Detection of Coherent Structures in Flows

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ABSTRACT

Title of Thesis: DETECTION OF COHERENT STRUCTURES IN FLOWS

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In this work, we have developed an experimental flow tank that can produce realistic ocean-like flows, including multi-gyre flows. By generating controllable ocean-like flow fields, we can study the flows to gain a better understanding of ocean dynamics. In particular, we use particle image velocimetry and finite-time Lyapunov exponents to determine the location of the Lagrangian Coherent Structures that determine transport in complex fluid flows. This understanding is useful for designing control algorithms and for optimizing the use of autonomous vehicles operating in the stochastic and time-dependent ocean environment.
DETECTION OF COHERENT STRUCTURES IN FLOWS

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Chapter 1

Introduction

The purpose of this thesis is to understand how one can take advantage of underlying ocean dynamics for a variety of applications involving autonomous vehicles. Autonomous surface vehicles (ASVs) and autonomous underwater vehicles (AUVs) have been used to study various biological and physical phenomena in the ocean, and to provide efficient and reliable monitoring of the uncertain ocean environment [1]. For example, autonomous underwater gliders have been developed as a sensing platform to improve weather and climate prediction. Operating in both deep ocean and coastal regimes, gliders have also been used for surveillance and reconnaissance. Gliders monitor water currents, temperature, and conditions that reveal effects from storms, and the quality of the water. This important information depicts a more complete picture of what is happening in the ocean. As another example, persistent surveillance has been used to study plankton assemblages and harmful algae blooms, as well as to measure salinity and temperature profiles [1]. In order to perform these tasks, it would be useful if the gliders could take advantage of the underlying structure found in geophysical flows to minimize the time or energy needed to perform the task [1].

Two specific and important problems related to using autonomous vehicles in
the ocean are (1) the station-keeping problem, and (2) the path-planning problem. Station-keeping refers to the ability of the vehicle to maintain position and orientation with regard to a reference object or particle. For example, an AUV may be required to remain in a region for an extended period of time for persistent environmental monitoring of the specific area. The success of this mission relies on the vehicle's ability to hold a precise station while maximizing its time on station. Environmental disturbances (wind, waves and currents) affect the station-keeping ability of the autonomous vehicles. In the path-planning problem, one is interested in how best to transition a vehicle from one region to another region. The goal is to find the paths of ASVs or AUVs which minimize travel time or energy in the presence of ocean flows. Autonomous underwater gliders are subjected to drift due to hydrodynamic forces [1]. To harness the ocean forces so that we minimize energy outflow during control actuation, we need to analyze the ocean structures from the correct dynamical viewpoint. The methods we will use to make such calculations are based on both deterministic and stochastic analysis techniques taken from dynamical systems theory. For vehicles operating in the stochastic ocean, noise can cause the vehicle to be displaced from one basin of attraction to another. The location of these basins is essential in determining how a particles relocates from the original basin. These basins can be found by computing uncertain sets, almost invariant sets, and Lagrangian Coherent Structures (LCS) given by the FTLE field. Knowledge of these allows us to determine appropriate control regions.

Knowledge of these control regions allows one to design set-based control methods. The control provides an increase in loitering time (i.e. it decreases the particles' probability to escape from the region). Lagrangian coherent structures denote boundaries of the control regions, and therefore it is useful to know where the LCS exist. One can compute LCS by using finite-time Lyapunov exponents [1,2]. But to do so, one needs global velocity information. Recently, researchers have developed a
method to track the LCS using a collaborative control strategy where autonomous underwater vehicles need only take local velocity measurements [4]. LCS essentially divide the flow into dynamically distinct regions that allows one to fully understand material transport. However, we still do not fully understand the dynamics of LCS in actual ocean flows. Therefore, we have developed an experimental flow tank that can produce realistic ocean-like flows. We use particle image velocimetry (PIV) to extract the velocity fields from the experimental flows, and these velocity fields are analyzed to understand the LCS and location of control regions. This way, we will understand better the underlying ocean dynamics, which will enable improved usage of autonomous sensing vehicles.
Chapter 2

LCS And FTLE

Crude oil and refined fuel spills from ocean drilling and tanker ship accidents have damaged vulnerable ecosystems in Alaska, the Gulf of Mexico, the Galapagos Islands, France, and many other places. The quantity of oil spilled during accidents has ranged from a few hundred tons to several hundred thousand tons. All these significant environmental events have one common theme: damaging material was released into the environment, from what was essentially a point source, which then can spread for hundreds of nautical miles in a thin oil slick. Predicting where this harmful material would be transported by the surrounding oceanic flow is of utmost importance.

Figure 2.1: A 300-km-wide view of the 2010 Deepwater Horizon oil spill in the Gulf of Mexico [9].
To predict the outcomes of such catastrophic events, the usual approach is to run numerical models of the ocean to forecast pollutant trajectories. Although this approach does positively forecast the behavior of individual fluid parcels, the predictions are highly susceptible to small changes in the time and location of release. One way to address this problem is by running several different models for the same scenario. However, the replication typically produces even larger distributions of advected particles – the particles transported by the fluid flow – and therefore the replication hides key organizing structures of the flow.

A new way of understanding transport in complex fluid flows is provided by the concept of Lagrangian coherent structures (LCS). The LCS approach provides a means of identifying key material lines that organize fluid-flow transport [9]. Such material lines account for the linear shape of the structure of the oil spill in Figure 2.1. Specifically, the LCS approach is based on the identification of material lines that play the dominant role in attracting and repelling neighboring fluid elements over a selected period of time [9]. Those material lines are the LCS of the fluid flow.

Throughout the 2000s, Haller determined specific criteria to distinguish between structures arising from different effects when identifying LCS [9]. Given well-established theoretical foundations, the popularity of LCS has quickly expanded and active research continues in multiple disciplines. Applications of LCS include determining flow structures in aeronautical weather data, transport in the oceans, computational fluid dynamics, and even human musculoskeletal bio-mechanics, blood circulation, and airway transport.

There are two different mathematical representations of fluid flow: the Lagrangian picture in which we keep track of the locations of individual fluid particles; and the Eulerian picture in which coordinates are fixed in space (the laboratory frame). The velocity field is a prime example of an Eulerian description [9]. It gives the instantaneous velocity of fluid elements throughout the domain under consideration. On the
other hand, the Lagrangian perspective is concerned with the identity of individual fluid elements [9]. LCS provide a new way of understanding transport in complex fluid flows. What makes these material surfaces special is the distinguished attracting or repelling nature. Notably, LCS are often locally the most strongly attracting or repelling material surfaces in the flow, and as such have a strong influence on the flow topology. More specifically, the LCS approach is based on the identification of material lines that play the dominant role in attracting and repelling neighboring fluid elements over a selected period of time. Those key lines are the LCS of the fluid flow.

We can use LCS to divide realistic ocean flows into dynamically distinct regions. As extensions of stable and unstable manifolds, LCS carry a lot of global information about the behavior of the flows. For two dimensional (2D) flows, LCS are analogous to ridges defined by local maximum instability, and are approximately quantified by local measures of Finite-Time Lyapunov Exponents (FTLE) [5]. In recent years it has been shown that LCS concur with optimal trajectories in the ocean [5]. Therefore, LCS may be used to reduce the energy and time needed to travel from one point to another.

To better understand the different physical, chemical and geophysical processes in the ocean, and to improve weather and climate predictions, there has been an increasing and significant interest in the use of autonomous sensors to measure a variety of quantities of interest. However, these autonomous sensors have one major disadvantage: they have to operate in time dependent and stochastic environments such as the ocean. Natural ocean dynamics cause the sensors to escape from their monitoring regions of interest. To prevent this escape, we can use knowledge of LCS location to our advantage.

The Finite-Time Lyapunov Exponent (FTLE) has been proven to be an effective metric for revealing distinct, bounded regions within a flow. In particular, ridges of
high FTLE values denote the Lagrangian Coherent Structures that separates phase space into different dynamical regions. The FTLE can be found for any model where the paths of neighboring trajectories can be computed using the governing differential equations. More on point, these calculating methods are especially useful when there is no model and we only have available experimental or measured data.

The computation of FTLE can be used to understand transport. In this model the FTLE quantifies localized sensitive dependence to initial conditions in a given fluid flow, and also gives, in deterministic settings, an explicit measure of phase space uncertainty. It is of great interest to determine how the particles that are initially infinitesimally close behave as time \( t \to \pm \infty \) in a dynamical system. The classical Lyapunov exponent provides a well known quantitative measure of this asymptotic behavior. If one restricts the Lyapunov exponent calculation to a finite time interval, the resulting exponents are the FTLE [3].

By using FTLE, we provide a quantitative measure of how much nearby particles separate after a specific amount of time. We consider a 2D velocity field \( \mathbf{v} : \mathbb{R}^2 \times I \to \mathbb{R}^2 \) which is defined over the time interval \( I = [t_i, t_f] \subset \mathbb{R} \) and the following system of equations:

\[
\begin{align*}
\dot{z}(t; t_i, z_0) &= \mathbf{v}(z(t; t_i, z_0), t) \\
z(t_i; t, z_0) &= z_0
\end{align*}
\]  

(2.1)  

(2.2)

where \( z = (x, y)^T \in \mathbb{R}^2, z_0 \in \mathbb{R}^2, \) and \( t \in I \).

To summarize, by using the FTLE method we are able to provide a fairly accurate local measure of sensitivity to nearby initial conditions and to measure the growth rates of the linearized dynamics about the trajectories. The FTLE derivation details along with the applications that utilize the FTLE have appeared in the literature, and in a short summary we will discuss the procedures [2, 3].
2.1 Procedures

The solution of the dynamical system given by equations 2.1-2.2 from the initial time $t_i$ to the final time $t_i + T$ can be viewed as the flow map $\phi_{t_i}^{t_i+T}$ which is defined as follows:

$$\phi_{t_i}^{t_i+T} : z_0 \rightarrow \phi_{t_i}^{t_i+T}(z_0) = z(t_i + T; t_i, z_0) \quad (2.3)$$

We consider an initial point located at $z$ at $t_i = 0$ along with a perturbed point located at $z + \delta z(0)$ at $t_i = 0$. Using a Taylor series expansion, one finds that:

$$\delta z(T) = \frac{d\phi_{t_i}^{t_i+T}(z)}{dz} \delta z(0) + O(||\delta z(0)||^2). \quad (2.4)$$

Dropping the higher order terms, the magnitude of the linearized perturbations is given as:

$$||\delta z(T)|| = \sqrt{\langle \delta z(0), \Delta \rangle}, \quad (2.5)$$

where $\Delta$ is the right Cauchy-Green deformation tensor and is given as follows:

$$\Delta(z, t_i, T) = \left( \frac{d\phi_{t_i}^{t_i+T}(z(t))}{dz(t)} \right)^* \left( \frac{d\phi_{t_i}^{t_i+T}(z(t))}{dz(t)} \right), \quad (2.6)$$

with $*$ denoting the adjoint. Then the FTLE can be defined as:
\[ \sigma(z, t_i, T) = \frac{1}{|T|} \ln \sqrt{\lambda_{\max}(\Delta)}, \]  

(2.7)

where \( \lambda_{\max}(\Delta) \) is the maximum eigenvalue of \( \Delta \).

For a given \( z \in \mathbb{R}^2 \) at an initial time \( t_i \), equation 2.7 gives the maximum finite-time Lyapunov exponent for some finite integration time \( T \) (forward or backward), and provides a measure of the sensitivity of a trajectory to small perturbations.

### 2.2 Computation of the FTLE

To begin the FTLE calculation, we define an initial grid of particles. The resolution of the calculated FTLE field is affected by the number and the spatial spread of the particles; if we use more particles, a better representation of the field is provided. Given that the FTLE will vary over time as well, it becomes necessary to re-calculate the entire FTLE field at various times in order to obtain a time-varying picture of the behavior of the flow. The particle trajectories from \( t_0 \) to \( t_0 + T \) are calculated using a Runge-Kutta numerical integration method.

### 2.3 Pendulum Example

Let’s consider the example of a simple pendulum as shown in Figure 2.2. A so-called "simple pendulum" is an idealization of a "real pendulum" but in an isolated system using the following assumptions: The rod or cord on which the bob swings is massless, inextensible and always remains taut, the bob is a point mass, the motion occurs only in two dimensions, i.e. the bob does not trace an ellipse but an arc, and does not lose energy to friction or air resistance. The differential equation which represents the motion of a simple pendulum is: \( \ddot{\theta} + \sin \theta = 0 \), where \( \theta \) is the angular displacement.
The FTLE field is computed and the results shown in Figure 2.3 indicate that the stable and unstable manifolds (LCS), which are highlighted in red, divide the flow into unique regions. At the center of the eye (the region enclosed by the two manifolds highlighted in red), circular trajectories correspond to the back and forth oscillation of the pendulum. The regions outside the manifolds (LCS), correspond to trajectories for which the pendulum constantly rotates in the same direction. These two dynamical behaviors, are qualitatively distinct, and the stable and unstable manifolds form sharp boundaries for them in the FTLE field. The ridges divide the flow into two dynamical distinct regions. As mentioned above, these high FTLE ridges correspond to the Lagrangian coherent structures.
2.4 Model

A more realistic model of an ocean-like flow is that of the wind-driven double-gyre flow. The model is given by:

\begin{align}
\dot{x} &= -\pi A \sin(\pi f(x, t)) \cos(\pi y) - \alpha x + \eta_1(t), \quad (2.8) \\
\dot{y} &= \pi A \cos(\pi f(x, t)) \sin(\pi y) \frac{df}{dx} - \alpha y + \eta_2(t), \quad (2.9) \\
f(x, t) &= \epsilon \sin(\omega t + \psi)x^2 + (1 - 2\epsilon \sin(\omega t + \psi))x. \quad (2.10)
\end{align}

The double-gyre flow is time independent when $\epsilon = 0$, while the gyre undergoes a periodic expansion and contraction in the $x$ direction when $\epsilon \neq 0$. In the equations, $A$ approximately determines the amplitude of the velocity vectors, the oscillation frequency is given by $\omega/2\pi$, the amplitude of the left-right motion of the separatrix between two gyres is determined by $\epsilon$, the phase by $\psi$, the dissipation is determined by $\alpha$, and a stochastic white noise with mean zero and standard deviation $\sigma = \sqrt{2D}$, where $D$ is noise intensity, is described by $\eta_i(t)$. The first and second order statistics are: $\langle \eta_i(t) \rangle = 0$ and $\langle \eta_i(t), \eta_j(t') \rangle = 2D \delta_{ij} \delta(t - t')$ for $i, j = 1, 2$. 

Figure 2.3: Snapshot of a) forward and b) backward FTLE fields for a simple pendulum with $T=8$ and a $36 \times 36$ grid.
Figure 2.4 shows the phase portrait of the double-gyre model for a time-dependent flow ($\epsilon = 0.15$) with $A = 0.1, \alpha = 0.005, \omega = 2\pi/20$, and $\psi = 0$. We consider the dynamics restricted to the domain $\{(x, y) | 0 < x < 2, 0 < y < 1\}$.

![Figure 2.4: Phase portrait for the double gyre given by Eqs. (2.8)-(2.10) with $A=0.1$, $\epsilon = 0.15$, $\alpha = 0.005$, $\omega = 2\pi/20$, $\psi = 0$.](image)

Figure 2.5 shows the forward time FTLE field calculated by using Eqs. (2.8)-(2.10) with $A=0.1$, $\epsilon = 0.15$, finite time $T = 20$, integration step size of 0.1, and grid resolution of 0.005 in both $x$ and $y$. We can see from the figure that there are ridges (in red) of locally maximal FTLE values. These ridges are the LCS, which separate the phase space into distinct dynamical region.

![Figure 2.5: FTLE field for the double-gyre given by Eqs. (2.8)-(2.10) with $A=0.1$, $\epsilon = 0.15$, integration time $T = 20$ and integration step size of 0.1.](image)
Figure 2.6 shows the forward time FTLE field calculated by using Eqs. (2.8)-(2.10) for a time-independent ($\epsilon = 0$) flow, with $A=0.1$, finite time $T = 20$, integration step size of 0.1, and grid resolution of 0.005 in both $x$ and $y$.

![Figure 2.6: FTLE field for the time independent double-gyre flow given by Eqs. (2.8)-(2.10) with $A = 0.1$, $\epsilon = 0$, integration time $T = 20$ and integration step size of 0.1.](image)

In this figure, the red ridges indicate higher FTLE values and the blue ridges indicate lower FTLE values. The higher FTLE values can be observed by the separation of the two gyres since the trajectories initially on opposite sides of the boundary diverge rapidly from each other.
Chapter 3

Experiment

In this section, we describe the development of our experimental flow tank built to generate controllable ocean-like flow fields that exhibit the transport-controlling LCS. The experimental tank is equipped with a $4 \times 4$ set of driving cylinders that can produce a variety of flows including gyre-like flows, and jet flows. In order to impose precisely formed perturbations onto controlled and realistic flows, we have designed a laboratory experiment in a low Reynolds number regime [3,4]. A total of 106 experiments were performed. The first experiments were done in pure water or a glycerine-water mix with very high Reynolds number, as well as pure glycerine (low Reynolds number) and all 16 discs ran together. For the second set of experiments, the tank was rebuilt to have two independent sides of eight discs each. In this set of experiments we changed both maximum and minimum values of angular velocity, as well as the length and time that each side would rotate. In some experiments these values were randomly assigned.
As shown in Figure 3.1, we use a 0.8 cm square tank driven by a $4 \times 4$ grid of rotating disks. A stepper motor is used to drive each side of eight disks with a specified rotational direction and velocity. We designed the experiment so that the two adjacent $4 \times 2$ sets of disks have separate stepper motors, controllers and gears, such that they may be independently driven at any time-dependent angular velocity function from a simple MATLAB code. We also can introduce perturbations to the steady flow case. By adjusting rotational direction, we can easily create multi-gyre flows, jets, and boundary layer flows. In our experiments a laser sheet is shined through the tank to illuminate micron-sized particles. A high speed camera is used to capture the resulting images, which are then used to create high-quality velocity fields using particle imaging velocimetry (PIV). We used both PIVLab and Davis software to perform the PIV analysis. The generated vector fields, are analyzed using FTLE to determine the LCS.
3.1 Particle Image Velocimetry

An important development of modern experimental fluid mechanics is the invention and development of techniques for the measurement of entire instantaneous fields of scalars and vectors [8]. Among these techniques is particle-image velocimetry (PIV). It has been more than twenty years since the term "particle image velocimetry" first appeared in the literature [7]. PIV is an optical method which measures space-and-time resolved flow velocities in fluids. Over time, many researchers became interested in PIV because it offered a new and highly promising means of studying the structure of turbulent flow. This goal influenced the choices made in the development of the technique. The accurate, quantitative measurement of fluid velocity vectors at a very large number of points simultaneously are defining characteristics of PIV. As we know, turbulence is a phenomenon that happens over a wide range of scales. Furthermore, it is impossible to determine a priori the direction of flow since one of the features of turbulence is randomness. To address the first feature, the measurement technique must be able to measure over a wide dynamic range of scales in length and velocity; and given the random character of turbulent flows, it must also be able to sense flows in all directions. Given that accelerations are large, the particles must be small enough to follow the flow in the presence of large local and randomly fluctuating accelerations. A basic PIV system consists of a pulsed laser with a light sheet illuminating particles a few microns in diameters in gases and, perhaps, a few tens of microns in liquids [7]. PIV calculates the velocity distribution within image pairs and also is used to derive, display and export multiple parameters of the flow pattern.
3.2 Particle experiment

The PIV we applied is a two dimensional technique used to retrieve highly detailed information about flow fields. PIVLab is an open source analysis tool used to analyze the particle movement. During PIV analyses, two images (1 and 2) of the illuminated plane are captured at $t_0$ and $t_0 + \Delta t$. Velocities in the sheet can therefore be illustrated from $\Delta t$ and the distance that the particles traveled from one image to another (particle displacement). In PIV, this distance known as particle displacement is computed for groups of particles by analyzing the cross-correlation of many small sub images known as interrogation areas.

The correlation yields the most probable displacement for a group of particles traveling on a straight line between image 1 and image 2 [21]. Before the image correlation takes place, it is best to enhance this image in order to get the highest measurement quality possible. Three pre-processing techniques that are implemented in PIVLab are: histogram equalization, intensity high-pass, and intensity capping.

Contrast Limited Adaptive Histogram Equalization (CLAHE) or shortened to histogram equalization was first introduced by Pizer et al. in 1987 for increasing the readability of image data in medical imaging. It is an adaptive contrast enhancement method. It is based on adaptive histogram equalization (AHE), where the histogram is calculated for the contextual region of a pixel. The pixel’s intensity is therefore transformed to a value within the display range proportional to the pixel intensity’s rank in the local intensity histogram (see Figure 3.3 (a)). By enhancing the image contrast locally, CLAHE improves valid vector detection by about 5%. Rather than the entire image, CLAHE operates on small regions within the image. This is an advantage because a uniform exposure of the complete image can not always be guaranteed.

The correlation signal is strongly affected by the in-homogeneous lighting caused by reflections from objects. A high-pass filter, which mostly sustains the high fre-
quency information from the radiance, can remove the low frequency background information. Consequently, the high-pass filter can improve the signal-to-noise ratio of the correlation signal. The high-pass filter highlights the particle information in the image, and it subdues any low frequency information in the image (see Figure 3.3(b)). To calculate, we apply a low-pass filter to the image, e.g. blur the image, and subtract the result from the original image (Figure 3.2 (a)).

![Figure 3.2: The effect of pre-processing techniques: a)original image and b) after pre-processing.](image)

In addition to the two aforementioned techniques, an intensity capping technique is used to suppress the increased impact of the bright particles on the correlation signal. The filter diminishes the presence of the bright particles while leaving the other particles unaltered (see Figure 3.3(c)). Much like CLAHE, the valid vector detection probability increases by about 5%. To recap, the CLAHE technique enhances the image contrast locally, while high-pass filtering of the image data suppresses low frequency background noise/light.
Figure 3.3: The effect of several pre-processing techniques: a) CLAHE, b) high-pass filter, and c) intensity clipping.

The most delicate part of a PIV analysis is the cross-correlation. This part also significantly affects the accuracy of PIV. Small sub images (interrogation areas) of an image pair are cross-correlated to draw the most probable particle displacement in the interrogation areas. Basically, through this statistical pattern matching method (cross-correlation), we can find the particle pattern from interrogation area 1 back in interrogation area 2. This numerical method is applied with a discrete cross-
correlation function (DCC). The DCC is well defined for finite regions.

The DCC approach uses interrogation areas of identical size and computes the correlation matrix in the two dimensional domain. In DCC, the interrogation areas 1 and 2 are of different sizes. When interrogation area 2 is chosen larger than interrogation area 1, a particle displacement of up to half the size of 1 will not result in any loss of information; furthermore, it provides a reliable correlation matrix with low background bright/noise. When using the DCC method we are expecting that both the systematic error and the random error of calculation decrease substantially. To speed up the computation time, we use the fast Fourier transform (FFT), which is well defined only for infinite domains. Therefore, in order to apply the FFT, the finite domain has to be extended to an infinite domain. By definition, the FFT method assumes recurring interrogation areas. Therefore they repeat themselves in all directions. When the displacement of the particles is larger than half the size of the interrogation area, the intensity peak in the correlation matrix is folded back into the matrix and will appear on the opposite side of the matrix [21] (see Figure 3.4, FFT with three interrogation areas).

Figure 3.4: Comparison of the velocity field using DCC with one interrogation area and the right one is found using FFT with three interrogation areas.
As a consequence, the displacement of the particles has to be smaller than half the size of the interrogation area. To keep the background information at a low level, it is recommended that we reduce the displacement further to about one quarter of the interrogation area. This approach was further enhanced and refined and the data will be analyzed in several passes: the first pass uses relatively large interrogation areas (64 x 32) to calculate the displacement of the image data reliably. A large interrogation area allows for large particle displacement, and a better signal-to-noise ratio, hence a more robust cross correlation. However, large interrogation areas will only give us a very low vector field. This is why we should decrease the size of the interrogation windows in the following passes. For the second pass, the area is reduced to half (32 x 16) and so is displacement. Finally, the third pass uses interrogation areas of 16 x 8, and as expected, it resulted in a reduction of the displacement. This multiple-passes approach yields a high spatial resolution in the final vector field, together with a high dynamic velocity range, and it increases the signal-to-noise ratio.

Some erroneous vectors will show up due to inadequately illuminated regions in the image flow. These defective vectors can be removed and interpolated by using a number of PIVLab image post-processing techniques that can significantly enhance the quality of the analysis. For example, we used velocity limit and manual rejection. The best results are achieved by a combination of both.

Figure 3.5 shows a sample of a snapshot image at one moment in time from a experimental run. Figure 3.5(a) shows the original image, while Fig. 3.5(b) shows the extracted velocity field after performing PIV analysis. The PIV analysis was performed using pre-processing filters and a fast Fourier transform (FFT) correlation with three passes and deforming windows. The first interrogation area we used is 64 x 32, the second is 32 x 16, and the third is 16 x 8. From this figure we can see the velocity vector field as a grid of 63 x 63 points.
Figure 3.5: Original image taken as a snapshot at one moment in time from an experimental run with associated velocity field extracted using PIV.

Figure 3.6 is an example of the computed forward FTLE field by using the velocity flow field found by PIV (see Fig. 3.5) for one of the flow tank's time-independent multi-gyre flow experiment. The finite integration time used is $T = 6$ and the gridsize is $126 \times 126$.

Figure 3.6: Forward FTLE field computed using PIV velocity field
The red ridges correspond to LCS which separate the phase space into distinct dynamical regions. The red indicates higher FTLE values and the blue indicates lower FTLE values. However, when we compare this result with Figure 2.6 for the theoretical time-independent double-gyre flow, we can see that we do not have definite high FTLE ridges (LCS) between gyres in the experiment. There are a few possible reasons for this, which we are now in the process of studying. One likely cause is due to poor experimental data from using low resolution imaging or a low frame rate. Another possibility is that the low Reynolds flow is somehow affecting the FTLE computation. In order to achieve the expected result, we are continuing to run new, higher quality experiments and are also investigating the effect of Reynolds number.
Lagrangian Coherent Structures (LCS) provide a new way of understanding transport in complex fluid flows. Specifically, the LCS process is based on identification of material lines that play the dominant role in attracting and repelling fluid elements over a selected period of time. It is known that the ridges of high value in the FTLE fields often times matched the LCS in flows. Another accomplishment of modern experimental fluid mechanics is the invention and development of particle-image velocimetry techniques (PIV) which measures space and time resolved flow velocities in fluids. This method provides a way to back out the velocity field from an experiment. Using the velocity fields, FTLE fields can be computed and used to find LCS. Knowledge of LCS leads to improved understanding of the flow dynamics and is tremendously useful in designing control schemes for autonomous vehicles. As noted, our FTLE fields are not as expected. Therefore, we will engage in comprehensive and detailed studies for a number of new experiments, including the testing of different PIV filters and the effect of Reynolds number on FTLE computations. As a result we will be in a stronger position to have more accurate computations of the FTLE fields. Needless to say, these experiments and explorations in the field are essential to the understanding of LCS in stochastic flows, which in return will lead to more
refined and enhanced understanding of a variety of geophysical flows.
Bibliography


