our aim was to investigate if and how the results of these previous experiments changed for time-dependent flows. We created a periodically switching double-gyre flow with different angular velocities in our flow tank and began to run through the same experiments we had performed in the time-independent case. First, we looked at passive tracer particles to again, set a benchmark for comparing the results we would obtain from inertial particles. Although we would have liked to get a cleaner FTLE field, we were able to notice that the separatrix had indeed shifted over towards the weaker gyre.

The next step was to perform inertial experiments in this time-dependent double-gyre flow. The first experiments we did were two experiments with an initialized grid of inertial particles, each experiment with different velocities for each rod. In the first experiment (lower velocities) we noticed that the clean gap where the separatrix was in the time-independent case was much smaller and appeared to be shifting slightly towards the weaker gyre. We did not see any particles crossing trajectories from opposing gyres or escaping one gyre region and entering the other. We did see some jagged trajectories which were not present in the time-independent case due to the on off switching of the gyres. In the next experiment (higher velocities) most of these things became more pronounced and we noticed some new behavior as well. This time the gap between the
gyres was no longer clear of trajectories. In fact it was inundated with trajectories crossing each other. In addition, we did see particles from the weaker region of attraction escaping and entering the stronger region of attraction.

In our last set of experiments we again compared shapes but this time, did the comparison in the periodically switching double-gyre flow. We noticed more pronounced behavior due to the shape of the particle and were also able to give further credence to the shift of the separatrix. We saw that the smaller shapes had become more likely to escape their initial region of attraction. By observing movies of the experiments, we were also able to notice that the particle that would have initially been straddling the separatrix had it been in a time-independent flow, was drawn into the stronger region of attraction. In addition, the particle immediately to the side of it (side of weaker attraction) was ejected out the top as a particle straddling a separatrix would be.

Moving forward, we would like to improve our methods to increase the accuracy of experiments and computations with the aim of providing more rigorous comparisons. We would like to do full grid experiments of different particle shapes and further expand the catalog of experimental time-dependent flows we aim to study. We also would like to move towards confirming some of Aucoin’s results involving different density ratios of particles with respect to the carrier fluid. In conclusion, we have provided a variety of new experimental results having to do with inertial particle dynamics. We have demonstrated that, in addition to numerical results showing that particle density affects trajectories [12], the surface area of the particle plays a role in its behavior as well. This can be valuable information to take into account when developing strategies to operate AUV’s in an energy and time optimal manner.
References


A Experimental Procedure

This is a manual describing how to setup and perform experiments as well as a guide meant to instruct you on how to use all the pre- and post-processing methods I have implemented in my research. DaVis is a very powerful program and much of what it is capable of will not be covered in this manual. Only the methods I specifically used will be covered. If you feel other methods may be of use to you consult the DaVis manuals in the lab. I will cover the two different types of experiments I performed (tracer and inertial) and explain the processing tools I’ve used for each.

A.1 Tracer Experiments

A.1.1 Setup

(i) Begin by taking out all necessary items to run the experiment. You will need the camera, its tripod, micron sized glass spheres, the 2-watt laser, laser stand, glycerin, discs or rods (if not already attached to the tank), and the cords for the camera.

(ii) Before I do anything else I run the tank for a short amount of time (15s) to make sure that all metal rods are spinning (refer to Appendix B for troubleshooting) and everything is running properly. Occasionally, some grey liquid will seep out from where the metal rods enter the tank due to friction. If this is the case, run for 1 minute and then clean with an alcohol wipe.

(iii) Once you have insured that the tank is functioning properly the next step is to setup the camera. First setup the tripod above the tank so that the camera will be able to aimed directly at the tank facing down. Then attach the camera to the tripod, attach the cables to the camera and aim the camera at the tank. Once this is done you can start up DaVis (do not start up DaVis before the camera is plugged in because DaVis recognizes the camera on startup). There is a specific order in which the cords must be plugged in. First plug the red power cord into the power 1 port. Then plug in FSYNC, Trigger and Ethernet into their respective ports, in that order. Each cord should be labeled. In DaVis, go to an existing project or make a new project you wish to record in. From there choose the recording tab on the top. Once in the recording section you are able to take a snapshot or live video to position the camera in the correct spot for your recording. I choose live video so I can continually make adjustments without having to constantly take new snapshots. There is a timing tab which allows you to adjust the number of images to be recorded and the capture rate (frames per second of the resulting video). I typically operated 100Hz. This is a very expensive camera so be careful when attaching/detaching to/from the tripod.

(iv) Now we are ready to make our tracer particle mixture by pouring glycerin into a water bottle and mixing in micron sized glass spheres. This is an area that is a bit of an art. We have not found a precise mixture that works perfectly but I encourage you to take the time and try out different amounts until you find what you think is best. We typically fill up a water bottle with glycerin between a fifth and a quarter full and then add two flathead screwdriver heads full of particles. We then slowly roll the water bottle head over end for 30 seconds or so to ensure the tracer particles become uniformly mixed without creating air bubbles in the liquid which can cause problems when imaging. Proper particle seeding concentration is an important part of the PIV method. Over-seeding can lead to noisy data...
and under-seeding can lead to rough data, both of which will be a detriment when computing FTLE fields. There is literature online with more information on sufficient particle concentration. Pour into tank evenly, amount varies. With discs aim to get the level of glycerin right above the discs and image the sheet of fluid in between the discs and the surface in hopes of maximizing the effects of the discs while minimizing 3-D effects. With rods the level is less of a concern and you can play with it to see what works best for you.

(v) The next step is to setup the laser. First turn on the laser light in the lab and place a sign on the door indicating that an experiment is in process and a high powered laser is being used. Take out both the 2-watt laser and the power supply. Plug the power supply into an outlet and the laser into the power supply. Then get the stand for the laser out and place the laser on the stand. Before you attempt to turn on the laser, ensure that either the push-pull button on the side of the laser is pulled out or that the laser is directed at an area that won’t reflect back into your eyes or on your skin. This is a very powerful laser that can damage your eyes rather easily and with enough time damage your skin. Never, under any circumstances, look directly at the source of the laser. Once you have ensured the setup is safe you can turn on the laser. Turn the switch on first*, wait a few minutes for it to warm up and then you can turn the key. If you have not pushed the push-pull button in yet you can do that once you have again ensured eyes and bare skin are out of the lasers path. Start turning up the knob on the laser to increase its intensity. Do not go above 1.5A (I usually operated the laser around 1.1A). You can turn off the lights and use the live capture mode of the camera to adjust the laser to attempt to get a uniformly illuminated image. If needed, move the laser farther or closer to the tank, adjust the prism on the front of the laser to adjust the orientation of the laser sheet, and change the intensity as needed (while staying below the 1.5A mark). This is an expensive laser so again, be careful. To be safe, I would raise the intensity slowly. Once you are satisfied with the image, you are ready to begin the experiment. Before moving forward I usually pull the push-pull button back out so you can move forward without fear of getting some unwanted laser rays.

A.1.2 Procedure

(vi) Now that you have setup the experiment you are ready to begin. Before moving further make sure you have the correct code up on the laptop to control the motors. The one that is most often used is quad_move_test_1direction.m. This code is setup to run the motors in one direction (meaning it runs until final position is reached then stops rather than running to position and then running backwards to 0, if you want this use quad_move_test.m) at a constant velocity. You will set the position, velocity, acceleration and current limit. Position will tell the motors what position to end at and velocity tells them how fast to spin. The measurements are all in 1/16 of a step (if position = 16, motor will move 1 step). Acceleration and current limit should be left alone unless you have a specific reason to change them. Other codes exist and you can write them as needed to create different types of time-dependent flows caused by periodic switching, a non-linear forcing function controlling the velocity or anything else you can think of.

(vii) Once more I check that the DaVis and MATLAB parameters agree. I usually tell the camera to capture for a little bit longer (few seconds) than the motors will be running to account for troubles with synchronizing their starts. When imaging
steady flows we typically run the tank for 15s to allow the flow to reach its steady state. Now, turn off the lights and get ready to begin. I run the MATLAB code to start the motors first (it is set to have a 1 second delay from the time you push run until the time the motors start, this can be changed in the code) and then immediately after start the camera by pressing the record button DaVis (be aware this usually takes about a half second to initialize). Now the experiment is in process. If you notice any problems you can either let the experiment run if the problem is not in danger of damaging the tank or cut the power to controllers if you believe the tank is at risk but this should be done with caution as it could damage the controllers. If there are no problems, just wait until the experiments concludes and DaVis will begin saving the images. You are now ready to process the data.

A.1.3 Processing

(viii) For tracer experiments we utilize quite a few built in operations provided by the DaVis software to aid in cleaning up our raw data.

(ix) Typically we first apply a min max filter for intensity normalization. This filter is meant to even out the light intensity of our images as the illumination is often not perfect. This filter is applied in hopes of smoothing out the light intensity to produce cleaner velocity fields.

(x) We then apply a large batch operation called PIV time-series. The time series indicates that we opt to record a time-series sequence of frames instead of the double exposed single frame. This batch operation performs preprocessing, vector calculations and post-processing. The first operation in the PIV batch operation is the time series cross correlation operation which sets the method we will be using.

(xi) The next operation is image pre-processing. This operation filters out large intensity fluctuations in the background which is meant to further smooth out the illumination of the images with the aim of smoothing out our velocity fields.

(xii) Next we apply a simple geometric mask so we are focusing only on the tank.

(xiii) The next operation is the Vector Calculation by means of a cross correlation method. This operation is essential to the PIV method and determines the local displacement between vectors in successive frames which is necessary to produce velocity vectors for each point in the domain. We perform multiple passes of this method with decreasing sized interrogation windows. There are many parameters that can be set within this operation that can remove and replace vectors if they are a certain number of standard deviations away from neighboring vectors. We often employ these methods to get rid of spurious vectors.

(xiv) Following this we perform Vector post-processing which applies further filters to the processed data again with the hopes of getting rid of and replacing any vectors that don’t belong.

(xv) Once all this is complete, we are able to extract velocity fields for each frame. These velocity fields are then run through our FTLE code to produce an FTLE field.
A.1.4 Cleanup

(xvi) To begin cleanup, disconnect and put away both the laser and the camera (do not disconnect the camera until the DaVis is finished saving the images or they may be lost).

(xvii) Then, take the large syringe and do your best to extract as much glycerin as possible from the tank. I usually put it in a paper cup and dispose of it when I am done.

(xviii) Once I have removed as much as possible with they syringe, I place small (roughly 4in\textsuperscript{2}) crumpled up pieces of paper towel into the tank to soak up the rest of the glycerin. It usually takes about a day for the paper towels to soak up the remaining glycerin. When using rods, make sure to place them in between rods to avoid the rods becoming detached from their screws. We do this instead up unscrewing all discs/rods to avoid getting glycerin in the screw holes and having to clean it later.

A.2 Inertial Experiments

A.2.1 Setup

(i) Begin by taking out all necessary items to run the experiment. You will need the camera, its tripod, inertial particles, tweezers, glycerin, discs or rods (if not already attached to the tank), and the cords for the camera.

(ii) Same as A.1.1 ii & iv.

(iii) For these experiments we will not be making a glycerin-particle mixture. Instead we will pour the glycerin in the tank and add particles in the configuration we desire. Add the desired amount to the tank by pouring in between discs/rods. The level to which you will fill the tank is dependent on the experiment and if discs or rods are being used. If discs are being used, I typically aim to just cover them so they are as close to the surface as possible and then attempt to image a thin sheet between the discs and the surface. We do this in hopes of maximizing the effects of the discs while minimizing 3-D affects. With rods this is less of an issue but depending on your configuration you may want a certain height. For example, I often do experiments and only focus on the four center rods (creating a Stokes four-roll mill). When I am doing this I take off all other rods and then make sure to make the glycerin level high. I do this because the metal rods which don’t have glass rods attached to them are still spinning and I want as little affect from them as possible in hopes of creating the four-roll mill flow with free boundaries.

(iv) The last step is to place the particles in tank. I typically place them very carefully in a grid configuration with tweezers. Sometimes we are interested in the different paths two nearby particles will take. Sometimes I just pour and aggregate in the center to see how it will evolve. This depends on the experiment you are attempting to run. Once you are satisfied with the placement of the particles, you are ready to begin the experiment.

A.2.2 Procedure

(v) The procedure is essentially identical to the procedure for tracer experiments. Refer to A.1.2 vi & vii.
A.2.3  Processing

(vi) The processing for these inertial experiments is quite different than the tracer experiments. We barely use DaVis for these experiments with the exception of a masking function. DaVis has PTV operations built in but we have not had any success with them and have had quite a bit of success using Dr. Ouellette’s suite of codes. In a nutshell, these codes work by looking for a contrast in brightness between particles and the background image. *

(vii) The first thing we do is apply a geometric mask to the images. This is done by selecting the mask operation, selecting geometric mask and then defining your mask/s. I begin by applying an enabling mask to the boarder of the tank (or whatever region you plan on focusing on). This mask will keep everything inside it but disregard everything outside it. I then, quite meticulously, apply sixteen disabling masks to each rod or screw (screw if the rod is detached). As I’ve mentioned above, Dr. Ouellette’s code works by looking for a contrast in brightness between particles and the background image. Our particles are dark (black) so we seek particles that are darker than the background image. The screws are dark as well so without the disabling masks, the code also thinks they are particles and tracks them producing unnecessary data we have to discard later. We now apply these masks, export the data and are ready to run the data through Dr. Ouellette’s PTV code. The codes have detailed explanations of how to use them and are accompanied by an example to further assist a new user.

A.2.4  Cleanup

(viii) To begin cleanup, disconnect and put away the camera (do not disconnect the camera until the DaVis is finished saving the images or they may be lost).

(ix) Next, remove the inertial particles with a pair of tweezers. I put them in small cup with water and swirl them around a bit to try to get the glycerin off them. Then I use place a paper towel or napkin on top of another cup to act as a filter. I pour the particles and water into the filter and then put the particles into a bag once all the water has strained. I’m sure there are better ways to do this so feel free to try different ways of doing this.

(x) Same as A.2.4 (x) & (xi).

B  Tank Maintenance

This is a guide on how to maintain the tank and keep it operational. Note: Some methods in this manual have been inherited from previous students, some I have come up with on my own. The methods may not always be the best way to do things so if you come up with better methods, consult your advisor and if he/she deems it safe, proceed with what works best for you.

B.1  Basic Upkeep/Cleaning

(i) Clean the tank regularly (once every two weeks or so). This involves taking off the top of the tank and cleaning both the top of the tank and the metal rods¹ and

¹I refer to two different types of rods in this manual. When I say rods I am referring to the acrylic/glass rods which are in the top of the tank, are bonded to the screws and create the gyre flow. When I say
shaft collars on the top of the bottom half of the tank (alcohol pads work best). Make sure to clean any grease that has seeped into the tank as this can cause problems when imaging. Plexiglass sides of the tank should be cleaned if there is anything on them that could possibly obstruct the laser sheet.

(ii) Discs/Rods should be checked often to see if they need to be re-bonded to screws. Failure to do so could result in letting days go to waste when experiments could be run but the discs/rods need time to bond.

(iii) Both discs and rods should also be cleaned (rods more so than discs) because failure to do so could, again, interfere with the laser.

(iv) Gears should also be checked to make sure epoxy bond is still strong (this can be done by running the tank for a couple minutes). Since we have switched over to the new epoxy this has not been an issue but it is still wise to pay close attention to them as failure to do so could again, result in wasted days.

(v) Be sure you are aware of how many of each part of the tank you have and what tools and cleaning supplies you have and inform your advisor if you are running low on anything.

B.2 Gears/Shaft Collars

(i) On the top of the bottom half of the tank there are 16 metal shaft collars - 4 large shaft collars (one on each corner metal rod) and 12 small shaft collars on all of the other metal rods. Maintenance on these is rare and straightforward (tighten the set screw if shaft collar is loose, replace if it breaks for some reason).

(ii) On the underside of the bottom half of the tank there are 2 different types of gears (4 angled gears and 12 flat gears) and the small metal shaft collars. I will denote angled gears as A, flat gears as F, and metal shaft collars as M. The combinations on the bottom are as follows: 2 - AMF; 2 - AFM; 6 - FM; 6 - MF (refer to picture for placement).

(iii) If a new gear is needed (you should have an almost full if not entirely full set of replacement gears in the lab) the gear needs to be drilled for a set screw and needs to be bonded to whichever part it needs to be bonded to (either another gear, a metal shaft collar or both).

(iv) First, we need to use our tapping bit to drill a hole through the base of the gear (I did this in a slightly unorthodox way because it was the simplest with what I had, if you find an easier way or way you like better then do that). Insert clamp A into clamp B, then insert the gear in clamp A. Drill over wood.

(v) The next step would be to bond the gear to whatever it needs to be bonded to (either another gear, a metal shaft collar or both). Regardless what it needs to be bonded to the procedure is essentially the same. You will need 2 clamp As, a spare metal rod, gloves, respirator, epoxy and paper towels. When bonding it is best to wear a long sleeve shirt and do this on a weekend if possible so not many people are in the building. The fumes from the epoxy are toxic and should not be inhaled (hence the respirator) nor should the epoxy itself get on your skin or eyes. If you feel either of these happened to a degree that could be damaging you metal rods I am referring to the metal rods that run through the entire tank and are attached to the gears and shaft collars. The metal rods drive the rods.
should seek help immediately. First setup for bonding by laying a paper towel under where you will be applying the epoxy and setting another aside to put the finished product on. Then get out 2 clamp As for each set that needs to be bonded, the spare metal rod and the pieces that needed to be bonded. I advise cracking the door (if the building is empty I would open the door all the way) and putting the STRONG FUMES DO NOT ENTER SIGN sign on the door. Before you take the epoxy out you should put on the respirator and gloves. From there you can take out the epoxy gun (should already be loaded, if not it is a simple load and should be explained on the epoxy case). For simplicity I will explain how to bond the bottom of a gear to a shaft collar but the same procedure will apply when bonding the top of a gear to a shaft collar and for gear to gear bonding. Now you are ready to begin; start by putting the shaft collar on the metal rod (set screw not tightened). Then apply a thin bead of epoxy around the bottom of the gear. Next, put the gear on the metal rod and squeeze it together with the shaft collar. Then attach a clamp A on both sides of the bonded set and slide off the metal rod. If you are doing this with more than one set make sure to clean off the metal rod with a paper towel in between bonding sets. Put the set or sets on your finished product paper towel and let dry for approximately 3-5 days. After the first 24 hours the fumes are no longer strong enough to be dangerous and you can come and go in the lab as you need.

(vi) There could be a problem with a gear if a single disc/rod is not spinning or if a whole quadrant is not spinning. Simply run the tank and look at its underside to see if the gear is the issue (alternatively, if a whole quadrant is not spinning this could be the result of a problem with one of the motors/controllers).

(vii) If the gear is the issue you will need to remove the gear from the tank and rebond it to whichever gear or shaft collar from which the bond broke. Removing the gear and shaft collar is typically simple, just unscrew the set screws of whichever parts you need to take off and they should just slide off (if they cant be slid off easily you can use needle nose pliers to remove the gear but make sure to grab the base of the gear and not the teeth). To bond the gear refer to A.2 step (v).

B.3 Miscellaneous Problems

(i) Do not remove metal rods unless it is absolutely necessary. It is very difficult to get them out, back in, and at the right height without damaging either the metal rods themselves or a ball bearing.

(ii) If it is absolutely necessary to remove a metal rod, first remove the tank from its housing. Then remove the top half of the tank. Remove the shaft collars and gears from each rod. Next you will need to hammer out the metal rod (I saw no better way to get it out seeing as the metal rods fit extremely tightly with the ball bearings). There is a rubber cork which can be placed on the side of the metal rod which will be hammered (rubber cork prevents the metal rod from being damaged). Once the rod is out, do whatever you need to do to fix your issue. For me it was a ball bearing so I will explain how I fixed that.

(iii) Ball bearing should come out with the rod (if it does not I would advise hammering the spare metal rod through the bearings until the one you need to replace comes

---

2It is imperative that you take the set being bonded off of the metal rod before the epoxy sets. Failure to do so will result in the set being stuck to the metal rod. This epoxy is an extremely strong adhesive and you will have a great deal of trouble getting the set off the metal rod (I know from experience).
off. I saw no better way to do this without damaging the bearing on the other
side). To put a new ball bearing in either use a rubber cork or small piece of
wood, place ball bearing in its housing and lightly hammer either the wood or
rubber cork on top of the ball bearing until it is completely in and is flush.

(iv) To get the rod back in you will need to use the wooden maintenance housing built
for the tank (4x6 block of wood with holes in it). Place tank in wooden housing
with the new bearing facing up (this is done to ensure the bearing fits snug in its
housing). Hammer as lightly as possible (with rubber cork in top of metal rod)
making sure the rod is completely straight. Check often to make sure both ball
bearings (top and bottom ball bearings) are staying in their housing. Continue to
hammer until rod is at correct height. Check to make sure the rod spins without
much resistance (should be roughly the same resistance as the others).