Video Case Materials and the Development of Collective Professional Knowledge

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VIDEO CASE MATERIALS AND THE
DEVELOPMENT OF COLLECTIVE
PROFESSIONAL KNOWLEDGE

A DISSERTATION

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by

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Abstract

VIDEO CASE MATERIALS AND THE DEVELOPMENT OF COLLECTIVE PROFESSIONAL KNOWLEDGE

by Victoria D. Bonaccorso

The dynamic nature of teaching means that teachers are making in-the-moment decisions on a daily basis. Video case study professional development can be used as a way to provide teachers an opportunity to analyze real teaching scenarios to prepare to make these decisions in practice. While work has been done to reveal the effectiveness of using case studies as a teaching tool, there has not been research conducted to determine if video case studies can be used to foster the development of collective professional knowledge. This study utilizes a particular professional development model using video case studies grounded in the Teaching for Robust Understanding framework (Schoenfeld, 2016) to determine how teachers in a community of practice can use these tools to develop collective professional knowledge. The study also attempts to understand the nature of the conversations teachers engage in during various parts of the professional development model. Data was collected during teacher engagement in the professional development sessions and during personal reflections after the sessions. During the professional development sessions, the teachers investigated the mathematical tasks of a formative assessment lesson, determined the mathematical ideas behind the lesson, watched a video of the lesson enacted in the real classroom, and analyzed the video for student understanding and teaching moves. The results of the study indicated that the community of practice had used the professional development model to approach the development of collective
professional knowledge. However, they were not able to meet all of the characteristics to claim that they were able to completely develop collective professional knowledge. In addition, the professional development model lent itself to different framings of problems given the activity teachers were engaged in throughout the model.

Keywords: Mathematics, video case study, professional development, collective professional knowledge, frame analysis, communities of practice
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Dedication

I dedicate this dissertation to the people that kept me grounded throughout this process – Francis, Joseph, and James. Francis, your support and understanding through the writing process provided me the motivation to complete this journey. Joseph and James, you inspired me to be better person daily and reminded me that we can do and be anything we want.
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CHAPTER 1: INTRODUCTION

In their 2000 *Principles and standards for school mathematics: A guide for mathematicians*, the National Council of Teachers of Mathematics (NCTM, 2000) details principles of mathematics teaching to promote student learning and highlight that for students to learn mathematics teachers must be supported in their development as professionals. While preservice teachers are intended to be prepared with a foundation as they enter the teaching force, the preparation is often outdated quickly in our ever-changing landscape of education. Therefore, NCTM claims that teachers need to “learn from their own teaching, from their students, from curriculum materials, from colleagues, and from other experts” (p. 370). With respect to teachers already in the field, research on effective professional development consistently finds that meaningful professional development occurs when it is connected to practice and collaborative in nature (Desimone, 2011; Garet, Porter, Desimone, Birman, & Yoon, 2001; Rogers et al., 2007). However, we know that to this day, teachers are still working in isolation (Hargreaves, 2019) and are not provided with effective and meaningful professional development (Zhang, Shi, & Lin, 2019). Hence, the role of mathematics teachers should be to build their mathematical and pedagogical knowledge in collaboration with their colleagues (NCTM, 2000).

**Teaching in Action**

Teaching mathematics is a dynamic endeavor with teachers making in-the-moment decisions about pedagogy in response to student understanding. For teachers to build their knowledge of mathematics teaching and learning, the nature of how classroom instruction occurs should be part of the development process. Cobb and Jackson (2011) presented a theory of action to propose a comprehensive view of improving mathematics teaching. The first component of
their theory is that teachers need a coherent instructional system to support their development. This component involves professional development with professional learning communities organized around high-quality teaching materials that provide teachers space to discuss and adapt their practice. Educational researchers have developed professional development models to attempt to meet these criteria. Using records of practice, or a collection of materials that represent an event, provides a platform for teachers to engage in discussions about mathematics teaching and learning (Ball, Ben-Peretz, & Cohen, 2014). Through these discussions, a community of teachers can begin to build collective knowledge related to their pedagogy. While the inclusion of records of practice assist in promoting discourse, not all records of practice incorporate high-quality teaching materials.

One type of record of practice to help teachers begin to build collective knowledge is classroom video. There is a community in mathematics education research that has asked a variety of questions about the use of video in teacher learning, both preservice and in-service. Much of the research has focused on the progression of mathematical noticing by teachers while watching video (Sherin, Linsenmeier, & van Es, 2009; Star & Strickland, 2008; van Es & Sherin, 2008), either of themselves or of others. Research has also been conducted to analyze how interaction with classroom videos can impact a teacher’s understanding of mathematics standards (Johnson & Cotterman, 2015; Mitchell & Marin, 2015), foster productive conversations about classroom practice (Roller, 2016), and whether video can assist viewers in connecting theory to practice (Koc, Peker, & Osmanuglu, 2009). While there is a lot to be done, the researchers in the field, having used mostly qualitative research methods, call for continued interrogations by the research community into the use of video in teacher learning.

One particular focus for the use of video in teacher learning is to look at how video can
be used as a tool to reflect on classroom interactions. Teachers can alter what they attend to and how they reflect upon classroom interactions. However, there must be a common understanding among participants to begin to build a joint understanding. Setting standards for common language about mathematics teaching and learning to use during teacher education is essential in laying the groundwork for teachers to understand each other and communicate effectively (Colestock & Sherin, 2009). For example, research indicates prior to engaging with video materials, teachers are more likely to notice who and what happened instead of analyzing the interactions (van Es & Sherin, 2002; van Es & Sherin, 2008). One way to create a common understanding and guidance for viewing and discussing videos is to situate the analysis of the video by providing a framework for effective mathematics instruction for teachers to use. In-service teachers can be provided these opportunities to interrogate and analyze video of mathematics teaching through professional development opportunities.

**Professional Development**

The National Center for Education Statistics (Rotermund, DeRoche, & Ottem, 2017) used data from 2011-2012 to report that short workshops continue to be the primary professional development model for teachers even though research indicates that effective professional development is prolonged and sustained. The authors of the report indicate that the most common professional development activities are related to content matter, use of computers, and reading instruction. However, secondary education teachers spent less time in professional development related to subject content then their elementary and middle school counterparts. Gathering data six years later, the 2018 Teaching and Learning International Survey (OECD, 2019) collected information regarding the types of professional development teachers engaged in and features of effective professional development. Through this international study, teachers
continued to report that effective professional development is content focused, collaborative in nature, sustained, and school embedded.

Research has sufficiently outlined key components of effective professional development. However, these do not consider the notion of analyzing teaching in action presented by NCTM (2000). Using real teaching scenarios, similar to their own experiences, coupled with features of effective professional development, can provide teachers the opportunity to learn to teach from teaching.

Professional development for in-service teachers takes a variety of forms and research has indicated that professional development is effective when it is ongoing, situated in practice, and collaborative in nature (Desimone, 2011; Garet et al., 2001; Rogers et al., 2007). With the advent of video of realistic classroom experiences, the ability to incorporate video into professional development has the possibility to impact teacher learning and professional growth (e.g. Borko, Jacobs, Eiteljorg, & Pittman, 2008; Sherin & Han, 2004; van Es & Sherin, 2008). Video case studies can provide teachers with genuine classroom scenarios to discuss (Bliss & Reynolds, 2004; Koc et al., 2009) and having a common platform to initiate discussion can lead to learning for those teachers involved (Borko, Jacobs, Seago, & Mangram, 2014). The collaborative nature of groups of teachers can vary among schools, and through these various collaborations the knowledge that is generated is situated in the context in which it occurs (Borko et al., 2000; Cobb & Bowers, 1999). By situating the professional development in the teachers’ context, they can engage with activities that are similar to what they do in the classroom. While Brown (1989) stressed the importance of authentic opportunities for students in the classroom, providing these authentic activities for teachers can be just as powerful to their professional learning (Putnam & Borko, 2000). The everyday activities that teachers engage in is
interlaced with what they know about teaching and learning. When a group of teachers analyzes video case studies as part of a professional development, they negotiate problems of practice and learn from common understandings of practice. The learning that can occur in this type of professional development has the potential to impact classroom pedagogy and in turn, student learning (Desimone, 2011).

For teachers to meaningfully collaborate in a professional development a common lens can bridge each teachers' individual experience. One way of unifying their focus is to use a conceptual framework as a guide for conversations. The use of a framework for effective mathematics teaching and learning to guide viewing and analysis of classroom videos is limited to only a few studies. In one example, Mitchell and Marin (2015) used the Mathematical Quality of Instruction (MQI) Framework to assist preservice elementary school teachers in improving their ability to notice key classroom features. While the framework benefited preservice teachers, the evaluative nature of the tool limited the potential to foster productive conversations.

However, researchers have indicated that the use of a framework for viewing and analyzing video in teacher learning can improve the reflection practices of preservice teachers (Barnhart & van Es, 2015). After an exhaustive search, no research could be found on a set of video case materials that includes the use of a framework for effective mathematics teaching from the inception of the case, through the planning of the lesson, the filming, reflection, and compilation of case materials.

**Rationale**

To address the lack of video case materials utilizing a framework for effective teaching,
Math for America\(^1\), in a partnership with Montclair State University, is currently creating a repository of video case materials that uses the Teaching for Robust Understanding (TRU, Schoenfeld, 2016) framework in development of video case materials. Research is needed to determine how engaging with such video case materials can inform how teachers discuss mathematics teaching and learning. The case materials that the partnership has created include carefully selected video clips, content materials for exploration, and discussion questions to use in the facilitation of the cases. Each part of the case materials encourages users to engage with the TRU framework and analyze mathematics teaching and learning through this lens. The formative assessment lessons used in the video case materials were also developed by the Mathematics Assessment Project (2015) to align with the TRU framework. The TRU framework grounds the video analysis in well researched lesson materials and provides a common platform for users of the case materials to have discussions about teaching and learning. While the connections between the TRU framework and the video case materials is evident, how and what teachers can learn from the video case materials needs to be investigated.

There have recently been calls by researchers for the generation of a collective knowledge base for mathematics teaching and learning (Hiebert, Gallimore, & Stigler, 2002). More recently, Ball et al. (2014) proposed the use of records of practice in the creation of this collective knowledge and claim that they provide the assistance needed to accomplish this task. They state, “Millions of teachers have professional responsibility to help students learn, and yet little structure exists for the development of collective professional knowledge” (pg. 331). When teachers come together to engage with video case materials in a collaborative environment, there

\(^1\) Math for America is an organization that provides 4-year fellowships to public school mathematics and science teachers. Their goal is to improve teacher retention and to help teachers impact their schools, communities, and the profession at large.
could be an opportunity for these teachers to create collective professional knowledge meaningful to their particular community. Using the video case materials as a record of practice for a community to use as a foundation to negotiate collective professional knowledge has not been researched.

**Significance**

Creators of video case materials hope that the incorporation of a set of these materials grounded in a framework for effective mathematics instruction can be utilized as a tool to answer the call for the creation of collective professional knowledge. However, research is needed to see if this call can be met by teachers collaborating while using the video case materials. “By creating a shared experience, video can serve as the focal point for professional development participants’ collaborative exploration of classroom interactions” (Borko, Jacobs, et al., 2014). Teacher collaboration is a primary factor in teachers’ ability to implement change in their instruction as they move toward more effective pedagogical strategies (Briscoe & Peters, 1997; Gajda & Koliba, 2008). Researchers have shown collaboration to be a key component to improving teaching practice and student achievement (Hair, Kraft, & Allen, 2001; Lick, 2000; Saunders, Goldenberg, & Gallimore, 2009). However, in order for collaboration to be effective, teachers should work with colleagues preparing to teach the same material and who have a shared vision on curriculum and instruction (Kanold, 2012). In fact, in a synthesis of the research related to the professional learning of practicing mathematics teachers, Goldsmith and her colleagues (2014) found that one key component to teacher learning was collaboration. Their results show that teacher collaboration can influence teachers’ knowledge and practice, but this influence is not inevitable. Teachers must also have the opportunity to build upon and interrogate ideas about mathematics content, teaching, and learning (Goldsmith, Doerr, & Lewis, 2014).
Analyzing and responding to student thinking is a daily task for teachers. Using video case materials as an object for analysis provides a common scenario for teacher discussion. This project aims to uncover how teachers in a community engage in professional development to analyze video case materials. In particular, the focus of the analysis is on how the community develops collective knowledge through collaboration and negotiation of understanding.

**Research Questions**

Even with all the research highlighting the power of teacher collaboration and effective professional development, there are still barriers in creating these learning opportunities for practicing teachers. While teachers may desire to collaborate, the time they have to do so during a school day can be limited (Collinson & Fedoruk Cook, 2001). Professional development can be an opportunity for a community of teachers to work together, but opportunities for teachers to be involved in such work is limited (OECD, 2019; Rotermund et al., 2017; Warren & Ward, 2019). The use of video within a teacher community can not only provide a window for teachers to view and discuss common lessons, but coupled with case study teaching methods, video case studies have the potential to impact teacher learning by changing what teachers notice about mathematics classrooms, providing a common focus to foster more productive conversations, and creating space for teachers to be more reflective about their teaching practices. To determine what types of knowledge teachers can build through engagement with video case materials, I am interested in answering the following research questions:

1. How do teachers use video case materials to develop collective professional knowledge for mathematics teaching and learning within their community?

2. What is the nature of the conversations about teaching and learning throughout professional development sessions while engaging with the video case materials?
CHAPTER 2: REVIEW OF LITERATURE

In this literature review, I first present research on effective professional development with a focus on the perceived impact from teachers. Second, I address the use of video in teacher learning, focusing on research conducted with both preservice and in-service teachers. Third, I present the advent of the video case study with a historical look at case study teaching methods and the evolution they have undergone to become video case studies. Then, I will present literature related to the development of collective professional knowledge for mathematics teaching and learning. Lastly, I introduce the situated learning perspective as a framework for learning and a way to understand and analyze research with respect to video case study professional development.

Professional Development

The goal of professional development for teachers is to improve teaching and positively impact student learning (Desimone, 2011). For this to come to fruition, teacher educators should attend to what constitutes effective professional development. Research indicates that features of effective professional development are content focused, active learning, coherence, duration, and collective participation (Desimone, 2011; Garet et al., 2001; Rogers et al., 2007). Even with these features, teachers perceive benefits of professional development in various ways depending on how it is implemented. Below, I discuss research on professional development and some of the ways teachers see the benefits of and learning from professional development.

Classifying and Researching Professional Development

Professional development for teachers can be difficult to research because of the numerous factors that can influence or impact the perceived effects of a particular program. A researcher’s beliefs about how learning occurs, individually or socially, can influence them to
collect specific types of data to be analyzed in specific ways. The manner in which teachers interact with a facilitator and/or the professional development program itself may have an impact on the effectiveness of a program. When participant interactions with others and a program are layered with personal beliefs and contextual factors for each setting, analysis of the learning that occurs in a professional development setting can be challenging.

Borko (2004) provides a review of literature on professional development and teacher learning with a structure for classifying research on professional development. While acknowledging that there is a wealth of knowledge on professional development, Borko claims that analyzing this research through the situative perspective will allow the field to see both the individual and a community of teachers as the unit of analysis. Borko states, “To understand teacher learning, we must study it within these multiple contexts, taking into account both the individual teacher-learners and the social systems in which they are participants” (p. 4). The situative theoretical perspective views learning as a change in participation connected to the context in which the individuals are encountering.

**What Makes Professional Development Effective**

Professional learning is a requirement for most in-service teachers. The types of professional development teachers obtain can range from learning about classroom tools to stress management to self-reflection practices. However, teachers are often quick to critique professional development they attend. Garet et al. (2001) conducted a large-scale research project to determine what makes professional development effective from a teacher’s standpoint. They used data from a Teacher Activity Survey, which are self-reports of experiences and behavior, as part of a larger project on the evaluation of the Eisenhower Professional Development Program. Using a national sample of 1,027 mathematics and science teachers and
empirical analysis, they found that there are three key features that have a perceived impact on teaching practice. The three features are a focus on content knowledge, being provided opportunities for active learning, and coherence with other activities. The researchers determined that the structures of professional development that create these features are (a) the type of activity, (b) the duration of the professional development, and (c) collective participation.

Garet and colleagues (2001) found that “time span and contact hours have a substantial positive influence on opportunities for active learning and coherence” (p. 933). That is, the longer the activity, the greater the opportunity for active learning for the participants. The length of the activity also has a moderate positive influence on the emphasis given to content knowledge. Each of the three core features of professional development were found to have a positive influence when it comes to enhancing participant knowledge and skills. If an activity is content centered, but does not extend the participants’ content knowledge, there is a negative impact on teacher practice. Teachers who reported enhancing their knowledge and skills through professional development also reported a positive impact on their teaching practice.

Rogers et al. (2007) sought to compare teachers’ and facilitators’ perceptions of effective professional development to each other and to the established standards of effective professional development. The research team used a phenomenological approach and conducted interviews with 72 teachers and 32 facilitators. Through data analysis they attempted to uncover similarities and differences in effective professional development for mathematics and science teachers and compare their results to professional development standards grounded in research. Teachers discussed three important characteristics of professional development: applicability to classroom lessons, experiencing learning in the same manner as students would, and the building of support systems and networks. Facilitators also believed the three characteristics that teachers
highlighted are important but included a fourth characteristic: the need to improve teachers’
knowledge of teaching mathematics and science. Both studies (Garet et al., 2001; Rogers et al.,
2007) confirm that from a teachers’ perspective, effective professional development needs to
have practical applications in classrooms and to be sustainable and collaborative.

While teachers’ perception of effective professional development includes continued
support, collaboration, and applicability, research has also shown that effective professional
development needs to be situated in practice (Whitcomb, Borko, & Liston, 2009). One way to
situate professional development in practice is through the use of video. In the next section, I
discuss the use of video in teacher learning to help unpack the nature of video, use of video, and
impact on teacher learning and practice.

**Video in Teacher Learning**

Using video of classroom interactions in professional learning provides a window into the
classroom without the constraints of in-person observations. For example, video can be viewed
multiple times in one sitting, paused for immediate discussion and reflection, and used as a
platform for conversations about actual classroom occurrences. Moreover, student responses in
class can open an important window into student thinking that may go unnoticed in the moment
of teaching. In this section, I discuss teacher learning with video across the professional
trajectory, starting with teacher preparation. I provide a brief historical perspective of video in
teacher learning and discuss the uses of video in teacher learning in various contexts including
mathematical noticing, video clubs, productive discussions and the problem-solving cycle, and
teacher reflection.

**Advent of Video**

Incorporating video into teacher professional development creates an opportunity for
teachers to discuss a common teaching scenario. Having a common scenario is important as each
teacher enters into a discussion focused on instruction with preconceived notions about what
teaching and learning mathematics looks like based on various individual classroom experiences.
These diverse experiences likely lead to differing interpretations of classroom events. Using
video as a baseline for discussions grounds the conversation and provides a common focus for
participants (Borko, Jacobs, et al., 2014).

Currently, there exists multiple publicly available databases of video from classrooms
that can be used in mathematics teacher development. As part of a large scale video study in
1999, the Trends in International Mathematics and Science Study (Hiebert, 2003) involved seven
participating countries recording more than 1,000 videos of eighth grade mathematics and
science classrooms. While not all of the videos are available for public consumption, there have
been a handful published from each country. These videos have been used in comparing and
contrast teaching styles used in different countries by selecting videos that contain key
features of lessons from a particular country. Viewers of the videos are provided transcripts,
copies of classroom materials, and timestamped commentary from both researchers and filmed
teachers. The benefits of the commentary are potentially meaningful to preservice teachers
looking for a window into what happens in a real classroom and how those teachers interpret and
react to situations (Sherin, 2004).

As another example, a public broadcasting service, WBGH Boston (1996) in conjunction
with the Annenberg Foundation produced a video library for secondary mathematics (grades 9-
12) using curriculum based on NCTM curriculum guidelines. They highlight the use of
collaborative work environments in the classroom as a model for effective teaching. These
videos showcase exemplar teaching scenarios and include narration from the teachers about their
lessons and commentary from the students to showcase the effectiveness of the pedagogy in the video. With a goal of focusing viewers to reflect upon effective teaching practices, they provide documents for exploration and discussion ideas. While these videos use a diverse group of settings, including a variety of geographic regions and socio-economic backgrounds, there are only a limited number of videos available.

The Mathematics Assessment Project ("Prototype Professional Development Modules," n.d.) provides five modules for professional development using video of classrooms from Great Britain. Each module includes materials for viewers including lesson plans, videos of teachers implementing the lesson plan, and discussions of reflections by the teachers after the implementation and recording of the lesson. Professional development modules are also provided for either the viewer or facilitator including guiding questions for reflection on the activities. The videos are limited in the demographics represented including the geographical location of the teachers and not representative of the larger teacher and student population.

Grounded in the belief that teachers should have a space to watch, share, and learn from each other, The Teaching Channel has created a video repository to assist teachers on their learning trajectories. This repository of over 1,300 videos is intended to help teachers build a space where they can experience new lesson ideas, examine pedagogical moves, and ponder discussion questions about the video. This repository of video clips shows students from Pre-K through high school and provides viewers with some supporting documents including transcripts (Teaching Channel, 2018).

The above resources provide only a brief selection of video materials available. These materials are highlighted in this literature review either because of the use of these videos in the research being reviewed or as examples of known commonly used databases for use with teacher
education.

**Uses of Video in Teacher Learning**

Video has been studied as a part of teacher learning in a variety of contexts. This review will highlight some of the notable contributions in research on video in teaching learning. I will start with research on video as a method for analyzing mathematical noticing. Then, I will discuss the format of a video club as a manner of facilitation for reflecting on video segments. Literature will be presented on the use of video in fostering productive discussions related to the problem-solving cycle. Lastly, I will present research on the impact video can have on teacher reflection.

**Mathematical noticing.** For the purpose of this literature review, mathematical noticing is the construct at the heart of the Learning to Notice Framework (van Es & Sherin, 2002). In this framework, mathematical noticing for teaching is defined with three components:

(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom events. (van Es & Sherin, 2002, p. 573)

While Sherin and colleagues (Sherin et al., 2009; Sherin & van Es, 2009) were studying video with respect to mathematical noticing with in-service teachers, other researchers analyzed the impact of video on preservice teachers’ ability to notice. Star and Strickland (2008) conducted research in a one-semester course with preservice teachers. They defined mathematical noticing based on the work from van Es and Sherin (2002); van Es and Sherin (2008) and used pre-and post-assessments to assess participants’ ability to observe various parts of the lessons with particular attention paid to their ability to notice. The assessments included
both single answer questions and opened ended responses which were graded on a rubric to
determine accuracy. Initially, Star and Strickland (2008) found that there was significant
improvement on a preservice teacher’s ability to notice in general. When what preservice
teachers were noticing was broken into more descriptive categories, the researchers observed
improvements in the categories of classroom environment, tasks, mathematical content, and
communication. Star and Strickland added to the literature on preservice teachers’ attending to
features of the classroom environment. Through participation in the methods course, the
preservice teachers were able to increase their score in this category from 44% to 86%. While the
preservice teachers did significantly improve in their ability to notice features related to the
mathematical content, the improvement was not as large as their improvement in the other
categories. The researchers noted a lack in attention given to important content issues in teacher
preparation.

Star, Lynch, and Perova (2011) conducted a replication study looking to mirror the
results of the first study. Similar to the original study, they used video from the TIMSS project
discussed earlier in this literature review. However, some of the findings from the initial study
varied from the replication study. Unlike the original study, they found that preservice teachers
did not make gains in noticing with respect to either mathematical content or task performance.
Also, while the initial study (Star & Strickland, 2008) did not produce statistically significant
changes in classroom management, the replication study did (Star et al., 2011). Preservice
teachers’ lack of attention to the mathematical content when discussing the lesson was actually
reinforced through the replication study (Star et al., 2011). Statistically significant improvements
in noticing classroom features about the classroom environment and communication converge
with the findings of the original study. Star and colleagues (Star et al., 2011; Star & Strickland,
2008) research shows that with explicit training on how to observe classroom experiences, preservice teachers can improve their ability to notice.

McDuffie et al. (2014) also found that preservice teachers were able to enhance their ability to notice salient classroom features and found an improvement in their ability to attend to equitable instructional practices. This was attributed to the facilitators’ activities and prompts focused on developing the preservice teachers’ ability to notice. Researchers analyzed recordings of the class discussions, teacher planning materials and journals from 73 preservice elementary teachers to determine how preservice teachers notice “equitable practices in mathematics teaching and learning” (p. 249). The research team developed four lenses for participants to use while viewing video: teaching, learning, task, and power and participation. They adapted the Learning to Notice Framework from van Es (2011) to categorize noticing into baseline, attention, awareness, and making connections. Preservice teacher noticing in video around equity at the beginning of the semester was focusing on attention and awareness. However, by the end of the semester, preservice teachers were able to move their noticing into awareness and making connections.

While the unfocused viewing of classroom video provides a window for a researcher to learn about a teacher’s initial interpretation of noticing important classroom features, such viewing can limit the opportunities for interactions among teachers. Researchers, therefore, frequently situate video use within particular frameworks for teaching. By using frameworks to focus viewers’ attention, teacher educators can develop a teachers’ ability to notice salient classroom features (Mitchell & Marin, 2015) and their ability to analyze a lesson (Santagata & Angelici, 2010).

Mitchell and Marin (2015) employed the Mathematical Quality of Instruction (MQI)
(Hill et al., 2008) analysis framework to examine how elementary preservice teachers use video to enhance their noticing. The MQI analysis framework was used as a tool to assist the preservice teachers in identifying key mathematical classroom features. The researchers used pre- and post- intervention comparisons to indicate changes in preservice teachers’ ability to notice. Before participation in collaborative video watching and discussion, participants coded outside video using the MQI framework to establish a baseline of what they attend to when watching the classroom video. After participation in the video clubs, they found that preservice teachers were able to attend to more salient classroom features. The use of the MQI framework focused the participants’ attention to quality mathematics instruction with focuses on mathematical thinking and pedagogical moves. Participants also self-reported that participation in the study encouraged them to be more deliberate about incorporating high quality instruction when planning lessons.

Mathematical noticing is prevalent in research on video in teacher education. The manner in which video is used in teacher learning varies. van Es and Sherin (2008) use their Learning to Notice Framework to investigate how in-service teachers’ thinking changes through participation in video clubs. In the next section, I discuss mathematical noticing in the context of video clubs to illustrate how video is used for in-service teacher professional development.

**Video clubs.** A video club is a collaborative meeting where teachers gather to watch and discuss video of their own classrooms (Sherin & Han, 2004). Wilson and Berne (1999) identify three salient features of video clubs: being designed as a community of teachers, promoting inquiry among teachers, and providing a space to be critical collaboratively. Video clubs are most often seen in practice with in-service teachers because the nature of the activity requires the participants to record their own teaching and select clips for discussion with each other (Sherin
& Han, 2004; van Es, 2012; van Es & Sherin, 2008). In the models of video clubs, the video segments selected are not meant to be exemplars of teaching, but rather to have the goal of inciting teacher inquiry and reflection on their teaching practice (Sherin & Han, 2004). Through work with in-service teachers in video clubs, researchers have found that participating teachers improved their analysis of pedagogy and student understandings (Sherin & Han, 2004), showed improvements in noticing through changes in interactions over time (van Es & Sherin, 2008), and the context of a video club promoted discourse about student thinking (van Es, 2012).

Sherin and Han (2004) participated, one as a facilitator and one as a participant observer, in video clubs with four middle school mathematics teachers to investigate what type of learning can occur in this context. While the initial design of a video club is for participants to show excerpts of their classroom, two of the four teachers opted to not videotape and share their classroom experiences. As the researcher was facilitating the video club, they sought to uncover what the participants attended to in the video with a focus on student thinking. Through a yearlong study with this video club, the data revealed that the participants focused on four issues: pedagogy, student conceptions, classroom discourse, and mathematics. They found that through participation in the video club, teachers were able to improve noticing around pedagogy and student conceptions. Sherin and Han concluded that participants did not only change what they discussed, but also how they discussed it. However, they did acknowledge the limited scope of this research and the lack of generalizability. They questioned whether other teachers would be able to connect pedagogical practices with student conceptions in the same manner they documented in the study.

In 2008, van Es and Sherin studied seven elementary school teachers as participants in a video club to analyze changes in thinking about reform-based mathematics pedagogy. They
found, throughout the course of the year, participants altered what they attended to when watching video and how they analyzed each of the events. Specifically, they found that teachers increased their attention on students’ mathematical thinking. While this research highlights how teachers view and attend to classroom features through video, van Es and Sherin acknowledge that research is needed to connect changes and teachers noticing to their classroom practice (van Es & Sherin, 2008).

In 2012, van Es continued to work with video clubs to delve into how teachers use video to collaborate on problems of practice. In particular, the ten sessions of this video club were centered on the implementation of new curriculum. She found that the teachers utilized the video club as a place to explore issues and problems that arose with implementing new curriculum. Through the use of video and transcripts as a support, the group was able to become more specific in their analysis of student thinking by transitioning discussion to be more focused and interpretive (van Es, 2012).

More recently, researchers have attempted to use the video club context with preservice teachers. The challenge faced in this attempt is the limited opportunities that preservice teachers have to teach and record those teaching experiences. Most preparation courses for preservice teachers occur prior to the student teaching experience. However, Johnson and Cotterman (2015) were able to apply the concept of video clubs during the student teaching experience with preservice secondary science teachers to analyze the impact that the video clubs might have on the initiation of discourse. Mitchell and Marin (2015) employed similar strategies with elementary preservice teachers conducting video club meetings during their fieldwork experiences. In addition to improving their noticing, it was also found that preservice teachers “adopted a more interpretive stance toward classroom components” (p. 571) and the group of
preservice science teachers developed stronger connections between content and pedagogy (Johnson & Cotterman, 2015). Similar to the video club studies on in-service teachers, both preservice teacher studies found improvements on the teachers’ ability to notice (Johnson & Cotterman, 2015; Mitchell & Marin, 2015).

Johnson and Cotterman (2015) hypothesized that connecting student teaching and video clubs would allow a space for preservice teachers to develop their knowledge of science teaching. They utilized five preservice secondary science teachers to discover how their conversations about video influenced their knowledge of the science content and how the content discussions could correlate to dimensions of teaching. They prepared the preservice teachers by using the TIMMS videos detailed earlier to focus their attention on becoming an interpretive viewer of the video instead of taking an evaluative stance towards the teacher. Through analysis of episodes of pedagogical reasoning, they categorized the discussions into four domains: understanding content, interpreting student thinking, analyzing instructional thinking and pedagogical moves, and integrating horizon content knowledge. They found that conversations were never started with the intent to deeply examine their understanding the content. Rather, these conversations occurred as a result of the need to make sense of the content to evaluate either interpretations of student thinking or analysis of instructional resources and pedagogical moves. Their main argument is that preservice teachers should be provided with more opportunities to question and discover the connection between their content knowledge and the implications this has for classroom practice. Merely providing preservice teachers with more content courses does not assist them in making the connections to teaching practice.

The research on video clubs with both preservice and in-service teachers indicates that teachers can improve their analysis of pedagogy and student understanding (Sherin & Han,
2004), transform their interactions to improve noticing (Mitchell & Marin, 2015; van Es & Sherin, 2008), and use the platform to promote discourse about student thinking (Johnson & Cotterman, 2015; van Es, 2012). Each of these findings is related to the type of discourse occurring in these learning experiences. One particular focus of some video-based research is on how the use of video can generate an environment where teachers can engage in productive discussions.

**Productive discussions and the problem-solving cycle.** Researchers have also focused their efforts on analyzing the way in which video can impact the productivity of discussions. Roller (2016) used self-video of preservice teachers as a catalyst for starting conversations with mentors around classroom interactions. She used preservice secondary mathematics teachers enrolled in a methods course as participants for her study. The goal of the methods course and its associated microteaching lab was to study teaching through collaborative planning, teaching, feedback and discussions. She analyzed the observation tools that 26 participants used while watching video and conducted six follow-up interviews. The observation tool provided a space for participants to list features of the video that they noticed. Later they were asked to rank what they noticed based on their level of significance. Preservice teachers’ observations were frequently grounded in “concern for students or teacher” and “level of inclusion of mathematics” (p. 484). While the preservice teachers noticed features of both the students and the teachers, through the ranking activity they placed more importance on concern for the student instead of the teacher. She found that based on the definition of noticing described above (van Es & Sherin, 2008), preservice teachers were able to identify important classroom features and connect what they were noticing to prior experiences.

The research on preservice teachers that Roller (2016) was conducting about productive
discussions was built from Borko et al.’s (2008) work with in-service teachers. Borko et al. (2008) grounded their research in the situated learning perspective to analyze the effect of a two-year professional development program. They investigated both the nature of whole-group discussions and how the discussions changed over the course of the study through participation in professional development. They also focused on maintaining a learning community and promoting productive discussions within the community. Participants used video of their own classrooms to initiate conversations using the Problem-Solving Cycle (PSC) for professional development. The PSC is initiated with participants sharing a common teaching experience. The PSC then allows participants, through the course of three workshops, to deeply analyze a single task through various lens of content, pedagogy, and student thinking. In the first workshop, participants spend time completing the mathematics of a particular task and addressing projected implementation issues. After the first session, participants are videotaped enacting the task with their classes and return to the second and third sessions to discuss their implementation. The researchers found that over the course of the professional development sessions, teachers’ conversations became more productive in nature, meaning that the productivity of the discussions is characterized by teacher conversations that were, “more focused, in-depth, and analytical…about specific issues related to teaching and learning” (p. 432).

To follow the previous study, Borko, Koellner, Jacobs, and Seago (2011) compared the aforementioned Problem-Solving Cycle program and the Learning and Teaching Geometry (LTG) program for professional development. The LTG project was initiated to create materials for professional development related to middle school geometry. The LTG model involves teachers exploring mathematics, viewing and analyzing video cases, comparing and contrasting video cases, and making links to their own instructional practices (Seago, Jacobs, & Driscoll,
Borko et al. (2011) had an underlying goal to use video as a central focus for collaboration around teaching and learning. They delved into the differences between the two programs and claimed that with proper selection of video and facilitation, the use of video will help teachers reflect upon teaching and learning. The two professional development programs they analyzed varied drastically in how adaptive they were. The LTG program was rigid in the goals, while the PSC was adaptive to the needs of the participants. While the facilitation would differ based on the structure or the program, the end result of more meaningful discussions was found in both instances.

**Teacher reflection.** Research has also been conducted to analyze teacher learning and the ability to reflect using classroom videos when viewed and discussed through a Lesson Analysis Framework (LAF) (Barnhart & van Es, 2015; Santagata & Angelici, 2010; Santagata, Zannoni, & Stigler, 2007). Santagata and Angelici (2010) use the Lesson Analysis Framework as an observational tool while watching video to determine if it can impact participants’ reflection on teaching. Researchers used two comparison groups of secondary preservice mathematics teachers. One group used the LAF to analyze videos while the second group used the Teaching Rating Framework (TRF) to analyze the same videos. The LAF focuses on the lesson as a whole and synthesized aspects of teaching, whereas the TRF focuses the observer to make judgments and separate aspects of teaching. Participants watched video clips, selected significant moments to them, evaluated the effectiveness of the teaching strategies, and were asked what they would change if they were teaching the lesson. Pre-and post-test repeated measures ANOVA were conducted for each of the comparative groups. The analysis indicates that participants using the LAF improved their analysis and reflections after the intervention, but participants using the TRF showed no improvement over the same time frame.
Barnhart and van Es (2015) grounded their research in the idea of lesson analysis from Santagata and Angelici (2010). They compared cohorts of students who participated in a course focused on lesson analysis (Learning to Learn from Teaching) to those who did not, to determine if there were differences in the attention and importance the preservice teachers paid to student ideas. They used responses from three prompts on the participants’ preservice teacher Performance Assessment for California Teachers. Using the Learning to Notice Framework (van Es, 2011), they compared participants responses in attending, analyzing, and responding to the events in the video. They found that preservice teachers who participated in the Learning to Learn from Teaching course were “more sophisticated in attending to specific instances of student thinking, analyzing this evidence, and commenting on adjustments to instruction in response to a student idea” (Barnhart & van Es, 2015, p. 88). These findings indicate that the use of frameworks and guidance when watching and analyzing video can improve a preservice teachers’ ability to reflect on teaching and learning.

Kleinknecht and Gröschner (2016) continued to address the concern about lack of focused reflection by providing an extremely structured video reflection program for preservice teachers to improve their ability to notice. This program aligned the facilitation to ensure that all participants entered into the discussion at the same point. After analyzing the impact of the program, Kleinknecht and Gröschner found that preservice teachers in the video reflection group increased their score on reflection of alternative teaching strategies in a video which was significantly different than the control group, indicating that their ability to notice and reflect had improved.

This section started with a brief overview of some of the video databases currently used in teacher learning and presented research-based literature on particular uses of video in teacher
learning. The use of video has been found to change what teachers notice (McDuffie et al., 2014; Mitchell & Marin, 2015; Sherin & Han, 2004; Star et al., 2011; van Es & Sherin, 2002; van Es & Sherin, 2008), and to foster more productive discussions (Borko et al., 2008; Johnson & Cotterman, 2015; Roller, 2016) and reflections (Barnhart & van Es, 2015; Kleinknecht & Gröschner, 2016; Santagata & Angelici, 2010; Santagata et al., 2007). A specific use of video in teacher learning is within the context of a case study. In the next section, I briefly discuss the use and history of case studies as a teaching methodology. Then I look at the use of case studies in teacher education. Finally, I discuss the advent of the video case study and its impact on teacher learning and practice.

**Video Case Studies as a Tool in Teacher Education**

Case studies have been used widely in law, medicine, and business programs (Merseth, 1996; Shulman, 1992). The purpose of each case has ranged from examples of exemplars to messages of ethical expectations to extractors of multiple solutions. The main goal of case study is to allow learners to live the reality of the case they are analyzing as though they might have been the person in that position (Shulman, 1992). In educational settings, case studies provide students an opportunity to explore real-world problems contextualized in their specific field before making these decisions in their own practice (Darling-Hammond & Hammerness, 2002; Shulman, 1992). In teacher education, case studies have been shown to assist in developing critical thinking skills, improving reasoning and decision making skills with participants, bridging the gap between theory and practice (Koc et al., 2009), and showing examples of ineffective practice (Cannings & Talley, 2003). By testing theoretical responses to cases, preservice teachers build upon their explanations of actions and reason collectively about a multitude of possible interpretations and reactions (Shulman, 1992). One of the most widely
agreed upon benefits of case studies is that by presenting realistic problems to students and asking them to respond as if they were more mature members of the profession, the discipline, or the policy community, case methods are seen as providing opportunities to practice ‘thinking like’ a professional (Shulman, 1992). Below I describe in more detail the use of case studies in education as it relates to both practicing teachers and prospective teacher candidates.

**Case Studies in Teacher Education**

Case studies can take on a variety of forms including written scenario descriptions which provide detailed narratives to describe practical situations (Levin, 1995; Merseth, 2008), or videotaped situations which provide visual documentation of a classroom (Cannings & Talley, 2003; Koc et al., 2009). Case studies are used in teacher education to “represent the problems, dilemmas, and complexity of teaching” (Levin, 1995, p. 63) that provide prospective or practicing teachers with the opportunity to consider real life problems. By their nature, cases are less abstract or decontextualized than other media for instruction. There is often more than one way to respond to a particular problem, and by considering cases, we can see how someone else responded to the problem, observe the consequences of that action, and ponder alternative approaches (Shulman, 1992).

Smith and Friel (2008) define a case as having to “make salient some idea, principle, or theory that is central to mathematics teaching and learning more generally – that is, the particulars portrayed in the case must be instances of larger, more generalized ideas” (pg. 1). While using case studies, learners are provided the opportunity to delve into content while analyzing problems from multiple perspectives and partake in in-depth discussions on the case materials (Merseth, 2008). The definition of a case described above allows one to use the materials to explore the world of teaching while also being able to relate the discussion to the
larger context of education. Moreover, the way case study materials are used in teacher education has a direct impact on how teachers learn from the case.

Levin (1995) examined how the reading of a case coupled with discussion among preservice teachers, early career teachers, and experienced teachers impacts teacher learning compared to reading without discussion. Analyzing discourse from 24 teachers, she found that incorporating a discussion component changed teachers’ thinking. Reading and writing about the case alone was not stimulating for participants and yielded poor reflections. The control group of teachers not participating in discussion continuously reiterated their same line of thinking with each new case they were presented. However, teachers who participated in a discussion about the case showed evidence of evolving their thinking after participation in discussions.

While a case is meant to shine a light on an important feature of mathematics teaching and learning, when coupled with discussion, the evolution of participation thinking can evolve over time. Findings from Brown and Kraehe (2010) mirror this while layering the research with sociocultural knowledge. They employed the use of case studies with a group of preservice teachers to determine if there was an impact on their ability to acquire sociocultural knowledge. The case materials used in this study were different from the traditional set of case materials. Researchers used autobiographic narratives, multimedia for realistic visual representations, and responses to ethnographic case study research. At the end of the semester, students pulled from their experiences to create their own video case. Students self-reported that the creation of the video cases helped them better understand the sociocultural factors of teaching. Participants’ reported that these activities allowed a space for “(a) guided participation, (b) dismantling and building connections, and (c) close introspection of schooling” (p. 320).

Cases provide teachers with the opportunity to consider real life problems. Often times
teachers are left to recall instances from their classroom interactions, but the memory of the teacher is fallible and only provides their personal interpretation of classroom events. For the purposes of instruction, cases are less abstract or decontextualized than other methods of instruction (Shulman, 1992). The open-ended nature of a case study provides for multiple responses to a particular problem. When analyzing a case, participants can see how others would respond, observe consequences of a particular response, and ponder alternative approaches. Darling-Hammond and Hammerness (2002) argue that cases layer theory with context, and in doing so, “allow the exploration of precepts, principals, theories, and perennial issues as they actually occur in the real world” (p. 125). Teachers who have participated in professional development utilizing cases self-reported a sense of safety when discussing a case that was not from their own teaching (Merseth, 2008). In the same research, it was also found that teachers reported enjoyment around using cases to engage in teacher learning.

While the benefits of case studies have been documented in research, Darling-Hammond and Hammerness (2002) also addressed the limitations of teaching using them. First, they addressed the limitations the writer of a case can impose on a case study. The writer may have a narrow focus or interest in which the case is pigeonholed into their ideas of teaching and learning. This would occur if a single person writes a case study. A second limitation is the possibility that the writer would not be able to integrate theory and pedagogical principles properly into a specific classroom instance. Finally, they conjecture that effective pedagogy with case studies can combat the limitations that case methods face.

Case studies are accessible for those who wish to use them in their practice or in teacher education. An additional layer to written case studies is the incorporation of classroom video as part of the case materials (Cannings & Talley, 2003; Koc et al., 2009; Llinares & Valls, 2009;
Incorporating video into teacher education has an impact for both preservice and in-service teachers.

**Video Case Studies**

Video has been used in professional development to help foster productive discussions about classroom practices (Borko et al., 2008). One impetus for using video over written materials has to do with limitations of written case study materials: “While written cases provide rich, linear descriptions of classroom life and opportunities for in-depth problem solving, they have a limited capacity to simultaneously show teacher actions and student responses” (Bliss & Reynolds, 2004, p. 31). Being able to avoid this limitation, and given the ease of now producing video to accompany a case study, many case studies in teacher education have become video case studies (Sherin, 2004).

Bliss and Reynolds (2004) investigated how the use of video case studies impact preservice teachers’ understanding of teaching standards. The researchers used a video-based curriculum that combines case study with video of nationally board-certified teachers. After surveying preservice teacher participants, they found 91% of the participants would prefer a video case over a written case. Responses were characterized by three main ideas, (a) “video enabled viewers to gain a richer understanding of the case by engaging their senses and emotions,” (b) “video keeps a viewer from imagining what is not warranted,” and (c) “video allows the viewer to see subtle communication and body language between teacher and student” (Bliss & Reynolds, 2004, p. 41).

Often when teachers reflect on video of classroom interactions, they are left with a feeling that they do not have the complete picture. They want to know what came before or after the clip they are watching, the backgrounds of the students, and the reasoning the teacher
underwent when either planning or reflecting on their own lesson (Koc et al., 2009). One approach to bridge the participants’ wonderings could be the incorporation of the recorded teacher and in-service teachers in discussion forums with preservice teachers to create a community focused on discussion and reflection on video cases (Koc et al., 2009).

Researchers used online discussion forums to determine if this manner of facilitation promoted a higher quality discussion. They found that the inclusion of the case study teacher in the discussion allowed the preservice teachers to get a window into what was going on behind the scenes and in turn enhanced connections between theory and practice. Koc et al. (2009) used video case studies with preservice teachers in methods courses with the intent of uncovering what teaching factors are discussed by preservice teachers, whether they can make theory-practice connections, and whether an online component joining in-service teachers and preservice teachers is an effective tool in using case studies during teacher education. The participants in this study covered both elementary and secondary preservice teachers, both undergraduates and graduate students. The researchers found that the most common discussion themes after watching the video cases were teacher role, student understanding, use of tools in the classroom, classroom culture and equity, and standards. As found in other research on video in professional development, these findings speak to the variety of lenses a viewer can use to analyze a case study. Koc’s research team also found video case studies to be an effective tool due to the accurate representation of a real classroom. In addition, they found that the online platform effectively provides a space for in-service and preservice teachers to collaborate around teaching and reflecting.

Koc et al. (2009) found that the incorporation of the taped teacher afforded participants the opportunity to answer any pressing questions they had about the lesson. This opened the
participants’ reflection to focus on the classroom practices instead of pondering the “what-ifs” of the teacher’s perspective. While this is an ideal situation, teacher learning and professional development needs to be scalable and sustainable (Borko, Koellner, & Jacobs, 2014).

Incorporating the taped teacher in every iteration of a professional development using the case they filmed is an unachievable expectation due to time and resource constraints. As previously discussed, other researchers have used the idea of self-video in the context of a video club to mitigate the problem of an insufficient picture of the classroom.

Llinares and Valls (2009) explored how participation in online video case studies engaged preservice teachers in meaning-making mathematics teaching. They used the situative perspective for learning within the context of an online community to examine the types of knowledge the preservice teachers were building. For the purposes of their research, they studied a subset of 15 preservice teachers from a larger data set to answer their research questions. They analyzed online responses from preservice teachers for two dimensions: participation and cognitive engagement. For the cognitive engagement dimension, they coded responses into four categories: description, rhetoric, integration and synthesis, and theorizing and conceptualization. A code of description was applied when preservice teachers were only describing features of the video without a connection to theory from their coursework. Data was coded as rhetoric if participants only referenced theory from their learning without connecting to the video. When participants could both identify features from the video and correspond them with theoretical ideas, it was coded as integration and synthesis. Finally, if participants “conceptualize opinions through a process of theoretical reasoning” (p. 255), the data was coded as theorizing and conceptualization. They found that preservice teachers were able to increase the type of conversation categorized as integration and synthesis or theorizing and conceptualization level
15% from the first online discussion to the second.

While Llinares and Valls (2009) analyzed the type of conversations that occur through the use of online video case studies, other sought to analyze the impact of using them for preservice teachers’ preparation. Using video case studies with preservice elementary teachers in preparation to teach science, Abell and Cennamo (2004) sought to analyze the impact of their created case studies on the preparation of preservice teachers. Through their design process, they intended for their case studies to “perturb students’ incoming theories of science teaching and learning and lead to changes in their beliefs and values regarding science education” (p. 109). During their training sessions, they required participants to view the cases from various lenses including the science student, the science methods student, and the science teacher. Through the use of the cases and the varying lenses the participants used, the researchers found that some cases encouraged participants to question their beliefs about science teaching and learning. Participants self-reported that the use of case studies incited a deeper thinking about teaching and learning while providing real world contexts to reflect on, left them feeling more prepared to teach science, and helped them understand student conceptions of science. Their findings indicate that the use of cases in teacher education provides a platform for preservice teachers to feel better prepared for entering the teaching field.

Video case studies allow teachers to see the complexity of a classroom that one is unable to glean from written text and, in turn, make deeper connections between theory and practice (Koc et al., 2009). These case studies have also been used with preservice and in-service teachers in methods courses as an opportunity for students to view and reflect upon their viewing through an online format with the case teacher being available for the online discussions (Koc et al., 2009). In literature, video is often framed through situated learning theory as a way of providing
a space for preservice teachers to participate in in a community of practice that they have not entered yet (Abell & Cennamo, 2004). The use of video can also be use in a community of teachers to foster the development of their own knowledge for mathematics teaching and learning.

**Collective Professional Knowledge**

The types of knowledge that teachers can develop has been researched, labeled and debated over time. There is a well-documented need for both content knowledge about mathematics and pedagogical knowledge for teaching mathematics (Hill, Ball, & Schilling, 2008; Shulman, 1986). The distinction between these two types of knowledge was first given prominence when Shulman (1986) described the shifts in educational accomplishments from content knowledge (absent of pedagogical knowledge) to pedagogical knowledge (absent of content knowledge) and called for the need to blend the two types of knowledge, coining the phrase *pedagogical content knowledge*, to better prepare teachers and enhance instruction.

Content knowledge is what we can ingest and regurgitate about mathematics. It includes topics and procedures for how to solve mathematical problems as well as knowing how to look at a mathematics question and determine what needs to be done to answer the question. A teacher must have content knowledge to teach mathematics. They need to understand the nature of mathematics including its procedural and conceptual knowledge. Pedagogical knowledge includes knowing how to write lesson plans, manage classroom procedures and behaviors, organize and structure lessons, and assess student understanding (Shulman, 1986). Shulman’s (1986) original description of pedagogical content knowledge includes an understanding of various representations of topics and common difficulties, preconceptions, or misconceptions. Magnusson, Krajcik, and Borko (1999) summarized pedagogical content knowledge as follows:
Pedagogical content knowledge is a teacher’s understanding of how to help students understand specific subject matter. It includes knowledge of how particular subject matter topics, problems, and issues can be organized, represented, and adapted to the diverse interests and ability of learners, and then presented for instruction. (p. 96)

The idea that a knowledge base for teachers may be different than just the compilation of subject content knowledge and pedagogical knowledge has been explored by many researchers to try to understand how teachers develop and utilize their various knowledge bases to improve instruction (e.g., Ball, Thames, & Phelps, 2008; Hill, Ball, et al., 2008; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012; Mishra & Koehler, 2006). Teachers have their own experiences in classrooms that build their knowledge in action. These experiences differ from teacher to teacher, and the insights that the experiences offer teachers can be invaluable. To help consolidate and share the knowledge and insights of individuals, the mathematics education community has recently called for collective knowledge for mathematics teaching and learning to be shared among all educators.

**Defining Professional Knowledge**

Hiebert et al. (2002) claim that teachers are not utilizing a shared knowledge to assist in improving their pedagogy. In addition, teachers are not utilizing mathematics education research to find ways for improving teaching. Instead, they are relying on their lived experiences from the classroom to develop their knowledge for mathematics teaching and learning (Stigler & Thompson, 2009). This knowledge from lived experiences, or practitioner knowledge, has three main characteristics: (1) it is linked with practice, (2) it is detailed, specific, and concrete, and (3) it is integrated around a problem of practice. Hiebert et al. (2002) claim that practitioner knowledge must be grounded in practice because it is generated from responses to problems in
teaching. The specificity of practitioner knowledge is also derived from the teaching scenarios in which this knowledge is developed. Practitioner knowledge is used to address a locally contextualized problem instead of the wider teaching and learning problems addressed by the research community. Practitioner knowledge integrates knowledge that includes content, pedagogical, pedagogical content, student, and so forth. While Hiebert and colleagues acknowledge that this practitioner knowledge is often generated in communities of teachers, they claim that there is potential power in transitioning this knowledge into a professional knowledge base. Hiebert et al. (2002) claim that to transition practitioner knowledge into a professional knowledge base for teaching, practitioner knowledge needs to have three characteristics: (1) be public, (2) be storable and shareable, and (3) have mechanisms for verification and improvement.

In order for knowledge to be public, it must be able to be communicated among colleagues. They claim that a key to making individual knowledge public is a strong sense of collaboration in a community. For knowledge to be shareable, it must be able to be represented in order for the communication among colleagues to occur. For example, when a teacher communicates their knowledge to colleagues, through some form of representation, it becomes public. To be shareable, the knowledge should be “represented through theories with examples” (Hiebert et al., 2002, p. 7). They go on to explain that these theories are hypotheses connecting classroom practice and student learning, including justification for the connections. These local hypotheses are shared when teachers are sharing their individual teaching practices with each other by proposing changes in pedagogy with justification for these changes. The locally developed hypotheses developed in one classroom can serve as resources to transform teaching and learning in others. Over time, the hypotheses can be tested and improved in other
communities. This generated knowledge that can be built upon by a community of teachers instead of the individual practitioner.

The second characteristic of professional knowledge is for it to be storable and shareable. Stigler and Thompson (2009) presented a “knowledge building process” (p. 442) with the purpose of improving teaching which entails creating, accumulating, and utilizing a shared knowledge. Stigler and Thompson (2009) claim the creation of knowledge comes in three forms: (1) a tweak to an existing instructional innovation; (2) borrowing a new instructional innovation; or (3) the invention of a new instructional innovation. They claim that knowledge is more easily shared within one culture instead of across various cultures. While the sharing of knowledge within one culture is deemed a more realistic endeavor, it does not occur without its own challenges. In particular, there are times when culture can change, but the traditions within a community persist. During instances like this, it is important to continue to build shared knowledge within a community to adapt dated traditions or adopt new traditions to meet the cultural demands of the community. To ensure that the knowledge developed is shareable it must meet three criteria: (1) be linked to theory and related to learning goals, (2) be described in detail, including the implementation, and (3) provide contextual description for generation and application (Stigler & Thompson, 2009). The link to theory does not imply larger theories on learning, but rather it can be as simple as justifications for teachers’ hypotheses about teaching practice. The generated knowledge can be represented and stored locally for a particular community through lessons plans, tasks, and curriculum (Hiebert et al., 2002; Stigler & Thompson, 2009). Stigler and Thompson claim that with the multimedia platforms available to teachers, accumulating and representing shared knowledge is easier than in the past. However, they are careful to note that the mere presence of the knowledge is not enough, but rather the
collection needs to be formatted to be organized and usable for teachers. According to Hiebert et al. (2002), the more difficult part of creating a professional knowledge base for teaching is determining how to store this knowledge to be shared with a broader population of teachers, outside of a particular community.

The last characteristic of professional knowledge is that it must contain mechanisms for verification and improvement (Hiebert et al., 2009). The dynamic nature of teaching and learning is an indicator that knowledge of the craft will change over time. To ensure that knowledge produced can be verified and improved if needed, a professional knowledge base for teaching needs structures in place. Hiebert, et al. (2009) insist that this can be achieved through expertise, implementation in other contexts, and repeated observations over multiple trials. Stigler and Thompson (2009) additionally call for teachers to utilize a shared knowledge base, which for them includes studying students, learning from others, developing hypotheses, and planning, implementing, and reflecting on instruction. Through these processes, teachers can share their experiences contributing to the development and improvement of the knowledge base.

Through the process of building professional knowledge, there is the generation, representation, sharing, storing, and revising of knowledge over time. Throughout each of these processes teachers and educators need to be included to ensure a collaborative effort in the development of professional knowledge.

**Making Professional Knowledge “Collective”**

More recently, Ball, Ben-Peretz, and Cohen (2014) support the calls for developing a useful knowledge base for teaching. However, they layer the previously defined professional knowledge as collective. To be collective knowledge, they argue that knowledge must be connected to practice, shared, and have mechanisms for building and improving knowledge.
To build on the concept of making knowledge public, connecting the knowledge to practice provides an opportunity for the knowledge to be collective within a community of teachers. Ball et al. (2014) draw on Orr’s (1990) work to specify the term “collective” as a “working knowledge shared by skilled members of a practice community” (p. 320). A community of skilled teachers making their individual knowledge public to each other is a step towards generating collective knowledge. By making this knowledge public, they are contributing to the shared knowledge of the community. Stigler and Thompson claim that shared knowledge needs to be related to learning goals (content) and contain detailed descriptions about implementation, generation, and application. Knowledge that contains these elements would also need to be connected to practice. Ball et al. (2014) add an additional layer to shareable knowledge by linking it with Shulman’s (1997, 1998) concept of a professional community. Ball and colleagues claim that the communities can be the place where construction of a shared knowledge can occur.

Drawing on Orr’s (1990) work with photocopier technicians, Ball and colleagues (2014) claim that “persevering and circulating knowledge in forms suited to the nature of a specific practice” should be extended to professional knowledge of teaching. The mechanisms for building this type of knowledge align with Hiebert and colleagues’ (2002) mechanisms for verification, which encompass teachers implementing professional knowledge in their own practice. Ball et al. (2014) believe that records of practice can help support the collective discourse to generate knowledge. Records of practice are a collection of materials that represent a teaching experience. For example, a teacher looking to reflect on their teaching may retain written reflections, lesson plans, assignments, assessments, and student work. These artifacts together can assist a teacher to reflect on an aspect of their teaching practice. Knowledge is
generated when a group studies records of practice. Members of the group provide interpretations and contribute to discussions on their investigation. Then the collective knowledge generated in a community through this process can be made public and be used by other groups for verification and improvement.

Similar to Hiebert et al.’s (2002) definition, for knowledge to be collective there must be mechanisms for improving that knowledge. However, records of practice can become either what is improved upon or evidence of this improvement. Ball et al. (2014) claim that “collective study can lead to a richer and more comprehensive kind of collective knowledge, created and shared by the group, based on diverse interpretations and points of view” (p. 328). The product of the individual studies on the same records of practice, when made public, can be part of and used to improve the collective knowledge. This implies that the collective study of records of practice through analysis and discussion is integral to creating collective professional knowledge.

**Trying it Together**

Hiebert et al. (2002) and Stigler and Thompson (2009) outline how knowledge is shared and how professional knowledge builds on and from practitioner knowledge. Ball et al. (2014) stress the importance that knowledge is collective and built by a community while proposing records of practice as a way of developing the knowledge. These conversations in mathematics education are echoed by other professional communities:

When professional activity is collective, the amount of knowledge available in a clinical unit cannot be measured by the sum total of the knowledge possessed by its individual members. A more appropriate measure would be the knowledge generated by the richness of the connections between the individuals. (Boreham, 2000).

For the purpose of this research, I define collective professional knowledge as having the
following characteristics: (1) connected to practice and public, (2) sharable and storable, and (3) contains mechanisms for building, verifying, and improving.

As stated earlier, literature on effective professional development also stresses the importance of collaboration. Throughout the literature, we find that when teachers are working together, whether in professional development or through departmental interactions, there is potential for effective professional development and the building of collective professional knowledge. How these components interact with each other has been understood by framing the research from the situated perspective, specifically with a community of practice (Wenger, 1998) lens. In the next section, I will discuss this learning theory and conceptual framework.

**Theoretical Framework**

**Situated Perspective**

Traditionally in the United States, learning occurs in social contexts. Classrooms are filled with dozens of students and recent best practices include fostering discourse in the classroom to enhance learning (Knuth & Peressini, 2001; Walshaw & Anthony, 2008). Due to the social nature of the educational system, research about how people learn can be seen to fit squarely into the situated perspective. A situated perspective to learning claims that learning, while occurring in social settings, is influenced by the activity and setting in which the experience is occurring (Cobb & Bowers, 1999; Greeno, Collins, & Resnick, 1996; Putnam & Borko, 2000). Within the research community, there has been a dichotomy between whether knowledge is developed by the individual or through a person’s interaction with their environment. The situated perspective attempts to break this dichotomy by viewing learning as a blend between how an individual interacts with/in their environment and participates with activities and others to build knowledge (Borko et al., 2000; Cobb & Bowers, 1999).
A situated perspective *situates* an individual as a component of a larger system. Examining the system as a whole also involves considering the individuals who are participating in that particular system and how their interactions can lead to learning and change. Greeno (1998) summarized the basis of situated research:

The situative strategy starts by investigating activity in intact multiperson, human-technology systems, asking how such systems function. This leads to conclusions about principles of coordination of interactive systems. With these conclusions, situative research can investigate the properties of the individual’s cognition and behavior that support their contributions to the functioning of the system in which they participate. (p. 7)

Teaching is a social activity. There are existing systems ingrained in the educational system, both physical and social, in which teachers participate (Greeno, 1998; Peressini, Borko, Romagnano, Knuth, & Willis, 2004). Proponents of social learning theories call for researchers to embed their research within such systems to study teacher learning in the multiple contexts in which it occurs. Greeno (2003) details two concepts of situated perspective: individual behavior and social interaction. He proposes that synthesizing the individual cognition with the social interactions leading to the cognition is the basis of situative research (Greeno, 2003).

Three main branches of learning theory classify learning in different ways. Behaviorism uses the attainment of skills to define learning and cognitive learning theories view learning as an evolution of conceptual understanding within an individual. However, situative learning occurs when participation in a system develops one’s understanding (Greeno, 1998; Greeno et al., 1996). Putnam and Borko (2000) detail the three themes of the situative perspective as, “(a) situated in particular physical and social contexts; (b) social in nature; and (c) distributed across
the individual, other persons, and tools” (p. 4). The third part of their description highlights the interactivity of the multiple parts of the system. In education, the system may be a community of practice in which teachers participate to generate and develop their knowledge about teaching and learning.

**Communities of Practice**

One of the themes detailed above by Putnam and Borko (2000) is that learning is distributed across a person or group of people and can include the tools they utilize in the learning process. The distribution of cognition implies, “when diverse groups of teachers with different types of knowledge and expertise come together in discourse communities, community members can draw upon and incorporate each other’s expertise to create rich conversations and new insights into teaching and learning” (Putnam & Borko, 2000, p. 8).

A community of practice contains three components: a joint enterprise, mutual engagement, and a shared repertoire (Wenger, 1998). The community works on a common goal that has been agreed upon by all members. While this enterprise may change as the group evolves, the members of the community are the ones accountable for developing it. Wenger (1998) defines a shared repertoire as routines, artifact, or styles. The resources that the community mutually creates over time are records of the knowledge that is being generated within the community of practice.

Grossman, Wineburg, and Woolworth (2001) proposed a model for determining if a group of teachers can successfully transition into a community of practice. Their model was developed out of the work of an 18-month research project of a teacher community comprised of secondary English and social studies teachers. For their study, teachers engaged in a daylong monthly meeting, biweekly afterschool meetings, and a five-day summer retreat. Their work
defines *professional* community as a group of teachers gathering together to focus on the well-being of the students. However, the focus for their community was on the teachers’ continuing intellectual development. Their model for identifying a community of practice has multiple dimensions (a) “the formation of a group identity and the development of norms for interaction” (Grossman et al., 2001, p. 64), (b) the negotiation of differences between members, and (c) “the willingness of its members to assume responsibility for colleagues’ growth and development” (Grossman et al., 2001, p. 65).

An alternative focus for research on professional development is proposed by Kazemi and Huberman (2008). They suggest transitioning away from examining the impact of professional development on teachers’ practice and towards examining the co-evolution between teacher practice and participation in professional development. Their driving force behind this transition is the varied implications professional development can have on individual teacher’s classroom practices. An argument is made that researchers should

(a) understand and elicit the diversity of teachers’ experimentation and incorporate depictions of that work in PD, (b) examine the situated nature of primary artifacts, and (c) explore how enactments, and specifically enactments of routine activities, can support the generation of new knowledge and ways of knowing. (Kazemi & Huberman, 2008, p. 435)

Communities of practice can occur when teachers gather with the intent to enhance their learning. The learning that can develop within communities of practice can be individual and fostered by the situation in which it is occurring. For this reason, research in the field of professional development can be viewed through the situative perspective with a focus on the role that the community of practice plays in the learning process.
CHAPTER 3: METHODS

This chapter presents the methods and methodology of my study, including a description of the research design, participants, context, development of materials, data collection and data analysis. I discuss data collected during the 2018-2019 school year within four professional development sessions where I engaged with secondary mathematics teachers in a structured analysis of video case materials to determine if and how teachers can build collective professional knowledge and how the nature of conversations change through engagement with the video case materials.

Research Design

This study examined how teachers use video case materials to develop collective professional knowledge for mathematics teaching and learning through participation in a professional development workshop series. I sought to understand what the nature of conversations about teaching and learning were throughout each of the sessions while engaging with the video case materials. Therefore, this research uses an intrinsic, exploratory, revelatory case study.

Case study research is used to investigate a phenomenon in its actual context, particularly when the boundaries between the phenomenon and the context in which it is occurring are not evident (Yin, 2018). An intrinsic case study is derived from the motivation of the researcher to closely analyze one case and the uniqueness of that one case (Stake, 1995). The goal of an exploratory case study is to explore the data collected by the researcher to find root causes or meanings behind the case (Yin, 1994). A revelatory case is designed to have the researcher

2 Note that “case study” is being used in two distinct ways. The video case study or video case materials refers to the model of the professional development for teachers. This allows learners to live in the reality of a specific real like example. The research design is also a case study. This type of case study refers to research methodology, which is a case study because it is investigating a particular phenomenon and its context.
uncover insights that will help readers understand a specific case more (Yin, 1994).

As mentioned in the theoretical framework, learning is situated in the context in which it occurs (Cobb & Bowers, 1999; Greeno et al., 1996; Putnam & Borko, 2000). While teachers are engaged in a professional development session, evidence of the phenomenon is occurring in the session and cannot be adequately captured in a survey. For this study, the case is capturing data from teachers engaged in real-life conversations with colleagues. The boundaries between the phenomenon (the knowledge developed) and the context are not clearly evident because in professional development within a community of practice the context can foster the development of the phenomenon. The nature of the relationships among colleagues and the community that they participate in cannot be separated from the manner in which they generate collective professional knowledge.

Therefore, I used case study methodology to investigate how collective professional knowledge develops in a particular community of practice while using video case materials. Case study research methodology allowed me to analyze the group within their pre-existing community of practice. By choosing case study research methodology, I expected to be able to produce results that can be interpretable and can capture unique features of this particular situation (Cohen, Manion, & Morrison, 2011). It is important for the study to accurately capture the features of this situation, because learning is situated in the context in which it occurs. Therefore, the specifics of the context leading to the development of the phenomenon needs to be interpretable to determine the larger impact of the study.

Case study research methodology requires the researcher to analyze a bounded system (Merriam & Tisdell, 2016). Merriam and Tisdell define a bounded system as “a single entity, a unit around which there are boundaries” (p. 38). Bounding the system in which the research is
occurring allowed me to analyze the relationships between the phenomenon and the context. The community of teachers participating in a professional development workshop is the bounded system of this research on the phenomenon of collective professional knowledge creation. While each individual will contribute in various manners, it is the way the nature of the conversation evolves through engagement with the video case materials that defines the boundaries for this study. This study is interested in how a particular community of practice negotiates common understanding through engagement with video case materials. By limiting the participants to the preexisting department members, it provided me with the opportunity to gather data without outside influences of supervisors or external teachers.

My motivation to closely analyze the uniqueness of one scenario also lends this research to case study research methodology (Stake, 1995). The case study in this research is intrinsic because the situation, person, place, or program studied is something that piques my interest as the researcher and is suited to a case study approach that aimed to discover more about that particular situation, person, place or program. When selecting a single case on which to focus, Yin (1994) claims that a single case should fall into one of the following rationales: the critical case, the extreme case, or the revelatory case. A critical case is framed to test a pre-existing theory. The extreme case is one that that has a striking quality that catches the researcher’s interest. A revelatory case is designed to have the researcher uncover insights that will encourage a deeper understanding of a specific case. The goal is to analyze a phenomenon that is occurring. For this research, I was interested in understanding how one particular community of practice uses video case materials to build collective knowledge base leading the research to fall into Yin’s revelatory case classification.
Context

This research study took place at a middle class, suburban high school, Pleasantville School District\(^3\), located in the northeastern United States. The school district has recently experienced a declining trend in enrollment over the last decade with a student population of 688 in the 2017-2018 academic year. This number has been slowly decreasing from 730 students in the 2010-2011 school year. Based on elementary school enrollment, the high school population is expected to further decline over the next decade. The school that is the context of this study is the only district high school, which is fed from one middle school and three neighborhood elementary schools. The school has 4.2% of the population classifying as economically disadvantaged. The ethnic breakdown of students is 88.2% White, 7.9% Hispanic, 2.3% Asian, 1.3% Black or African American, 0.1% Pacific Islander, and 0.1% two or more races. To serve the high school student population, there are seven secondary mathematics teachers and three special education teachers with a mathematics concentration. I am one of the seven members of the mathematics department. Another one of the seven teachers alternates between being a mathematics teacher and the special education teacher in the classroom. When assuming the role of a mathematics teacher, she is the only teacher in the classroom, but when fulfilling duties as a special education teacher, she serves as the in-class support teacher to another mathematics teacher. The department is managed by a district K-12 mathematics supervisor with a background in elementary education. The teachers have not been provided with a professional development opportunity that engages them in secondary mathematics teaching and learning. Most of their district professional development has focused on using technology in the classroom and building positive relationships with students, parents, and the community.

\(^3\) All names are pseudonyms.
The teachers in the mathematics department have met in district required professional learning communities in a variety of forms over their tenure at the school. When the district-mandated communities were first formed, the department met twice a week as a whole group during designated non-teaching time. After this iteration, for two years the school required teachers to meet daily in cross-curricular teams with the purpose of improving students’ reading comprehension. Most recently, teachers are required to meet twice a week as a non-teaching duty with a small fraction of the mathematics department in an attempt to group teachers by similar course loads. At the time of data collection for this study, department members were required to meet in small groups of two or three department members twice a week. The school attempts to group them with other teachers based on common subjects being taught during the school year. Despite the institutional constraints that department members have to meet during the day, the collaborative nature of the department is ingrained in their routines. The teachers took it upon themselves to meet beyond the prescribed times regulated by the school. This usually occurs informally before school, after school, or during common lunch time. During these meetings, the teachers in the department create common assessments throughout the school year. They have been doing this independently prior to administrative mandates for the previous five years. In addition, the teachers collaboratively built and maintained a shared database of lessons, plans, and resources. During the creation of these shared documents, the community appears to have more closely aligned their teaching styles as well. Based on my participation in this community, teachers have used similar ways of explaining mathematical concepts to create a cohesive learning environment for students as they move from one class to the next. In addition to their shared resources and collaborative planning, the community of teachers usually chose to align their personal learning goals to each other as well. Members of the department voluntarily chose
the same professional development sessions during in-district professional development days to work towards a common goal as a department. The teachers in the department took time to work collectively to create shared resources and common learning goals that contribute to similar teaching styles. This mathematics department constitutes a community of practice because they are working towards a collectively negotiated goal through shared routines, artifacts, and styles (Wenger, 1998).

**Participants**

The population of the research study was five secondary mathematics teachers at Pleasantville High School. Table 1 details the characteristics of each of the participants.

Table 1

*Table of department members and their experience.*

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Teaching Experience</th>
<th>Teaching Experience at Pleasantville High School</th>
<th>Type of Experience</th>
<th>Courses Currently Taught</th>
<th>Courses Previously Taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barbara</td>
<td>34 years</td>
<td>10 years</td>
<td>High School, College, International (Ukraine &amp; Mongolia)</td>
<td>AP Calculus, Precalculus, Algebra II</td>
<td>Calculus, Statistics, College Algebra</td>
</tr>
<tr>
<td>Jackie</td>
<td>15 years</td>
<td>15 years</td>
<td>High School</td>
<td>Algebra I, Geometry, Geometry Honors</td>
<td>Algebra II, Precalculus, Calculus, Math Applications</td>
</tr>
<tr>
<td>Rachel</td>
<td>12 years</td>
<td>12 years</td>
<td>High School</td>
<td>Algebra II, Precalculus Honors</td>
<td>Geometry, Calculus, Math Applications, SAT Math</td>
</tr>
<tr>
<td>Name</td>
<td>Years</td>
<td>Experience</td>
<td>School Level</td>
<td>Courses Offered</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>------------</td>
<td>--------------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>Ashley</td>
<td>10</td>
<td>10</td>
<td>High School</td>
<td>Geometry, Geometry Honors, Algebra I, Algebra II, Pre-calculus, Calculus, Statistics, Math Applications</td>
<td></td>
</tr>
<tr>
<td>Tania</td>
<td>8</td>
<td>3</td>
<td>High School, Special Education Certified</td>
<td>Algebra I, Calculus Honors, Algebra II, Pre-calculus, Calculus, Statistics</td>
<td></td>
</tr>
<tr>
<td>Facilitator <em>a</em></td>
<td>12</td>
<td>12</td>
<td>High School</td>
<td>Algebra II, Algebra II Honors, Statistics, Algebra I, Geometry, Pre-calculus, Calculus, Statistics</td>
<td></td>
</tr>
<tr>
<td>Carly <em>b</em></td>
<td>8</td>
<td>3</td>
<td>High School</td>
<td>Statistics, Algebra I, Geometry, Pre-calculus</td>
<td></td>
</tr>
</tbody>
</table>

*Note. a I am the facilitator. b Carly had previous obligations and could not attend the sessions. Background information on Carly is provided because her colleagues do mention her as a team member throughout the professional development sessions.*

Six of the teachers have worked together for ten years with Tania joining them three years ago. Prior to joining this department, Tania and Carly spent five years teaching together at another public high school. I provided the opportunity to participate in afterschool professional development sessions to each member of the mathematics department.

**Positionality.** My first teaching position was at Pleasantville High School starting in 2006 and I have been working with each participant for between three and 12 years. I have experience as a participant in professional development sessions with them and I shared many conversations regarding the perceived effectiveness of these professional development sessions.

I have also participated in professional learning communities with each of the members of the department where we collaborated on achieving school and department goals. There was a collaborative working environment to prepare lessons, lesson plans, assessments, and teaching
strategies.

In addition to my connections with the participants as their colleague, I have in the past also assumed the role of mathematics department team leader and professional development facilitator. Through these two roles I have previously facilitated conversations regarding our practice as educators. As stated earlier, the professional development that I facilitated had a different purpose than this research project. Those professional development sessions were to prepare teachers to use new technological tools in their classroom. There was not a focus on our ability to reflect on teaching practices.

Through this research project I assumed the role of participant observer. I was both an insider and an outsider. As a colleague of the participants, I was a part of the community of practice (Wenger, 1998), collaborative discussions, and daily interactions. However, I was still an outsider as the facilitator of the professional development and the only researcher (Herr & Anderson, 2015) engaged in data collection and analysis.

A participant observer needs to be aware of their ability to guide a conversation in a particular manner to suit their research needs (Kluckhohn, 1940). I used a structured facilitation guide (Appendix A) for each professional development session to mitigate my bias and my potential to lead the discussion in a particular direction.

**Professional Development Structure**

The professional development was offered as a voluntary opportunity to engage in conversations about teaching and learning mathematics with colleagues. All of the sessions were held after school hours and none of the participants were obligated to attend as part of their teaching duties. As mentioned earlier, the participants have a history of attending the same professional development sessions when provided the opportunity to choose. The collaboration
exhibited as a community is filled with support, interrogation of teaching and learning practices, and the desire to work towards a common goal.

**Collaborative Research: Analyzing Instruction in Mathematics using the Teaching for Robust Understanding Framework (AIM-TRU).** The video case materials used in each professional development session were created in collaboration with researchers in the field of mathematics education, currently practicing teachers and doctoral students (including myself) through a research practice partnership (RPP). The materials were part of a larger project titled Analyzing Instruction in Math and Science using TRU (AIMS-TRU). The project involved Montclair State University, Math for America (MfA), SUNY Buffalo State and the New York State Master Teacher Program. Together these partners have been creating a repository of video case studies of secondary mathematics teaching. As stated in the literature review, one of the limitations of case study teaching methods is the possibility of a narrow field of vision for the case writer (Darling-Hammond & Hammerness, 2002). By incorporating a range of professionals in the writing, the team mitigated this limitation during the development of the video case studies.

One goal of the project was for the repository to be consumed broadly by practicing teachers, pre-service teachers, teacher educators and facilitators of professional development to improve mathematics instruction. The research team chose to ground the case study materials in the context of a research-based framework characterizing the dimensions of high-quality instruction. In doing this the goal was to create a set of materials that could be used across various contexts while also maintaining the value of the case materials. After considering a number of alternatives, the group settled on the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2016). The TRU framework delineates five dimensions necessary and
sufficient to support high-quality instruction: The Content, Cognitive Demand, Equitable Access to Content, Agency/Ownership/Identity, and Formative Assessment. The first co-designed project for the team was to create a process by which TRU could be used to plan lessons, collect video, select excerpts, generate necessary background information, and combine these materials as a final case study product. This process was prototyped, tested, and iteratively improved in the six-month period spanning August 2016 to January 2017.

The video case studies include a selected video clip of secondary mathematics instruction, the publicly available lesson and tasks, contextual information for the lesson in each video, transcript of the video, and discussion questions focused on specific TRU dimensions. As stated in the literature review, research has demonstrated that utilizing a framework as a lens for teachers to view video can assist in the development of their ability to notice salient classroom features (Barnhart & van Es, 2015; Mitchell & Marin, 2015). These findings provided a rationale for the choice of TRU. The team collaborated with teachers to plan lessons to be taped for the purpose of creating the video case studies. The case creation process was developed so that teachers could collaboratively plan around common, well-used set of lessons aligned with the TRU framework (Schoenfeld, 2016). These publicly available lessons were created as part of the Mathematics Assessment Project, a collaborative effort between the Shell Center for Mathematical Education, University of California at Berkeley and the University of Nottingham (Mathematics Assessment Project, 2015). The lessons incorporated in the video cases are called formative assessment lessons. Such lessons are intended to be used two-thirds through a curricular unit as a way for teachers to determine how student knowledge is progressing. When preparing for collaboration among the participants in our RPP, the research team felt that having common lessons that have been developed using research-based pilots and revisions for
refinement would benefit the project. Research on the implementation of these lessons indicates that they assist with the implementation of the Common Core Standards, support student engagement, enhance students’ mathematical knowledge, and raise teachers’ expectations (Research for Action, 2015). Incorporation of these publicly available formative assessment lessons was fruitful for the larger research team and shifted the focus to the teaching practice and off of the lesson itself.

Details of each session. Using materials created by the developers of the TRU framework and videos of teachers in secondary mathematics classrooms, my research participants joined in professional development using the AIM-TRU case study materials to critically analyze particular parts of mathematics teaching and learning. I conducted professional development sessions that each lasted between four and five hours. In each session, teachers first read the lesson plan and identified the intended big mathematical idea(s) of the lesson. The teachers then completed the mathematical task that was used in the video case materials. This part of the professional development included a group discussion about the content including anticipating student solutions. After discussion about the mathematical content of the lesson, the participants were provided some context for the lesson they watched. After watching the video clip, the group used one or two of the TRU dimensions based on the case materials to critically analyze the mathematics teaching and learning that is occurring in the video clip. A summary of the session details is provided in Table 2. An example protocol for the facilitation of the professional development can be found in Appendix A.
Table 2

Summary of professional development sessions.

<table>
<thead>
<tr>
<th>PD Session</th>
<th>Date</th>
<th>Name of FAL (Video Case #)</th>
<th>Details of Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>February 26, 2019</td>
<td>Introduction to norms for watching video, TRU Framework, and FALs.</td>
<td></td>
</tr>
<tr>
<td>Session 2</td>
<td>March 12, 2019</td>
<td>Representing Quadratics Graphically (Video Case 1)</td>
<td>Completed protocol for Video Case 1.</td>
</tr>
<tr>
<td>Session 3</td>
<td>March 26, 2019</td>
<td>Evaluating statements about Radicals (Video Case 2)</td>
<td>Completed protocol for Video Case 2.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Properties of Exponents (Video Case 3)</td>
<td>Completed the mathematics and the discussion about the mathematics for Video Case 3.</td>
</tr>
<tr>
<td>Session 4</td>
<td>April 9, 2019</td>
<td>Properties of Exponents (Video Case 3)</td>
<td>Watched and analyzed the classroom video for Video Case 3.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Representing Conditional Probabilities (Video Case 4)</td>
<td>Completed protocol for Video Case 4.</td>
</tr>
</tbody>
</table>

Below I describe each of the four video cases that we analyzed in the professional development sessions. Research has indicated that when professional development is situated in practice and applicable to classroom practices, it has the potential to be more effective (Whitcomb et al., 2009). When choosing these lessons to focus on, I selected them to reach as many of the participants and the content they were currently teaching. When all teachers could not be reached with a particular content matter, I chose the topic due to connections the teacher could make to their current courses.

Video Case 1: representing quadratics graphically. This lesson’s goals were to promote
understanding of the three forms of a quadratic function and the key features that can be identified from each form. This lesson was chosen because each teacher in the community was currently teaching a course that included a unit on quadratic functions. While they were all teaching different levels, it was a lesson that all participants could relate to and that had practical meaning for their current teaching practices.

As an opening to the lesson, students are asked to sketch two quadratic curves that are different from each other. After a class discussion about the key features of quadratic functions, the lesson required students to use a set of cards that connect like dominos, matching equations from one card to a graph on a different card, to form a circle by matching quadratic equations to graphical representations. An example of the card sort is illustrated in Figure 1. Participants in the professional development engaged in both of these mathematical activities prior to watching a video segment of a real classroom implementing the lesson. The video segment shows a whole class discussion sharing out student results from the opening activity. Students were debating whether every quadratic function has a y-intercept.

![Figure 1. Example card set from Representing Quadratics Graphically](image)

**Video Case 2: evaluating statements about radicals.** The main activity of this lesson requires students to sort cards with radical statements into three groups: always true, sometimes true, and never true. The goal of this activity is for students to be able to distinguish between equations and identities. Examples of cards are illustrated in Figure 2. Similar to the lesson on quadratic functions, this lesson was connected to units in each of the participants classes. Jackie and Ashley did not have units explicitly teaching solving radical equations but do work with
radical expressions in their Geometry sections. Both of these teachers also taught courses last year in which they were solving radical equations. The video segment for this lesson illustrates the teachers’ launch of the task and his interaction with students in small group work while they are completing the card sort.

![Figure 2. Example of a card set from Evaluating Statements about Radicals](image)

**Video Case 3: applying properties of exponents.** This lesson is intended to be used in the 8th grade. Although the expectation is that students enter the participants classroom with an understanding of the properties of exponents, I, as the researcher know that there are student misunderstandings about properties of exponents that have implications in higher level mathematics classes. The participants have previously expressed concerns about these misunderstandings. Therefore, using this video case as an object for discussion was intended to invite the participants into a conversation about a known problem of their practice.

The mathematical goals of the lesson are to use properties of exponents to generate equivalent expressions. To accomplish this goal, the students were provided two card sets: expressions (E) and single exponents (S). Examples of the two types of cards are shown in Figure 3 and Figure 4. The students were to sort the cards into groups of equivalent values. There can be more than two cards in each group. During the video segment as part of the video case study, the students worked together in a group of three to determine if two cards have the same
value.

| E1 | \(2^2 \times 3^2\) | E2 | \(3^2 - 2^3\) |
| E3 | \(2^2 + 2^3\) | E4 | \(2^2 \div 2^3\) |
| E5 | \(6^8 \div 6^4\) | E6 | \(2^3 - 2^2\) |
| E7 | \(3^2 + 3^3\) | E8 | \(4^2 \div 2^3\) |

*Figure 3. Example of Expression Card*

| S1 | \(2^1\) | S2 | \(2^5\) |
| S3 | \((-2)^1\) | S4 | \(2^{-1}\) |
| S5 | \(2^0\) | S6 | \(2^6\) |
| S7 | \(6^4\) | S8 | \(6^2\) |

*Figure 4. Example of Single Exponent Card*

**Video Case 4: representing conditional probabilities**. Probability and statistics have a historical tendency within this school district to be the first topic cut when making decisions about curricular changes in the school year due to time constraints. Four of the five participants have taught a probability unit in either a statistics course or a math applications course. However, their interaction with the content has been limited. The choice to use this lesson as an object of analysis was to provide an opportunity to see teachers interact with content that they are not as familiar with.

The lesson itself is grounded in assisting teachers in assessing their students’ understanding of conditional probability through representations of samples spaces and communications of their reasoning. Each part of this lesson revolves around the task presented in Figure 5. This question provides students with a scenario of two children playing a game and asks students to determine if the rules make for a fair game or not. The teachers in the
professional development session engaged in answering the prompt and justifying it with an explanation and a visual to support their answer. In the video segment the teachers analyze, the students had previously responded individually to the prompt. Based on their responses, the teacher had given them an additional question or prompt to further their thinking which they, in turn, needed to justify their solution to the group. The video segment is of a group of students sharing solutions including their visuals with each other.

![Lucky Dip](image)

**Figure 5.** Task for Representing Conditional Probabilities 1

**Data**

Data collected and analyzed to answer the research questions included transcripts and artifacts from the professional development sessions, as well as personal reflections from the participants. Table 3 provides an overview of how the data collection and analysis answered each of the research questions.
Table 3

*Research Table for Data and Analysis*

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Sources</th>
<th>Data Analysis</th>
</tr>
</thead>
</table>
| RQ1: How do teachers use video case materials to develop collective professional knowledge for mathematics teaching and learning within their community? | ➔ Video and transcripts of teacher meeting of collective investigation of video case studies  
➔ Artifacts from meetings (e.g., pictures of math, on target documents)  
➔ Personal reflections from before and after the professional development sessions. | ➔ Frame analysis of transcripts  
◆ Looking at how teachers take different perspectives on lessons and negotiate a common understanding.  
➔ Selective coding of personal reflections for connections to video case materials. |
| RQ2: What is the nature of the conversations about teaching and learning throughout professional development sessions while engaging with the video case materials? | ➔ Video and transcripts of teacher meeting of collective investigation of video case studies  
➔ Artifacts from meetings (e.g., pictures of math, on target documents) | ➔ Frame analysis of transcripts  
◆ Focus on how teachers frame problems of mathematics teaching and learning. |

**Data Collection.** Data collection methods can vary greatly in a case study. Some common data sources include observations (including field notes, videos and other recordings), interviews, archival records, and artifacts (Cohen et al., 2011). The goal of a case study research design is to study one unit of analysis with enough depth to answer the research question(s). This may involve including a wide range of data collection techniques and tools to paint a good picture of the bounded case.

Due to the nature of this research project, data collection techniques included recording
and transcribing the professional development sessions, collecting the artifacts created by participants during the professional development sessions, and obtaining personal reflections before and after the professional development.

The situated perspective, coupled with the boundaries of the case, places an individual as a component of a larger system. Examining the system as a whole also involves considering the individuals who are participating in that particular system and how their interactions can lead to learning and change. The data collected needed to not only represent the collective knowledge that the group created, but also the individual ideas that surfaced in the development process. The video recordings of the professional development sessions held the potential to represent the collective meanings that are being negotiated. The personal reflections and artifacts were meant to be more representative of the contributions of each individual to the group.

The artifacts and transcripts from professional development sessions assisted me in determining how the teachers began to create collective professional knowledge and how the nature of the conversation evolved throughout engagement with the materials. However, the personal reflections added an additional layer to help me determine if the video case materials were an important factor in the development of collective professional knowledge.

**Data Analysis.** To answer my research questions, I used frame analysis to analyze the data. Frame analysis has recently been borrowed from research on social movement organizations. “Frames organize teachers’ collective attention by shaping and structuring existing meanings in a group and positioning teachers as agents in the problems that they face” (Bannister, 2018, p. 132). A frame is a concept that helps to organize and interpret information that a person is sharing. By framing their interpretations of information, people are able to “locate, perceive, identify, and label” events they are experiencing (Goffman, 1974, p. 21). By
viewing conversations as frames, I can understand how teachers “render events or occurrences meaningful and thereby function to organize experience and guide action” (Benford & Snow, 2000, p. 614). I utilized collective action frames to understand how a community of teachers framed problems in their practice as a way to garner bystander support and develop collective professional knowledge.

By characterizing the discussion using frames, I was able to analyze possible changes within and across teacher frames as evidence of changes in participation throughout the duration of participant engagement in the professional development. Changes in participation are reified by specific actions the community is taking. Changes in participation and the reification provided by teachers in a community of practice are indications of learning taking place within the community of practice (Wenger, 1998). I used this evidence of learning, identified through changes in frames, as evidence of the participants’ development of collective professional knowledge for mathematics teaching and learning. Similarly, by analyzing changes in frames within the professional development, I was able to conceptualize how the nature of the conversations changed in relation to the video case materials.

The analysis of the multiple data sources and triangulation of data to support findings is what helps form successful case study research (Stake, 1995). Using three data sources, video of professional development sessions, artifacts, and reflections, I triangulated the data to “increase the credibility” of my findings (Merriam & Tisdell, 2016, p. 245). To do this, I triangulated my personal notes, the transcripts from the professional development sessions, and the artifacts collected from each session. I also watched the video recordings multiple times during analysis to ensure my interpretations were aligned to the teachers’ statements.

Pulling again from the theoretical framework, learning is situated in the context and
through the activities in which people partake (Borko et al., 2000; Cobb & Bowers, 1999).

Within a community of practice, learning is occurring when there are changes in participation and reification patterns from participants. Bannister (2015) connects changes in participation to the mutual engagement of the community of practice and changes in reification to the shared repertoire in which the community is engaged. She highlighted how collective action frames (Benford & Snow, 2000), specifically diagnostic and prognostic frames, can be used to determine changes in participation and reification (see Figure 6).

Collective action frames are constructed in part as movement adherents negotiate a shared understanding of some problematic condition or situation they define as in need of change, make attributions regarding who or what is to blame, articulate an alternative set of arrangements, and urge others to act in concert to affect change. (Benford & Snow, 2000, p. 615)

For this research, the shared repertoire that the group negotiated is the creation of collective professional knowledge. In the bounded case of this study, the collective professional knowledge is the common understanding within this one school’s mathematics department. To help investigate the creation of collective professional knowledge in this setting, I used frame analysis, which employs three types of collective action frames: (1) diagnostic framing, (2) prognostic framing, and (3) motivational framing. Diagnostic framing allows people to identify a problem and its attributions. Prognostic framing “involves the articulation of a proposed solution to the problem, or at least a plan of attack, and the strategies for carrying out the plan” (Benford & Snow, 2000, p. 616). For example, in Bannister’s (2015) research, she characterized how teachers frame the struggling student. When teachers claimed students are “choosing to fail” by not completing any work, she coded these interactions as a diagnostic frame because they
assigned blame for the problem while attempting to uncover the attributes of the problem. Bannister also provides an example of a prognostic frame when teachers suggest reinstating communication with other teachers who also have the same student. This is coded as a prognostic frame because the teachers are attempting to formulate a solution to the previously diagnosed problem. A motivational frame is the rationale for the proposed action.

Figure 6. Connections between Communities of Practice and Frame Analysis. (Bannister, 2015, p. 351)

Case study research methodology, while allowing the researcher to capture unique features, produces a large amount of data and can become overwhelming.

Analyzing shifts in participation and framing patterns over time fosters more manageable units of interactions for empirical analysis of collaborative teacher learning (Evans, 2002; Russell & Munby, 1991; Wenger, 1998), generating analytic purchase on how teachers’ engagement in these negotiations of new meanings shape their ideas and guide community action. (Bannister, 2018, pp. 133-134)

My data analysis procedure (see Figure 7) began with transcripts of the professional development sessions. Similarly to the work from Johnson and Cotterman (2015) examining
secondary science preservice teachers’ knowledge development using video case analysis, the initial round of analysis was to section the data into episodes of pedagogical reasoning (EPRs).

Horn (2007) developed and defined an EPR as “units of teacher-to-teacher talk in which teachers exhibit their understanding of an issue in their practice” (p. 46). These episodes are characterized by teachers discussing the groundwork of a problem and include some elaboration on the reasons or justifications for their statements. Each EPR is an instance of the community framing an event occurring in their practice. By first dividing the data into EPRs, I was able to look for common themes across all the EPRs. Once these larger themes were identified, I analyzed each EPR with the purpose of assigning a frame for each episode.

To determine what the teachers were talking about in each EPR, I first used open coding (Merriam & Tisdell, 2016) to create descriptions of the conversations. During this first reading of the data, I described the topic of conversation for the community of teachers. For example, some of my initial descriptions were: students will take too long to complete task, multiple ways of approaching the task, vocabulary connecting domain and definition of function, and students
unable to connect content among various units. The descriptions were specific to the mathematical and pedagogical content of each EPR. I then used axial coding, or the process of grouping my descriptions, (Merriam & Tisdell, 2016) to look for common themes across EPRs. I created categories or themes which were “conceptual elements that ‘cover’ or span many individual examples (or bits or units of the data you previously identified) of the category” (Merriam & Tisdell, 2016, p. 206). Finally, within each EPR I applied frame analysis to determine if the EPRs were diagnostic, prognostic, or motivational and coded them as such.

After categorizing the data based on the frame in which teachers are discussing mathematics teaching and learning, I then looked for changes in how they discuss either what the problems are or how problems of practice can be addressed. Through the data analysis, I looked for the ways in which participation with video case materials can foster the change in these conversations over time. Changes in participation are defined in two ways, either within or across frames. A change within frames occurs when the nature of the type of frames change over time. A change across frames occur when teachers transitioning among the three collective actions frames.

The change in participation across frames (see Figure 8) is evidenced by a teacher or community of teachers being able to transition through the framing spectrum, from diagnostic, to prognostic, to motivational frames in their discussion of mathematics teaching and learning. By looking at data in this manner, I was able to find times when the teachers were sharing their personal practitioner knowledge with the community (Hiebert et al., 2002; Stigler & Thompson, 2009).
Figure 8. Transitioning among frames as changes in participation.

Another change in participation within frames occurs as the nature of one particular frame shifts over time (see Figure 9). For instance, how the community of teachers diagnoses a problem of practice in the beginning of the professional development session compared to how they diagnose a problem after interaction with the video case studies. Bannister (2015) provided an example of how teachers initially offered a diagnostic framing of student failures as students choosing to fail, and then later spoke about failing students by describing a disconnect between student grades and work habits.

Figure 9. The evolution of a frame as a change in participation.

Through data analysis, I hoped to uncover how teachers used video case materials to develop collective professional knowledge and how the nature of their conversations changed over their time engaged with video case analysis. I was looking to find changes in participation as evidence of learning within a community of practice and indications that knowledge was being developed. Reification of the changes in participation were evident in the specific actions
and events the community of teachers were discussing. I employed frame analysis as an analytic method to identify possible changes in participation and reification. To understand the types of knowledge the community is sharing, I looked at the teachers’ collective action frames within identified themes to discern these changes. This analysis allowed me to create stories about how this community of practice was able to develop collective professional knowledge for mathematics teaching and learning through their engagement with video case studies.
CHAPTER 4: RESULTS

To investigate the datum, I conducted a tiered analysis. As explained above, I first grouped the data into smaller segments by identifying episodes of pedagogical reasoning (EPRs). From these segments, I used open coding and axial coding to find categories or emerging themes throughout the professional development sessions. To understand how teachers built collective professional knowledge, I then applied frame analysis techniques to determine the nature of the conversation around three emerging themes through community engagement with the video case materials. After assigning frames to each EPR, I conducted a second iteration of frame analysis to determine how the nature of their framing patterns changed over time and in relation to the professional development model. The first round of analysis focused on what the teachers discussed within the reoccurring themes, while the second focused on patterns across and within frames throughout the professional development sessions. The results presented address the following research questions:

1. How do teachers use video case materials to develop collective professional knowledge for mathematics teaching and learning within their communities?

2. What is the nature of the conversation about teaching and learning throughout professional development session while engaging with video case materials?

The Development of Collective Knowledge

I divided the transcripts from four professional development sessions into more manageable pieces for coding purposes by identifying EPRs. As defined in the previous chapter, EPRs are “units of teacher-to-teacher talk in which teachers exhibit their understanding of an issue in their practice” (Horn, 2007, p. 46). While coding the data, I found that EPRs were separated by changes in the topic of the conversation. These changes in conversation were noted
by either a change in the mathematical content, pedagogical move, or object of focus being discussed. An example of a change of content is the teachers transitioning from a discussion on graphing quadratic functions to a discussion on graphing conics. An example of a change in conversation about pedagogical moves is when teachers transition between talking about questioning techniques to task design. And an example of changing the object of focus is when teachers transition from discussing the video case materials to a discussion about their personal experiences.

After identifying EPRs, I used open coding to create descriptions of the conversations during each of these EPRs. Through this coding, I found that multiple descriptions were related to each other throughout the sessions. Examples of these topics are vocabulary use, student understanding of definitions, implementation strategies for the FALs, anticipation of various approaches for solving tasks, anticipating student solution strategies, visualization of mathematical concepts, various mathematical representations, maintaining cognitive demand, process-oriented students, and student perseverance. After identifying these initial descriptions, I began to combine similar EPRs based on the descriptions to categorize the data. This process was done to reorganize data into similar clusters of “smaller, larger, and meaning-rich units” (Leavy, 2014).

EPRs described as vocabulary use and student understanding of definitions were grouped together under the label *precision of language*. Through analysis, I noticed that when teachers were discussing vocabulary use and student understanding of the vocabulary, there was often a concern about the precision of language being used in the classroom. I combined EPRs about anticipation of various approaches, anticipations of student solutions, visualization, and various mathematical representations into *multiple solution strategies*. In each of these instances, the
teachers were focused on identifying, connecting, and understanding the implications of various pathways to complete a mathematical task. Lastly, I grouped maintaining cognitive demand, process-oriented students, and student perseverance EPRs into productive struggle. Schoenfeld (2016) uses productive struggle to define the level of challenge students should be held to while learning mathematics. I was able to combine these themes because when teachers were discussing the single pathway nature of student problem solving, they were doing so to highlight the lack of perseverance students have to productively struggle through a task.

The syntheses and creation of the themes of the EPRs yielded three categories: precision of language, multiple solution pathways, and productive struggle. To answer the first research question, I used these themes to determine what the teachers are discussing and in turn, beginning to develop collective professional knowledge around. Within each category, I assigned a frame to each EPR. The frames of the EPRs informed me on how the teachers were discussing problems of practice. By focusing on the content of the teachers’ conversations, I began to understand how they may have been developing collective knowledge for mathematics teaching and learning. I looked for changes both within and across collective action frames to determine if there were changes in participation and reification, which together constitute evidence of learning within this community of practice. I aligned these changes in frames with the characteristics of collective professional knowledge.

**Precision of Language**

The ways in which the teachers discussed vocabulary use varied, but the significance of using correct terminology and understanding each term was interwoven through the discussions during all four video cases. One aspect of content knowledge is knowledge of mathematics terminology in order to partake in discourse about mathematical ideas (Shulman, 1986). The
importance of this type of content knowledge is often seen as an essential feature of effective teaching (e.g. Ball, Thames, & Phelps, 2008; Shulman, 1986). In a joint meeting of mathematicians and mathematics educators, participants agreed that “the ability to reason about and justify mathematical statements is fundamental, as is the ability to use terms and notation with appropriate degrees of precision” (Ball et al., 2005). Therefore, because of the teachers’ experiences teaching and within other professional learning opportunities, it was not surprising that while analyzing the video case materials, this community of teachers paid particular attention to the manner in which the students were using vocabulary to discuss the mathematics of the task. One way the teachers discussed the precision of language was to identify inconsistencies in vocabulary use as a problem in their practice. They sought to determine the underlying cause of the inconsistencies and attempted to negotiate ways to address the problem. The teachers also discussed student language and vocabulary use as a window into understanding student thinking. To present my findings within this category, I first present the EPRs focused on defining and addressing student language. I then present the EPRs focused on teachers using language as a window to student understanding. Through both, I give evidence of changes in participation and reification through changes in frames throughout the EPRs.

**Defining and addressing student language.** During the first video case study analysis, the teachers watched a video segment of a class of students discussing whether or not every quadratic curve contains a y-intercept. The discussion arose when a student in a small group was describing their picture of a parabola and indicated that the parabola was in only one quadrant. To further probe student thinking, the teacher asked, “Can a parabola ever stay in one quadrant? That’s a good question. I am going to leave you with that question to discuss.” The video clip then transitioned to the teacher prompting all students in the class to consider the same question
in their small groups. After students engaged in small group discussions around the question, the teacher gathered the groups for a teacher-facilitated whole-class discussion.

Some of the language that students used in this video segment were, “it’s only going straight up,” “although it’s thinner, eventually it’s going to get wider,” “a perfect U,” “the window,” and an “asymptote on a graph is a specific place where you can’t go any higher or lower.” Teachers Tania, Ashley, and Rachel, in a small-group setting, within the community of practice, considered these statements while discussing vocabulary use by students.

```
1  Rachel: So, it seemed like they understood the general shape of a parabola.
2    Like they just kept saying “u” shape, “u” shape, “u” shape, which
3    is like, to me, very babyish way of describing it.
4  Tania: Right, they are juniors, it’s not freshmen, you know what I mean?
5 Facilitator: They could be seniors, too.
6  Tania: Right, right, sorry. Juniors and seniors, not freshman in high
7    school.
8  Rachel: They’re not in middle School. They don’t have to just say “u”
9    shape, but that's not really sufficient because they keep saying
10   straight up where some kids know like, well yeah, it's going very
11   steep, but it's eventually, eventually, eventually going to get, there.
12   But then there were other ones were like, no, but what did you go
13   straight up? It's like can't go straight up. So, then we were tying it
14   all back into vocabulary, um, you know, end behavior of the
15   function. Um, asymptotes, domain, range, all that kind of stuff. If
16   they have a core understanding of that or misunderstanding of
```
In this EPR, teachers highlighted student language and pointed to instances where the students could improve their language use (lines 2-3; 8-13). The teachers spent time identifying a problem of practice as students using imprecise language when they discussed mathematical concepts. From this EPR, I noticed that teachers indicated that they thought the language being used was too elementary for their grade level. Both Rachel and Tania indicated that the student language of “U shape” was typical of a middle school student or high school freshman but should not be used by a junior or senior. Rachel then continued to build on their defined problem of practice by providing examples of vocabulary words she believed the students should be using at this stage in their studies (line 15). I identified this EPR as a diagnostic frame because the teachers identified a problem of practice and attributed causality to a lack of understanding of mathematics terminology (lines 15-18).

The second video case analysis was focused on a lesson that required students to complete a card sort activity, among other activities. In the card sort, students were asked to group radical statements by determining if they were sometimes true, always true, or never true. Prior to watching the video segment, the teachers engaged in the mathematical task of the lesson. Rachel responded to my prompt for the teachers to consider what the big mathematical idea of the lesson was:

I mean I see that they need to know the basic skills and they need to be literate and fluent at speaking like eight is really two to the third power and things like that, so they can be efficient at solving a problem. (Video Case 2, 3/26/2019)

Rachel identified the problem of practice as the importance of literacy and fluency with
mathematical vocabulary. I take this to mean that she believed the reason mathematical literacy and fluency are important is because these language skills are essential for efficient problem solving. During this exchange, the rest of the teachers did not respond to Rachel’s statement because immediately after making this statement she transitioned the conversation to another topic. However, the diagnostic frame within this EPR indicates that the idea of mathematical language continued to arise throughout the professional development sessions through this type of framing.

For the third video case analysis the teachers again completed the mathematical tasks from a lesson on properties of exponents prior to watching the video segment. While the teachers engaged in conversation about the mathematical task, Rachel continued to expand upon her previous ideas regarding literacy and precision of language in the mathematics classroom. She connected the misuse of vocabulary words to experiences in her own classroom and attempted to identify a cause for the students’ behavior.

1 Rachel: But I also think that, I don't know where it's happening, but there's
2 so many teachers using wrong words or they don't use the right
3 vocabulary and the kids have this poor understanding of what …
4 "Oh, you do the opposite thing." I'm like, "No. It's the inverse
5 operation. It's not the opposite operation" or they're saying, "half of
6 it." There's someone telling them it's okay to say the square root is
7 half or something. They're learning that and I don't know if the
8 elementary people are not math certified and they're just using
9 their own vocabulary, especially when they're dividing and they're
10 using things with powers. I'm sure the word ‘power’ has been
threw around.

Facilitator: But do they know what it means?

Rachel: In the wrong context. They are associating power with, like, factor
trees or factors and everything's just getting overlapped. Too much
of the language is meshed together. We can't separate. (Video Case
3, 3/26/2019)

Rachel identified a problem of practice as students’ misuse of vocabulary, but this time
she referenced experiences from her own practice (lines 4-6). By connecting the conversation to
practice and making it public, she is contributing to the first characteristic in the generation of
collective professional knowledge within this community. During this EPR, however, Rachel
added an additional layer to her characterization of the problem of practice by attributing
causality to previous educational experiences including teachers (lines 1-3; 6-9). During the first
video case analysis, the teachers determined that the cause of imprecision of language was
because of a lack of student understanding. During this EPR, two sessions later, Rachel extended
the attribution of causality for the lack of understanding of mathematics terminology. This is an
example of a change in participation in terms of the attribution of causality. That change is
reified in the shift Rachel makes in the attribution of causality from the students to the students’
prior educational experiences. This is an example of a change in reification because, although the
problem of practice remains, the attribution of causality is shifting as they are trying to identify
the cause. She hypothesized that this may be due to a lack of mathematical content knowledge on
the part of the elementary school teachers or a teacher’s desire to use personalized vocabulary to
provide equitable access for students. These two previous instances are indicators that Rachel
seemed to value vocabulary and precision of language as essential to mathematical activity.
Because of the way in which Rachel identified the problem of practice and provided a diagnosis of misuse of vocabulary by students, this EPR as the previous two, can be identified as a diagnostic frame.

During the fourth video case analysis the community of teachers revisited Rachel’s ideas from the previous sessions. Specifically, the teachers again addressed the problem that the imprecision of language in earlier grades can impact students’ current knowledge of the vocabulary. In this exchange, Jackie attempted to relay a personal experience to the group.

1 Rachel: I mean, I still think that those elementary school teachers are often
2 not math-certified people. Or they'll--
3 Ashley: They're teaching the steps and the process--
4 Jackie: Oh, my God, you know what bothered me so much the other day?
5 Jackie: [My son] came home with something, and it said something about
6 an equation, and it was--
7 Ashley: An expression?
8 Jackie: A fricking expression. I hate that.
9 Facilitator: That's a big problem.
10 Jackie: I know.
11 Ashley: But the thing is, when you're a teacher of all subjects like that, I
12 feel like it's like master of-- what is it? Master of--
13 Jackie: Jack of all trades, master of none.
14 Ashley: Exactly. I feel like they can do a little bit of everything, but do they
15 really know the math that well to be able to get them to this level.
16 That's what we have teaching them this. That's the problem. (Video
Rachel continued her identification of the problem of practice from the previous video case analysis that previous educational experiences, including the imprecision of language by teachers could be the cause of students’ misuse of vocabulary (lines 1-2, 11-15). Jackie added a personal experience for the community to relate to. Her personal experience is important to note because in addition to being a high school mathematics teacher in Pleasantville School District, Jackie also resided in Pleasantville and her two children attend elementary school in the district. Jackie highlighted a specific example of an elementary school teacher in Pleasantville School District using the words *equation* and *expression* interchangeably. By adding this personal experience, she verified Rachel’s claim that there may be issues around precision of language in the younger grades (lines 5-8). Ashley then interjected by adding her own personal experience about her daughter who is in kindergarten. As a mother, she encouraged her daughter to write notes to her teacher to address misunderstandings. Ashley indicated that she thinks it is important for her daughter to address these issues with her teachers in an attempt to bring the problems to light for the individual teacher. Ashley’s additional personal anecdotes from her daughter’s experiences built on the consensus of a possible lack of mathematical content knowledge of elementary school teachers. This is an example of the community of teachers having a mechanism for building collective professional knowledge. Multiple members considered previously discussed problems of practice and returned to the community with their own interpretations from their experiences to contribute to the discussion. This is an example of the community connecting their knowledge to practice, making it public and sharable, and creating mechanisms for building knowledge. In addition, by creating contextual scenarios to better define the problem of practice, the participation patterns are changing in terms of the community’s definition of the problem of
practice. This is reified by them now using personal contextual experiences to describe and define the problems of practice when they had not before.

The community of teachers then built on these problems to find a way to address this concern.

1 Facilitator: If you're not going to say something, who is?

2 Jackie: I know. I think I should. You're right.

3 Facilitator: In a totally non-confrontational manner.

4 Ashley: It's just the fact that

5 Jackie: I actually forgot about it because what I was thinking was, maybe I should call [supervisor] because [supervisor’s] in charge, but then she's going to know that it came from me.

8 Facilitator: Yeah. I feel like it might just be easier for you to be like, "Hey, we've discovered at the high school, this is a big problem, them not being able to distinguish between expressions and equations. I noticed on [my son’s] homework, it came home, so I was thinking that maybe it's stemming--""

13 Rachel: Was it handwritten or typed?

14 Jackie: It was not a worksheet it was her work.

15 Tania: Maybe say, I think you used the word equation by accident.

16 Ashley: Maybe it's better not to approach it as Ryan's mom or from her supervisor, but, "Hey, math high school teacher here," right? I'm getting your product. Maybe it's something to think about. So, then it's not like you're going to her supervisor.
Rachel: Yeah, I feel like that would look better. (Video Case 4, 4/9/2019)

Together the teachers developed a course of action for Jackie to address the concern of the misuse of vocabulary by an elementary teacher. Jackie is worried that going to the district mathematics supervisor will result in ill feelings on behalf of the teacher. As the facilitator, I offered a way for her to talk to the teacher as a colleague instead of addressing it through the supervisor or as a parent of a student (lines 8-12). This hypothetical solution was restated by Ashley (lines 16-19) in support. This EPR is a prognostic frame because the community of teachers worked towards the generation of a solution to address vocabulary misuse in the elementary schools because they identified this as a problem in their own practice. By the last video case analysis, the community of teachers changed the way they were discussing the precision of language for mathematics teaching and learning. This indicated changes in the participation patterns of the community because they transitioned from framing conversations diagnostically to, by the last session and EPR, prognosticating a course of action to address the misuse of mathematics vocabulary by students.

Through each of these EPRs, there is evidence of a change in participation patterns throughout the four professional development sessions. The teachers transitioned from identifying a pedagogical problem from a video of classroom teaching to connecting it to specific classroom instances from their individual practice while extending their attribution of causality. Through Rachel’s continued reminders about the importance of student language, the community collaboratively developed a plan to address mathematical language use with elementary teachers. This shift from diagnostic to prognostic frames indicates a change in participation, which promotes the development of collective professional knowledge. This change is reified by the community when they finally proposed a way to address their identified problem of practice.
Using language as a window to student understanding. In the previous section, I presented evidence of the teachers defining and negotiating a response to the use of vocabulary by students. In this section, I build on students’ vocabulary use by presenting evidence of teachers using student language as a window into student understanding. During the first video case analysis, the teachers completed the mathematical tasks and watched a video of the lesson enacted in a classroom. This video segment, as described previously, showed the teacher posing a question to the class, “Can a parabola ever stay in one quadrant?” During the segment, the teacher facilitated a student discussion about the posed question.

In this first session, the teachers expressed their belief that a review of relevant vocabulary words could assist students in making the mathematical connections between the various concepts related to graphing (e.g. intercepts, domain, function). One of the suggestions from the teachers was to review vocabulary by building a lesson similar to the dominos activity in the original lesson plan. Rachel stated, “the activity like dominos and have a word and definitions. An asymptote and what an asymptote is and a picture of an asymptote because you can have it three ways. Picture of, word of,” (Video Case 1, 3/12/2019). Ashley finished Rachel’s thought by adding, “with a description and then an example of something” (Video Case 1, 3/12/2019). Rachel provided support for her development of a new activity by claiming that she thinks that they need to “strengthen their definitions of function, domain, range, [and] asymptote.” Earlier in this professional development session, the teachers identified students’ misuse of mathematics terminology as a problem of practice and attributed causality for this misuse on the students’ lack of understanding of the vocabulary. This EPR is a motivational frame because the teachers hypothesized changes to pedagogy by incorporating a review of vocabulary prior to the implementation of the formative assessment lesson. The motivation for
using this additional task was to enhance student vocabulary use and understanding prior to their
engagement with the lesson.

During a discussion with a partner, Barbara and Jackie had an exchange related to
vocabulary when attempting to make sense of the mathematical content and student
understanding in a video clip. They began by clarifying their understanding of the mathematics
when Jackie stated, “It would it be impossible for a quadratic function to remain in one quadrant.
Because the only way [it] would be, if the function was straight up and down, but then it’s not a
function” (Video Case 1, 3/12/2019). In this statement, Jackie used the video segment to draw on
the vocabulary used by students. Jackie’s statement is the first indication that she is pushing
towards the desire to have students connect their understanding of multiple vocabulary words to
make connections and determine what is occurring in the graph.

Before Jackie and Barbara could suggest connecting student understandings, they also
tried to make sense of exactly what students understood about the vocabulary while analyzing
the type of vocabulary students used.

1 Barbara: Right, yeah. That is very good. I would say, I would think, I would
ask the domain here. There was no word about domain.

2 Jackie: No, they did mention domain.

3 Barbara: Well it was a couple, a little bit.

4 Facilitator: Yeah, there was a back and forth a little bit. So, like when she talks
about the “window”.

5 Jackie: Which is a reflection of the domain and range.

6 Barbara: But I think it’s a little bit like…

7 Jackie: So, you would want to talk about it a little more.
Barbara: Yeah, I would do it, fix it when I start with a different question about domain and range.

Jackie: Well that ties into mine about how the domain is.

Barbara: One was talking about asymptotes.

Jackie: So then, I know, so then we are going to combine both of ours. If the U was straight up and down, it’s not going to pass the vertical line test. Therefore, the domain would not be all real numbers.

Barbara: Meaning you would have a whole discussion about that.

Barbara: I would discuss more about that.

Jackie: Because one of them didn’t understand the terminology to explain why it doesn’t stay in one quadrant, then they’re not understanding. Anybody that really understands would say that.

Barbara: Exactly, exactly. So, the students do not have enough terminology about [motioning for the domain of the graph].

Jackie: Because they understood like pieces. But they didn’t understand the big picture.

Barbara: To make a big conclusion about the domain is negative infinity to infinity. (Video Case 1, 3/12/2019)

Barbara and Jackie referenced instances of vocabulary use in the video segment (lines 1-3; 7; 13). This is an example of how the teachers used the video case materials to share their knowledge about student understandings in relation to their vocabulary use. They identified that the students were discussing domain and range through their own terminology, “window”. Barbara also identified a student in the video referring to asymptote as an area of concern. These
instances implied that Barbara and Jackie defined the problem of practice as the misuse of student vocabulary due to misunderstandings of the terminology. Streamed throughout this interaction, the two teachers began to express their ideas for addressing the student understanding (lines 9-11, 17). To effectively address student understandings, Barbara and Jackie determined the gaps in understanding the students were experiencing based on their vocabulary use. The teachers determined (lines 22-27) that the students do not have an understanding of the terminology to apply the concepts in order to make sense of the big picture through connections among domain, intercepts, and function. They proposed teacher questions to foster discussions about vocabulary meaning because they believed that the students only understood pieces of the vocabulary and therefore were unable to make the larger connections. This EPR is a motivational frame because the two teachers were hypothesizing changes in pedagogical moves to further student thinking and then justified these changes.

After Barbara and Jackie were able to assess the understanding of the students’ vocabulary, they came to a consensus about how they would approach the situation as a teacher. I asked the teachers to share responses, both written and verbal with each other through a task called a reflection rhombus. In this activity, the teachers first wrote their individual answers to the question outside the drawn rhombus. After sharing their responses, the two teachers negotiated a common response and placed it inside the reflection rhombus. Their resulting response, which they later shared with the entire community of teachers, appears in Figure 10. Jackie indicated in the activity that she would question students about the meaning of a function and specifically connect the definition of function to the teacher’s question from the video. Barbara indicated that she would ask more specific questions. However, it was through their discussion to build their combined answer for the rhombus that they formulated how the
mathematical ideas are connected and in turn, how to address student thinking.

Figure 10. Jackie and Barbara’s negotiated response to student understanding

Barbara and Jackie developed a prognosis for the problem of practice by determining that they needed to extend the whole class discussion to include the definition of function and the relation of this definition to the domain and range of a function. The motivation for this proposed action was that Barbara and Jackie hoped that the students can make the connections that the domain of a quadratic function is all real numbers and, in turn, no matter the location of the parabola, it would have to contain a y-intercept. The previous two EPRs, which occurred simultaneously, were both examples of motivational frames. While they had the same problem of practice, attribution of causality, and motivation, the prognoses were different. Rachel, Ashley and Tania proposed a review task prior to the formative assessment lesson and Jackie and Barbara proposed possible teacher questioning techniques to address student understanding in the moment. When Jackie shared her and Barbara’s response with the whole group, the
community of teachers combined the ideas of task creation and discussion to further student vocabulary understanding and use.

1  Barbara:  If we have a, like a was feeling like about the domain was not much talking about domain. So like baby language, like parabola stuff. Well we should be like domain is from negative infinity to infinity. Parabola don’t stop, right.

5  Facilitator:  So, continue that discussion.

6  Barbara:  It was not enough, yeah.

7  Ashley:  Extend the vocabulary usage a little bit.

8  Rachel:  I mean, they could have even done like a warmup of the domino lesson.

10  Barbara:  The vocabulary was poor. Vocabulary was poor.

11  Rachel:  With like matching cards. Like the word domain, then then like here’s a picture of a graph, here’s the written-out domain of that graph, here is a blank card for you to describe it in your own words. And then here’s another graph, here’s a parabola, what’s the actual domain, how do you write out the domain, what’s a picture of it look like, or something when they like learn to use the matching game, but they use it to strengthen their vocab for functions, domain, range, asymptotes, infinities, end behavior, then they can do the parabola lesson after that. (Video Case 1, 3/12/2019)

This is an example of how, during the first video case analysis, the community of teachers
worked together to build an understanding of the vocabulary use by the students in the video segment (lines 1-4, 10). The teachers investigated the problem of practice in smaller subgroups and returned to the community to utilize the mechanisms for building knowledge within the community through interpretations and contributions about their investigations. The teachers then determined how they would respond to student thinking (lines 7-9, 11-19). In small groups, then as a whole group, they illustrated how they believe that focusing on developing student vocabulary understanding would benefit the students in making connections among the various mathematical concepts.

There is a common understanding among the teachers that precise language and conceptual understanding of terminology is integral to student understanding. This was discussed again in the last video case analysis. The content covered in the last video case study, conditional probability, what least familiar to the participants. Due to the lack of expertise in teaching conditional probability, the teachers spent more time in this session making sense of the mathematics for themselves. The next EPR occurred during the discussion about the big mathematical ideas of the lesson before they solved the lesson task.

1    Facilitator: So, if we think about conditional probability, what do we think is important for our students to take away from it?
2
3    Rachel: Well, they have to understand to recognize that there is a condition.
4
5    Ashley: Like, something has to happen in order for this other thing to happen.
6
7    Facilitator: So, like kind of a sequencing?
8    Rachel: Yeah. I feel like, what’s it in geometry, if-then, the conditionals.
Barbara: Conditional statements.

Ashley: If this happens, then this will happen. What is the probability if this happens, then this happens? It's the same thing.

Rachel: So yeah, that they have to recognize that language even. The other thing I feel like with those problems, and I do this on—the SAT. It comes up again. I feel like I never stop talking about it. But when I have them read word problems, I'm like, sometimes you have to translate it. Like when you write an essay in Spanish class, you're not going to take an essay that would have written in English class and write the same way in Spanish. You change how you speak. So, when you read a word problem or you're describing a scenario, you might have to re-language it to math language. "This is this percent of that thing." It's not going to be written that way or, "If this, then that," you have to pull that out from it. You have to be a good critical reader and find that language or rephrase it into that language. (Video Case 4, 4/9/2019)

This EPR is a diagnostic frame because the teachers identified the problem of practice as the importance of literacy and fluency with mathematics vocabulary, specifically, in this case, the use of the word conditional. They believed that an understanding of the vocabulary would help illuminate the big idea of the lesson. This is a shift in the reasoning for this problem of practice. During the second video case, Rachel indicated that the reason for this problem of practice was for efficient problem solving, however, here, they shifted to the need for vocabulary understanding in order to understand the big mathematical goals of the lesson. This is a change
within the diagnosis of the same problem of practice, the important of literacy and fluency. They focused on the word conditional and the implication it has on the big ideas of the lesson (line 3-4, 8). Ashley made the claim about the importance of the sequencing of events as the condition (line 5-6, 9-11). Rachel related the big idea back to conditional statements in geometric proofs. The teachers again claimed that an understanding of vocabulary can help students organize the mathematical implications of conditional probability. Lastly, Rachel attempted to connect the way we read and write mathematics to translating between writing in different languages. Rachel built on the importance of literacy with mathematical language by connecting to examples outside of the mathematics classroom. These EPRs are evidence of changes within frames as well as a change in the way the teachers are connecting the mathematics to contextual situations. Therefore, the changes within frames are instances of learning within the community of practice.

Through this progression of EPRs, there is evidence of changes in participation about language as a window to student understanding. During the first video case analysis, the video segment itself provided the community a focal point to make connections between the language students use and their understanding of mathematical concepts. It was here the EPRs from smaller groups of teachers contained motivational frames, which were condensed through whole group discussions. Their motivation prompted different prognoses which were shared within the community. By the last video case analysis, the teachers are able to relate this connection to the mathematical goals of the lesson. By doing this the teachers are acknowledging the interwoven nature of each component of mathematics teaching. The community’s change in participation is evidenced by their change in the attribution of causality. This change is reified when they shift from defining the problem of practice as specific student misunderstandings of terminology to the importance of mathematics literacy and fluency to align with the lesson goals.
Multiple Solution Pathways

Research in mathematics education has shown that students conceptualize mathematics in their own individual way (von Glasersfeld, 1995). Due to the ways a person can make sense of mathematics, students may have various methods for approaching a task, solving a problem, or representing their thinking. Through the data, the discussion of multiple solutions manifested itself in a variety of ways. At times the community of practice focused on the influence of multiple representations of a function highlighting the connections between equations and graphical representations and how these can lead to various solution paths for students. At other times, the teachers focused on the task setup and the access it provides students of multiple ability levels and their idiosyncratic methods for solving. Below are the details of how this community of teachers discussed multiple solutions in each of the four video case studies. To understand the types of knowledge the community is sharing, I looked at their transitions among and within collective action frames within identified themes. To determine how the community may generate collective professional knowledge, I looked for changes in participation and reification as demonstrated by changes either within or between collective action frames.

Multiple representations for student thinking. The teachers discussed various mathematical representations and solution pathways at different moments of the professional development model. During the first video case analysis, as in each session, the teachers began by discussing the big mathematical ideas of the lesson. For this session, the teachers discussed the importance of making connections among various forms of quadratic functions. During this discussion, Ashley, Tania, and Rachel began to talk about three different forms for representing quadratic functions. This EPR is from a small group conversation among these teachers.

1 Ashley: You basically want to understand why all three forms are
important.

Tania: Right.

Ashley: And what they're used for and what they look like rapidly.

Tania: Right. But also, that they are the same function in all three forms.

Rachel: It’s the same result in the end, but you approach it in different ways.

Tania: Yeah, like Rachel said, you can approach it one way and find different things. Or you do it in a different order…

Ashley: They all work the same.

Tania: Yeah, the big picture is like comparing and contrasting. (Video Case 1, 3/12/2019)

Ashley indicated that it is important for students to be able to utilize three forms of a quadratic equation to quickly identify key features of a graph (lines 1-2). Tania added that she believed that while the three forms appear different, students need to understand that the forms actually represent the same function (line 5). This introduced the idea that Tania believed there needs to be a focus on the connections between the various solutions and representations. Rachel then connected the various forms as indicators of the multiple ways students can approach graphing quadratic equations (lines 6-7). The small group of three teachers appeared to be developing a common understanding that recognizing and utilizing multiple solution pathways is a big mathematical idea of the lesson through this diagnostic frame. This EPR is identified as a diagnostic frame because the teachers identified a problem of practice, students’ understanding of multiple representations of a function. The cause of this problem of practice is the need to identify key features of a graph.
As Rachel, Tania, and Ashley partook in their discussion about the multiple representations for a function, Jackie and Barbara had a parallel conversation where they too considered the problems of practice related to the lesson goals and the ability of students to identify key features of a graph.

Jackie: So, the big picture is to recognize what form you have and what it’s telling you, right?

Barbara: Very important, x-intercept in factored form. Very important for factoring.

Jackie: So, we want them to, when they see a certain form, automatically think, oh intercept or zeros.

Barbara: Yes.

Jackie: Or maximum and minimum, vertex.

Barbara: With vertex form, maximum and minimum right away. (Video Case 1, 3/12/2019)

Jackie claimed that the big picture is the combination of not only recognizing the three forms, but also identified how to utilize each to identify key information about the graph (lines 1-2; 5-6). Barbara reframed Jackie’s ideas by providing specific instances of what students should be able to identify from each form of a quadratic equation (lines 3-4). This EPR is diagnostic because they defined the need to understand multiple representations of a function as the problem of practice because of the need to identify key features of a graph.

Following the separate conversations, all of the teachers came together to share the ideas they generated about the big mathematical ideas of the lesson.

Tania: Each different form can identify different parts of the function,
Rachel was saying. Then you approach how you graph it differently.

Rachel: And, in calc they want to find maybe max and mins, but when they're doing polynomials, so you'd want to graph by intercepts, so like with quadratics you could learn both things. So then later on when we're highlighting the max and mins, we talked about the vertex more. We talked about this optimum point more. And then when we graph just polynomials, you showed all the x intercepts. I like that for more…

Tania: Well, with the freshman right, I want them to know that like it doesn't matter what the form is that they can pick on information. Well, they're making tables because they can't handle the graph like patterns. And then be able to find the vertex no matter what form it is.

Barbara: Like in Algebra 2, very important vertex, x-intercept, right?

Rachel: Even just the, I hate when they don't understand the input and the output relationship.

Tania: Yeah.

Rachel: They don't understand like if you plug in this value for x,

Tania: Well that's why we are making tables with them.

Rachel: Yeah. Then precalc honors, now we're doing parabolas like as conics and they're asking you how wide is the bridge as many feet above the ground or whatever and be like, don't understand the
correlation between inputs and outputs that well. So, they might not understand that they should double the x value after they get it because it's how wide is the bridge you but the vertex there. All right, so good. Sometimes that big part like, yeah, sure. They found the vertex in the right place and they found the x intercepts and they have this part sketched but they don't even understand the input and output applications. So, it's like they have to connect all the x and y.

Jackie: I also think the big picture of what I would want my students to gather is no matter what form your quadratic is in, you can get all the information, right. So yes, one form gives you specifics, but then you can turn it into another form and get other things.

Ashley: Right, you can alter them.

Jackie: No matter what form you're giving, you can graph.

Tania: Right, right.

Barbara: You can work with whatever points if you can look, whatever.

Rachel: Yeah. If it's factorable and your vertex is going to be at like 1.5 maybe you'd rather just find your x intercepts.

Tania: Yup.

Barbara: Yeah. That's a vertex, x-intercept, and y-intercept, you can be able to graph. (Video Case 1, 3/12/2019)

Tania and Rachel started this conversation by explaining the problem of practice, understanding multiple representations of a function to identify key features of a graph. Rachel
claimed that the various methods are essential to student understanding when they reach calculus and students need to distinguish between extrema and intercepts (lines 4-11). She believed that providing them the ability to see the various key features of a graph now will facilitate their understanding in later mathematics courses. Tania, Rachel, and Barbara reiterated specific instances of student expectations in relation to their understanding of the content because graphing quadratic equations is seen across the high school curriculum. These three teachers had a variety of students in their classes across the curricular spectrum. Tania related her expectations of algebra I students (lines 12-16), Barbara repeated this is for her algebra II students (line 17), and Rachel concluded with her expectations of precalculus students (lines 23-34). In each of these instances, the teachers indicated specific connections to their individual practice, one component for the development of collective professional knowledge. Even though each teacher’s expectations varied for students depending on the course being taught, the teachers did not disagree with each other’s expectations of student understanding for their specific courses. To conclude this conversation, Jackie restated the ideas discussed in each of the earlier small group conversations, with agreement from the other teachers (lines 40-47). This EPR, similarly to the two proceeding, is an example of diagnostic framing because the teachers have identified the problem of practice and the reason for this problem. Through these EPRs the problem of practice and attribution of causality does not change. However, the ways in which the teachers investigated their ideas separate and then came together to discuss and share their investigations is an example of the mechanisms for building knowledge within this community, specifically around the definition of a problem of practice.

Later during the third video case analysis, while the teachers were engaged in the mathematical task, they began to discuss the various methods for approaching the task.
Specifically, the community was focused on considering any misconceptions the students might face when completing the task. For example, student struggles with multiple representations of division. Tania shared her belief that students might revert to evaluating all of the expressions through expansion instead of utilizing the properties of exponents. The conversation then turned to the various ways in which division can be represented.

1 Rachel: But I never write like this anymore. I never write, "divided by"

2 next to it either. So, because I don't write like that-

3 Tania: Well, actually. Right. So, I feel like that was hard for us to.

4 Facilitator: Yeah, so I feel like it was harder for us to look at the division ones

5 just because we're not used to seeing it that way.

6 Rachel: Oh, my gosh. Katie makes the kids write it like that in class all the

7 time and I'm like "Ugh." (Video Case 3, 3/26/2019)

Tania and I agreed on a problem of practice, students’ struggle with multiple representations of division, in particular, the one provided in the card set (lines 3-4), (see Figure 11). They believed that this could be problematic for their students because they are not accustomed to seeing this symbol for division as high school students (lines 1; 5). Rachel expressed that the only time her students see this representation is when her special education, in-class support teacher, Katie, has the students represent division with an obelus (÷) as a standard practice. This EPR is a diagnostic frame because the community identified the problem of practice and attributed a cause for the problem.
Figure 11. Expression card from formative assessment lesson showing division with the obelus symbol.

The brief mention of the formatting of division occurred prior to watching the video segment. However, the conversation about the multiple representations of division continued in the analysis after watching the video segment. Upon completion of the mathematical task, the teachers watched a video segment. During the video segment, students are working in groups to complete the task by matching cards. The card in Figure 11 is one of the cards that is a focus of conversation for the students.

1 Facilitator: And I wonder if he would have taken this four divided by eight and
2 written it as a stacked fraction versus like four divided by eight—
3 Ashley: I know. That bothered me that he kept writing the division sign
4 instead of writing it as a stacked fraction.
5 Rachel: I know, I never make them write—
6 Facilitator: I think that’s an eighth-grade thing.
7 Rachel: Katie always makes them write like that.
8 Ashley: Every time she does it that way.
9 Facilitator: Even the lesson itself is set up like that.
10 Rachel: Is there any benefit to that?
11 Tania: Because that’s how they’re—right, that’s how they’re taught when they’re little.
12 Ashley: But why?
Tania: I don’t know.

Ashley: I feel like even when we talk about—if I say, “Write this over—or calculate this over this,” they don’t—a lot of them still don’t connect fraction to division. Do you know what I mean? Like they see a fraction, and they’re like “Well, how do I put the fraction in here?”

Rachel: I’m like, yeah, division button.

Ashley: But they don’t connect the two thoughts as one thing. They see division and then fraction.

Rachel: Yes, and they don’t—yeah, it’s not considered the same. (Video Case 3, 3/26/2019)

The teachers diagnosed the problem of practice as students’ struggle with multiple representations of division and pondered if these multiple representations can have an impact on the students’ understanding of the mathematical operation. The teachers believed that the problem is caused by the students’ lack of connections between the various representations (lines 16-17; 21-22). The teachers indicated that the student in the video segment and the lesson material represent division as two terms with an obelus symbol. They identified this as common notation in younger grades and with the special education teachers at the high school. However, the teachers are extending their definition and description of the problem of practice to connect with similar student misunderstandings connected to their own practice. Students’ struggles with multiple representations of division were manifesting themselves in the absence of a connection between fractions and division. Ashley and Rachel stated that their students do not recognize them as the same concept (lines 16-23). As the facilitator, I provided a personal experience (line...
2), from my classroom when representing division horizontally with an obelus was hindering student visualization. I did this to provide a concrete example from practice to inform on the diagnosis of the problem of practice.

But today I was actually doing simplifying division of rational expressions and they had it written horizontally, and they couldn’t come up with the fact that the numerator of the second fraction is going to produce domain restrictions, or restrictions in general. That if the numerator of the second fraction, if that was equal to zero, they didn’t see that as a restriction. Because they’re not seeing it as being a denominator. So, they’re not seeing that numerator as being part of what they’re dividing by. So as soon as I wrote it as a stacked fraction — as a fraction over a fraction — they were like, “Oh, because if the numerator of the denominator is equal to zero, that—” so I felt like that almost helped them visualize it. (Video Case 3, 3/26/2019)

The previous three EPRs diagnosed the problem of practice as students’ struggle to be mindful of the multiple representations of division. In each of the EPRs the community has identified the same problem of practice, but their attribution of causality changes over time. To start the discussion on this topic, Rachel expressed concern about writing division horizontally as it caused an issue for students due to the elementary nature of the representation. She believed that the cause of the problem of practice was the student lack of familiarity with this particular representation. After watching students in the video case segment struggle with this type of expression, the teachers further defined the problem of practice and changed the attribution of causality to the students’ struggle to make connections among the multiple representations of division. Finally, I presented a personal experience from classroom instruction bridging the ideas and connecting the problem of practice to the Algebra II curriculum. These instances are ways in
which the teachers diagnostically framed problems in their classrooms. They identified a common problem of practice and provided real classroom evidence to support their concern. By connecting it to practice, the community displayed changes in participation again in terms of their attribution of causality for the same problem of practice. This is reified when the community shifts their attribution of causality from the elementary nature of the representation to lack of space for students to make connections among multiple representations of division.

The EPRs presented in this section were related to the various ways mathematical statements can be represented. In the first video case analysis the teachers were connecting the various representations of a quadratic equations to the identification of key features of a graph. They determined that knowing multiple representations and the variety of ways to graph is a big mathematical idea for quadratic functions. During the first video case analysis, the diagnostic frames did not change as the community utilized their mechanisms for building knowledge. In the third video case analysis, the teachers again discussed multiple representations of mathematical statements with respect to promoting student understanding by making connections among representations. During this video case analysis, the community of practice did experience changes within their diagnostic frames through changes in the attribution of causality. The changes in frames indicated that there are changes in participation when discussing multiple representations.

**Anticipating student responses.** In the first video case analysis, the teachers completed the mathematical task from the lesson. For this task they were connecting domino tiles to match quadratic graphs to quadratic equations (see Chapter 3). After the teachers identified the big mathematical idea of the lesson, they transitioned to anticipating how students might approach the task. Tania, Ashley, and Rachel first discussed this idea separately from Jackie and Barbara.
Following their small group discussion, the teacher shared their ideas through a collective discussion.

1 Rachel: Well, I think like whatever one's in front of you and you look at it.

2 Like if a kid looks at one, they decide, you know, if it has x-

3 intercepts, they're probably gonna want to do factored form. But if

4 they do one by factored form, they're going to try to do all of them

5 by factored form. Cause I feel like there's with one track mind,

6 right? Like where are the other bubbles? Let’s find the other

7 parentheses and they're going to try all of them until they hit a

8 roadblock because there's only so many that you could do like that.

9 Ashley: Without work.

10 Rachel: Yeah. And then they have to force themselves.

11 Facilitator: So, the activity almost forces them out of being reliant on one

12 form.

13 Ashley: Yeah, they can't just use one form over and over or memorize one

14 process.

15 Tania: And we said we put them in a circle first and then wrote the

16 equations, they might try to expand it or factor it right in the vertex

17 form, which will take longer.

18 Rachel: Right, right. But they just wanna get something on paper.

19 Facilitator: When I did it, I actually had to stop that.

20 Tania: I'm going to tell them no notebook.

21 Facilitator: And the way the lesson is structured, it tells them, do not write on
Rachel began the discussion by identifying the problem on practice, students not recognizing multiple solution strategies. The stated cause for this problem of practice is that students would take a “process-oriented,” single pathway approach to matching graphs with equations. For example, she claimed that if they match the first graph to an equation using the x-intercepts from factored form, they would want to do this for each of the following cards (lines 1-6). In turn, this could limit the students’ use of various representations to complete the task. She also stated that the nature of the task itself safeguards students from falling into this mindset. Ashley agreed with Rachel (lines 13-14), stating that the students cannot use one form repetitively to complete the task. Tania (lines 16-17) indicated that students might be inclined to take each equation and algebraically manipulate each equation to rewrite them in all forms before matching to graphs. To prevent this from occurring she suggested having students complete the task without pen and paper by taking their notebooks away (line 20). As the facilitator, and with experience teaching this lesson, I shared that I needed to monitor my students from falling into the same pitfall (line 19) and restated that her suggestion aligns with the instructions of the formative assessment lesson (lines 21-23). This EPR is a prognostic frame because the community developed a proposed action, not allowing students space to algebraically manipulate functions, to address the problem of practice. I was able to draw on my implementation of the lesson to contribute and support Tania’s prognosis. This is an example of a mechanism of the verification for improvement. Through implementation, I was able to verify
that Tania’s prognosis was necessary to maintain the integrity of the task and to elicit multiple solutions strategies.

I then prompted the teachers to consider the various ways in which the students could approach the task. The conversation that ensued involved teachers hypothesizing about various student solution pathways. They started by sharing their personal methods for approaching the tasks. During this interaction, they discovered that most teachers fell into two different approaches for completing the task. The teachers either looked at the graph first and tried to match an equation to it or they started with an equation and located the matching graph. In all cases, the teachers identified one card and didn’t move on until it’s match had been located. Jackie questioned if going from equation to graph was the cause for her to take longer to complete the task. Instead of addressing Jackie’s question, the teachers attempted to make sense of what the students might do when engaging with the cards.

1 Barbara: They see a x-intercept. I think they do better.

2 Jackie: If they see the x-intercepts though. I was looking at the equation and seeing what information I got from there and trying to match up with the graph.

3 Ashley: I went back and forth between like what, what was in front of me. Like I was like, okay, we matched this one. Then whatever one ended up in front of me next. I’ve just looked at both sides and decided which one was easier to figure out.

4 Rachel: How do the kids decide what's easier?

5 Ashley: Yeah.

6 Rachel: I feel like they just go with what’s familiar at first.
Barbara: I think every one’s kids will do different way like we did.

Jackie: I guess it really depends upon like the dynamics of the group.

Ashley: That’s what I’m saying. It depends on, honestly who takes control of the work. You know.

Barbara: Some kids use a different knowledge. Like some kids, very good with vertex. Some kids do it from the equation like they know it’s be 25 and not even deal with.

Ashley: Just look at that?

Barbara: Yeah. Just looking and they know it be three, five just from here, right? Yes. They know that they using negative or positive. They knew it. Some of them. So different kids

Ashley: Depending on the level. (Video Case 1, 3/12/2019)

This interaction shows the teachers diagnosing the problem of practice, anticipating student solution strategies. The cause of this problem is to be prepared to respond to student thinking given a particular solution strategy. They are attempting to build an understanding of how students might approach the task. They used their personal methods, graph to equation or equation to graph, to compare and contrast the manners in which students would approach the task. Barbara indicated that she thinks the students will focus on the x-intercepts of the graph, causing students to need to compare the graphs to the intercept form of a quadratic function (line 1). Jackie indirectly agreed with Barbara by stating that she also focused on x-intercepts which caused her to first analyze the equations to find a match (lines 2-4). Four of the five teachers agreed that students are going to take a variety of approaches (lines 11-15), whether it is because of their familiarity with a certain form or the power dynamics of the group. Barbara and Ashley
concluded by building on this idea and attributing the various approaches to the level of understanding of the content a student has (lines 20-23). This EPR is a diagnostic frame because the teachers discussed a problem of practice, anticipating student solutions, and began to define the variety of solution pathways that students may take to complete the task.

During the second professional development session, teachers continued discussions around the problem of practice, anticipating student responses because of multiple solution pathways to complete the tasks. For the lesson at the root of this session, teachers completed a task in the FAL: a card sort where they analyzed various radical statements in order to determine if the statements were sometimes, always, or never true. As they completed the task, the teachers noted that to determine which group to place a card in, they solved the equation for the unknown variable. For cards that they found to be always true, the teachers analyzed the possibility of restrictions on the variable to determine if there were limitations on always being true. I then asked them to consider any issues their students may have in completing the task in order to clarify the problem of practice or diagnosis.

1  Rachel:  What number would I put in for x to make the…they don’t think
2  algebraically solve. They never isolate x.
3  Tania:  They would only plug in like one, zero.
4  Facilitator:  So, they never think about solving for x. They think about like---
5  Tania:  But one of them had to.
6  Barbara:  They are doing all the time.
7  Rachel:  They don’t use algebra. (Video Case 2, 3/26/2019)

Rachel, Barbara, and Tania indicated that students have a tendency to approach this task by simply “plugging in” random values to each statement to determine if the statement is
sometimes, always, or never true (lines 1; 3). In doing this method, Rachel highlighted and more clearly diagnosed the problem of practice that students are not mindful of the multiple solution pathways available to solve the task, specifically in this task, students were not considering algebraic methods (line 1-2; 4) to complete the task. They attributed causality for this problem of practice to students’ reliance on guess and check methods. I then prompted the teachers to think about this idea more deeply by questioning if this particular solution pathway could be beneficial for some students.

1 Ashley: It would be hard for the ones that are always true.
2 Barbara: For some kids probably. Because some kids working with the actual numbers could be. But in general, for conceptual, I would say not.
3 Facilitator: So, some kids that have good number sense might make their way through it that way.
4 Barbara: Yes.
5 Facilitator: Would it cause some of them a problem do you think?
6 Rachel: Well yeah. The second that the answer is a fraction or decimal. Oh, you're going to have a meltdown because they want it to be 0, 1, 2.
7 Ashley: Or they want to do some order of operation. Anyway.
8 Rachel: They'll find a way to make it work like they're going to like; I'll bend this square peg into a round one.
9 Rachel: Make up a rule.
10 Ashley: It's supposed to equal four so now that equals four.
11 Rachel: Now it does, done. (Video Case 2, 3/26/2019)
This interaction shows divergent diagnoses among the participants in the community. Barbara claimed that some students with strong number sense might be able to rationalize when each of the statements is true (lines 2-3). However, Ashley and Rachel stated that students will struggle with plugging in values because they will not be willing to evaluate the expression for a value other than a whole number (lines 9-15). Based on prior experience, Barbara has consistently taught high honors level students while Ashley and Rachel have more experience teaching middle and lower track courses. This EPR is a diagnostic frame because they are still diagnosing the problem of practice as students not considering algebraic methods to complete the task. However, Barbara offered another attribution of causality in this EPR. She suggested that some students might utilize this method effectively based on strong number sense. The last two EPRs demonstrate a pattern in the teachers’ definition of the problem of practice and student reliance on guess and check methods.

In both of the EPRs from the video analysis on radical statements, the teachers identified various solution pathways that they took or that their students may take. They began to consider the implications for students who take certain solution strategies by discussing whether a pathway may be helpful or harmful to students’ mathematical development. These two EPRs show how the community of teachers identified one problem of practice, although members in the community were attributing causality differently. While they did not make it explicit, each of the teachers is connecting the diagnosis to their practice by trying to understand how their particular students might approach the task. In this instance, the teachers did not reach a consensus on whether guess and check as a solution method is beneficial for the learning objectives of this lesson.

The last video case analysis utilized content less familiar to the community of teachers.
Due to their lack of familiarity with teaching conditional probability, the teachers spent more time in this session making sense of the mathematics for themselves. The teachers spent a large amount of time developing various methods for solving the problem and presenting them to each other. During the three previous video cases, the teachers spent no more than five minutes completing the task before conversation was turned to student thinking, misconceptions, or pedagogy. In this video case, the teachers spent approximately fifteen minutes discussing the various solution pathways for the task. The teachers took time to explain each of their methods and compared and contrasted each of the methods. In the following EPR, the problem of practice the teachers are discussing is the anticipation of student solutions, because in order for them to respond to student thinking, they must be able to predict the multiple solution strategies that students may take when completing the task.

They shared three different ways for approaching the problem. (See Figure 12). Tania, who has the most formal experience teaching probability, approached the problem by taking into consideration that the lesson goal was related to conditional probability. She calculated that the probability of pulling the same color (Amy wins) was $\frac{2}{5}$. She solved the problem by saying the probability of pulling a white ball on the first draw is $\frac{3}{6}$, and the probability of pulling another white ball is $\frac{2}{5}$. Then, using the product rule for probability, she claimed that the probability of pulling two white balls was $\frac{1}{5}$. She stated that the probability of pulling two black balls would be identical. Then, applying the addition rule, the probability of pulling the same color, regardless of color is $\frac{2}{5}$. She then used the concept of complements to determine that the probability of selecting two different colors would be $\frac{3}{5}$, making the conclusion that the game is not fair.
Rachel has the least experience formally teaching probability and has never taken a formal college course in probability. She presented a similar, but alternative method for determining if the game was fair. Rachel calculated the probability for pulling two white balls, indicating that it was $\frac{3}{6} \times \frac{2}{5} = \frac{6}{30}$. Then she compared this to the probability to selecting a white ball, then a black ball, which she calculated as $\frac{3}{6} \times \frac{3}{5} = \frac{9}{30}$. Through this comparison she also agreed that the game was not fair.

Ashley added a much more simplified explanation in agreement with the other two teachers about the fairness of the game. She stated, “The first choice doesn’t matter because whether you pick black or white, then the probability is just whatever you pick second. The first thing doesn’t matter because that is all that I care about.” She expanded upon this idea by indicating that the pulling of the first ball is a fair event, and each color has an equal chance of being pulled, but due to the fact that they are not replacing that ball, the only impact on the fairness of the game is the probability of drawing a particular color for the second ball.

Jackie, who solved the problem similar to Tania with a tree diagram, questioned the validity of Rachel’s solution, indicating that she did not understand how different values can be found for the same probability. She claimed that all the solutions lead to a comparison of $\frac{2}{5}$ and $\frac{3}{5}$. 
but Rachel’s reasoning produced different numerical values to represent the probability. The community of teachers engaged in a conversation about the relationship between these two results to assist Jackie in making sense of the differences. Rachel explained it in the following way, “Mine is not right for probability, no, because mine doesn’t do black and white. Mine only does a color and then the other color, just a circumstance.” Through this exchange, Jackie was building her content understanding in relation to a variety of ways to represent the solutions for the task.

In this EPR from the last video case analysis, the community of teachers shifted the way they were discussing the problem of practice, anticipating student solution strategies. While the teachers were doing the mathematical task, they were prompted to consider various solution methods for students and how these methods connect to each other. For this reason, it is a diagnostic frame. Due to their lack of experience with the content and the need to be able to respond to student thinking, the ways in which they discussed the multiple solutions changed indicating changes in participation. These changes were reified when the teachers shifted their focus from entry points for students to approach a task to actual solution methods. In this session, the teachers were focused on creating and making public the various solution strategies to answer the question from the task. They presented three various ways to answer the question, then engaged in conversation about the validity of one of the representations.

In this section on anticipating student responses, the evidence indicates that the teachers focused their attention on determining ways in which students would approach the mathematical tasks in each lesson. In the first video case analysis, the teachers discussed the single pathway nature of student solutions and related their solution pathways to the individual understanding of each student. In the second video case analysis, the evidence provided indicates a similar pattern,
however, the change in the task itself lead the teachers to discuss mathematical pathways the
students might attempt instead of more general statements like “everyone’s kids will do a
different way”. The teachers identified the easiest pathway students might try (guess and check)
and identified concerns with this method. This is evidence of changes in participation that were
reified by changes within their diagnostic frames. In addition to identifying solutions pathways,
they are now identifying possible student misconceptions with these pathways. In the last video
case analysis, the conversation appeared to transition from the community focusing on student
responses to focusing on developing and comparing various solutions to a mathematical task
without direct connections to student solutions. This is another change in the teachers diagnostic
framing and evidence that there were changes in participation when the teachers were discussing
possible student solution strategies.

**Productive Struggle**

Maintaining high cognitive demand is a balancing act that is acknowledged as an
indicator of effective teaching (Schoenfeld, 2016). Moreover, one of the Standards for
Mathematical Practice (National Governors Association Center for Best Practices, 2010) is
making sense of problems and persevering in solving them. While teachers can choose a high-
demand task and work to maintain the cognitive demand through implementation, the students
also need the perseverance to struggle with the mathematical concepts. Further, providing
students a space to productively struggle with mathematics can be a challenging task for
experienced teachers. The community of teachers at the center of this research recognized the
importance of maintaining cognitive demand and encouraging perseverance in the mathematics
classroom, whether through task design or questioning techniques. Below I provide EPRs from
each of the four video case analysis sessions where the topic of productive struggle was brought
to light through discussions about maintaining cognitive demand and supporting student perseverance.

**Maintenance of cognitive demand.** In the first video case analysis, the teachers watched the video segment in which students engaged in a debate over whether or not every quadratic curve has a y-intercept. The class discussion arose when a student from the video segment, in a small group, described their picture as a parabola in only one quadrant. To further probe student thinking, the teacher in the video asked, “Can a parabola ever stay in one quadrant?” The video clip then transitioned to the teacher prompting the whole class in their small groups to consider the same question. After the students had time to discuss in small groups, she gathered them for a teacher facilitated whole class discussion.

After watching the video segment, I prompted the teachers to consider what moves they might take as a teacher if they could go back to that moment to address student thinking. In this case, the student believed that the quadratic graph may not have a y-intercept. In response, one of the teachers indicated her admiration for the moves that the recorded teacher did make.

1 Jackie: …she let the students talk. Which you know she could have went in and use all these fancy words.

2 Ashley: Right. Like the second somebody says something incorrect, it’s corrected.

3 Jackie: Exactly.

4 Rachel: Cause they said window cause they were thinking calculator and she was okay, but window is representative of the what?

5 Facilitator: She didn’t say it.

6 Rachel: She’s like coaching them. (Video Case 1, 3/12/2019)
Through this interaction, the teachers analyzed the moves the recorded teacher made as they grappled with the problem of practice of how a teacher can maintain the cognitive demand of a task. Maintaining the cognitive demand of a task is an integral part to mathematics teaching and learning (Stein & Smith, 2011). They indicated that allowing the students to talk (line 1) and acting as a coach (line 9) for the whole group discussion was a successful teacher move because it provided a space for students to productively struggle with the mathematical concepts. In the video the teachers could tell the students were struggling to make sense of the vocabulary and the mathematical concepts. The recorded teacher did not give immediate responses to students’ questions, but rather prompted them to think and discuss more. In this EPR the community of practice indicated that she assisted them while the students struggled with material, but through her careful facilitation moves, she ensured that the struggle was productive. This EPR is a diagnostic frame because they collectively diagnosed a problem of practice. They used the video segment as a tool to define and describe that problem of practice by acknowledging that addressing student thinking through this questioning is one way to address maintaining the cognitive demand of the classroom.

During the second video case analysis, the teachers began to assess their own curricular materials in relation to the previously diagnosed problem of practice, how a teacher can maintain cognitive demand. Through engagement with the mathematical task, the teachers predicted student responses in the lesson about radical statements. In the task, students were expected to sort a card set of radical expressions into piles that are sometimes true, always true, and never true. As presented in the section on multiple solution pathways, the teachers were trying to determine if certain pathways would lead students to draw incorrect conclusions.

1 Facilitator: And likewise, I think both of them are going to tell me that this
[Figure 13] is always true. Right?

Rachel: Yeah, well even because in calculus when I do limits with them at infinity that's what messes them up at the positive and negative infinity, because they don't put the absolute value bars.

Facilitator: Right.

Ashley: Yeah. So, like that has to be a lesson before they would get that. Like they would have to. They wouldn’t get that by just plugging in values. (Video Case 2, 3/26/2019)

![Figure 13. Card from formative assessment lesson, Evaluating Statements about Radicals](image)

During the following EPR, the teachers referenced a particular group of cards in which there will be restrictions on the values of the variable. In the example above, Figure 13, the statement is sometimes true. Specifically, the statement is true only for positive values of $x$ because when evaluated, $\sqrt{x^2}$ becomes $|x|$, which equals $x$ for those values. Rachel connected statements about radicals to her experiences teaching calculus and having students find limits of functions. While this discussion was inspired by their analysis of Figure 13, the teachers refer to limits of basic radical functions. Because the statement in the card is only true for positive values of $x$, she is reminded of previous experiences with limits of radical functions. This interaction led to an analysis of the cognitive demand of the calculus curriculum.
Rachel: Right, like our old Calc book the limit examples never had limit examples like this at infinity. The old black book.

Tania: Specifically, at negative infinity.

Rachel: Right. That's the one, and the book even asks, even our new one, only asked to approach positive infinity, then I have to manually go in and change the test generator to make it approach negative infinity.

Ashley: Yeah, the old one was not doing that.

Tania: The honors calc is not, I like the other book you have.

Facilitator: But you have to make it.

Rachel: You have to put it in. It’s not in the book.

Ashley: It's funny because obviously the person that made up that question knows…

Tania: Knows about that.

Ashley: …that like, they know enough to know we should leave this way because that way would be harder for them then why wouldn't you include that? Because you’re learning calculus.

Tania: Right, because there's no challenge.

Facilitator: Why aren’t we challenging them?

Ashley: Why aren't we?

Ashley: Why aren’t we [publisher] expecting them to do that? I feel like even these writers at the Calc books are assuming that these kids don't know that. Which is like very strange.
After previously identifying the need for conceptual understanding of variable restrictions with radical functions in calculus, the teachers identified concerns with the current resources for the calculus course. The teachers used the content of the lesson from the video case materials to make connections to their practice. Rachel, Tania, and Ashley indicated that the expectation from the calculus resources is that students do not need to have an understanding of the restrictions associated with the statement in Figure 13 because the resource never asked the students to address this limit from negative infinity (lines 1-2; 4-5; 8). This EPR is diagnostic because the teachers are discussing a previously diagnosed problem of practice about how teachers can maintain the cognitive demand of the lesson based on research. They believe that by not questioning students about the limit of a radical function at negative infinity, there is a missed opportunity for students to build mathematical connections.

In this section I provided data from the professional development sessions where teachers engaged in conversations about productive struggle and it’s connection to maintaining the cognitive demand of a task, lesson, or course. In the first session, the teachers identified instances of the videotaped teacher maintaining the demand of the lesson through her choice of questioning in the moment. In the second video case analysis, the teachers began to question their own curricular resources and whether they maintain a level of cognitive demand acceptable for the course. While both EPRs follows the same diagnostic framing patterns with respect to the problem of practice, there are changes within the diagnostic frames. For the first EPR, the teachers used the video segment as a focus to highlight teaching moves that maintained the cognitive demand. In the second EPR, the teachers began to reflect on their own curricular materials and their cognitive demand. This change within their diagnostic frames provides
evidence of changes in participation which is reified when the teachers transition from focusing on the video to their own curricular materials.

**Student perseverance.** During the first video case analysis, the teachers completed the dominos activity by matching quadratic equations and graphs. During this activity, the community of practice began to focus on how students approach mathematics problems. While the teachers were analyzing the video case materials, they discussed the students’ motivation to struggle with mathematical concepts. To have the community consider the connections between student solution strategies and lesson goals, I asked the teachers to consider how the various ways of solving the task can illuminate the big mathematical goals of the lesson. The next EPR is their response to that prompt.

1. Ashley: Is it going to take them this long to solve the problem? Well, I
2. think they are going to be more process oriented. Like they are
3. going to be like, ok, I have this, I have to try to find the vertex and
4. then, you know what I mean.

5. Rachel: Yeah, once they do one by the vertex, they’re going to try to do
6. them all by the vertex until they can’t.

7. Ashley: Yeah.

8. Rachel: Then they are going to be like oh, oh, alright. And then they will
9. decide x intercepts, factored form. And then they will try to do all
10. of them like that. Let’s find the other parentheses and they're going
11. to try all of them until they hit a roadblock because there's only so
12. many that you could do like that.

13. Ashley: Without work
Rachel: Yeah. And then they have to force themselves.

Facilitator: So the activity almost forces them out of rely on being reliant on one form.

Ashley: Yeah, they can't just use one form over and over or memorized one process. (Video Case 1, 3/26/2019)

This EPR is the first time the community started to define a problem of practice that students are lacking the desire to struggle with the concepts. The attribution of causality was that students rely on using only one solution method, without attempting other methods. Rachel supported Ashley’s claim by providing an exact iteration of what she expects students to do when attempting to complete the task (lines 60-64). Rachel thought the students would pick one process and not deviate from it until they hit a “roadblock”. She did not indicate that she believed the students would be able to move past this roadblock. Rachel, Ashley, and I indicated that the design of the task will force students out for being reliant on only one form of quadratic function. The teachers restated that the students are focused on the end product instead of the ways in which they arrive there. This EPR is an example of a diagnostic frame because the community defined a problem of practice and the underlying cause for the problem. After discussing the topic for a second time in the session, the community agreed that the students are lacking the desire to struggle with the concepts. Instead, the students are more concerned with the result as an example of showing what they do know, and a desire to move to the next task.

In the second video case analysis, as part of the discussion before watching the video segment, the teachers continued to discuss the ways in which students are focused on an end product instead of the process to get there. While discussing the ways in which students interact with exponents and radicals, Rachel claimed, “kids can only kind of memorize one way to do it
and then they do it this long way every time.” As described earlier in this chapter, the teachers
discussed the need for students to be well versed in the multiple solutions pathways for each
problem and be aware how the multiple solution pathways are connected. Described in the
section on multiple solution pathways, when attempting to identify a cause for this problem, the
teachers made connections between multiple solutions and student perseverance when solving
tasks. In the following EPR, the teachers attempted to define local problems related to the lack of
student perseverance and find ways to address them.

1  Rachel:  But again, I think that comes from life. We don't spend enough
2  sequential time with them in a day. These 46 minutes go.
3  Barbara:  No, yeah.
4  Facilitator:  Yeah. I think that that's…
5  Rachel:  They're not getting…they don’t have supported practice time. Go
6  home and try to practice and then like even here, where it is, it says
7  persevere. Yeah well, this one “Make sense of problems and
8  persevere in solving them” but when they don't have enough time
9  to practice in class and they get stuck at home then they're done.
10 Tania:  Right. Well there's no way, there is no perseverance.
11 Ashley:  They don't get stuck at home.
12 Barbara:  Because they don't do that.
13 Rachel:  We used to be able we had longer periods.
14 Facilitator:  Right. You feel okay giving them more time to persevere and to
15 have that productive struggle in class. Because if you had the time
16 to play with in order to give them to do that.
Two years prior to this professional development this community of mathematics teachers had 60-minute periods with the students every day. Now teachers only see their classes for 46 minutes a day. This reduction of 14 minutes a day was a loss of 42 hours of contact time throughout the year with every class. Rachel diagnosed the problem of practice as the students’ lack of perseverance in problem solving (lines 5-8; 10) and attributed causality to a lack of time in class connecting teachers and students (lines 8-9). After Rachel voiced her opinion, the community of teachers not only agreed with her, but some also expressed that having more time with students in class might provide an opportunity for more students to engage in productive struggle with the assistance of a teacher as a facilitator (liens 14-16). This EPR is a diagnostic frame because the teachers identified a lack of perseverance from students as the problem of practice and attributed causality to the current institutional constraints of instructional time.

During the last video case analysis teachers began a discussion on the recent state mandated computer-based exam. Tania reported that her students expressed difficulty they had reading directions on the computer screen during the exam. The students claimed that the prompts for the tasks were longer than those they were accustomed to receiving in class. Ashley echoed Tania’s students’ beliefs by adding that a student said, “The words actually went blurry and I couldn't read. I couldn't focus on what this was actually saying, and I was so bored by it
that I couldn't.” During that EPR, the teachers acknowledged that the students were struggling to read long passages about mathematics, and this might be impacting their performance on standardized assessments. In the following EPR the teachers continued discussion about their diagnosis of the problem of practice, lack of student perseverance because of their single pathway nature of problem-solving.

Rachel: But one thing I was thinking for us, with what we were just saying. They don't read. And then when we give them a worksheet, it'll say the directions, but then there's 6, 10, 25 examples doing the same thing. I think we have to re-write the directions every time and jumble up the questions, so they have to read it to know what they're doing on that example. I think we have to stop doing these directions and giving three problems that match it because they don't associate the directions with each one. They just mimic what they just did over and over.

Barbara: And they do mistake. For example, we ask them to factor and they solving equation, was no equation given to you. How the heck you solve it?

Rachel: I think we have to stop doing that. No more giving directions and then multiple questions. Every question has to have its own directions. You know what I mean?

Facilitator: I think we should change this.

Ashley: Less problems, separate.

Jackie: Wait, so what are you saying?
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19 Rachel: The kids read the directions once. They kind of look around and
decide what they have to do for this set and then they just do the
whole set the same way. And then they go to the new set of
directions because we keep giving them a set of directions and then
many examples. But we should just be giving directions for every
question and shuffle them up--

25 Jackie: And mix up the directions.

26 Rachel: --and make them read every time, you know?

27 (Video Case 4, 3/26/2019)

Before offering a suggestion for addressing the problem of practice, Rachel first attempted to better define the problem and determine the attributing factors. She shifted the attribution of causality away from the students and claimed that the current structure of curricular resources they choose to use allows for students to read a short set in instructions and apply this one type of solution method to a problem set (lines 2-3). Barbara added that the students have trouble distinguishing between directions and their perception of what they are supposed to be doing (lines 10-12). For instance, when her algebra II students are given a set of quadratic expressions and asked to factor, the students produce solutions for an equation instead of following the instructions. Rachel proposed a prognosis to this newly identified attribution of causality to assist students in building their perseverance when solving problems. She stated the desire to change the structure of curricular resources by indicating that each question in the curricular resources should contain directions to break students to their single pathway problem-solving techniques (lines 13-15; 23-24). She believed that the students have this single pathway nature of problem solving because the structure of the curricular resources reinforces this
behavior. The community believed that this has the potential to encourage multiple pathways when completing tasks because it is requiring students to analyze each problem or task independently and does not allow them to apply one procedure repeatedly to a set of problems. By doing this, she hypothesized that the students would break away from their linear, single pathway nature to solving problem and be able to make connections among mathematical representations. Jackie, Ashley, and I agreed by echoing her statements throughout her suggestion. Tania contributed to Rachel’s definition of the problem of practice and prognosis in the following excerpt.

1. Tania: But that's like even when we tell them to solve and they have to
2. use square roots or quadratic formula or factoring, and they can't
3. choose. I mean, it took weeks for them to choose. They were trying
4. square roots on something with an x-term. You can't do that.
5. Jackie: And you said it took weeks, but if you gave it to them right now,
6. would they know?
8. Rachel: So, I think every question needs to be able to stand on its own. We
9. can't make them part of a set unless it's from one picture and we're
10. asking three questions. Otherwise--
11. Tania: Or if the instructions are vague. Solve. I'm not telling you how.
12. Rachel: But even that, they're going to factor. They don't know the
13. difference because they're not associating the directions with each
14. individual problem. We're giving them repetitions of factoring.
Ashley: But then, if one is factoring, if one they have to use quadratic-- if one they have to-- if they have to use different things for each one, they will be like, "Well, I can't factor this one, I factored the last one, so I can't do this." Because they don't understand that it's like, "Maybe I have to solve this one differently than the last one."

Tania: Right, right. (Video Case 4, 3/26/2019)

Tania attempted to expand Rachel’s original prognosis, altering the curricular materials, by providing context related to the multiple solution strategies for solving quadratic equations. She indicated that most of the directions will tell the students the method(s) in which they will need to use to solve the particular problem. The community identified the problem of practice as the teachers giving explicit directions that are leading students to a single pathway solving process and students are not being expected to analyze a quadratic equation and determine the appropriate methods for solving it (lines 1-4). Rachel and Ashley attempted to connect the two concerns when they stated that the single pathway problem-solving of students will still prompt them to want to solve all problems grouped together in the same manner (lines 13-21). This EPR is a prognostic frame because the community identified the problem of practice, their structure of curricular materials does not provide students space to persevere when solving problems. They attributed causality to the fact that the directions are too explicit or leading and suggested restructuring their curricular resources to account for this problem of practice. In addition, this EPR has connections to the following characteristics of collective professional knowledge: (1) connected to practice and public, and (2) sharable and storable. The prognosis addresses changes in their current practices which they made public through their community of practice. Due to the shared nature of their curricular resources, these changes will be storable within this community.
of practice.

The three previous EPRs are evidence of a change in framing patterns across and within frames. The first EPR was a diagnostic frame with a problem of practice defined as students lacking the desire to struggle with the concepts because of their single pathway solution methods. The second EPR was a diagnostic frame with the lack of perseverance as the problem of practice caused by the limited time teachers had with students. In the last EPR, the community changes both the way they framed the diagnosis and the way they were able to transition to a prognostic frame. The community transitioned from attributing causality for the problem of practice from the students to their own teaching materials. They identified the problem of practice as the structure of curricular materials used does not promote student perseverance because the directions were too explicit and leading. This was also a change across frames because the teachers proposed a hypothetical solution to the problem by changing the structure of their curricular resources. These changes across and within frames are evidence that community learning is occurring by changes in participation which are reified by: (1) focusing on their own teaching practices instead of the students and (2) developing a prognosis to address the problem of practice. The community was utilizing their mechanisms for building knowledge about student perseverance by reflecting on their own teaching materials and proposing changes to those materials based on their personal investigations with their students.

In this section I presented evidence of the development of collective professional knowledge. This community of teachers used personal experiences, in connection with the video case materials to share their understandings of mathematics teaching and learning. The results provided above highlight three particular areas of conversation: precision of language, multiple solution pathways, and productive struggle. To determine how teachers used video case
materials to develop collective professional knowledge for mathematics teaching and learning, I looked for changes within and across collective actions frames. These changes within and among frames were evidence of changes in participation which indicated that learning was occurring within this community of practice. What the teachers are learning is becoming part of their collective professional knowledge through the process of making it connected to practice, public, shareable, storable, and through developing mechanisms for building knowledge.

The Nature of Conversations

To determine what the nature of the conversations was, I used my previous analysis of each EPR to categorize them into diagnostic, prognostic and motivational frames. This analysis was similar to the analysis from the first research question because I initially divided data into EPRs and used frame analysis to determine how the teachers were discussing problems of practice. However, the analysis for this section did not use the open coding and identified themes because, to answer this research question, I was interested in the nature of the conversations instead of what the teachers were learning and building knowledge about. To answer the second research question, I focused my analysis to look for patterns in how teachers framed their conversations within the different parts of the professional development model. Specifically, I focused on the parts of the model pre-video watching (pre) and post-video watching (post). I identify these parts of the model as follows: identifying big mathematical ideas (pre), doing the mathematics (pre), anticipating student solutions (pre), watching the video case, and critical analysis of teaching and learning in the video segment in relation to TRU dimension(s) (post). During the critical analysis of teaching and learning, the professional development model focused on two types of analysis: analysis of student understanding and analysis of teaching moves. This process allowed me to understand the nature of conversation throughout each
professional development session as teachers engaged with the video case materials. In the sections below, I provide evidence of changes in framing patterns when comparing the conversation pre and post video watching. Further, because the nature of the conversation was similar in the first three sessions but changed in the fourth, I present data from sessions one through three together and end with data from the fourth session.

**First Three Sessions**

**The nature of conversation pre-video watching.** Prior to watching video segments of real classrooms enacting formative assessment lessons, the community of teachers partook in conversations that were more diagnostic than prognostic. That is, the teachers were more likely to identify problems of mathematics teaching and learning and determine the cause of these problems without discussing potential solutions. Rather than discuss solutions, they identified common issues related to the mathematics of the lesson, student understanding from their own classrooms, and/or institutional constraints that hinder teaching and learning. I identified the only change in this pattern of framing during the fourth video case analysis. In particular, during the last video case study the teachers’ framing pattern was more prognostic than diagnostic prior to watching the video segment.

In this section, I present evidence from each session prior to watching the video segments. I have divided the data into two parts of the professional development model: identifying mathematical goals and anticipating student solutions. These two sections align with the two of the parts of the professional development model prior to watching the video segment. To follow the facilitation guide, I used the first portion of the professional development sessions to guide the teachers in determining the mathematical goals of the lesson and completing the mathematical task while anticipating student work.
Identifying the mathematical goals. Relying on consistent facilitation throughout the four video case professional development sessions, I first asked the teachers to read through the lesson plan and determine the big mathematical goals of the lesson. The teachers first discussed their ideas in small groups, then the community shared their ideas as a whole group. During these discussions, I also asked them probing questions about the key components of the topic for them in their classrooms in the attempt to connect the lesson back to their practice. Conversations about the mathematical goals occurred in all four sessions.

In the first session, while the community was identifying the mathematical goals of the lesson, I identified only five diagnostic frames. There was no EPR that could be coded as a prognostic or motivational frame. During the diagnostic EPRs the teachers drew on the predetermined lesson goals and extracted a big mathematical picture in relation to their own practice. In each EPR, the teachers used the lesson goals and connected them to their practice using specific examples from their experience. Below are two representative examples of diagnostic frames in the first video analysis about quadratic functions. The examples provided show a small shift in what the teachers connected the mathematical goal to, the content or the lesson task.

The mathematical goals were first discussed in a small group setting initiated by Ashley when she stated, “You basically want to understand why all three forms are important and what they are used for and what they look like.” Ashley was referring to the three forms of a quadratic equation: standard, vertex, and intercept forms. The following EPR is an example of how these ideas were then discussed in the whole community.

1 Tania: Each different form can identify different parts of the function,

2 Rachel was saying. Then you approach how you graph it
 differently.

4 Rachel: And, in calc they want to find maybe max and mins, but when
5 they’re doing polynomials, so you’d want to graph by intercepts,
6 so like with quadratics you could learn both things. So then later
7 on when we’re highlighting the max and mins, we talked about the
8 vertex more. We talked about this optimum point more. And then
9 when we graph just polynomials, you showed all the x intercepts. I
10 like that for more.

11 Tania: Well, with the freshman right, I want them to know that like it
12 doesn’t matter what the form is that they can pick on information.
13 Well, they’re making tables because they can’t handle the path like
14 patterns. And then be able to find the vertex no matter what form it
15 is.

16 Barbara: Like in Algebra 2, very important vertex, x-intercept, right? (Video
17 Case 1, 3/12/2019)

The teachers determined, both in small groups and in the whole group discussion, that the
importance of this lesson is the power of comparing and contrasting the three forms of a
quadratic function. Through this comparison, they noted the additional importance of utilizing
the structure of each form to identify key features of quadratic graphs. The teachers are drawing
on experiences from their particular classroom contexts to highlight the importance of the lesson
for them. This EPR is diagnostic because the teachers have identified a problem of practice, the
need to understand multiple representations of a function, and provided an attribution of
causality that students are struggling to easily identify key features of graphs. This idea was
echoed later in the discussion by Jackie when she stated, “No matter what form you’re given, you can graph.” The teachers revisited the problem of practice and expanded on their diagnosis about the ability to transition between multiple representations of a quadratics function.

Jackie: I also think the big picture of what I would want my students to gather is no matter what form your quadratic is in, you can get all the information, right. So yes, one form gives you specifics, but then you can turn it into another form and get other things.

Ashley: Right, you can alter them.

Jackie: No matter what form you're giving, you can graph.

Tania: Right, right.

Barbara: You can work with whatever points if you can look, whatever.

Rachel: Yeah. If it's factorable and your vertex is going to be at like 1.5 maybe you'd rather just find your x intercepts.

Tania: Yup.

Barbara: Yeah. That's a vertex, x-intercept, and y-intercept, you can be able to graph. (Video Case 1, 3/12/2019)

Like the previous EPR, this is a diagnostic frame related to the problem of practice, the need to understand multiple representations of functions because they need to be able to easily identify key features of a graph, in particular, quadratic functions. The teachers are focused on diagnosing this problem by understanding the lesson and identifying important mathematical content, as well as the connections and conceptual understandings that they want their students to make. During these interactions, the teachers did not propose changes to lessons or pedagogy, which would allow me to define the frame as prognostic.
Through my data analysis of the second video case, I found three diagnostic frames and no prognostic or motivational frames. In these three EPRs, the community used the lesson goals from their formative assessment lesson and made connections to their practice and student misconceptions. Below is one example of the community’s diagnostic framing prior to watching the video segment where they address one lesson goal and made connections to the mathematical task. In alignment with the professional development model, I first prompted the teachers to consider the mathematical goals of the lesson. One lesson’s stated mathematical goals was to “Distinguish between equations and identities.” Ashley made a connection between the stated mathematical goal of the lesson and the manner in which her group completed the task. They had taken each radical statement and algebraically solved for the unknown variable. Then, they interpreted their answers to determine whether the statement was sometimes, always, or never true.

1  Ashley:  So, the second one, the equation is an identity. Since the equation
2                              is always true, that's an identity then, cause like this.
3  Jackie:  Right. That's what I was thinking.
4  Rachel:  Right. Because those are the only rules
5  Ashley:  Because the always true is the identity.
6  Ashley:  And then these were equations because we solved them and got
7                              answers. (Video Case 2, 3/26/2019)

In this diagnostic EPR, the community’s problem of practice is the alignment of classroom tasks and lesson goals. The attribution of causality is whether or not the task in this lesson illuminates the lesson goals. The teachers ensured the task was aligned but did not look for ways to better align the task to the stated goals. The reason for the need for alignment was raised later when I
stated, “The struggle is having them identify when a statement is always going to be true versus when is it something that I can actually find a value for.” Drawing on this struggle, the teachers discussed how the task aligned with the goal of understanding the difference between equations and identities. For this reason, the EPR is coded as a diagnostic frame.

In the third video case analysis, I found two diagnostic frames and one motivational frame while the community was identifying the mathematical goals. In the EPR below, I provide one representative example of the diagnostic frames from this session. This is representative of the diagnostic frames as it illustrates the way the teachers discussed the big mathematical ideas. After teachers were prompted to discuss what they thought the mathematical goals for the lesson were, they began to identify the lesson’s stated goal that they believed was most important: “Identify the appropriate property to use and apply it correctly.” In the EPR below, the teachers used this lesson goal as a catalyst to define a problem of practice, the need to understand multiple solution pathways to simplify expressions using properties of exponents.

1 Ashley: The appropriate property and applying it correctly. So, like when
to use what. When to use what because they need to, basically,
look at a problem, evaluate what's going to help me the most right
now? Because like Deb was saying before, they don't know when
to do this versus when to do this. They need to look at it and say,
"Okay, what's the best property for me to use at this moment?"

7 Tania: First.

8 Jackie: First.

9 Ashley: First.

10 Jackie: Yeah, they're still in their instincts.
During this EPR, the teachers identified the mathematical goal they found most important and used it to define their problem of practice. The reason for this problem of practice stemmed from their experiences with students’ application of properties of exponents. This EPR is a diagnostic frame because the teachers diagnosed the problem of practice as the need to know multiple solution pathways (lines 1-2) because based on their experiences, students do not know when to rely on a certain pathway (lines 5-9; 23, 25-27). In addition to this attribution of casualty, the
community also claimed that the problem of practice is important, because when students know multiple solution pathways for simplifying expressions using properties of exponents, then they can then engage in conversation about efficient methods (lines 11-14). This EPR is diagnostic in nature because the teachers are not offering suggestions for changes in pedagogical practice to address identified problems.

In one of the conversations around the big mathematical ideas, I coded only one EPR as a motivational frame. In this EPR, Rachel discussed the big mathematical ideas from the lesson Evaluating Statements about Radicals:

I mean I see that they need to know the basic skills and they need to be literate and fluent at speaking like eight is really two to the third power and things like that so they can be efficient at solving a problem. So, when they're doing things with like exponents and radicals and all that like you don't need to cube something before you take a square root you can have good numbers sense and break it apart and get to the answer quicker. Sometimes you reduce before you multiply. Sometimes you subtract before you add make the number smaller before you make the numbers bigger. So, I feel like you know there's a common theme across everything but that to me makes me crazy. When we work on exponents and radicals. When kids can only kind of memorize one way to do it and then they do it this long way every time. But really there needs to be like a hundred examples done so that they can do case by case basis like what's, what makes most of this time and why? (Video Case 3, 3/26/2019)

Rachel’s problem in her practice is not explicitly detailed in this conversation, but I inferred that she is concerned about students’ lack of efficiency when problem solving, specifically related to properties of exponents. I made this connection from the discussion in the previous example of a
diagnostic EPR when she attributed causality for the problem of practice to the students’ need to “be efficient”. In this motivational EPR, she identified the singular solution pathway of her students’ mathematical thinking as the cause of the problem of practice when she said, “Kids can only kind of memorize one way to do it and the they do it this long way every time.” Her prognosis to remedy this problem of practice is for students to see many different examples. She indicated her motivation for the prognosis was that by providing them many illustrations of properties of exponents students will be able to build patterns of efficient methods to simplify expressions.

The results in this section outline interactions around the identification of the mathematical goals of the lesson. These interactions occurred prior to watching the video segment. During this portion of the professional development model, all of the EPRs were diagnostic in nature, with one motivational EPR as an exception. In the next section I present more evidence prior to participants watching the video segment while they are completing the mathematical task and anticipating student solutions.

**Anticipating student solutions.** The second part of the professional development model had the teachers engaged in doing the mathematical tasks. Following the completion of the tasks, the teachers were asked to consider the following questions: What are different ways to solve the problem? How do different ways to solve the problem connect to one another? What approaches are students likely to take when trying to solve the problem? During this part of the professional development model, the teachers were still thinking about the mathematical goals of the lesson while also connecting these goals to their practice. At times, the teachers shared examples from their experience connected to the content of the lesson. There was an abundance of diagnostic frames while teachers shared multiple solutions and anticipated student solutions. Throughout
this section, the diagnostic EPRs provided are representative examples of diagnostic frames in each of the first three video case analyses. I chose the following examples to illustrate the various ways teachers were diagnosing problems of practice. However, it is important to note that there were some motivational frames as well during this portion of the professional development. I have also provided these motivational frames prior to watching the video segment.

In the first video case analysis, teachers discussed common student misconceptions based on their experiences teaching quadratic functions. Rachel indicated that her students struggle to recognize the input and output relationship of functions in general. She used an example from her honors class to provide a context for her colleagues:

Then precalc honors, now we're doing parabolas like as conics and they're asking you how wide is the bridge as many feet above the ground or whatever and be like, don't understand the correlation between inputs and outputs that well. So, they might not understand that they should double the x-value [of the vertex] after they get it because it's how wide is the bridge… (Video Case 1, 3/12/2019)

This is a diagnostic frame because Rachel identified her problem of practice as students not being able to connect the input-output relationship to a contextual situation. She used a specific instance from her classroom experience to attribute causality, students not being able to recognize the relationship between the value of the vertex and the x-intercept. She shared an example of a quadratic task which asked students to find the length and height of a bridge. Her students were finding the vertex of the parabola but could not relate the values of the vertex to the contextual situation to determine the length of the bridge.

During this first video case, Rachel introduced another misconception she experienced in
her teaching when she identified that students often question whether or not a quadratic graph has a y-intercept. This was prior to watching the video segment, while anticipating student solutions, so the teachers were not aware this question was about to be addressed in the upcoming video segment. Jackie was skeptical of this student misconception and questioned whether or not the other teachers believed that students thought a quadratic could only remain in one quadrant. She seemed generally surprised that Rachel, Ashley, Barbara and I shared a consensus that this is a common student conception. However, with all the discussion about this misconception, no teachers provided a possible teaching move to counter this issue during this part of the session. In this EPR, coded as a diagnostic frame, the problem of practice is identifying important characteristics of a function in its graph. The attribution of causality is that the students might claim that a quadratic graph does not have a y-intercept. In turn, the teachers related this to students’ struggle to make connections between mathematical concepts around graphical features of the graph. In this discussion, the teacher attributed the cause of this misconception as students relying on a portion of a graphical representation of a function. For instance, if the students see a graph that only appears in one quadrant, they struggle to connect the domain of the function to understand that the graph will eventually have a y-intercept.

Another example of a diagnostic EPR in this session occurred when Rachel, Ashley, and Tania indicated that students will often confuse task cards with similar characteristics. For example, there are three graphs which contain x-intercepts of (3,0) and (5,0). Rachel said, “They are so similar, so they have to distinguish what makes them different.” This led the group of teachers to assume students would connect each graph and equation using the same reasoning. Ashley shared, “I think they are going to be more process-oriented. Like they are going to be like, I have this, I have to try to find the vertex.” Ashley was referring to the single method
pathway to problem-solving she experienced with her students. Rachel finished Ashley’s sentiment by adding, “Yeah, once they do one by vertex, they’re going to try to do them all by the vertex.” While having this discussion, the teachers believed the structure of the task might elicit the need for students to use multiple solution pathways, breaking their tendency to only look for a singular path. The problem of practice for this diagnostic frame is being able to understand multiple representations of functions for students to make mathematical connections. The teachers attributed causality for this problem of practice to their students' tendency to only look for a single pathway to complete similar problems. Once students solve a problem using one method, the teachers believe that the students will continue to rely only on that method, thereby hindering their ability to build connections among the multiple representations.

These three EPRs from this session are examples of the diagnostic frames during conversations throughout the professional development session when the community was anticipating student solutions. In all three instances, the teachers introduced and collectively defined possible student misconceptions. I chose these as representative examples, because they display the various ways in which teachers discussed the mathematical tasks and students’ possible solutions. In particular they represent instances when teachers made references to task design, connected the content to their current practice, and highlighted student misconceptions from experience. However, in their EPRs, the teachers did not provide suggestions for teaching to counteract the identified misconceptions.

During the second video case analysis, the teachers discussed personal experiences about how students resist struggling with the mathematical concepts and persevere in solving them. These examples are evidence that the diagnostic framing patterns continue from the first to the second video case analysis. In this EPR, Rachel and Barbara raised current classroom issues they
are faced with related to the content of the formative assessment lesson in the video case. At the time of the professional development session these two teachers were teaching radical equations and functions in their respective classes.

1 Rachel: So, I feel like you know there's a common theme across everything but that to me makes me crazy. When we work on exponents and radicals. When kids can only kind of memorize one way to do it and then they do it this long way every time. But really there needs to be like a hundred examples done so that they can do case by case basis like what's what makes most sense this time and why?

2 Barbara: Especially when we're talking about fraction exponents right.

3 Rachel: Exactly that. That's what I mean

4 Barbara: And, and when it's a negative fraction, you need to do a lot of examples because they use the same strategy for all of them.

5 (Video Case 2, 3/26/2019)

The teachers were quick to correlate the mathematical task to current issues in their practice and classrooms. During this diagnostic frame they identified the problem of practice as the single pathway problem-solving nature of their students. This is evidenced with Rachel’s statement when she indicated that the students will memorize one solution pathway and use this for all similar problems (lines 3-4). After more discussion identifying specific instances of student struggles with properties of exponents, Rachel attributed causality for this problem of practice to a lack of student perseverance related to the local constraints of their schedule. She believed that a reduced amount of contact time with students is limiting their opportunities to persevere in solving problems and in turn students are not be able to build the mathematical connections to
foster their development and to consider multiple solutions pathways when solving tasks.

In the same session, the community anticipated other student solutions that they experience specifically related to the task in the lesson. This EPR is related to the previous because it also addressed the single pathway nature of students when problem solving, but this time the teachers discuss the students’ reliance on guess and check. During this part of the professional development model, the teachers engaged in the mathematical task by sorting the cards to identify which cards were sometimes true, always true or never true. (An example of the cards can be found in Chapter 3, Figure 2)

1  Rachel: What number would I put in for x to make the…they don’t think algebraically solve. They never isolate x.

2  Tania: They would only plug in like one, zero.

3  Facilitator: So, they never think about solving for x. They think about like…

4  Tania: None of them do it.

5  Barbara: They are doing this all the time.

6  Rachel: They don’t use algebra.

7  Barbara: They are doing all the time this mistake (see Figure 14). My AP kids do it.

8  Facilitator: Barbara’s mistake right.

9  Barbara: I cannot take it.

10  Facilitator: So, everybody, look at Barbara’s. So, what are our students thinking when they write this?

11  Tania: Distributing the square root. (Video Case 2, 3/26/2019)
Figure 14. Card referenced during discussion on student understanding.

As seen in this EPR, Rachel and Tania believed that students rely on plugging in various quantities to determine if a statement is true instead of utilizing their algebraic solving techniques and analyzing the solution (lines 1; 3). Rachel and Tania continued the conversation from the last EPR about the single pathway nature of students’ problem solving (lines 1-5). Then, Barbara identified another student misconception she experienced in relation to Figure 14. She indicated that her students usually make the mistake illustrated by the card when manipulating an algebraic expression. She added that this is a mistake that she encountered in all levels of her classes, including her highest performing AP Calculus class. When I prompted the teachers to consider the reason students make this error, Tania indicated (line 15) that students may think they are distributing the square root over the operation of subtraction. Ashley extended on this diagnosis indicating that she thinks this misconception is related to a similar misconception that her students exhibit.

1   Ashley:  I think they are the same thing that squaring instead of foil.
2   Rachel:  Yeah, well I was going to say I try to show them like oh x plus two
3                                 squared \((x + 2)^2\) isn’t x squared plus four \((x^2 + 4)\) and they are
4                                 like, yeah it is.
5   Ashley:  Because no matter how many times they say it, they still make that
6   x squared plus four. It doesn’t matter, like write it twice and
During this continuation of the previous EPR, Ashley and Rachel agreed with Tania’s identification of a possible student misconception. In addition, they connected the misconception to a similar one experienced with quadratic functions. The problem of practice for these two EPRs is student misuse of the distributive property. The attribution of causality of this was discussed, after multiple varieties of this misconception were shared, when Ashley, Barbara and I identified that students experience confusion with mathematical symbols. We discussed that the students might be confusing the radical symbol with a numerical value to incorrectly apply the distributive property.

In each of the above examples from the second video case, the community of teachers diagnosed problems that arise in their practice related to anticipating student solutions. Generally, all EPRs in this portion of the professional development model involved problems of practice directly related to the activity of anticipating student thinking. For example, Tania and Rachel indicated that students will plug in values to determine the truth value of a mathematical sentence. Barbara, Ashley, and Rachel identified a common mistake of applying a function over addition or subtraction.

The teachers predicted additional student solutions to the task. They expressed that seeing patterns and structure could be powerful in determining which cards were grouped together. I asked how the teachers believed their students would match cards. Initially, Rachel, Ashley, and Tania shared that students would group cards based on visual appearances of the statements without concern for operations. For instance, they would group the cards in Figure 15 together because all cards show a variable and number joined by an operation under a radical on the left side of the equality symbol. On the right side of the equality symbol, the variable and numeral
are under an individual radical symbol and separated by the same operation. The teachers believed that the visual pattern elicited by these cards will cause students to group them together expecting similar outcomes without regard for conceptual understanding. In this diagnostic EPR, the community identify another problem of practice, the assumption that similar symbolic representations mean similar mathematical procedures. This example is similar other problems of practice because the attribution of causality is confusion related to mathematical symbols. Their cause for identifying this problem of practice is based on the three cards below and the indication that students will not take into consideration the differences between multiplication and addition/subtraction.

![Figure 15. Select cards from Evaluating Statements about Radicals lesson.](image)

In addition to the diagnostic frames during the second professional development session, I identified some motivational frames as the teachers worked to anticipate student solutions. In particular, the community was able to generate a motivational frame related to the previously diagnosed problem of practice, students’ misuse of the distributive property. The community began to discuss ways to address student understanding.

1. Rachel: I mean I put on the board I always do three squared plus four squared equals five squared.
2. Jackie: Yup, me too.
3. Rachel: Then, I show that nine plus sixteen is twenty-five. Twenty-five equals 25. Check. Then I go again, and I take the square root the
Barbara: That is what we write down for my class.
Ashley: That is literally how I do it.
Jackie: I was going to say the same.
Tania: Yeah. That’s what I do.
Rachel: OK. So, I prove by showing by counterexample. It can't be true because it a counterexample exists.
Facilitator: Right. Anybody do it differently?
Tania: Or even just saying, the square root of three plus four, like how would you do that, the square root of seven, is that the same as…

Rachel started to transition the conversation to a prognostic frame by sharing a method for addressing the incorrect application of the distributive property. The rest of the community agreed that this has been a method they have also used. During this interaction, the teachers mutually agreed upon a solution to this problem, or a prognosis, which was to provide a numerical example to the students as to why their thinking will not hold (lines 4-6). This EPR is a motivational frame because the community also expressed a motivation for their prognosis when they indicated that a proof by counterexample would help students see why their misuse of the distributive property is inaccurate (lines 11-12).

Another motivational frame occurred in this part of the professional development model during the second video case analysis. The teachers engaged in discussion about distribution of a constant into a function as a common student misconception. For example, students would manipulate the expression $2\sqrt{x} + 3$ to be $\sqrt{2x} + 6$. This example inspired Rachel to discuss an
experience from her precalculus honors class in relation to graphing conic sections and the way graphing transformations is taught in the school.

Rachel: In precalc honors now when I'm doing all this graphing stuff, after they learn conics, you know how like when the coefficient is out front it's a vertical stretch so it's a two. But, if you have a two inside, it's a compression. So, when I teach it to them, in precalc, after I do conics where they look at everything is ellipses and hyperbolas where the a squared and the b squared are underneath I rewrite everything like that so, I feel like I can't go back in time and teach it this way. But, if I have like, y equals two times one third x plus four squared minus seven $y = 2\left(\frac{1}{3}(x + 4)\right)^2 - 7$, right. There it's a vertical stretch of two horizontal stretch of three, left four, down seven. Because they just memorize everything backwards here [referring to 1/3 and 4] and everything is kind of normal here [referring to the 2 and the 7]. But I bring everything back, so I have $y - 7 = \frac{2}{3} = \left(\frac{x + 4}{3}\right)^2$ So, I'm like, all right well where's your vertex moving. Left four. Down seven. They're both backwards because they're like in their respective realm. And then this is a three. So, there's a stretch of three in the x direction and it's a stretch of two in the y direction so there's no underneath is greater than one. So, it's a stretch apply the y direction this number underneath is it's getting the ones the stretch applied in the x
direction. So, instead of them thinking that, “oh it’s the opposite here, it’s the same here”. I just have to put everything back where it belongs. And look at it that way. So, same thing with like square root functions with everything even lines whatever you mean like the slope of lines. So like y minus two equals two thirds x plus five. Well they could do the same exact thing. I could divide by two. So, over the three is here and over the two of is here. So, now that line is moved left five up to, I have a stretch of three and the x direction and I have a stretch of two and y direction. So, there's a rise there's a run there's a rise there's a run so you rise is under the y and your run is being run from your ex so, I always tell them to do that. And rise and y sound alike. Because then they did it with ellipses and stuff like. But we can't go back and change how they were lines, so they learn everything from middle school.

Barbara: Yeah but for conic sections it makes sense.

Rachel: After they learn on conics. It's easy but like if they had it like that the whole time conics would be easier. Like conics wouldn’t be so hard if they just did everything like that. But we only solve for y. Really. Why do we do that? Because the calculator needs it. Why do we do it? We don't need to do that. We don’t even let them use a calculator half the time. So, why the hell do we care? They don't need to solve y to graph in Demos. They can write y right in the equation. (Video Case 2, 3/26/2019)
The diagnosed problem of practice for this EPR was the need to build student understanding about the connections between the values of a function and the graphical representation of that function. The attribution of causality for this problem of practice was connected to their individual practice and experiences with student confusion when graphing transformations of functions. The ways in which students are typically taught to graph transformations of functions was to consider the values of a function and whether they are “inside” or “outside” of the function. The prognosis was to change the way in which graphing transformations is approached to more closely align to the methods of graphing conic sections. Using ideas from graphing conic sections in her precalculus honors class as her catalyst for graphing using this method, she suggested that all graphing can occur in the same manner as conics does. The department has been consistent in the way they present graphing transformations of functions to maintain consistency throughout a student’s career. Rachel theorized that using a method similar to that which is used to graph other conic sections has the potential ease the confusion when students reach this part of the curriculum. Her motivation is that it eliminates the common misconception from students about how a transformation has an inverse impact from the value in the equation (lines 22-24). More specifically she thinks the nature of graphing transformations can sometimes confuse students (line 22-23). Rachel continued to support her hypothesis when she claimed that the students are not always certain how each value effects the graph or why there is a different effect depending on the placement of the value. Using the form detailed above (line 14), the value of 4 and 7 have similar graphical impacts on their respective direction of transformation. She also referred to the need to write functions as “y equals” as antiquated. Technological updates in recent years, including the most commonly used in school, Desmos, do not require students to isolate the y to input an equation
into the program. The community of teachers listened diligently to make sense of Rachel’s argument. All of the teachers could be heard either saying “right” during her presentation or “I like that.” However, the conversation did not continue past this point. I attempted to push the teachers’ thinking by probing them about the impact of implementing this new method of graphing in their classrooms.

1  Facilitator: Do you think it would make more sense for us to do it like that?
2  Rachel: I love doing it like that.
3  Facilitator: But like from the beginning?
4  Ashley: I feel like then like then with algebra one you like set up tables then that's the only thing.
5  Rachel: Because they know automatically if they are given something on the SAT. They would just organically change into the form they're used to. And then they could pull out. Well what's the parent function you know when x is being squared it's a parabola in that direction and y is being squared like there wouldn't be a problem for them to do as sideways parabola. Because, it's like the same thing because it gets confusing the stretches and compressions when they have y squared instead of x squared.
6  Tania: Right. Right.
7  Rachel: They get confused, but if they just look at your vertical and horizontal pieces all kind of clumped together like that, I feel like it is easier to understand. (Video Case 2, 3/26/2019)

Ashley presented an issue for implementation, citing the way students start graphing
linear relations in algebra I, by first using tables. Rachel’s main teaching focus has been in algebra II and above. Her knowledge of the algebra I curriculum and the pedagogical choices used are only surface level. Ignoring Ashley’s concern, Rachel provided an additional motivation related to formal standardized assessments (lines 6-13) and to ease students’ misconceptions with graphing transformations (lines 15-17). While the teachers appeared to agree that Rachel identified an issue with graphing transformations, they did not appear to readily accept or deny the proposed change. A lull in the conversation seemed to indicate that the teachers were contemplating the idea. This EPR is an example of Rachel providing a motivation for a proposed solution.

Another motivational frame occurred during the same video case. The following EPR is an instance where the community of teachers identified a misconception, proposed an intervention and explained the motivation for using this intervention.

1. Facilitator: My algebra 2 honors students today. They wanted to divide by log.
3. Rachel: Oh yeah of course.
4. Ashley: Well we divide by tangent all the time.
5. Jackie: Well it’s like dividing by sine.
6. Facilitator: I was like, would Ms. Jackie be okay with you dividing by sine last year and I said like sine of theta equals zero point three. How would Mrs. First feel if you divided by sine and they're like, ‘I don't know’.
7. Rachel: I divide by square root to prove them wrong. The square root of x equals 10. Solve for x. Oh, I’m going to divide by square root
symbol. So, I divide by square root symbol, x equals ten over square root symbol. That's what I show them because that one they know well enough to know that this isn't a function operation. You don’t divide by you do the inverse right. You do divide by plus two. You subtract two going when you divide is multiplication. (Video Case 2, 3/26/2019)

In this motivational EPR, I shared a recent problem from my classroom experience. When teaching solving logarithmic equations, a student wanted to divide by the “log” symbol (line 1). The diagnosed problem of practice is students’ confusion using mathematical symbols. The attribution of causality was students dividing by a mathematical symbol. Jackie, Rachel, and Ashley agreed that this was something they have experienced before and related it to solving trigonometric equations when students want to divide by the symbolic representations of “sin” and “tan” (lines 2-5). Rachel proposed a prognosis with motivation to address this misconception that students are having. She claimed that she would attempt to divide by the symbolic representation for the square root ($\sqrt{}$; lines 12). She stated that by using this symbolic representation, the students can visualize the concept of inverse functions (lines 13-14).

For the second video case analysis, I presented all of the motivational frames and an example of their diagnostic frames. I chose these examples to represent the types of EPRs that were diagnostic within this session. I also provided examples where teachers were later able to use the same problem of practice but provide prognoses and motivation for the problem of practice.

In the third video case analysis, the community again had an abundance of diagnostic frames while the teachers were anticipating student solutions and misconceptions that arise when
they have taught properties of exponents. Through these conversations, the teachers integrated topics of precision of language and multiple solution pathways while also relating student understanding to the previous video case materials on radical statements. Rachel dissected student understandings related to properties of exponents by immediately assigning causality to students’ previous educational experiences for a lack of precision of language.

1 Rachel: But I also think that, I don't know where it's happening, but there's so many teachers using wrong words or they don't use the right vocabulary and the kids have this poor understanding of what ...
2 "Oh, you do the opposite thing." I'm like, "No. It's the inverse operation. It's not the opposite operation" or they're saying, "half of it." There's someone telling them it's okay to say the square root is half or something. They're learning that and I don't know if the elementary people are not math certified and they're just using their own vocabulary, especially when they're dividing and they're using things with powers. I'm sure the word power has been thrown around.
3 Facilitator: But do they know what it means?
4 Rachel: In the wrong context. They are associating power with like factor trees or factors and everything's just getting overlapped. Too much of the language is meshed together. (Video Case 3, 3/26/2019)

This EPR highlights Rachel’s focus on precision of language as an important feature of mathematics teaching and learning. She detailed commonly misused vocabulary words (lines 1-7). After this comment, I asked her if she thought her students understand the meaning of those
terms. Rachel responded by relating the importance of precision of language to students’ understanding of mathematical concepts. In line 13 she claimed that their understanding of the terms is not properly associated with the correct context. This EPR follows the pattern of continued diagnostic framing by teachers prior to watching the video segments in each session because they named the problem of practice as the imprecision of language use by students and attributed causality to previous educational experiences, including teachers, that foster improper language development.

Another example of a diagnostic frame in this portion of the professional development model occurred when Ashley presented a recent student concern from her class associated to computational errors. I chose this EPR as an example because it shows a repetitive diagnosis of a problem of practice from early sessions, misuse of the distributive property.

1 Ashley: Well, I even do that when we're doing square roots. Like leave it in simple radical form. They don't know if ... They're doing area of a trapezoid and they're doing half. Eight plus 18 times the square root of four, square root three $\frac{1}{2}(8 + 18)(4\sqrt{3})$. They don't know how to handle all that with the square root of three because they want to multiply; they want to distribute, multiply in the square root, or what to do first.

2 Jackie: How often are you seeing students have something like one half times eight times six and they want to distribute the half-

3 Ashley: Distribute the half to both? All the time.

4 Rachel: My kids want to do it with integers. So, I'm not surprised that happens with one half. (Video Case 3, 3/26/2019)
Ashley, Jackie and Rachel conveyed a concern about students inappropriately applying a distribution property (lines 6; 9-10). This concern was also discussed in the previous video case analysis when the teachers indicated that students might distribute a radical sign to the sum of two terms under a radical. This is a diagnostic EPR because the teachers identified the problem of practice as student misuse of the distributive property. Here they connected this misconception to number computation. Ashley and Jackie both shared examples of their students using distribution when simplifying an expression of $\frac{1}{2}(8)(6)$. Jackie claimed that her students are multiplying both 8 and 6 by $\frac{1}{2}$. Rachel agreed that her students make a similar error with integers. Jackie and Ashley have taught Geometry for most of their tenure and have knowledge of students simplifying similar expressions often in class. However, they both agreed that this is a new misconception that has become more apparent this year. Rachel made a statement that echoed earlier statements about misunderstandings from previous grades.

It's not even high school, it's even slipping through the cracks because if they were honors drop downs, anyway, then they probably just wrote it off as they were confused because they were in honors but now that they're in academic and they're still…There's that gap…That was before high school. (Video Case 3, 3/26/2019)

During this EPR the teachers attributed causality for students’ misuse of the distributive property (lines 5-8) to the students’ previous educational experiences, including teachers. The two examples from the third video case analysis show similar patterns as the rest of the EPRs from the first three sessions while the community is diagnosing problems of practice.

In each of the EPRs above, the teachers diagnosed problems in their practice either related to the mathematical goals or student understandings. During these video case analyses, the teachers were not able to hypothesize solutions to the defined problems. Each time the
teachers made claims based on experiences they have had with students, but, at this point in session, they did not investigate ways to address their identified problems. Following the part of the professional development model when teachers completed the mathematical task and shared various solution strategies from the students’ perspective, they watched a video segment of the lesson being enacted in a classroom. The conversations that followed watching the video segment displayed changes in the teachers’ framing patterns.

The nature of conversations after watching the video segments. After discussing the mathematical goals of each lesson and engaging in the mathematical tasks, the teachers watched a video segment of the lesson enacted in a real classroom. The teachers were asked to analyze students’ mathematical understandings from the video segment and decide how they might respond to student thinking. During this part of the professional development, the community of teachers began to hypothesize about teaching moves that could be made to address their diagnosed problems of practice. Some of their hypotheses were related to problems identified prior to watching the video segment, while other hypotheses were based on new problems of practice identified after watching the video segment. All of the EPRs presented below are of conversations about students’ mathematical understandings. The EPRs that had similar problems of practice to those pre-video watching show how the community was able to use the video segment as a catalyst to think about prognoses to the problems of practice. This likely occurred because the professional development model called for the teachers to analyze students’ understandings and teacher moves after watching the video segment. The results are presented as a progression of the video case analyses in chronological order.

In the first video case analysis, the teachers found that one of their identified anticipated student misconceptions prior to watching the video segment was highlighted through the video
segment. When anticipating student solutions the community of teachers acknowledged that students may believe that a quadratic curve can remain in only one quadrant and does not contain a y-intercept. While the teachers agreed on this as a common student misconception prior to watching the video segment, they did not offer a pedagogical move to address the misconception. After watching the video clip, Rachel claimed, “I felt like if they refresh vocabulary the day before of what domain is, what range it, and what a function is because they said it goes straight up, then it fails the vertical line test.” Through this talk turn, Rachel began to offer changes that could be made prior to the lesson to address the anticipated student misconception. Rachel, Tania, and Ashley debated shortly whether the cause of students’ confusion is the lack of vocabulary understanding or knowledge about the shapes of graphs.

1 Rachel: Yeah, they use the word eventually a lot. And I feel like that word eventually kind of ties into end behavior and can be tied into domain.

2 Tania: Yeah, well why don’t we say vocabulary to start, and then, the end behavior of the function. The end behavior that, you know, is either going straight up or is it going up and out. (Video Case 1, 3/12/2019)

This group formulated a response to the students’ mathematical understanding from the video segment in the form of teacher questioning. They decided that the teacher should address two different elements of the content. First, Rachel indicated that she would address the vocabulary understanding prior to the implementation of the formative assessment lesson. Then Rachel and Tania claimed that they would attempt to make connections between the vocabulary and the shape of the graph by incorporating the concept of end behavior (lines 4-6). Rachel specifically
referenced the word “eventually” (line 1) as an example of the vocabulary that needs to be developed and connected to the end behavior. As previously presented in this chapter, this is a prognostic frame because the teachers previously identified the lack of precision of language as a problem of practice. To strengthen the students’ vocabulary knowledge and understanding, the teachers posed two different hypothetical solutions, incorporating a new task to review terminology and facilitating a discussion to help build connections between various concepts related to the graphs.

At the same time, Barbara and Jackie developed a response to student thinking in their small group. Barbara echoed the other group’s consideration for focus on facilitating a discussion about vocabulary by stating, “I would ask the domain here. There was no word about domain.” They discussed a direction for teacher questioning at the conclusion of the video segment.

1 Jackie: If the “U” was straight up and down, it’s not going to pass the vertical line test. Therefore, the domain would not be all real numbers. Meaning you would have a whole discussion about that.

2 Jackie: Because one of them didn’t understand that terminology to explain why it doesn’t stay in one quadrant, then they’re not understanding. Anybody that really understands would say that.

3 Barbara: I would discuss more about that

4 Jackie: Because they understood like pieces. But they didn’t understand the big picture.

5 Barbara: Exactly, exactly. So, the students do not have enough terminology about [motioning for the domain of the graph]

6 Jackie: To make a big conclusion about the domain is negative infinity to
Jackie and Barbara decided that they would address the perceived vocabulary deficiencies (lines 5-6, 8-9), then make connections in the vocabulary to develop the big mathematical picture for the students (lines 10-13). This is a prognostic EPR because the teachers proposed suggestions for changes by facilitating discussions to assist students in making mathematical connections. Together, in the last talk turns, Jackie and Barbara claimed that they believed if the students understood the terminology of the smaller pieces, they could make a conclusion about mathematical concepts that the domain of the function is all real numbers and the graph must have a y-intercept (lines 10-13).

After the teachers concluded their parallel conversations, they shared their strategies to address the students’ mathematical understandings in the video as an entire community. During this discussion, Rachel suggested creating a task, similar to the dominos activity, to help reinforce vocabulary understanding in students. I mean, they could have even done like a warmup of the domino lesson…With like matching cards. Like the word domain, then then like here’s a picture of a graph, here’s the written out domain of that graph, here is a blank card for you to describe it in your own words. And then here’s another graph, here’s a parabola, what’s the actual domain, how do you write out the domain, what’s a picture of it look like, or something when they like learn to use the matching game, but they use it to strengthen their vocab for functions, domain, range, asymptotes, infinities, end behavior, then they can do the parabola lesson after that. (Video Case 1, 3/12/2019)

Based on the community’s earlier agreement that the domino task elicited students to use multiple representations to graph quadratic equations, Rachel sought to develop a similar task to
address the perceived lack of vocabulary understanding. This is an extension of an EPR presented previously from the partner discussion when Rachel suggested reviewing vocabulary prior to the beginning of the formative assessment lesson. During this EPR, Rachel is providing a more detailed prognosis for the whole group to consider.

Based on these interactions following analysis of the video segment, the teachers transitioned from merely identifying student misconceptions to posing possible pedagogical moves to address the misunderstandings. The evidence presented above from the first video case analysis details the way in which teachers presented hypothetical solutions to their identified problems of practice. This example highlights how teachers used the video as tool to focus their prognosis on particular students’ mathematical understandings. This supports the idea that teachers used the video case materials, specifically the video segment, as a way to focus discussion around pedagogical moves to address problems of practice instead of just identifying what the problems are.

During the second video case analysis, after watching the video segment, the teachers again were able to generate prognostic frames. The EPRs presented below are of conversations among teachers when they attempted to make sense of students’ mathematical understandings in the video segment. For the first EPR, teachers identified concerns that their students tend to look only for a single pathway when approaching a task. They discussed the single pathway nature of their students' problem-solving when they said they believed that their students might struggle to easily identify when the solution to an equation would be zero. For example, Figure 16 illustrated work performed by a student in the video segment to find a solution to the statement $\sqrt{x} + 2 = \sqrt{x} + \sqrt{2}$. Tania shared with the group that the students in the video “showed every single step”. Ashley responded to this statement, “at some point, you have to look at it. You have
... Yeah, you've isolated x pretty much”. Tania and Rachel continued to agree with Ashley and claimed that if students are understanding the mathematics of their process, there should be a point where the process can be replaced with reasoning. Tania indicated this by adding, “So, she should be able to look at this and think, ‘All right. This is all going to be zero. What's going to make this true?’”.

Tania suggested that there needs to be a focus on mathematical reasoning when solving equations. While analyzing the classroom video Tania indicated that she believes a student should be able to reach a point in the algebraic procedure when they can deduce that the value of the variable is zero. Her suggestion was that the students be able to realize this when they have the equation as $0 = 2\sqrt{2}x$. Tania identified the problem of practice as students are linear in the problem solving strategies and unable to break their tunnel vision in the solving process. This EPR is an example of a prognostic frame because Tania provided a prognosis about how she would address this pedagogically instead of just stating what the problem is. She presented teacher questioning options to further student thinking and help students build mathematical connections. This example illustrates how teachers used the video segment to make connections to their own personal teaching experiences. Through these connections the community found problems of practice which were linked to their own practice and then the community was able
to provide a prognosis to address that problem.

Another example of a prognostic frame in this video case analysis occurred when the teachers were determining if a student was going to divide by a variable to algebraically solve an equation. Using the same video clip from the previous EPR, the community focused on an alternate problem of practice. This example indicates how one video segment can elicit two different problems of practice and prognoses from one community. In the video segment the community watched, a group of students was solving a radical equation. They manipulated the equation $\sqrt{2 - x} = \sqrt{2} - \sqrt{x}$ to be $x^2 = 2$. At this point in the video, the students asked the teacher how to proceed in solving the equation. Tania stated, “Yeah, and her question was the question that everyone is going to have. She was going to try to divide by $x$ and just get $x = 2$, probably”. I asked the community to consider how they would respond to the student thinking in the video.

1 Ashley: Can you talk about the fact that if its $x$ squared and it’s a quadratic, you can't just get rid of an $x$. There should be two solutions.

2 Rachel: One thing that I always say to them is I stress all year the difference between a constant and a variable. So, like a constant is a constant. It means it doesn't change. It's constantly that number but a variable can vary. So, being that $x$ is a variable, you're not allowed to divide by $x$ because it can vary. What if $x$ was zero?

3 You're not allowed to divide by zero. So, how do you work your way around that? So, if you can't divide by $x$, what else can we do to solve? Instead of telling them, "Memorize when this is this, you set it equal to zero." Well, why aren't we allowed to divide by $x$?
Tania: Or even dividing to get rid of it, though. Because when we're doing algebra one and we're doing ... Like if I give them any of the PARCC test questions and they're solving or literal equations, I always say, "You can't divide by a variable" and then they're dividing by R or something without canceling it, you know what I mean?

Ashley: You can't get rid of it. You can divide by it but that's. You can't eliminate.

Tania: Like you can't cancel because then you're losing ...

Rachel: Yeah, that one to one nature of it.


Jackie: Well, what about when you have the x squared or the x is equal to radical 2x. What about asking the question? Having them get into the habit of asking, "Is this always, sometimes, never, true? When is it true?" So, they recognize that there are, hopefully, are two values and then, "Okay, well, how would we get to that? If we divided by x, we wouldn't have two values. We'd only have one."

Then it kind of forces them to think, "Oh, how are we going to get to ..." I don't know, right? (Video Case 2, 3/26/2019)

In this EPR the problem of practice is a student misconception that it is mathematically sound to divide by a variable to solve an equation. The cause of this problem is the lack of mathematical connections and conceptual understanding the students built because they are focused on the solution instead of the process (lines 10-11) to understand that a quadratic
equation will have two solutions. This EPR is a prognostic frame because Rachel proposed a pedagogical move to further student understanding when students divide by a variable to solve an equation (lines 6-10). The questioning techniques she proposed attempted to further student understanding instead of propagating the singular pathway of problem solving exhibited by students. She attempted to develop conceptual understanding about why one cannot divide by a variable instead of reinforcing student memorization of a process. Tania countered Rachel’s statement by connecting the topics typically covered in Algebra II to her Algebra I students and their understanding (lines 12-17). The community of teachers collectively developed a line of teacher questioning to promote students’ conceptual understanding.

In the two previous video cases, the teachers transitioned from mostly diagnostic frames prior to watching the video to prognostic frames after watching the video segments. These are representative of the larger group of prognostic EPRs because they exhibit how the community used the video and connections to their practice to make prognoses for the identified problems of practice. The pattern of prognostic frames post video watching appeared again after the third video case analysis about properties of exponents. Teachers partook in discussions including the hypothesis of teaching moves to address the perceived problem of practice. The topics teachers discussed in relation to this case were also centered around precision of language and students’ mathematical understandings. The community of teachers had repeatedly stated that they believed that precision of language can either help or hinder students’ mathematical understanding.

In the community’s analysis of the video segment, the teachers compared the various methods students took to complete the task. A change in the video segment elicited a slightly different nature of prognoses in this video case. The segment showed two different students’
mathematical understandings. For this case, the teachers need to compare multiple student understandings before presenting a prognosis. When engaged in these conversations, the teachers used the video segment to make sense of particular student understandings and determine ways to address student thinking.

1  Jackie: So, the boy understood how to work things out and plug things into a calculator.

2  Ashley: I feel like the girls had a better basic understanding of the properties. Where the boy was kind of trying to, just calculate.

3  Barbara: Right, and he understood his math how he does and it’s probably easier for him, but he doesn’t use properties.

4  Ashley: Yeah, like he wasn't working harder-- I mean, he was working harder, not smarter, you know. And she was like, "No, we have these properties let’s confirm them."

5  Tania: But he was the only one that did the subtraction problem.

6  Rachel: I almost feel like it almost would have been better if they did this before they did properties and asked them to see if there's any patterns where they can discover the property.

7  Barbara: We said the boy was working hard too, but he didn't know he was working, it was a calculator--

8  Tania: Oh, he was doing it the long way.

9  Facilitator: So, what do you think he didn't understand?

10  Tania: Well, I don't know that he knew the properties.

11  Ashley: The properties.
This EPR’s problem of practice is the importance of understanding multiple solution pathways for students. This was one theme identified to answer the first research question and also representative of the ways in which the community diagnosed problems of practice. While making sense of what the student solutions indicated about their mathematical understanding, the teachers determined that the boy in the video did not have an understanding of the properties of exponents (lines 5-6; 18-19), but rather that he only knew how to use the calculator to evaluate each statement (lines 1-2; 4; 6; 14-15). The attribution of causality for the problem of practice is that one student has an understanding of only one solution pathway, using the calculator, and is resistant to learning other methods. To address this student’s lack of understanding about properties of exponents, Rachel proposed a prognosis to use this task as a way for students to build patterns of expanding exponential expressions to discover the properties of exponents (lines 11-13). Based on her suggestion for the problem of practice, this EPR is a prognostic frame. This example illustrates how the teachers hypothesized changes to the task structure and implementation to address students’ mathematical understandings.

The next EPR, also from the third video case analysis, shows how the community was able to address multiple students’ mathematical thinking in one video segment.
Rachel: But I don't know if she knew why she was doing that.

Tania: She knew to use it, not why.

Barbara: She probably memorized it and that's it.

Ashley: Yeah, it seemed like she was very like, "This is the property, we just learned this, this is what we're doing." And he was like, "Well, no, this is what I got”.

Rachel: And she was just pleased with the calculator too and she's like, "See, look. I'm right."

Ashley: Yeah. She might have not understood why, but she definitely was like all about the properties, where he probably-- I don't know, he must have missed that lesson or something.

Rachel: But I don't think either of them understood it well enough because neither one of them could convince the other to believe them, and they were both kind of right with what they were talking about, right?

Ashley: Just different processes.

Rachel: But neither of them could convince the other one. So, you don't know it well enough, because you're not selling what you're doing, you know? (Video Case 3, 4/9/2019)

During this interaction, the teachers dove deeper into making sense of the student understanding.

Drawing on the same problem of practice, understanding multiple solution pathways for students, the teachers discussed that two of the students in the video insisted on using the properties of exponents to simplify an expression. The community acknowledged that these
students have a basic understanding of the properties and how to apply them (lines 3, 10-11). However, they indicated that these students do not appear to understand why the properties work (lines 3, 10, 13-15) based on their explanations of their thinking. In this part of the conversation, the teachers diagnosed the problem of practice as the importance of understanding multiple solution pathways for students because the students are lacking the deep conceptual understanding to be able to explain their solutions to each other. I then asked the teachers how they would respond to student thinking in the video.

1 Rachel: I would have probably just said to them, "It seems like we have a bit of a disagreement of how we approach this problem. So, what do we know? What does four to the fourth power mean? What does four to the eighth power mean?"

2 Tania: Like show what it is, right.

3 Jackie: "Do you need the calculator? Can you do it without a calculator?"

4 Tania: Right.

5 Ashley: Yeah, how would we do this if we didn't have the calculator?

6 Barbara: If we changed the base from six to smaller, let's do it without calculator and see-- because they will figure out. Pretty sure they will figure it out because, huge base they didn't have. (Video Case 3, 4/9/2019)

This EPR is a prognostic frame because the teachers started to consider teacher questioning moves that could further student thinking related to their lack of connections between solution strategies. Through this line of questioning, they hoped to have students see the connections between expanding an exponential expression and the use of properties to simplify that
expression. Rachel and Ashley developed their understanding of the problem of practice by creating an additional prognosis.

1  Rachel: Yeah, even what if I had a hidden number? I put a cover on it. A
2               number to the fourth divided by a number to the eighth.
3  Ashley: Well, because, make it like x even.
4  Jackie: Yeah.
5  Rachel: Why don't they do x right away? How old are they when they do
6               x? I don't remember.
7  Ashley: I feel like it's easier to prove when it's numbers, which is I feel like
8                      why it's probably good for them, but at the same time, maybe some
9                      with numbers and then some with variables to connect.
10  Facilitator: I think that-- I feel like from sometimes with our kids when they
11                        get properties of exponents, they do them so often with variables
12                        that they forget that they work with numbers.
13  Ashley: Yeah
14  Tania: Agreed. They don’t apply a power to a power to a number. (Video
15                        Case 3, 4/9/2019)

This EPR is a prognostic frame because Ashley and Rachel presented an alternate prognosis for the same problem of practice, understanding multiple representations for students. They suggested introducing variables to the expressions, but not eliminating the numerical base examples (lines 1-3). I acknowledged experiences, from my practice, when students struggle transitioning between numerals and variables as bases (lines 10-12). Ashley specifically indicated that she wanted to include both variables and numbers to help build this connection.
The EPRs presented from the third video case analysis, similarly to the first two, are representative examples of how the community of practice prognostically framed conversations when addressing the students’ mathematical understandings from the video segment. In addition, during the third case, the community used various students in the video segment to analyze their mathematical understandings and propose multiple prognoses to address the students’ struggles with properties of exponents.

**Fourth Video Case Analysis**

The community of practice appeared to have different discussion patterns during this last video case analysis. For the previous three cases, the teachers' conversations consisted of diagnostic framings around particular problems of practice. Contrary to the previous cases, when the teachers engaged with the mathematics about conditional probability, the conversation included suggestions for changing practice to counter diagnosed problems of practice. This difference resulted in the occurrence of more prognostic frames prior to watching the video. Of the five participants, only Tania and Ashley had recent experience formally teaching probability and statistics. I hypothesize that the community’s lack of familiarly with the content lead to more prognostic frames prior to watching the video. In this section, I provide two representative examples of how the community developed prognostic frames prior to watching the video segment in the fourth session. The first example occurred during the initial activity, identifying mathematical ideas and the second example occurred while they were anticipating student solutions.

As in previous cases, after an initial reading of the lesson, the teachers were prompted to determine the big mathematical ideas addressed in the lesson. The following EPR is an example
of a prognostic frame.

1 Facilitator: So, if we think about conditional probability, what do we think is important for our students to take away from it?
2 Rachel: Well, they have to understand to recognize that there is a condition.
3 Ashley: Like, something has to happen in order for this other thing to happen.
4 Facilitator: So, like kind of a sequencing?
5 Rachel: Yeah. I feel like once they're in geometry if, then, the conditionals.
6 Barbara: Conditional statements.
7 Ashley: If this happens, then this will happen. What is the probability if this happens, then this happens? It's the same thing.
8 Rachel: Yeah, so that they have to recognize that language. (Video Case 4, 4/9/2019)

Following the professional development model, the teachers identified the goals of the lesson as understanding what conditional probability is; in particular, the impact a condition can have on determining the probability of an event occurring. Rachel and Ashley stressed the importance of understanding vocabulary related to conditionals (lines 3-6). In the continuation of the EPR below, Rachel connected the idea of vocabulary use and reading comprehension. She then provided a prognosis to the community of teachers that she utilized in her classroom.

But when I have them read word problems, I'm like, sometimes you have to translate it. Like when you write an essay in Spanish class, you're not going to take an essay that would have written in English class and write the same way in Spanish. You change how
you speak. So, when you read a word problem or you're describing a scenario, you might have to re-language it to math language. (Video Case 4, 4/9/2019)

By sharing a teaching move she makes, she focused the teachers’ attention on the problem of practice as reading comprehension in the mathematics classroom. Rachel, Tania and Barbara supported Rachel’s indication by relating this problem of practice to recent standardized exams, the SAT, the AP Calculus exam, and NJSLA-M. Tania shared a specific student concern which was echoed by other teachers.

And I feel like it's a problem that our kids struggle with. They even were telling me today with an SLA exam, whatever exam they took today. They were ‘I was reading—’ they're like, ‘the words actually went blurry and I couldn't read. I couldn't focus on what this was actually saying, and I was so bored by it that I couldn't." And these were my honors kids. (Video Case 4, 4/9/2019)

Ashley added, “today we were doing practice problems for PARCC in Honors. And my kids were like, ‘Can you just read it?’ One of the kids was like, ‘Can you just read it to us?’” This EPR is a prognostic frame because the teachers decided that reading comprehension is a problem of practice and attributed causality to students’ lack of perseverance to read longer passages. They presented a prognosis to address this issue; teachers suggested having students read out loud in class to collectively help the students make sense of the mathematical language. This example was chosen because it illustrates how the teachers used the lesson goals to make connections to their practice, identify a problem of practice, and propose a solution to that problem.

Another example of a prognostic EPR prior to watching the video segment occurred when the teachers questioned their own structure of assignments as a possible cause of the
students’ lack of perseverance. I am including this EPR to illustrate how the community addressed a previously diagnosed problem of practice through analysis of their own curricular resources. The community discussed reformatting their assignments to have directions for each problem instead of grouping similar questions into a larger problem set. By negotiating this idea collectively, the teachers were able to present pedagogical changes for their identified problems of practice and justify why these solutions may be effective. Evidence of this prognostic frame was presented previously in the section about productive struggle. As described in that section, the community of teachers identified their current curricular resources as a potential source hindering students’ reading and desire to persevere by lowering the cognitive demand of the assignments. The teachers diagnosed the problem of practice as students’ lack of perseverance when completing problem sets. They attributed causality for the lack of perseverance to the structure of their curricular resources because the current structure of resources allows for students to read a short set in instructions and apply one type of solution method to a problem set.

In this section I presented evidence on what the nature of conversations is throughout the community’s engagement with the video case materials. I aligned the results with the various parts of the professional development model and found patterns in how teachers framed conversations depending upon the part of the professional development they were engaged in. I found that the community’s frames when identifying math goals and anticipating student solutions were heavily diagnostic. While the teachers were able to diagnose problems of practice, including student misconceptions, they could not regularly develop prognoses during this part of the professional development model. However, after they watched the video segment and were asked to interpret student thinking and decide how to respond, they were able to develop
prognoses, either for previously diagnosed problems of practice or newly defined ones. Most motivational frames also occurred prior to watching the video segment. It was during these times when teachers offered suggestions from their practice for other teachers. The last video case gave way to a different framing pattern than the previous three video cases. During this session, the teachers were able to provide more prognoses prior to watching the video segment. These prognoses were for problems of practice from their own experiences instead of based on the video segment as the object of analysis. This evidence not only indicates that the model influenced the nature of the conversation during the professional development, but also that those initial patterns may be changing through longer engagement with video case analysis.
CHAPTER 5: DISCUSSION

The results presented in the previous chapter provide evidence I used to determine if this community of teachers was able to build collective professional knowledge and what the nature of their conversation was while engaging with the video case materials. The first set of results identified instances of teacher conversation about three reoccurring themes: precision of language, multiple solution pathways, and productive struggle. Within these three themes, the teachers made their practitioner knowledge (Hiebert et al., 2002), or knowledge generated through participation with their own practice, public. The preexisting norms of the community of practice allowed their knowledge to be storable and sharable through daily lesson plans and shared resources. In some EPRs I also identified changes in frames consistent with the mechanisms for building knowledge. The second set of results detailed the patterns of diagnostic, prognostic, and motivational frames throughout the four video case analyses. I aligned the framing patterns of the community to the different parts of the professional development model, pre and post video watching, to determine the nature of the conversation during engagement with the video case materials. In this chapter, I first discuss the results from each of the research questions sequentially. I then discuss limitations and implications for future research.

The Development of Collective Professional Knowledge

To determine how teachers use video case materials to develop collective professional knowledge, I first identified three recurring themes and found evidence of some development of knowledge within each of these themes. When discussing collective professional knowledge, it is important to recall that the teacher participants entered the professional development sessions with prior pedagogical knowledge based on their personal experiences as a teacher. Often referred to as craft knowledge (Grimmett & MacKinnon, 1992), this type of pedagogical
knowledge has been developed through years of teaching experience. As discussed in the literature review, recent calls from the mathematics education community include ways for teachers to share their individual practitioner knowledge with the goal of developing collective knowledge for all mathematics teachers (Ball et al., 2014). Research indicates that the knowledge the teacher participants entered the professional development with is grounded in concrete experiences and classroom practices (Hiebert et al., 2002). Through their own concrete experiences, the participating teachers have developed individual knowledge and were poised to share their experiences within this community of practice. Part of the reason the teachers were poised to share their experiences stems from the fact that this community of practice had been established prior to this research, so the preexisting norms developed within this community had provided the space for the teachers to be open and willing to share their individual practitioner knowledge in this setting. Through their involvement in the video case study professional development sessions, the teachers made their practitioner knowledge about precision of language, multiple solution pathways, and productive struggle public within their community of practice. The nature of their norms and open dialogue are indicators that the sharing of knowledge could continue past the professional development sessions.

During discussions around the three themes, the teachers elicited their practitioner knowledge, which was linked to their practice, detailed, concrete, specific, and integrated around a problem of practice. Through the participation in the professional development, the community made their knowledge public within their community of practice. They then used their existing norms, in addition to collective lesson plans and resources, for sharing and storing knowledge to achieve the second requirement to be considered collective professional knowledge (Hiebert et al., 2002). Moreover, in some EPRs, within the precision of language and multiple solutions
pathways themes, the community displayed mechanisms for building knowledge. However, they were not able to develop mechanisms for verification and improvement of their knowledge. By recognizing the possible limitations of local knowledge, Hiebert et al. (2002) stressed the importance that shared knowledge has means for verification and improvement based on teachers’ use of the knowledge. While the knowledge developed in this community of practice has the potential to become collective professional knowledge, this research study did not have evidence of the final characteristic occurring. Researchers claim effective professional development should be sustained and ongoing (Desimone, 2011; Garet et al., 2001). The longer teachers have to engage in the activities of a professional development, the more effective it can be. The scale and duration of this research project did not allow for prolonged contact time in the professional development setting. However, I argue that the changes in participation over this short time frame could potentially be developed if participants were to continue engaging in video case analysis. In particular, if members of the community engaged in their own investigations of innovations discussed and return to the community to share their experiences to verify and improve the community’s knowledge.

There were multiple ways the development of collective knowledge might be determined in future research. For example, a researcher might find methods for verification and improvement through teachers engaging in the implementation of the formative assessment lessons in their classrooms. During discussion in the professional development sessions, Tania and Ashley indicated that they had intentions of using these lessons in their classes. They asked questions during breaks about grouping strategies and timelines for implementation. Tania, Rachel, and Barbara self-reported in their reflection surveys that the formative assessment lessons were believed to be most useful material from the video case materials for them to use in
their classrooms. This indicates that the teachers see value in the formative assessment lessons and this professional development could have encouraged them to use these high quality teaching materials in their classroom practices. Based on self-reported reflections after the professional development, Tania and Rachel also indicated that they have used the materials in their classrooms. This is evidence that these teachers have the potential to implement these lessons in their classrooms. If provided time to reflect on their experiences as a community of practice after implementing these lessons, the structures for verification and improvement of their knowledge might occur within this community. Mechanisms for verification and improvement can be achieved through expertise, implementation in other contexts, and multiple observations of those implementations (Hiebert et al., 2002). If teachers implement the lessons in multiple contexts and continue to share their individual investigations, they can begin to develop these mechanisms for verification and improvement within this community.

While the community of practice was not able to create mechanisms for verification and improvement, the results presented in Chapter 4 indicate that the knowledge being shared, stored, and built in this community of practice was able to meet the first two characteristics for collective professional knowledge: connected to practice and public and sharable and storable within this community of practice. There were also instances where they began to develop the mechanisms for building knowledge as a community. In the next three sections, I will discuss the knowledge that they began to develop in each of the three themes: precision of language, multiple solution pathways, and productive struggle.

**Precision of Language**

Research in math education states that, “the care with which terms are introduced, defined and employed will, of course, be crucial to the child’s understanding of them” (Austin &
Howson, 1979, p. 182). The community of practice in this study also recognized the care with which terminology should be used in mathematics through their discussion of the video case materials and reflection on their own practice. The community discussed their beliefs that students have superficial or incorrect understandings of mathematics terminology. They connected the misunderstanding of vocabulary by considering it as a hinderance for students to build mathematical connections and conceptual understanding which is integral to student learning.

**Defining and addressing student language.** Throughout the four video cases analyses, the community of teachers engaged in multiple conversations about student use of vocabulary and the importance of precision of language. All teachers in the community engaged in discussions about student vocabulary use in the first and last video case analysis sessions. However, Rachel raised concerns about this topic during all four of the sessions. During video cases two and three, Rachel elaborated on concerns presented in the first session and sustained conversation about student language and vocabulary even if the whole community did not respond. Through these discussions, the community mutually developed a response to this locally identified problem of practice.

During the first video case analysis, the teachers were asked to make sense of students’ mathematical thinking in the video segment. During this interaction, the teachers focused on the student language use in the video as a window to understand the students’ mathematical thinking. They noted, in multiple instances, how the language use in the video was too elementary for high school students and indicated that their students should be using more advanced vocabulary to make mathematical connections. During this interaction, the teachers shared their individual practitioner knowledge, which was grounded in practice, and developed a
problem of practice thereby meeting the first characteristic of the development of collective professional knowledge. The teachers used the video to guide their definition of the problem of practice, student misuse of vocabulary, when they referenced specific instances in the video as evidence for poor language use by students. In the second video case analysis, Rachel began to connect the previously identified problem of practice, student misuse of vocabulary, to the larger mathematical goal of the lesson by indicating the importance of being able to speak mathematics fluently is an integral part of the lesson. For the third video case, Rachel changed the way she was diagnosing the problem of practice, specifically around student language use. In this session, Rachel drew on her personal experiences connected to her practice to share her practitioner knowledge with the community. She highlighted the confusion of “inverse” with “opposite” and “half” with “square root.” Rachel changed her attribution of causality for the problem of practice, student misuse of vocabulary, and claimed that the problem could be a result of previous educational experiences including teachers’ misuse of mathematical terminology.

The first three EPRs indicate that the community is concerned with student misuse of mathematical vocabulary. During the last professional development session, Rachel explicitly referenced her earlier attribution of causality that previous educational experiences including teachers contribute to students’ misuse of mathematical vocabulary. In this fourth session the community of teachers engaged Rachel in conversation to collectively build their understanding about the problem of practice. Both Ashley and Jackie echoed and supported Rachel’s concerns about the elementary school teachers’ vocabulary use. Jackie shared a specific encounter with one teacher to build the knowledge about this problem. Together, the teachers developed a course of action to address this concern with the elementary teachers, indicating a framing pattern change when addressing student misuse of mathematics vocabulary. In the first three
EPRs, the teachers' conversations consisted of diagnostic framings, but by the last video case analysis, they were able to develop a prognosis to address the problem of practice. The changes between frames indicate that there are changes in participation within this community of practice. Changes in participation in a community of practice indicate that learning is occurring and in turn knowledge can be developed.

Similar to findings documented by Sherin and Han (2004), the teachers in this study changed the ways in which they were discussing the problem of practice over time. They transitioned from defining the problem, to locally contextualizing the problem, to attributing causality, and finally to proposing a solution to the problem. Through these changes, the community was creating mechanisms for building knowledge, part of the last characteristic of collective professional knowledge. Through group interactions over time, the members of the community provided their interpretations of the problem of practice. With the evolution of the problem of practice and attribution of causality, the community revisited this problem of practice and continued to contribute in new ways to their investigation.

**Using language as a window to student understanding.** Language and vocabulary use were discussed as critical components to mathematics learning by this community of practice. During the first video case analysis the teachers were able to share practitioner knowledge connected to their practice, represent this knowledge through artifacts, and collectively generate ways to respond to student understanding. Through this process, I claim that they were also using mechanisms for the building of knowledge, however they were not generating mechanisms for verification and improvement. During the analysis of student understanding from the video segment, the community determined that the students had an understanding of some mathematics terminology, but students were either not clear on the meaning of the vocabulary or were having
trouble making connections between the various concepts. During these EPRs, the teachers were referring to mathematical connections as links between distinct prior knowledge to help build mathematical relationships (Eli, Mohr-Schroeder, & Lee, 2013). Rachel and Ashley indicated that to assist students in building the connections, they would review vocabulary to help build students’ understanding of terminology. They suggested creating a parallel task to help develop a stronger understanding of vocabulary. Jackie and Barbara also focused on the development of vocabulary use but suggested doing this through facilitating discussions to assist the students in building connections. They indicated the importance of the students seeing the bigger picture, and the bigger picture to them is understanding the relationship between domain, range, function, and asymptote.

In the EPRs within the theme of precision of language, the teachers were using instances from the video segment, a record of practice, to identify students’ mathematical understanding linked to vocabulary. They made suggestions for addressing specific student understandings which arose in the video segment. During this video case analysis, the community was able to make public their practitioner knowledge, connect it to practice, and share their various prognoses for addressing student understanding based on student vocabulary use. The teachers investigated a defined problem of practice in smaller subsets and returned to the larger community to build knowledge within the community about how to address the problem of practice. There were changes in their framing patterns throughout their investigations. While the teachers only diagnosed a problem of practice earlier in the session, using the video as a focus later in the session, they did develop a prognosis and motivation for the same problem of practice. The changes between frames here indicate learning and knowledge creation for the community of practice.
During the fourth video case analysis, the teachers built on their previous conversation, drawing on student language as a window into student understanding, by connecting these ideas to the big mathematical idea of the lesson. During this session, the teachers indicated a problem of practice was the importance of literacy and fluency with mathematics because it contributed to students’ problem-solving efficiency. During this video case, the community considered the vocabulary of conditional statements and conditional probability important to the conceptual understanding related to the sequencing of events. Using the same problem of practice, they shifted the causality from student efficiency to the need to understand vocabulary to achieve the mathematical goals of the lesson. This was an example of a change in participation by teachers in the community of practice which is reified by changes within diagnostic frames. The “thingness” (Horn, 2007) that they are using to conceptualize the problem of practice had shifted from the video segment to their own teaching practices and the lesson goals. During this session, the community is connecting their defined problem of practice to their teaching practices, making it public, storable, and sharable. The community showed that they can continue to generate conversations around the same problem of practice over time, however, they constitute changes within frames instead of between.

**Conclusion.** “Collective knowledge must be represented as theories with examples. Theories are teachers’ hypotheses with explanations for connections” (Hiebert et al., 2002, p. 7). In each of the two instances detailed above, the community of teachers created the theories Hiebert and colleagues suggested by hypothesizing changes and providing explanations. In the first instance, the teachers determined a way to address a locally identified issue related to language development during elementary school mathematics instruction. The hypothesis was to discuss the impact mathematical language in their classrooms has in secondary classrooms with
elementary teachers. In the second case, the community hypothesized classroom activities to assist students in developing stronger mathematical language skills in the hope of promoting meaningful mathematical dialogue in the classrooms and a stronger conceptual understanding. These instances illustrate the beginning of collective professional knowledge since this knowledge around precision of language was sharable and storable through existing norms in the community. Additionally, this indicates that the community of practice is approaching the development of collective professional knowledge, however, they have not met all of the requisite characteristics.

**Multiple Solution Pathways**

Using and connecting mathematical representations is an important aspect of mathematics teaching (Stein & Smith, 2011). Acknowledging student solution pathways and creating a culture where student voices are heard and their mathematical ideas are built upon are integral to their opportunity to learn mathematics (Cengiz, Kline, & Grant, 2011). During professional development sessions using video case analysis, this community of practice recognized the importance of these aspects of mathematics teaching and learning, discussed the integration of multiple representations into lessons, and hypothesized ways to structure tasks. Moreover, during their only prognostic frame in this theme, the community showed one way that mechanisms for verification could be achieved.

**Multiple representations for student thinking.** The teachers discussed various mathematical representations to promote student thinking during different parts of the professional development model and during each of these instances, they maintained a diagnostic frame while engaged in these discussions. During the first session, while teachers were sharing initial thoughts about the big mathematical idea of the lesson in pairs, each set of teachers
identified the big mathematical idea of the lesson as the need for students to understand multiple representations of a quadratic function because it is essential to identifying key features of a quadratic graph. After sharing their initial ideas, the community agreed that making connections between multiple representations is an integral part of mathematics learning. To do this, the teachers drew on their concrete practitioner knowledge to provide relevant examples in their own classrooms. They investigated the lesson goals in small subsets of teachers, then came together to provide interpretations and contribute to discussion on their investigations. This set of EPRs does not show changes either within or between frames, however it details how the teachers can, in small groups, investigate problems of practice and then as a community convene to share and make their investigations public while explicitly connecting their investigations to their practice. This is another example of how this community has mechanisms for building knowledge within the community.

During the third video case analysis the community focused discussion about multiple representations on the tasks being implemented in the lesson. Teachers discussed the impact of multiple representations of division on student understanding. Through these EPRs, within this video case analysis, the diagnosed problem of practice remained the same: students did not consider the relevance that division by zero is undefined when they encountered a division expression in conventional form (i.e., featuring the obelus). However, there were changes within the diagnostic frame throughout the conversation that provided evidence of learning in the community of practice. The teachers used the cards from the task in the lesson as examples to discuss one representation for division, the obelus symbol. They hypothesized that students might struggle with this symbolic representation because it is not often seen by students in the high school curriculum. After their initial investigation, the community readdressed the problem
of practice, but shifted their attribution of causality. Instead of the lack a familiarity, the community attributed the cause of the problem to the difficulty students have in connecting representations (and symbolic forms) of division. Drawing on a specific detailed example connected to practice, I shared how one representation was problematic recently for my students as they were identifying variable restrictions in complex fractions. Changes in participation were occurring during these EPRs which were reified by changes in the diagnostic frames over the course of the conversation. Teachers transitioned from discussing multiple representations as defined by the learning goals of the lesson to drawing on personal practitioner knowledge to share and make sense of the various representations in relation to the tasks. This is another example of the teachers using mechanisms for building knowledge by providing their interpretations, contributing to discussions, and deciding on a problem of practice collectively through their investigations and discussions.

The examples within the themes of precision of language and multiple representations are representative of the community’s mechanisms for building knowledge. I use these examples as evidence to substantiate the claim that the community has developed mechanisms to build knowledge. While every EPR or video case did not show evidence of this development, these examples indicate that they have the potential to engage in these practices. I hypothesize that with prolonged engagement with video case analysis, the mechanisms for building knowledge could develop more regularly through the professional development model.

**Anticipating student responses.** One of the five practices for facilitating productive discussion in mathematics classes is to be able to anticipate student solutions (Stein & Smith, 2011). For teachers to engage in meaning mathematical discourse, they need to be able to anticipate how their students could approach a mathematical task and prepare to address their
understandings and misunderstandings. The teachers in this research study engaged in this practice during the four video cases analyses.

During the first video case, teachers used the cards in the mathematical task to anticipate student solution strategies for the dominos activity on quadratic functions. The teachers shared that the students would look at one card, determine how to connect an equation to a graph and then attempted to complete all the other matches in the same manner. This is where the teachers first alluded to the single pathway of student solutions while problem solving. To address this singular problem-solving pathway, the community suggested limiting the resources (pen and paper) students have to write and algebraically manipulate functions. Their motivation was that by not allowing them the space to do this, the students would be forced to consider the multiple representations and identify the key features based on each representation. Through this one motivational EPR, the community was building knowledge about the implementation of the task through their interpretations and discussions connected to their own practice. In this instance, I identified an indication that the community could develop mechanisms for verification and improvement of this knowledge within the community. Prior to the professional development session, I had implemented the formative assessment lesson from the video case study in my class. During the implementation of the lesson, I found that my students, exactly as Tania anticipated, wanted to algebraically manipulate the functions to avoid analysis of the multiple representations of a quadratic function. I was able to verify her shared knowledge about the anticipations of student solution pathways. While this only occurred once throughout the professional development sessions, it is the only time one of the teachers had personal experience implementing a formative assessment lesson used in the video case. It is possible that if teachers continued to use the formative assessment lessons in their class and engaged in video
case analysis, they could use the lessons as a record of practice (Ball et al., 2014) to verify and improve their knowledge for teaching and learning mathematics.

During the second video case analysis, the teachers shared their anticipation of student solutions as the previously identified singular solution pathway when problem solving. They indicated that a problem of practice they experience is students not considering an algebraic method when solving a problem. The community of teachers experienced their first divergence in diagnosis during discussion about this task. Some teachers indicated that students would use the guess and check method instead of algebraic methods. Through the anticipation of student misconceptions, they indicated that students would not be able to solve the task using guess and check. Barbara, who typically teaches honors, upper-level students, disagreed by claiming that some students with strong number sense might be able to use this method to accurately complete the task. When she refers to her students with a strong number sense, other teachers in the community may have had a hard time relating to this idea because their typical student does not possess the skills Barbara was referring to. At the end of this conversation, the teachers collectively decided that although some students might be able to solve the task using guess and check, the mathematical connections between the statements is what is most important. Through this example, it appears that the development of collective knowledge can be limited by the practitioner’s connection to their practice. While the community was able to agree upon a common problem of practice, the cause and outcome of the problem of practice varied among community members based on the level students they teach. In this instance, the community was only able to connect the knowledge to their practice and make their individual knowledge public.

In both the first and second video cases analyses, the teachers spoke in more general terms about the solution strategies of students. They discussed the singular solution pathway of
student problem solving, guess and check methods, and initial student approaches. The community’s teaching experience could be a reason they focused on task approaches to answer this question instead of the mathematical approaches and solutions. For example, they considered how a student might enter a task, instead of predicting mathematical solutions to the task. Through years of teaching together, they have developed the practitioner knowledge of common student misconceptions. In their years together as a community, the teachers have shared common student understandings and misconceptions, specifically in algebra and geometry courses. They might not have felt the need, because of the established norms, to restate and name all of the mathematical solution strategies and misconceptions during the algebraic tasks. I found a change in the fourth video case analysis in the way the community participated around the anticipation of student solutions.

The content of the fourth video case forced the teachers out of their content comfort zone. Conditional probability is not a topic regularly taught by the members of the community. The only member of the community currently teaching conditional probability was myself. During this session, the teachers spent more time completing the mathematical task and discussing the possible various solution methods. The five teachers presented three different solution pathways for students to answer the original task. However, Jackie then questioned the validity and connection of Rachel’s solution strategy. The teachers took the time to share how the various methods are connected and build their own content knowledge about conditional probability. During this last video case analysis, the problem of practice was anticipating student solutions because, I inferred, the teachers felt the need to be prepared to respond to student thinking. However, the way they were anticipating those solutions changed in this video case. The choice of lesson and lack of familiarity in teaching the content led the teachers to think more about the
mathematical approaches the students would take. The teachers needed to build their own
content knowledge collectively before they could address student thinking.

Stein and Smith (2011) offer that anticipating is the act of predicting the ways in which a
student might approach a task. During conversations focused on the anticipation of student
solution pathways, there were changes in the nature of teacher conversation. In the earlier video
case analysis sessions, the teachers were more concerned with detailing entry points for students to
approach a task or how the students might rely on guess and check methods. However, in the last
session, the teachers were more focused on the variety of ways the students could
mathematically solve the task. They worked to build an understanding about how the various
solution methods for the conditional probability task were connected. To properly execute the
anticipation of student solutions, teachers must solve a task in as many ways as possible, often
with colleagues to compare and review methods (Stein & Smith, 2011). The last video case
analysis is the most closely aligned to this picture of what anticipation of student solutions means
and began to display this through the nature of the conversation when the community needed to
spend more time exploring the content instead of the pedagogy.

**Student perceptions.** There were instances throughout the professional development
model where the community did not use their mechanisms for building knowledge. Through data
analysis, I could not draw conclusions about why the community chose to investigate some
problems while brushing over others. This example is a prognostic EPR where the community
highlighted important aspects of mathematics teaching and learning, student beliefs. However,
the community of practice never revisited this problem of practice throughout the professional
development.

With all the discussions about multiple solution pathways and representations, on one
occasion Jackie questioned how the students perceive multiple solutions pathways. She shared her concern that students might view another student group who is taking longer to complete a task as being inefficient. These students might not value the other students’ solutions or open themselves up to understanding the solutions because they are aware that it took longer for one particular group to complete it.

1  Jackie: I also thought it was interesting, I didn't bring this up before, but I want to bring it up now. Is that like when you guys were doing that circle dominoes thing, you guys did it faster than us, right? We took longer and we made a mistake and had to go back. So then when we were discussing it afterwards, now I'm picturing students in the classroom like okay, so say that there was a group that's like quick, quick, quick, they know what they're doing, everything. And then there's this group that you're struggling a little bit more, they're taking more time, they're making mistakes, now when you're having a class discussion, it is what we bring to the session discounted a little bit because we, I'm just saying because we are slower. Like when you say, “What's the best method?” and then we're putting our two cents as well. Do other students who are there going, “You guys were, like, slow.” So, I just think of those things.

14  Facilitator: Yeah. Um, so how can we, like, counteract that? So instead of saying like, what's the best method, what are things that we can say instead of what's the best method?

17  Tania: What's your method?

18  Jackie: Or when you were making a mistake, did you really, did you, did you
think of it like, did you have a method you were using that you realized
you needed to like, let go of because it wasn't working for you?

Rachel: And maybe you came back to it later because it’s more appropriate for
another moment.

Jackie: Right. Sometimes the people who take longer, like get more out of it.
Sometimes they have to rationalize it out in their head.

Tania: Absolutely.

Ashley: Yeah, yeah.

Jackie: But some of the kids are sometimes immature and they don’t realize that,
and they might -

Ashley: Because they are more about like end product. Okay, done.

Jackie: Yup.

Ashley: And not about like the understanding and really taking it all in and they
are more about like finished products. (Video Case 1, 3/12/2019)

Jackie began to voice this problem of practice as the students not recognizing the importance of
considering multiple strategies for solving a problem. The attribution of causality was personal
feelings about completing the task in two smaller groups and her group finishing slower than the
other. Tania, Jackie and Rachel offered suggestions to advance student appreciation for all
approaches (lines 21-27). Jackie justified the need for supporting the students who take longer by
acknowledging that they may actually be getting more out of the task because they are spending
the time rationalizing their decisions. To counter this, Tania suggested focusing on “your”
methods instead of “the best” method as a way to value all the pathways taken to solve the task.
Jackie then provided a prognosis by altering the teacher questioning to draw the mathematical
thinking out of the students. She did this by posing questions that would have students reflect on their method and determine if they need to alter course, how they altered their course and why they thought it was beneficial to do so. This EPR concluded with agreement from community with statements like “absolutely” and “yeah”.

This conversation is an instance where the teachers were able to share personal experiences but did not build on their shared knowledge over the course of the video case analysis. This EPR provides insight into the ways in which teachers have the potential to be concerned about student beliefs as mathematics learners. Research indicates that student beliefs about mathematics learning can impact achievement (Mason, 2003; Schoenfeld, 1989). It is important for teachers to be aware of student beliefs and create a culture where their voices are heard and respected. This is an instance where the community might benefit from utilizing the research on student beliefs and cultivating an environment in which to consider multiple solution strategies as important in improving their practice. Given a longer professional development, this has the potential to be a developing conversation connected not only not multiple solutions, but also other integral pedagogical concerns. In this research project, the community of teachers was not ready to use this as a catalyst for examining their pedagogy. However, through careful facilitation and choice of new video case materials that highlight this problem of practice, the community could revisit the concern and build mechanisms for knowledge, verification, and improvement.

**Conclusion.** To build collective knowledge, teachers must hypothesize about their pedagogy and explain their hypotheses (Hiebert et al., 2002). Through discussions on multiple solutions pathways, the teachers hypothesized about student understandings related to their singular solution pathways, symbolic representations of division, and anticipating student
solutions. Based on their teaching experiences, the teachers believed that certain symbolic representations could be barriers for student understanding and discussed various causes for these barriers. They were able to begin building knowledge around understanding multiple representations of a function in multiple EPRs by making individual knowledge public, contributing to discussions, investigating their diagnoses or prognoses, and building on their investigations. The community changed the nature of their diagnostic frames and how they were anticipating student solutions when completing unfamiliar content tasks. The teachers worked to define solution strategies and build their own understanding of how the various methods are connected. This is an example of how the teachers built their own content understanding but lacked a connection back to classroom practice. I hypothesize that this change in frame was due to their lack of familiarity with the content knowledge.

While discussing multiple solution strategies, I also found two interesting EPRs. First, there was one instance of mechanisms of verification. This only arose when I was discussing my implementation of the formative assessment lesson in my practice and another teacher’s prognosis aligned to my experiences. I hypothesize that through prolonged engagement with video case analysis coupled with the implementation of the formative assessment lessons, a community of practice might be able develop mechanisms for verification and improvement. Secondly, the community hypothesized that students might have belief barriers that restrict them from appreciating multiple representations and that it is important for teachers to mediate these barriers. I found this EPR interesting because the community focused on a more pedagogical characteristic of their practice. Much of their other conversations were grounded in the mathematical content, but during this EPR, the community began to consider other factors, which indicated that there is a change within their frames. The changes within the frames are
evidence of learning in a community of practice, and in turn the beginning of them developing collective professional knowledge.

**Productive Struggle**

Students struggling to make sense of mathematical ideas is an important part of their knowledge and understanding of the subject (Hiebert & Grouws, 2007). During discussions while teachers were analyzing video case materials, they highlighted the importance of creating these opportunities for students. The teachers discussed the maintenance of cognitive demand witnessed in video segments, providing appropriate access to the mathematical concepts, and student perseverance when faced with struggle. The teachers partook in conversations that were either directly related to their classroom practices or the video case materials. I found changes within and between collective actions frames while teachers were discussing productive struggle in the mathematics classroom.

**Maintenance of cognitive demand.** Providing students a space to productively struggle with mathematics can be a challenging task for the most experienced teachers (Schoenfeld, 2015). This community of teachers also recognized the importance of maintaining the cognitive demand in the mathematics classroom by allowing students to productively struggle with the concepts. During the first and second video cases, the community of teachers discussed a problem of practice, ways in which a teacher can maintain the cognitive demand in a classroom. The community agreed that the teacher from the video segment was successful at maintaining the cognitive demand in her class through her questioning techniques. Three of the teachers highlighted that the teacher was promoting student struggle by allowing them opportunities to grapple with the mathematical concepts themselves by “letting them talk” and “coaching them” instead of “telling them.”
The second video case analysis contained evidence of a shift in the way the community was discussing how teachers can maintain the cognitive demand in a lesson. During this session, instead of drawing on the video segment to discuss the maintenance of cognitive demand, the teachers used connections from their own practice to reflect on the cognitive demand of their curricular resources. They expressed their concern that the resources did not ask students to find limits at negative infinity of radical functions. Therefore, they shared that they have modified curricular materials to include these additional questions, providing students a space to struggle and possibly make mathematical connections.

While both of these frames were diagnostic, the focus used to define and describe the problem of practice was different. In the second EPR, the community made connections to their practice, which they did not do in the first EPR. This shift within the diagnostic frame is an example of a change in participation and evidence of learning occurring within the community of practice. The community’s lack of prognostic frames regarding the maintenance of cognitive demand indicates that the community did not meet the standard of developing theories in the generation of collective knowledge. However, the community was able to meet the first two characteristics: (1) connected to practice and public and (2) sharable and storable. Within other themes the community was able to move past these two characteristics and create mechanisms for building knowledge. The maintenance of cognitive demand was only discussed in the first two video cases. The content of the last two lessons may have attributed to the halt in this conversation. The third video case focused on an eighth-grade lesson and the last on conditional probability. These two topics may have hindered the development of knowledge with respect to the maintenance of cognitive demand. If video case studies were chosen to highlight cognitive demand in the classroom, the community may have continued their investigations around
maintaining cognitive demand in the classroom. However, with prolonged engagement in this professional development model, with different content, the teachers might be able to build, verify and improve knowledge around the maintenance of cognitive demand as well.

**Student perseverance.** Student perseverance is necessary for them to struggle through towards the development of a solution. The community identified the lack of perseverance to struggle in students as a problem of practice. Discussion around this problem of practice appeared in three of the four video case analyses. As early as the first video case analysis, the community of teachers identified their students’ problem solving as constrained to a single pathway. The teachers believed that if a student completed a problem, they would continue to use the same method to solve all following problems even if it was the least effective method. This idea of rote application of taught procedures was reinforced when teachers shared that the students are more concerned with the end product of a mathematical task instead of the ways in which they arrived at the solution. During the second video case analysis, they changed the attribution of causality to institutional time constraints with their students. However, by the last video case, they returned to their original attribution, the students’ single pathway nature when problem solving. While revisiting this attribution of causality, they were able to change between frames from diagnostic to prognostic and propose a restructuring of their curricular materials.

Through these interactions, changes in participation occurred which were reified by changes between frames. The community began by identifying a problem of their practice by making it public and connecting it to practice. Over time they shared newly acquired practitioner knowledge, which is storable through curricular materials, by contributing to discussions and continuing to investigate this problem of practice. This pattern of investigation, sharing of interpretations, and contributions to discussions indicated that they were using mechanisms of
building knowledge within their community. Similar to previous themes, I believe that if the
community of teachers continued the professional development model, they would have the
opportunity to verify and improve this knowledge within their community because they proposed
immediate changes to curricular tasks. Through investigations in multiple contexts, the
community can contribute to discussions on their investigations to improve their suggested
changes.

**Conclusion.** Teacher’s recorded hypotheses are how collective knowledge can be
represented (Hiebert et al., 2002). Through the theme of productive struggle, the teachers shared
their practitioner knowledge related to maintaining cognitive demand and student perseverance.
Discussions within this theme were centered around defining and describing the problem of
practice. They spent all four video case analyses attempting to make sense of why they are
experiencing issues related to productive struggle. Their hypotheses in this theme were
hypotheses about the cause of their problems of practice and a prognosis for changes to their
curricular materials. The results indicated that the teachers are prepared for the development of
collective professional knowledge but need to continue to develop mechanisms for verification
and improvement.

**Conclusion**

This community of teachers was able to make progress towards the development of
collective professional knowledge. The teachers were able to define their practitioner knowledge
and make it public and sharable within this community. The knowledge they shared was
grounded in their practice and contextual to their school. Their departmental and community
norms made the knowledge they developed storable through lesson plans and curricular materials
jointly stored on shared drives. In some instances, the community was able to use mechanisms to
build knowledge collectively, however, they had no mechanisms for verification and improvement. This can be attributed to the shortened length of the professional development session and not looking outside of their community for additional knowledge to build their community knowledge. Since there were only four video case analyses, the teachers were still generating, sharing, and storing their own practitioner knowledge. However, the teachers may be able to attain methods of verification and improvement through longer engagement with the video case materials as evident from the one example, which occurred because I had implemented the formative assessment lesson in my class and had a personal investigation to discuss. Members of the community discussed making changes to teaching strategies and curricular materials. Over time, it would be important for the teachers to revisit these suggested changes to investigate the implementation and determine if their developed knowledge can be verified and improved.

The Nature of Conversations

The second research question sought to determine what the nature of conversations about teaching and learning mathematics was throughout the professional development sessions while the teachers engaged with video case materials. I used frame analysis as an analytic method to uncover how teachers participated in diagnostic, prognostic, and motivational conversations over time. Through the analysis, I found patterns in each of these types of conversations, which were presented in the previous chapter. In this section, I will discuss the results related to each of the three frames. In particular, I will discuss the relationship between the frame of each EPR and the activities occurring in the professional development session. I will also address the results from the last video case analysis and the relationship between the video case materials and the conversations the teachers engaged in.
**The Nature of Diagnostic Frames**

Through the first three video case analyses, this community of teachers exhibited a pattern of diagnostic framing in discussions prior to watching the video segment. The professional development model prescribed that this time was for the teachers to determine the big mathematical ideas of the lesson, complete the mathematical task in the lesson, and anticipate student struggles that might occur during the implementation of the lesson in a classroom. While the teachers engaged in these activities, they were often diagnosing problems in their own practice. The teachers identified problems of mathematics teaching and learning and determined the cause of these problems but did not work to hypothesize solutions for the identified problems.

**Identifying the mathematical goals.** While the teachers were identifying the mathematical goals of the first lesson, they discussed how the lesson goals are integral in mathematics teaching and learning. Setting learning goals is a critical component of preparing to teach a lesson (Stein & Smith, 2011). In their framework for analyzing teaching, Hiebert, Morris, Berk, and Jansen (2007) emphasize that, “formulating clear, explicit learning goals sets the stage for everything else” (p. 51). The professional development model emphasized this aspect of mathematics teaching and learning by first having the teachers reflect on the lesson goals. During video case analysis, the teachers are drawing on the predetermined lesson goals and extracting a big mathematical picture in relation to their own practice. I chose the example EPRs in the previous chapter as representative of the nature of the diagnostic frames occurring while the teachers were discussing the mathematical ideas in the lesson. In each of the examples, the teachers used the lesson goals and connected them to their practice using specific examples from their experience.
There were shifts in the discussion, but there were not enough changes within frames to hypothesize whether it was a result of the professional development model or the content the teachers were discussing. In the first video case, the community focused on specific content related to the lesson goals, but in the second video case analysis, they made an effort to use the mathematical task from the lesson as support for the lesson goals. Specifically, the teachers highlighted the ways the task illuminated what it means for a mathematical statement to be an identity. During the third video case analysis, there was another shift in the way the teachers discussed the mathematical goals of the lesson. They began to incorporate and connect student thinking and understanding to the mathematical goals. In each of the three video cases, the teachers had diagnostic frames when identifying the mathematical goals of the lesson prior to watching the video segment. I believe this happened because at this point in the professional development model the community was orienting themselves with a new lesson. They were attempting to unpack the lesson, including it’s mathematical goals. During the first three video case analyses, this part of the professional development model did not foster the development of prognoses to their identified problems of practice.

**Anticipating student solutions.** Prior to watching the video segment, the teachers completed the mathematical task associated with the formative assessment lesson. Through this activity, the teachers anticipated student mathematical understandings and misunderstandings. Contrary to the identification of mathematical goals, there were a few EPRs when the community was able to provide a prognosis for their problem of practice. However, with 33 diagnostic frames and only six prognostic frames, I determined that there was a diagnostic pattern in the ways the teachers were framing problems of practice during this part of the professional development model.
In the first video case analysis, the teachers pointed out specific instances where students might struggle in the completion of the task. For instance, they defined possible areas of student struggle as (1) understanding the relationship between inputs and outputs of a function, (2) determining if a function has a y-intercept, and (3) attempting to use one solution pathway to solve the task. During these discussions, the teachers built on each other’s concepts of student understanding, often agreeing and sometimes questioning each other. However, they often did not push each other to consider making pedagogical changes to address their identified problems of practice. During the second video case analysis, the community of teachers continued their diagnostic framing while anticipating student solutions. Initially addressed as a lack of student ability to see multiple solution pathways and the desire of students to guess and check, conversation turned into a way for teachers to connect these student understandings to other contexts. By the end of this part of the second session, the teachers formed connections in student misunderstandings between radical, polynomial, and trigonometric expressions. While the topic covered in this video case analysis is not taught by all of the participants, this connection allowed a space for all participants to engage in the conversation around this particular student misunderstanding. For the third video case the teachers addressed precision of language as a problem in their practice and attributed causality to students’ previous educational experiences. While they introduced new student misconceptions related to the content of this particular lesson on the properties of exponents, they also were able to expand on the misconceptions about the distributive property from the previous video case analysis. During these three video case analyses, the teachers were, again, discussing student understandings through diagnostic frames.

There was only minimal change in the diagnostic frames of student understandings
throughout these professional development sessions. This change came in the second video case analysis. The teachers changed their attribution of causality for student misunderstandings, which is consistent with other research. In particular, Sherin and Han (2004) found that through participation in the video club teachers were able to improve noticing around pedagogy and student conceptions. They concluded that participants did not only change what they discussed, but also how they discussed it. This research project found similar patterns; however, it was not always occurring after the teachers watched video segments. In this study, the teachers were changing how they discussed the problems of practice prior to watching the video segments. In many of the diagnostic frames, the teachers attributed causality to previous educational experiences, citing the lack of mathematics content knowledge in elementary classrooms. In the previous chapter, I chose EPRs that illustrated this change around the problem of practice of students’ misuse of the distributive property. In the second session, the teachers' attribution of causality shifted from previous educational experiences to questioning whether the students’ cognitive ability was hindering them from visualizing the symbolic representations. I hypothesize that through prolonged engagement with video case analysis, teachers can change how they are discussing problems of practice. This indication that there is potential for change in how they were discussing student misconceptions in three sessions of the professional development could be an indicator that longer, consistent participation in this professional development model could illicit more changes in the ways teachers diagnose and attribute causality for a problem of practice.

**The Nature of Prognostic Frames**

Following the segment of the professional development where teachers watched a video segment of the lesson enacted in a classroom, the teachers changed the way they framed their
discussions. After the video segments, there were more instances of prognostic frames in comparison to before watching the video segment. Each of the conversations that were coded as prognostic were centered around student understandings. This aligned with the professional development model because the teachers were prompted to consider student understandings and misunderstandings and determine how they would address them. In many of the EPRs, the teachers not only referenced the student understandings both from the video segment, but they provided additional support with personal experiences from their classrooms. This supports the idea that the professional development model may have effectively helped teachers to connect to their practice and transition from diagnostic to prognostic frames.

The examples I presented show the range of teachers’ prognoses throughout the video case analysis. The prognoses included changes to task development, implementation, and teacher questioning. In the first video case analysis, the community suggested teacher questioning techniques with the intent of promoting student discussion centered on the students’ mathematical thinking. Drawing on task design from the formative assessment lessons, the teachers also proposed including new tasks in the lesson to address students’ misconceptions around terminology related to quadratic functions. In the second and third video case analyses the teachers continued to suggest teacher questioning techniques to further student thinking. Throughout the first three video case analyses the teachers appeared to make suggestions with the intent of assisting students in building mathematical connections, especially between multiple solutions pathways. Mhlolo, Schafer, and Venkat (2012) found that students are losing opportunities to build meaningful mathematical connections because oftentimes the ways in which teachers present the connections are superficial. They suggest that building a teacher’s repertoire around multiple representations could aid in their pedagogical practice to enhance
students’ ability to make mathematical connections. Since the community consistently discussed multiple solutions pathways and multiple representations, this pedagogical concept could provide an opportunity for the community of teachers to hone their pedagogical skills around fostering discussions that promote meaningful mathematical connections.

The way teachers hypothesized about changes in pedagogical moves, typically around questioning techniques, is consistent with prior research using video as a focus for teacher discussion. Sherin and van Es (2009) found that with the use of video, teachers had a larger capacity for noticing and attending to students’ mathematical thinking. Similarly, Barnhart and van Es (2015) found that through a video based professional development model, preservice science teachers increased their sophistication in attending to, analyzing, and responding to students’ mathematical thinking. In this research, I also found that using video provided the teachers the opportunity to make sense of students’ mathematical thinking and develop responses to their thinking. Joining the ideas of building mathematical connections and responding to student thinking, using video case analysis in professional development can lead a community of teachers to hypothesize ways to respond to student thinking while fostering essential mathematical connections for the students.

**The Nature of Motivational Frames**

A motivational frame occurs when teachers proposed a change in pedagogical moves and detailed their rationale for this change. Through data analysis, I found that most of the motivational frames appeared in the portion of the professional development when teachers were completing the mathematical task of the lesson. In these EPRs, teachers were more willing to share their motivations for pedagogical choices while relating the discussion to personal experiences. This aligns with the literature and provides an opportunity to see how engagement
with these video case materials can become a sustained practice for this community.

Prior research on professional development indicated that key features of effective professional development include focus on content, active learning, coherence, duration, and collective participation (Desimone, 2011; Garet et al., 2001; Rogers et al., 2007). Rogers et al. (2007) specifically indicated that teachers value professional development that is applicable to their classroom practices, provides a space to learn in ways similar to their students, and builds support systems. This professional development model utilized a preexisting community of practice, which positioned the teachers to develop their support system. Combining their support system with opportunities to learn, which model student learning experiences, and having it be applicable to their practice created a space for the teachers to share changes in classroom practice and reasons for making those pedagogical moves. Research has also indicated that teachers find professional development to be more relevant when it is linked to teachers’ daily responsibilities (Flores, 2005; Tate, 2009). In this instance, certain content was inextricably linked to their classroom practice. The pattern of motivational frames when teachers were sharing specific classroom practices they have used is also an indicator that if the teachers were to use the formative assessment lessons in their classrooms and return to the community, they would be poised to continue providing motivational frames. These motivational frames about their investigations would also contribute to the previously discussed development of collective professional knowledge.

The first three video case analyses contained the most motivational frames. More than half of the motivational frames throughout the whole professional development model occurred during these three video case analyses while the community was anticipating student solutions. While all teachers were curricularly connected to properties of exponents and quadratic
functions, some teachers were not teaching a course with radical functions and equations in the curriculum. However, the community of practice was able to relate student misconceptions in the radical lesson to trigonometric, polynomial, and logarithmic functions, allowing more members of the community to make connections to their practice. The teachers noted that they were experiencing similar misconceptions across all of these functions. This provided a platform for all teachers to engage in the conversation around this content and student misconception. The teachers each had tangible, concrete, and recent experiences with the content to relate to and engage in conversation about. It was during these conversations that the teachers shared changes they had made to their practice to address the identified student misconceptions and provided rationale for the changes. The scope of this project did not follow the teachers after their participation in the professional development to determine what that impact could be. However, there is a possibility for the discussions to have an immediate impact on their teaching practices of quadratic functions.

Stigler and Thompson (2009) stressed the importance of building a shared knowledge within a community to either adapt dated traditions or to adopt new traditions. The three criteria they defined to characterize shared knowledge are (1) linked to theory and related to learning goals, (2) described in detail, and (3) contain contextual descriptions. Each of the motivational frames from the previous chapter meet these three requirements to become shared knowledge. They claim creation of this shared knowledge occurred through multiple forms including tweaks to current innovations, borrowing of innovations from others, and inventing new innovations. The occurrence of motivational frames in this research was in two of the forms, tweaks and borrowing innovations. The community of teachers suggested tweaks to their instruction on properties of exponents by having students complete many examples to boost their ability to
identify efficient methods for simplifying. Another motivational frame proposed borrowing an innovation, proof by counter-example, to address students’ misuse of the distributive property. Changes to graphing transformations was a tweak to their current instructional innovation by aligning methods to conic sections. Another borrowed innovation was found within a motivational frame when the community identified a way to address students’ confusion with mathematical symbols. Through these motivational frames, it is apparent that the teachers are effectively sharing their knowledge about mathematics teaching and learning. Since most of the motivational frames occurred when the community was discussing contextual problems related to their practice, I believe that if they implemented the formative assessment lesson while participating in the professional development analyzing video cases, I would find more instances of motivational frames. The compilation of formative assessment lessons, video case materials, and teachers’ experiences could be the records of practice Ball et al. (2014) claim could foster the development of collective professional knowledge.

**Video Case Four: Anomaly or the beginning of change?**

The first three video case studies had evidence of a preponderance of diagnostic frames prior to watching the video segment and prognostic frames after watching the video segment. During the last video case analysis, there was a shift in the ways the teachers framed discussions before and after watching the video segment, in that the community had similar framing patterns both pre- and post-video watching. The teachers partook in equal amounts of diagnostic and prognostic frames prior to and proceeding the video segment.

The shift in framing patterns can be an indication that through engagement with video case analysis professional development, the community of practice was changing their discussion patterns over time. In the fourth case, prior to watching the video, the teachers were focused on
implementation and making content connections to their practice. Post-video watching they were focused on addressing particular student understandings brought to light from the video segment. If this last video case is an indication of changes, you could expect to see these teachers engage in more prognostic conversations prior to watching the video segment as they continue engagement in the professional development model.

Alternatively, this case can be an anomaly to the pattern from the first three video cases due to the content of the lesson, conditional probability. The content of the lesson was the least familiar to the community of teachers. This was evidenced by the change in the way they engaged with the mathematical task. In the previous sessions, the teachers simply completed the task and looked for possible student misconceptions. However, in this case, the teachers took more time solving the task and comparing and contrasting their individual methods for solving. Johnson and Cotterman (2015) found that preservice teachers enter discussions without the intent of understanding the content, but rather, time spent understanding the content occurred out of a natural need for the teachers to make sense of the content for themselves. The conversations that occurred in this video case analysis were similar to those described in Johnson and Cotterman’s work. While preservice teachers had a naturally occurring need to make sense of the content for themselves, the teachers in this community were provided time to collectively investigate the content. The teachers were afforded the opportunity to break out of their isolation and enter conversations around mathematical content. The teachers in this community of practice diverged from their previous patterns during the fourth video case analysis and dove deeper into understanding the mathematics, but I believe this only occurred when they were not familiar with the content.
Conclusion

Through engagement with video case study analysis, this community of practice’s framing patterns were aligned with particular parts of the professional development model. In the first three video cases, teachers consistently used more diagnostic frames when identifying the mathematical ideas of the lesson and anticipating student solutions but transitioned to prognostic frames after watching the video when they were analyzing student understandings and proposing pedagogical moves to address those understandings. I found motivational frames when teachers were engaged in conversations about mathematical tasks and their individual practitioner knowledge. The last video case analysis fostered wider changes in framing patterns which could be attributed to their time in the professional development model or to the content of the lesson being analyzed. The professional development model appeared to have elicited particular framing patterns in teacher conversation depending on the activity in which they were engaged. This indicated that the professional development model may have been successful at promoting teachers to prognosticate about addressing students’ mathematical understandings from video cases. More interestingly though is that most motivational frames appeared when the teachers were connecting the mathematics to their own practice, indicating a need to align their teaching practices to the resources from the professional development model.

Limitations

A key feature of effective professional development is providing sustained participation to determine if change is taking place (Garet et al., 2001). This study provided teachers the opportunity to engage in four video case analyses. While some changes in participation, reified by changes in frames, were documented and the community indicated that there is potential for them to develop collective professional knowledge in this short time frame, a continuation of this
professional development model with this community of practice is needed to determine if the changes continue or could be sustained. Sustained professional development would provide the opportunity to determine if the changes evidenced here are typical during video case analysis and if the changes in participation are sustainable over a longer time while engaged in video case analysis. In addition, increasing the length of the professional development series in conjunction with the implementation of the formative assessment lessons would allow more time to determine if mechanisms of verification and improvement could be reached. The professional development spanned over three months, which was not enough time to determine if the community could enact a model to verify their developed knowledge around precision of language, multiple solution pathways, or productive struggle. The length of the professional development also hindered the ability to determine if the changes in framing patterns found in the last case analysis would continue. Changes in participation and reification over this short time frame can potentially be strengthened with a longer engagement with video case materials.

While the length of the study possibly stunted the development of collective professional knowledge, there were indicators that it could be possible. In conversations with participants, they expressed the desire to use the formative assessment lessons in their classes and to review video of themselves teaching for purposes of improvement. In reflections after participation in the professional development, two teachers self-reported that they implemented a formative assessment lesson. In her reflection Ashley stated, “As a group, we always spoke a great deal about what was going on with our students in regard to their learning. This influences planning and curriculum.” Ashley is referencing changes to lesson plans and curriculum to be enacted. Through this enactment, she can return to the community to share her interpretations of the investigation to refine the community’s knowledge utilizing mechanisms for verification and
improvement. Since plans and curriculum are shared and stored artifacts within this community, making changes to them would allow for verification in her classroom and others. Thus, this is an example of how this community is beginning to consider mechanisms for verification and improvement. In Barbara’s reflection she indicated that she believed they were afforded the opportunity to “reflect on the effectiveness” of their professional practices. This also suggests that she is open to continued reflection and verification of their developed knowledge of mathematics teaching and learning.

This study was conducted using a professional development model which was currently in development by a larger research team. Based on feedback from participants and facilitators in this and other implementations of the model, the facilitation of the professional development model has undergone changes to see if the length of engagement could be a factor in the development of mechanisms for verification and improvement. The next round of research on this professional development model is occurring with communities of teachers over the course of an entire school year.

Another limitation in this work would be my role as a researcher and participant as a facilitator. There were benefits to including an “insider” as the facilitator. For example, it provided a level of comfort for the teachers to interact in their normal community of practice. This allowed me to clearly define this group of teachers as a community of practice. However, due to this familiarity within the community, at times the teachers began to stray from the professional development model and delve into current issues in their practice. This was not the initial intent of the professional development model. Even so, through these conversations, I found moments where the teachers connected the lesson from the video case materials to their current classroom practices. With an outside facilitator, the sessions might have aligned more
closely to intended professional development model, but that might have sacrificed connections
to classroom practices that the community might not share with an outside facilitator. Similarly
to Sherin and Han (2004), due to the nature of the preexisting community of practice and situated
nature in which this learning occurred, I question the generalizability of this work. It remains an
open question whether the outcomes would be similar if this professional development model
were used in a different community.

I also believe that as an internal facilitator, I may not have pushed the teachers in the
community to interrogate their own practice and provide motivational frames for their prognoses
as much as I could have. The results detailed how most EPRs after watching the video segment
were prognostic frames. As a facilitator I could have continued to push their reasoning behind
these prognoses to garner their motivations for their suggestions. The community had generated
norms for providing motivational frames driven by their own classroom practices, so I believe
with a facilitator probing their motivations for other prognoses, they might be able to transition
between these frames.

My insider perspective may have also hindered the community’s ability to assign
causality to themselves in some instances. I wonder if an outside facilitator may have been able
to push the community to more closely analyze their own classrooms and rituals as causes for the
defined problems of practice instead of consistently looking to assign causality to others. The
professional development model itself, using video case study as an object for teacher reflection
could also be a way to mitigate the outward facing attributions of causality. While discussing the
video segments, the community was focused on attending to students’ mathematical thinking and
understanding and what questions they could ask students to further their understanding. Since
the teachers were analyzing another teacher’s classroom, one for which they did not have
intimate knowledge, they needed to focus on student understanding. I believe this led the
teachers to attribute causality for identified problems of practice to the students. While this is
difficult to avoid in the model, their prognoses are changes in teacher moves and pedagogy. This
is an indication that they have the potential to look within to propose changes in practice.

Next Steps

This research filled the need to investigate how video case materials can assist in the
development of collective professional knowledge and how conversations evolve through teacher
engagement with the materials. It added to the newly emerging research in mathematics
education utilizing frame analysis as an analytic method to understand how teachers generate
knowledge and learn in a professional setting. This is only the beginning of determining if
utilizing this professional development model can be effective in eliciting the development of
collective professional knowledge. To answer these questions, more research would need to be
conducted to analyze a community of practice engaged in regular meetings for a longer period of
time. More sustained engagement coupled with alterations in the facilitation could lead to the
community’s development of mechanisms for verification and improvement. Only then would
one be able to determine if there is the possibility of using this model to generate collective
professional knowledge.

This research illuminated the power of frame analysis to determine if learning was
occurring in a community of practice. I found instances where changes within and across frames
were evidence of this particular community’s ability to produce meaningful changes in
participation within the professional development model. While this study found small changes
in the community’s participation, using frame analysis as an analytic method in other
communities of practice can possibly reveal more meaningful changes. For example, this
community of teachers was often able to change their attribution of causality. However, if a community could determine a new attribution of causality to more clearly define their problem of practice, they could possibly then use these new realizations to develop prognoses.

The changes evidenced in this study were often around the attribution of causality to help the community define their problem of practice. Given these changes, the community of teachers often assigned blame to others and rarely focused on themselves as contributors to the problems within their practice. Based on previous research, a community should be able to break this deficit mindset for mathematics teaching and learning and be able to be more introspective about their practice and how they can make meaningful pedagogical changes.

In prior research in mathematics education, after defining the problem of practice, researchers have looked to the attribution of blame. In this study, I chose to use the word causality instead of blame in an attempt to lessen the deficit thinking perspective. My belief is that the use of the word blame contributes to the deficit thinking perspective. Adiredja (2019) describes deficit-narratives in research related to students’ mathematical thinking as focused “on misconceptions and failures to conform to normative ways of thinking” (p. 406). However, by using the word causality in future research using frame analysis we can look for root causes of problems of practice. By doing this, we may also see mathematics educators either take ownership of problems in their practice instead of looking to blame someone or something else, or collaborate to find solutions to problems of practice that are not their own.

This community of practice showed promise for the generation of new knowledge through the suggested changes to classroom practices. A community showing this progress needs continued research efforts to determine if they hold the ability to implement these changes and bring their investigations back to the community for verification and improvement. Following
this community beyond their engagement in the professional development model could allow researchers to determine the lasting impact of these discussions over time. Hiebert et al. (2002) state the “support and interest in new forms of professional development make a new research and development system a more realistic goal” (p. 13). Positive responses from the teachers in this professional development echoed this statement. They appreciated professional development that was content based and encouraged them to unpack mathematics teaching and learning as a community to strengthen their practice. There was one EPR that indicated there is a potential to create mechanisms for verification and improvement to build collective professional knowledge using the formative assessment lessons as part of a record of practice (Ball et al., 2014). As teachers in the community are implementing the formative assessment lessons in their classes, continuing the professional development model to reflect in the implementation can facilitate the development of mechanisms of verification and improvement and generate collective professional knowledge within their community of practice. Also, by taking a particular community’s generated collective professional knowledge and verifying and improving that knowledge within other communities, the collective professional knowledge has the potential to become more globally generated and defined. Since the development of collective professional knowledge, in this research, is predicated on learning that is demonstrated by changes in participation, future work will need to consider how various communities of practice engage and participate with the same collective professional knowledge. Through deeper research on the professional development model both within this community and within other communities, we can determine if it is valuable in the creation of collective professional knowledge locally and globally.
Conclusion

This research sought to determine the mechanisms for developing collective professional knowledge within a specific context and community of practice. While the community appeared to be becoming prepared for developing collective professional knowledge, the next step would be to determine if this local community’s collective professional knowledge could be analyzed, verified, and improved on a more global scale, including additional research in this context and in other contexts. In addition, consistent reflection upon facilitation is needed to ensure the community is being provided opportunities to develop and share more motivational frames and interrogate their own attributions of causality.

This research has shown that video case analysis within this professional development model can be a powerful tool for teachers to use to analyze mathematics teaching and learning. Although this community’s lack of experiences truly interrogating teaching practice may have hindered the ability for them to develop more meaningful changes, the fact that they were able to change within four sessions demonstrates a potential for the development of more meaningful change to occur if the professional development model consistently became part of their mutual goals.
REFERENCES


Leavy, P. (2014). *The oxford handbook of qualitative research*


APPENDIX A

Protocol for Professional Development

- Identify the Mathematical Ideas (10 minutes):
  - What are the mathematical goals?
  - What do you see as the “big picture” from your experience?
- Do the Mathematics (15 minutes)
  - Do the math that students will do in the video clip individually.
  - Compare your work with a partner at your table. Try to understand how the other person was thinking.
  - Each person shares how their partner was thinking about the problem with the rest of the group.
  - Write a joint solution(s) for the group on the chart paper next to your table.
- Group discussion about The Mathematics (10 minutes)
  - What are different ways to solve the problem?
  - How do the different ways to solve the problem illuminate the big mathematical idea(s) in this lesson?
  - How do different ways to solve the problem connect to one another?
  - What approaches are students likely to take when trying to solve the problem?
- Get context for the lesson we are analyzing (10 minutes):
  - Go through the abridged lesson outline as a group to get a sense of the timeline and key activities in the lesson. If anyone is unclear about the structure of the lesson or the activities that take place, consult the lesson guide.
  - Get context for the video
- Watch and discuss video
  - Review norms and framework for video
  - Watch the video once, making brief notes about anything you notice.
  - Watch the video again, this time thinking about it from the point of view of a specific TRU dimension(s).
    - Answer and discuss questions related to the selected dimension(s) as a group.
    - Compare your thinking with the collective responses that have been produced over time by other teachers who have considered this case. Do you have anything new to add?