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A Community of Learning in an Elementary School Mathematics Classroom

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A COMMUNITY OF LEARNING IN AN ELEMENTARY SCHOOL MATHEMATICS CLASSROOM

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Abstract

A COMMUNITY OF LEARNING IN AN

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by Megan Louise Roeder

The goal of this study was to investigate opportunities for cultivating a community of learning in an elementary school mathematics classroom using four guiding principles for productive disciplinary engagement. A community of learning involves teachers and students participating equally in negotiating, sharing, and producing knowledge as co-learners, co-teachers, and co-collaborators in the classroom. Characteristics of a community of learning align with effective teaching and learning practices described by national governing bodies and researchers in the field of mathematics education. The essence of a community of learning is beneficial in an elementary mathematics classroom because it invokes deep learning about disciplinary content. Based on the lack of naturalistic studies related to investigating aspects of a community of learning in elementary school mathematics classrooms, this study was conducted in the natural setting of a first-grade elementary school classroom and employs a qualitative case study design. Opportunities for cultivating a community of learning were investigated in the classroom, with no interventions. Data was collected in the form of observations, interviews, and artifacts. After several rounds of data analysis, the results of this study demonstrated missed opportunities for cultivating a community of learning in this first grade classroom based upon the principles. Data from this study illuminated difficulties for the teacher in enacting a curriculum as intended, eliciting student thinking, and maintaining a high level of cognitive demand in the classroom. Implications from this study include investigating ways to support teachers in enacting a curriculum and facilitating discourse in ways that can build on opportunities to cultivate a community of learning in elementary school mathematics classrooms.

Keywords: community of learning, classroom culture, cognitive demand, curriculum, enculturation, elementary mathematics, mathematics education.

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Dedication

I dedicate this dissertation to the most important people in my life, my incredible husband, and amazing children. Ryan, your love and support are immeasurable. You continue to be my rock, you inspire, motivate, and teach me daily about perseverance, commitment and overcoming challenges. You make me strive to be a better person and help me see the best in every situation. To my children, you have been patient, kind and supportive. You inspire me to work harder and do better each day. We can do hard things. I love you.

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CHAPTER 1: INTRODUCTION

In mathematics education, teachers have been encouraged to shift their teaching practices toward helping students develop deeper understandings of mathematical ideas, relations, and concepts (Kazemi & Stipek, 2001). This shift has been influenced by the adoption of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, 2010). The standards were intended to help improve mathematical learning and achievement in the United States. These standards stress conceptual understandings of key ideas and set forth a specific sequence of topics to provide focus and coherence across grade levels (National Governors Association Center for Best Practices, 2010). Over the past several decades, the National Council of Teachers of Mathematics (NCTM) and researchers have developed tools and sought ways to help teachers implement standards and help students achieve deeper levels of mathematical understanding and higher levels of mathematical achievement. In 2014, NCTM shared a set of research-informed actions for all teachers to take to ensure students' mathematical thinking and achievement based on the standards in their publication *Principles to Action*. NCTM (2014) found that despite the adoption of the standards, too few students were achieving high levels of mathematical learning. Among the issues identified that were affecting students' mathematical learning was too much focus on learning procedures without any connection to meaning or understanding, and too many students being limited by low expectations.

To address the issue of mathematical learning being too focused on learning procedures without any connections to meaning or understanding and students being limited by low expectations, NCTM (2014) recommends ways to improve the teaching and learning of mathematics through certain types of student experiences and teacher practices. Students should have experiences learning mathematics which enable them to do the following:

- Engage with challenging tasks that involve active meaning making and support meaningful learning.
- Connect new learning with prior knowledge and informal reasoning and, in the process, address preconceptions and misconceptions.
- Acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations.
- Construct knowledge socially, through discourse, activity, and interaction related to meaningful problems.
- Receive descriptive and timely feedback so that they can reflect on and revise their work, thinking, and understandings.
- Develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance. (NCTM, 2014, p. 9)

In addition to these experiences recommended for students, NCTM (2014) described eight teaching practices to promote effective mathematics teaching and learning (Figure 1). These teaching practices involve teachers providing opportunities for students to think deeply and learn to reason effectively in mathematics classrooms, while making student inquiry and student thinking central to learning. These practices emphasize that mathematics classrooms should be places where students engage in meaningful discourse and share ideas. According to these practices, teachers should pose purposeful questions and give students opportunities to grapple with mathematical ideas. Teachers should provide students with opportunities and time to engage in productive struggle. Additionally, teachers should elicit and use student thinking to guide mathematical teaching and learning.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Figure 1. Eight mathematics teaching practices which can promote effecting teaching and learning (NCTM, 2014, p. 10).

These research-based descriptions of how students should experience learning mathematics and the eight practices recommended for teachers set forth by NCTM (2014) provide evidence of what mathematics teaching and learning should look like in the classroom, and centers on ways to build upon students' thinking and deepen students' learning. This work is supported by other research in mathematics education that describes similar experiences for learners and teaching practices for effective mathematics teaching and learning (Boaler & Brodie, 2004; Carpenter, Fennema, Franke, Levi, & Empson, 2000; Franke & Kazemi, 2001; Lampert, 1990; Schoenfeld, 2016; Stein, Grover & Henningsen, 1996). Student-centered mathematics classrooms are not a new idea, and the experiences and practices described herein apply to the teaching and learning of mathematics across all grades.

Given this information regarding effective teaching practices and student learning experiences, and the shift for teachers to engage with practices aimed at broadening students' conceptual understanding and meaning making in the classroom, I began investigating the culture of elementary school mathematics classrooms. As a former elementary school teacher, I was most interested in understanding the teaching and learning in elementary school mathematics classrooms. Motivated by the knowledge of what effective teaching and learning should look like in mathematics classrooms, I wondered if students were experiencing the kinds of learning described in, and if teachers were engaging in the types of teaching practices set forth by, NCTM (2014) when students begin learning mathematics in school. I sought to investigate the culture of the mathematics classroom in the earliest grades. I found few studies aimed at investigating the culture of the mathematics classroom in early elementary classrooms, kindergarten through second grade as well as research studies conducted related to classroom culture in general.

During my search, I encountered the concept of a community of learning (Brown, 1992; 1997). This concept was attractive to me because it reinforced what NCTM (2014) described in *Principles to Actions*. The conceptual ideas underlying a community of learning focused on deepening students' understanding of concepts (Brown, 1992). One of the ideas of a community of learning is that all members of a classroom community, teachers and students alike, participate in the teaching and learning as co-learners, co-teachers, and co-collaborators in the collaborative construction of knowledge.

Bruner (1996) describes this sort of inquiry-based pedagogy as one of mutuality which "presumes that all human minds are capable of holding beliefs and ideas which, through discussion and interaction, can be moved toward some shared frame of reference" (p. 56). Student inquiry, knowledge and thinking is built upon in a community of learning, which honors my beliefs on teaching and learning and aligns with the effective teaching practices and student learning experiences (NCTM, 2014). Student thinking and student ideas are the centerpiece of a community of learning (Brown, 1997). In a community of learning, students have opportunities to participate in the collaborative construction of meaningful knowledge and to engage with the learning of mathematics in the same ways set forth by NCTM (2014).

Rationale

In reflecting on my own personal experiences learning mathematics as an elementary school student, I have a vivid memory of missing recess in third grade because I needed to learn the standard algorithm for multiplying 2- and 3-digit numbers. I recall that the procedure was meaningless to me, yet I was asked to practice this procedure repeatedly until I mastered it. I was a naturally curious child who loved learning, and genuinely loved learning in school, but I hated the replication of meaningless procedures in math class. I developed angst and ambivalence about learning mathematics, and it depleted my confidence as a student of mathematics. This circles around to my experiences as a teacher.

When I began teaching elementary school mathematics, I encountered those same feelings of angst and ambivalence (likely having roots in the experiences I described above). As I gained experience teaching mathematics, I became more knowledgeable about the practice of doing mathematics, which eventually led to a change in my perspective. I sensed a similar angst and ambivalence about learning mathematics among many of the students with whom I shared the classroom. I also heard similar sentiments among colleagues about teaching mathematics. Based on these experiences, I found myself wanting to understand more about the teaching and learning of mathematics, and the experiences of students and teachers in elementary school mathematics classrooms, thereby contributing to the focus of this dissertation research.

As a teacher and parent, I have had experiences with children which helped me to recognize the natural inquisitive dispositions of children. Children are vessels of inquiry and we are all born with a natural sense of curiosity. Piaget (1964) observed this curiosity as a natural human quality that begins at infancy. According to Piaget (1964), infants acquire knowledge through sensory experiences and manipulating objects, much like scientists, and this curiosity carries through stages of cognitive development as they grow and mature. Anyone who has ever been around young children has likely witnessed this themselves. I recall watching my own young children, when they were first trying to make sense of their world, as they would investigate and explore. They would observe, probe, test, fail and try again. This natural curiosity and inquisitive nature of children can be built upon in the classroom. Philosophers and psychologists such as Bruner, Dewey, Piaget, and Vygotsky have acknowledged and investigated how children learn through interacting with objects, their environment, and those around them. Dewey believed inquiry to be the basis for discovery and learning (Artigue & Blomhøj, 2013). Ball (1993) recognized that the naturally curious disposition of young children can provide special insights in mathematical learning environments and the role of the teacher should be to listen to their ideas. According to Ball, “the things that children wonder about, think about, and invent are deep and tough, and learning to hear them is, I think, at the heart of being a wonderful teacher” (p. 374).

When I was teaching elementary school, I noticed that my students seemed to slowly abandon their natural curiosities and inquisitive nature in the classroom. I thought about how students experienced mathematics in the classroom during their first few years of school. I thought about how important inquiry is in doing and learning mathematics. Inquiry is an important aspect of doing and learning mathematics, as it invites students to work in ways similar to mathematicians (Artigue & Blomhøj, 2013). Inquiry relates to what Schoenfeld (1992) describes as thinking like a mathematician: observing, making a conjecture, testing, failing, and trying again. Mathematics classrooms should be environments in which students are encouraged to discuss ideas with one another, take intellectual risks, feel that their thinking is valued, and be provided with sufficient time and opportunities to explore and explain their thinking (NCTM, 2000). Mathematics classrooms should not be places in which students simply practice demonstrated algorithms, much like what I experienced myself as a third-grade student. Rather, students should be presented with tasks that demand inquiry and build on their thinking, as advocated for in many documents from national governing bodies (e.g., NCTM, 2014). Students should engage in tasks which require them to construct meaning and decide what to do and how to do it, and then be provided with opportunities to reflect on the reasonableness of their actions. Classrooms that support this kind of learning become “communities in which mathematical sense-making of the kind we hope to have students develop is practiced" (Schoenfeld, 1992, p. 13). Supporting student inquiry and student thinking in mathematics classrooms is important for the teaching and learning of mathematics.

I believe that the role of the teacher is not to impart knowledge on students, rather to create opportunities for students to engage in meaningful learning. Teachers can create opportunities for students to engage in meaningful learning by providing tools, experiences, and space for students to investigate and discuss their own ideas. Teachers need to cultivate a classroom environment in which students can develop the skills of asking questions and seeking their own answers, a culture of student learning through inquiry and exploration. My experiences as a teacher, parent, and student have contributed to an idyllic vision I hold of a classroom culture that fosters a child's natural inquisitions, builds upon their own ideas for investigation, gives them tools they need to creatively explore those ideas, and helps them develop authentic intellectual curiosity, while providing challenge and creating opportunities for students to experience productive struggle. In many ways, the effective mathematical teaching and learning practices described by NCTM (2014) align with my vision. I believe fostering a community of learning in an elementary school mathematics classroom may help to achieve this type of teaching and learning. I designed this study to help me better understand how an early elementary classroom could foster a community of learning to support students in retaining and developing their intellectual curiosity around mathematics. This understanding is meaningful to me because of my own experiences as a student of mathematics, elementary school teacher of mathematics, and parent of small children. If we can better understand early elementary mathematics teaching and learning environments, perhaps we can help teachers to cultivate communities of learning in their classrooms.

Significance

Learning to think deeply is a beneficial skill for all students, across all grade levels. For students to think deeply, they must become productively engaged in disciplinary issues (Engle & Conant, 2002). Engle & Conant (2002) devised four guiding principles for fostering productive disciplinary engagement in a community of learning classroom: (1) problematizing content, (2) giving students authority, (3) holding students accountable to others and to disciplinary norms, and (4) providing relevant resources. Classrooms that promote productive disciplinary engagement reinforce these four principles. Similar to NCTM (2014), these principles may also support current goals in mathematics education that focus on creating learning environments in which students are engaged in disciplinary work and have opportunities to explore and explain their own mathematical ideas.

The *Principles to Actions* (NCTM, 2014) provide us with a clear picture of what mathematics teaching and learning should look like across grade levels, including the experiences students should have when learning mathematics and the practices teachers should engage in while teaching mathematics. These principles also help illuminate what effective learning and teaching should look like in mathematics classrooms to improve student understanding and achievement based on the standards of mathematical practice (National Governors Association Center for Best Practices, 2010). We also know that young children have a natural inquisitive disposition and bring a natural curiosity to the classroom. We should build on this inquiry, which is important to the learning of mathematics. Moreover, student thinking should be a centerpiece of the classroom and teaching and learning should build on that thinking. What we have yet to fully understand is what is happening in early elementary mathematics classrooms (e.g., Ball, 1993; Lampert, 1990; Wood, Williams & McNeal, 2006; Yackel & Cobb, 1996).

What is happening in mathematics classrooms when students start school? With no tools or interventions, without involving a teacher in a certain kind of professional development program, without imposing a reform-based curriculum, what does the teaching and learning look like in the natural setting of an elementary school mathematics classroom? This study seeks to investigate the culture of an elementary school mathematics classroom and the experiences of the teacher and students in that classroom in their natural setting, with no interventions. I do this by leveraging the idea of cultivating a community of learning based on the four principles for fostering productive disciplinary engagement (Engle and Conant, 2002) that aligns with the experiences and practices for effective mathematics teaching and learning presented herein (NCTM, 2014). To accomplish this goal, my study seeks to answer the following research question: What opportunities are there for cultivating a community of learning in an elementary school mathematics classroom based upon the principles for productive disciplinary engagement?

CHAPTER 2: REVIEW OF LITERATURE

Bruner (1996) describes the living context of education, the classroom, as the place where teachers and students convene to pursue the “crucial but mysterious interchange we call education” (p. 44). To investigate the teaching and learning in an elementary school mathematics classroom, it is important to consider all elements of the classroom environment and situate the investigation in its naturalistic setting. I begin this chapter with a summary of the evolution of social learning theories which characterize learning as a social process. Thereafter I provide a description of a community of practice and bridge that description with what has been referred to as a community of learning, summarizing various ways that researchers have conceived of a community of learning in mathematics classrooms. Finally, I leverage various characteristics of communities of learning with the goal of invoking a specific focus for further investigations of teaching and learning in elementary school mathematics classrooms, maintaining the integrity of engaging with mathematics as a dynamic process of discovery.

Constructivism and Social Learning Theories

Learning is the active process of constructing new knowledge and building upon prior knowledge. According to von Glasersfeld (1991), “knowledge is not passively received but actively built up by the cognizing subject” (p. 232). Constructivist learning perspectives are rooted in several main ideas. First, knowledge is actively constructed by the learner, it is something done *by* the learner not imposed *on* the learner. Second, learners bring their own existing knowledge and ideas about phenomena to a learning situation. Third, although learning is individual, the way knowledge is constructed by an individual is through their interactions with others and the physical world (Sjoberg, 2007). A constructivist perspective on learning embeds knowledge with belief which can create meaning for learners (Confrey, 1990).

Mathematical learning should be meaningful, and a constructivist-oriented mathematics classroom can promote meaningful mathematical learning (Confrey, 1990). In a constructivist-oriented mathematics classroom, students actively participate in constructing knowledge and creating meaning through experiencing, exploring, observing, questioning and thinking, as well as making connections, probing, making mistakes, conjecturing, collaborating, and interacting with people and phenomena (Brown, 1992; Lampert, 1990; Schoenfeld, 1992). During learning, the construction and building of knowledge draws upon our interactions with others and the world around us. Piaget described this interaction as a process of decentering our thinking from the egocentric view to consider our thinking from the perspective of others (Brown, Metz & Campione, 1996; Piaget, 1947/2001).

Interacting with people and phenomena as part of the learning process illustrates that learning is in fact a social activity. During the 1970's and 1980's, a shift occurred in the field of mathematics education and researchers began to consider learning mathematics from a social perspective rather than from an individual cognitive perspective. Lakatos (1976) discussed this logical progression of teaching and learning mathematics in *Proofs and Refutations.* This work is important because it challenged formalism in mathematics and emphasized mathematics as a dynamic process of discovery. This work also represents the beginning of a shift in the conception of the teaching and learning of mathematics. Concerned with the philosophy of mathematics, Lakatos presented his publication as a series of dialogues between a teacher and students. In the teaching and learning of mathematics, this was a new idea, moving from an individual cognitive to a social view of learning that involves both teacher and students. Lakatos' work represented a humanistic view of mathematics as "a quasiempiricist enterprise of the community of mathematicians over time rather than a monotonically increasing body of certain knowledge" (Lerman, 2000, pg. 22).

A few years after Lakatos' (1976) *Proofs and Refutations*, Vygotsky (1978) presented a sociocultural theory of cognitive development, which described how learning can *only* occur in conjunction with one's social experience and asserted that the cognitive development of children is advanced through social interactions with other people, particularly those who are more skilled. Vygotsky believed that children constructed knowledge actively and described learning as a social experience requiring great mental effort of internalization. More specifically, "learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (Vygotsky, 1978, pg. 35). Vygotsky identified the *zone of proximal development* of a child, the space between a child's actual development level (based on their ability to problem solve independently) and their potential development level (under the guidance of a teacher or in collaboration with more capable peers). The zone of proximal development depends upon a child's social interactions and initializes as a collaborative interaction, gradually becoming internalized to the child's independent developmental achievement (Brown et al., 1996). The work of Lakatos regarding the nature of learning mathematics, and the work of Vygotsky regarding the zone of proximal development, influenced the shift from an individual cognitive view of learning to a social view of learning.

Building on the idea that learning is social, Lave (1988) presented a new theory on learning referred to as *situated learning theory*. Lave's theory on learning described how knowledge and learning was dependent upon the community, culture, participants, and their motives, and meaning. Lave's theory relates to the work of Vygotsky (1978), since both theorists recognized that learning depends on one's social interaction with the environment and with others. Lave's work also relates to the work of Lakatos, in that both Lakatos and Lave focused on the idea of learning in authentic ways. While Lakatos was focused on learning mathematics in authentic ways, as a process of discovery, Lave emphasized learning through social interaction and collaboration in an authentic context. Lave supposed the idea of cognition in practice, that knowledge is in a constant state of change based on social, cultural, and historical systems of activity. Lave focused on situated knowing, or knowledge as a form of situated experience, wherein learning is seen as increasing participation in practice, gradual attainment of mastery and development of identity.

This changing view of learning from an individual cognitive process to learning as a social process was applicable across all disciplines including mathematics. Lerman (2000) refers to this changing view of teaching and learning in mathematics education as "the social turn" (p. 19). Lerman characterizes this movement in 1988 as a time when mathematics education researchers began to see meaning, thinking, and reasoning as products of social activity. It was a time when anthropology, psychology, and sociology converged to create a new lens for viewing teaching and learning. This lens for viewing the teaching and learning of mathematics makes sense as one considers the principles for effective teaching and learning of mathematics (NCTM, 2014). These principles are supported by mathematics education research that was conducted over the past two decades related to the ideas of learning as a social activity. The principles are focused on helping students develop meaning, thinking, and reasoning skills in the social context of the mathematics classroom. Principles such as facilitating mathematical discourse, eliciting student thinking, and engaging students in collaborative learning and collective productive struggle, serve as a guide for mathematics teaching today. In the following sections of this paper, I explore two concepts that emerged from these social constructivist perspectives on learning:

communities of practice and communities of learning.

Communities of Practice

An important product of the social turn is the idea of cognition in practice and learning as social participation in a community of practice. A series of publications beginning with Lave's *Cognition in Practice* (1988), followed by Lave and Wenger's S*ituated Learning: Legitimate Peripheral Participation* (1991) and leading to Wenger's *Communities of Practice* (1998) shows the development of the researchers' ideas that emphasized the social nature of human learning and exploration of learning as participation in a community. Lave's conceptions of the situational aspect of learning led to the emergence of yet a new theory of learning centered around the idea that learning occurs through engagement in social practice known as a "community of practice." While Lave and Wenger each credit the other with coming up with the term "community of practice" in the late 1980's, Wenger has been committed to developing the concept more extensively over the past two decades (Wenger, 2010).

The underlying theory of a community of practice is that learning is a social system that occurs at the intersection of four necessary components: *community, social practice, meaning* and *identity* (Figure 2).

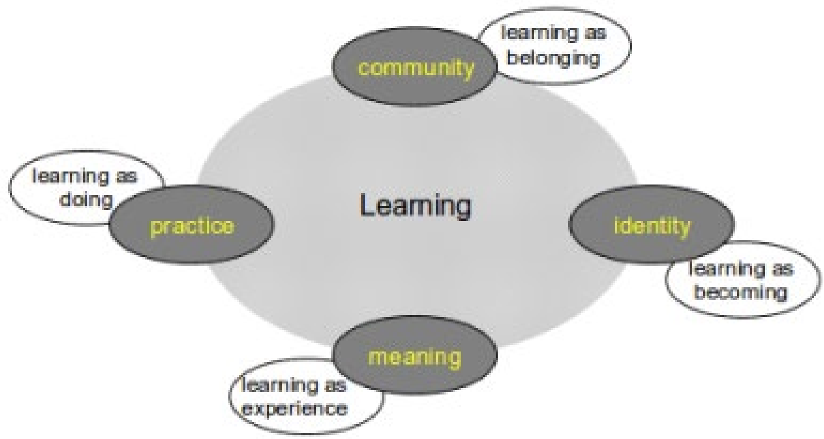

Figure 2. Wenger's components of a social theory of learning (Wenger, 1998, p. 5).

A community of practice is such a social learning system, and Wenger's theory primarily focuses on the patterns of participation and reification that constitute learning. Wenger (1998) describes participation as the experiential process of taking part in the community of practice, while reification gives form to that experience through "objects that congeal this experience into thingness" (p. 58).

Wenger (1998) describes learning which involves membership in a community of practice as personally transformative. A group of people who are mutually engaged in an activity, sustained by a joint enterprise, and who have a shared repertoire of customs for practice, cohere to form a community of practice. These are the three characteristics that define a community of practice. *Mutual engagement* refers to how a group's interactions shape the group's culture and practices. It is concerned with the ways in which members establish norms and build collaborative relationships through their participation in the community. These relationships bind the members of the community together as a social entity. A *joint enterprise* is the collective process of negotiation and mutual accountability in pursuit of a common goal, like rhythm to music. Rhythm is what makes music move and flow, and a joint enterprise is what makes a community of practice move and flow. Members create shared understandings through negotiations with one another and through this process, the community establishes mutual accountability. Finally, a *shared repertoire* is a set of communal resources, such as words, tools, ways of doing things, or concepts that the community has produced, which have become part of their practice. A shared repertoire is used in pursuit of a community's joint enterprise and can be symbolic, such as a gesture that is meaningful to the practice, or literal, such as a certain word that is meaningful to the practice.

Guided by these three characteristics, a community of practice involves learning through social participation which can occur almost anywhere. A community of practice can exist in many different settings such as an office, a factory floor, even in a virtual community. Communities of practice exist in government organizations, educational institutions, and professional associations. The locus of learning for a community of practice does not have to be a classroom (Wenger, 2010). In the next section, I discuss how the classroom can be considered the locus of learning through social participation in a community. While Lave and Wenger were conceptualizing a community of practice, Brown (1992) was conceptualizing the idea of a community of learners within the classroom, placing a focus on the complexities of learning as a social system within the classroom.

Communities of Learning

The birth of the term *community of learning* is attributed to Ann Brown (1992) when she embarked on her ground-breaking research known as the "Community of Learners" project. Brown's research project will henceforth be referred to as FCL (Fostering Communities of Learners). The goal of FCL was to design and study a classroom in which students could become actively involved in their own learning. Brown partnered with her husband, Joseph Campione, for the latter years of this project, but initially began her research solo in her laboratory. Inspired by learning in situ, the idea that "knowledge is situated, being in part a product of the activity, context, and culture in which it is developed and used" (Brown, Collins & Duguid, 1989, p. 32), Brown decided to relocate her research from the laboratory. She relocated her research to urban classrooms that included children between the ages of 6 and 12. As the FCL project developed, Brown committed herself to conducting research that situated learning in context. Brown (1992) describes this work as being "devoted to the study of learning in the blooming, buzzing confusion of inner-city classrooms" (p. 141). In fact, Brown (1992) states that this change in her research from earlier approaches of conducting research with individuals in a laboratory to the classroom was "an awakening to the fact that real-life learning inevitably takes place in a social context, one such setting being the classroom" (p. 144). Brown's commitment to situating her study in the context of the classroom demonstrates the shift in research to studying teaching and learning from a social learning perspective.

FCL involved a series of studies that focused on guided instruction and assessment in the social context of the classroom. A fundamental tenet of FCL is that "students have a right to understand, evaluate and orchestrate their own learning" (Brown & Campione, 1994, p. 270). Brown's work commenced with studying individual children's metacognition one-on-one in the laboratory and progressed to studying children in groups in resource rooms outside the classroom. After this work, Brown moved to studying naturally occurring reading groups within the classroom, and finally arrived at studying reading comprehension groups within the science classroom (Brown, 1992; 1997). The early part of this research investigated how an instructional technique called *reciprocal teaching* could help students with reading comprehension. Reciprocal teaching is an intervention that involves providing students with four specific reading strategies to promote comprehension: questioning, clarifying, summarizing, and predicting. These four strategies promote dialogue between teachers and students which help them to construct meaning (Brown, 1997). As the study progressed and Brown moved her research from the laboratory to the classroom, her goal morphed into investigating how fostering a community of learning in the classroom could enrich the knowledge base of the whole classroom community (Collins, Joseph & Bielaczyc, 2004).

Brown (1997) describes four crucial ideas underlying the FCL project: (1) agency, (2) reflection, (3) collaboration, and (4) culture. These crucial ideas were pointed out by Bruner (1996) after he visited the FCL classrooms which Brown (1997) had designed. *Agency* refers to the consistent ways in which teachers, students, researchers, and children actively engaged in learning by searching for understanding, putting forth effort and finding meaning. *Reflection* refers to the insight learners developed regarding their own strengths and weaknesses, a type of knowledge and control over their thinking that has often been referred to in research as metacognition. Reflection involved learners developing the ability to appreciate good questions and critically evaluate their answers. The third idea, *collaboration*, refers to how expertise was distributed and how members of the learning community depended on one another, which created an atmosphere of shared responsibility and mutual respect. The fourth idea related to the *culture* of learning, negotiating, sharing, and producing. Brown (1997) describes the culture as a way of membership into a community, with shared discourse structure, goals, values, and belief systems.

FCL was primarily concerned with the design of learning environments that would allow students to engage with deep disciplinary content and be active in their learning. This project additionally resulted in creating a culture of learning, negotiating, sharing, and producing work that would help students to become "independent, self-motivated critical thinkers able to take responsibility for life-long learning" (Brown & Campione, 2002, p. 120). By using methods such as reciprocal teaching and the jigsaw method for cooperative learning,[1] Brown sought to distribute expertise among all learners within the community, including teachers and students. She remarks that the children were responsible for mastery of content: "Expertise did not rest with a single authority figure, the teacher; it was distributed throughout the classroom" (Brown & Campione, 2002, p. 124). While working with different teachers and students during their FCL project, Brown and Campione (1994) described characteristics for their ideal classroom. These characteristics included the following:

- Individual responsibility for learning coupled with communal sharing
- Ritual, familiar participant structure
- A community of discourse
- Multiple zones of proximal development
- Seeding, migration and mutual appropriation of ideas

The first characteristic, individual responsibility for learning coupled with communal sharing, refers to how students and teachers each have ownership of expertise, but not one person has it all. This means that not one person possesses needed knowledge, rather, members of the community each share individual responsibility for finding out and uncovering knowledge through collaborative learning. Individual responsibility coupled with communal sharing resonates with the idea of teamwork, that members are responsible for their own individual expertise but it is the pooling of that expertise that benefits the community and leads to a

[1]The jigsaw method refers to a research-based cooperative learning technique invented and developed in the early 1970s by Elliot Aronson and his students at the University of Texas and the University of California that positions each student as an expert on a subtopic (See jigsaw.org for more information).

richness in collaboratively constructed knowledge. The second characteristic, ritual, familiar participant structure refers to repetitive, ritualistic activities which are an essential aspect of a classroom and help children recognize a participation structure. Although there is a familiar participant structure in terms of activities, students and teachers understand that within this structure, there is an expectation of discovery. A community of discourse, the third characteristic, is essential to a community of learning classroom in that this is the way in which constructive discussion, questioning and criticism lead to the common voice of the community, the negotiation of terms and definitions, thoughts and ideas in pursuit of the collaborative construction of knowledge. Multiple zones of proximal development refer to the inclusion of people, adults and children with varying levels of expertise. Finally, the seeding, migration and mutual appropriation of ideas refers to how the learning environment is first "seeded" with ideas, concepts that the community values and harvests. These ideas or concepts migrate among participants and persist over time. At any time, members of the community can appropriate these ideas or concepts, transforming them via interpretation in multiple directions (mutual appropriation). Specifically, the idea of mutual appropriation refers to how ideas and concepts are not limited to the process of the child learning from the adult, rather learners of all ages and expertise, seeding the environment with ideas, and contributing to the migration and appropriation of those ideas with one another in the collaborative construction of knowledge (Brown & Campione, 1994). These characteristics embody the elements of a social theory of a learning (Wenger, 1998) and align with how researchers began to view teaching and learning from a social lens after Vygotsky (1978) presented his theory on learning through social interaction. There is continuity in the research of looking at teaching and learning from the social perspective and both the concept of a community of learning and the concept of a community of practice emphasize learning as a social practice.

Similarities and Differences

Although there are connections between the constructs of a community of learning and a community of practice, there are also important distinctions. In particular, the ways in which learning, responsibility, shared values and beliefs, and identity play roles in both communities has been key to my theoretical perspective for this research. During my investigation of communities of practice and communities of learning, I have identified some similarities and differences.

1. Learning - Learning in a community of practice is not intentional. A community of practice is defined as groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly (Wenger, 1998). Though learning may occur, it is not an intended outcome. In a community of learning however, learning *is* the intended outcome. The goal of establishing a community of learning is described by Brown (1992) as "setting up a classroom ethos that would foster self-reflective learning" (p. 143). At the heart of the FCL project is the concept of "learning to learn" or helping students develop the ability to engage in intentional, self-directed learning (Brown, 1997).
2. Responsibility - Both a community of practice and a community of learning emphasize the responsibilities of its members. There is an expectation within both communities that its members (students, teachers, practitioners) exercise their right to understand, evaluate and orchestrate their own ideas and the ideas of others. No member is given more responsibility than others, everyone is equally responsible for meaning making in the community. Members of both communities have a responsibility as stakeholders in the

community. Meaning is negotiated among members of the community, so each participant is responsible for their role and engagement within the community.

3. Shared values and beliefs - As social systems, both a community of practice and a community of learning seem to inevitably lend themselves to the natural establishment of a set of shared values and beliefs. Though the community may not start out this way, through the process of engagement with one another, members of the community establish these shared values and beliefs. These may be referred to as norms, or patterns of behavior practiced within the community, or even an established discourse within the community (modes of questioning, discussing, and critiquing) that reflect a set of values and beliefs.
4. Identity - While a community of practice and a community of learners are focused on the collective, the collaborative and the community, each participant has their own identity. According to Brown (1997) the very nature of interdependence which exists within a community promotes “joint responsibility, mutual respect and a sense of personal and group identity” (p. 411). The community contributes to the development of each participants’ individual identity, just as each participants’ individual identity contributes to the evolution of the community.

These two concepts, a community of practice and a community of learning, are similar yet different. Similarities include that they both relate to learning, emphasize the responsibilities of participants, lend themselves to establishing shared values and beliefs, and consider identity in relation to the collective group. The main way that they are different is that within a community of practice, the learning is not intentional. In a community of practice, learning happens naturally through changes in participation and reification. On the other hand, in a community of learning,

learning is intentional. In fact, learning goals and learning outcomes play a major role in the classroom.

The four areas (1) learning, (2) responsibility, (3) shared values and beliefs, and (4) identity, intertwine to contribute to what Bruner (1996) described as the living context of a classroom community. Since learning is an intended goal in the classroom and learning is an intended goal when fostering a community of learning, this study focuses on the opportunities that exist for fostering a community of learning in the mathematics classroom. In the next section, I share literature regarding how aspects of a community of learning have been conceptualized and investigated in mathematics classrooms.

Mathematics Classrooms as Communities of Learning

To continue Brown's (1992) work, Engle and Conant (2002) built upon the FCL project by devising four guiding principles for fostering productive disciplinary engagement within a community of learning classroom. These four principles for fostering productive disciplinary engagement have appeared in research in mathematics teaching and learning in several ways and under different guises. The four principles Engle and Conant describe abstract features of a community of learning in order to explain student engagement. The four principles include (1) problematizing content, (2) giving students authority, (3) holding students accountable to others and to disciplinary norms, and (4) providing relevant resources.

Engle and Conant (2002) devised these principles through their analysis of a case of productive disciplinary engagement in an FCL classroom. Their goal was to abstract a set of underlying principles from the case that would help account for productive disciplinary engagement within the case and be general enough to be used for understanding other cases and to help guide future design efforts. Engle and Conant (2002) emphasize the potential role of these principles in informing decisions of practitioners as they work to engage students with the ideas and practices of a discipline.

Engle and Conant (2002) presented examples of each of these principles using case studies involving two fifth-grade environmental science classes that were observed over the course of four months. The classrooms were designed and organized in accordance with Brown's FCL classrooms. The researchers observed and videotaped groups of students to trace their learning throughout an "Endangered Species" unit that was created in accordance with FCL. The unit was designed to focus the students' attention on the guiding question, "How do animals survive?" This work occurred in the environmental science classrooms included an intervention, the FCL-based science curriculum. Engle and Conant (2002) note in these classrooms, student engagement was fostered by a teacher who recognized the value of student engagement and was supported by certain aspects of the FCL design. In these classes, evidence of student engagement was seen through students' development of sophisticated arguments, students' process of using evidence to support their claims and awareness of the importance of using evidence to support their claims, students posing deep, new questions about disciplinary content, and students engaging in ongoing discussions and passionate debate. Engle and Conant hypothesize that the embodiment of the principles in these classes helped foster this sort of student engagement.

For the remainder of this section, I will showcase research in mathematics education that are related to these four principles. It is important to point out that the research projects I discuss do not fit singularly into one principle and not another, that is, the research projects I discuss exemplify several principles simultaneously.

Problematizing Content. The first principle, problematizing content, refers to the ways in which teachers "encourage students' questions, proposals, challenges and other intellectual contributions rather than expecting that they should simply assimilate facts, procedures, and other answers" (Engle & Conant, 2002, p. 404). This ties into the notion of learning mathematics through a dynamic process of discovery, which is built upon student inquiry. Teachers can help students problematize what they are studying by encouraging them to elicit their curiosities and sense-making skills. Teacher actions described in the principles for effective teaching and learning (NCTM, 2014) relate to the principle of problematizing content. First, NCTM (2014) describes that through the teacher action of posing purposeful questions, teachers can assess and build on students' sense making about mathematical ideas. Encouraging students to explain and reflect on their thinking can lead to meaningful mathematical discourse, which in turn relates to students being actively involved in reasoning, making sense of mathematical ideas, and problem solving in a way that allows them to develop deep understanding. Discussing strategies for solving, providing justifications for their solutions, connecting new ideas with prior knowledge and thinking about and attending to the reasoning of others, are other ways students become actively involved in their learning and in problematizing content. When teachers encourage students to engage with mathematical learning in these ways and encourage students to consider varied approaches and strategies when making sense of and solving mathematical problems, teachers give students responsibility for making sense of problems and for making connections among mathematical ideas. Teacher actions of eliciting student thinking, facilitating student discourse and giving students time to grapple with mathematical ideas, can be beneficial to students in helping them to develop their abilities to reason, make sense of mathematical ideas, and engage in meaningful mathematical learning. The principle of problematizing content in a community of learning classroom relates to research conducted by Lampert (1990), Boaler and Greeno (2000), Schoenfeld (1992), and Franke et al. (2009).

Lampert (1990) focused on creating a classroom community that would help students make mathematical learning meaningful and achieved this through the development of moral qualities and virtues among her students. Lampert does not use the term community of learning in her research, rather she uses the terms culture and classroom community. Lampert's research relates to a community of learning in that the culture and classroom community Lampert describes in her study share many of the same constructs as a community of learning. In her work, Lampert emphasized developing a mathematical community of discourse in which students learned to appreciate and construct good questions and evaluate their answers. This relates to one of the tenets of a community of learning, reflection, described by Brown (1992). Reflection according to Brown (1992), involves learners developing the same abilities described by Lampert (1990) (abilities to ask good questions and evaluate answers). Brown describes the classroom of her FCL project as an intentionally metacognitive environment with an atmosphere of wondering, querying, and worrying about knowledge. In creating a mathematical community of discourse, with students as shared stakeholders, with a shared set of values and beliefs, engaged in the negotiation of mathematical meaning making, Lampert (1990) was cultivating what Brown described as a community of learning.

Lampert (1990) conducted her research as a case study in her own fifth grade mathematics classroom, in which she attempted to bring the ideas of what it means to do mathematics in the discipline with the learning of mathematics in elementary school. In particular, she focused on helping her students to develop "the moral qualities of the scientist" as described by Pólya (1954). Pólya (1954) described the inductive attitude of an individual regarding science and in doing mathematics. According to Pólya (1954) we need three things for an inductive attitude:

> First, we should be ready to revise any one of our beliefs. Second, we should change a belief when there is a compelling reason to change it. Third, we should not change a belief wantonly, without some good reason (p. 8).

These three things, as Pólya (1954) states, requires unique qualities. To be able to revise any one of our beliefs, requires us to have “intellectual courage” (p. 8). To be able to change a belief when there is good reason to change, requires us to have “intellectual honesty” (p. 8). And lastly, to not change a belief wantonly, without some good reason requires us to have “wise restraint” (p. 8). This intellectual courage, honesty and wise restraint relate to the idea of metacognition. Students need to be aware of their learning within a discipline and how they participate in that learning (Brown, 1992). These qualities can support students in a community of learning, especially in terms of the principle of students problematizing content. When students have intellectual courage, honesty, and wise restraint they are positioned to make meaningful intellectual contributions to the community. Lampert (1990), drawing upon the ideas of Pólya, as well as the ideas of what it means to know mathematics in the discipline and notions of how new knowledge is developed in the discipline (Lakatos, 1976), identified courage and modesty as being essential to engaging in mathematical activity in the classroom and attempted to develop these qualities in her students. Courage and modesty in the context of the mathematics classroom refers to traits needed for students to share and explore their thinking, admit their vulnerabilities and to accept that they need to revise their thinking alongside others in the pursuit of determining a mathematical truth. Lampert asserted that the culture of the classroom community should resemble the mathematical practice of a community of mathematicians. She states:

> My research examined whether it was possible to make knowing mathematics in

> the classroom more like knowing mathematics in the discipline. My organizing ideas have been the "humility and courage" that Lakatos and Pólya take to be essential to doing mathematics. I have treated these as social virtues, and I have explored whether and how they can be deliberately taught, nurtured, and acquired in a school mathematics class. I concluded that these virtues can be taught and learned. What has been described here thus is a new kind of practice of teaching and learning, one that engages the participants in authentic mathematical activity. (Lampert 1990, p. 59).

Lampert (1990) explicitly states that she was able to engage her students in authentic mathematical activity. In other words, Lampert was able to help her students to engage with mathematics in the classroom in ways consistent with the discipline of mathematics. Lampert was able to do this because it was her deliberate intention as a knowledgeable instructor of mathematics. Lampert's work can be directly correlated with Engle and Conant's (2002) principle of problematizing content. Problematizing content is described as eliciting students' sense-making skills. Engle and Conant state that in the process of trying to resolve substantive problems, students become engaged in the process of refining ideas, generating questions and reorganizing understandings. For Lampert, when the teacher and students, all members of the classroom community, shared the same set of values (e.g., courage, modesty, open-mindedness, vulnerability, and respect among participants), the classroom community was rich with meaningful mathematical learning. She states that "the students had learned to regard themselves as a mathematical community of discourse, capable of ascertaining the legitimacy of any member's assertions using mathematical form of argument" (Lampert, 1990, p. 42). Lampert's focus on creating this mathematical community and drawing on this community of discourse enabled her students to problematize the content. In her classroom community, using the words "know, think, revise, explain, problem, and answer" was a normative practice among students. She required the use of the word "hypothesis" and students were encouraged to challenge one another's thinking in respectful ways. According to Lampert, classroom culture involves not just teaching content, but teaching students how to participate or engage with content. This participation or engagement with content is what Engle and Conant describe in this first principle.

Participation or engagement with content was also the focus of Boaler and Greeno's (2000) research where they studied how students made intellectual contributions in mathematics classrooms in the form of questioning, challenging, proposing, and critiquing as opposed to assimilating facts. Boaler and Greeno (2000) focused their research on the practices of learning mathematics and how these practices define the knowledge that is produced in mathematical classrooms. They studied this by investigating high school students' perceptions of their mathematics classroom environments and characteristics of the kinds of mathematical learning with which they engaged. Through this work, the researchers also measured students' beliefs about mathematical positioning in their worlds and attempted to understand students' identifications with mathematics. Their research was motivated by wanting to investigate the disparity in numbers of students pursuing mathematical related fields after completing high school. They wanted to see if there was any information about students' identification and experiences with mathematical learning that might relate to the disparity. Based upon their findings, they recommend engaging students in discussion-based mathematical problem solving processes rather than creating a community where students are "excluded from the negotiation or development of procedures," "restricted in their application of selves," and to create a community where students' ideas, inventiveness, and agency are valued (Boaler & Greeno, 2000, p. 189). The relationship between this work and a community of learning lies in the principle of problematizing content, which focuses specifically on creating opportunities for students to participate actively in resolving substantive problems.

While Boaler and Greeno's (2000) study involved high school students, the implications of their work may apply to other mathematical learning environments. Boaler and Greeno demonstrated in their work that the nature of student participation is a major factor in how students experience mathematical learning. Their findings call upon researchers to further investigate social practices in mathematical learning communities and they believe that studying mathematical learning environments could provide researchers with important insight regarding the development of identity and agency in mathematics classrooms. Schoenfeld (1992, 2016) also refers to engaging students in a mathematical learning environment in the classroom that gives students practice in thinking mathematically. Schoenfeld generalizes these ideas to all school mathematics, elementary through high school and even beyond. Schoenfeld contends that if we want all students of mathematics to develop habits and dispositions of interpretation, sense-making, and mathematical thinking, then we need to create environments, or communities of practice, which support that kind of learning (at any grade level).

These ideas presented by Schoenfeld (1992, 2016) relate to the principles of problematizing content, accountability, and authority. In 1992, Schoenfeld discussed problem solving, metacognition and sense making in terms of all school mathematics. He outlined concepts and summarized research around mathematical thinking and problem solving that are applicable across grade levels yet drew these conclusions based largely on research conducted in secondary mathematical learning environments. Specifically, Schoenfeld considered making mathematical learning meaningful and authentic for students. This is a concept that could also be considered in elementary classrooms, however I have not been able to identify much research that relates to studying mathematical learning environments in elementary school other than intervention based studies cited previously and in this chapter. Schoenfeld has looked at how students' experiences with mathematics as they engage in problem solving ties to their ability to think mathematically and influences their point of view. Schoenfeld has suggested helping students make sense of important mathematical concepts by using a broad range of open-ended, exploratory type problems or exploratory situations, related to the principle of problematizing content. He also suggested deflecting teacher authority to the student community, related to the principle of authority, and developing a critical sense of mathematical argumentation within the classroom community, related to the principle of accountability, where students can reflect or reject proposals made by class members as a community. Schoenfeld has contended that such a classroom community garners meaningful learning among students and enculturation into mathematical thinking. Schoenfeld uses the "community of practice" term to describe this environment. A community of practice in which students are actively involved in sense-making during problem solving, as described by Schoenfeld (1992), promotes productive disciplinary engagement and relates to the four guiding principles of Engle and Conant (2002). The process of making sense of a problem and attempting to solve it involves eliciting curiosities, refining ideas, generating questions, reorganizing initial understandings and testing conjecture. This process is what Engle and Conant describe in the principle of problematizing content, which is giving students the opportunity to actively participate in resolving substantive problems.

When students have opportunities to actively participate in problem solving, they also apply the important aspect of sense-making (NCTM, 2014). Mathematics classrooms that promote effective mathematics teaching and learning should include teaching that "engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 5). The ways in which teachers elicit student thinking and build on student thinking in the classroom enables students to develop mathematical understandings and make sense of mathematical concepts (Franke & Kazemi, 2001). Developing mathematical understanding and making sense of mathematical concepts involves students having opportunities to problematize content. When students have opportunities to refine their ideas, generate questions and reorganize their understandings, they become productively engaged in understanding disciplinary content (Engle & Conant, 2002).

An example of a mathematics program that centers on creating opportunities for meaningful problem-solving and learning in elementary school mathematics classrooms is cognitively guided instruction (Franke et al., 2009). Carpenter, Fennema and Franke (1996) developed CGI to create a model of eliciting and using student thinking to inform teachers' knowledge of how to build on student thinking during instruction. Franke et al. (2009) investigated the effectiveness of this model by looking at how teacher questioning elicited student thinking in the classrooms of three elementary school classrooms (second and first grade). According to Franke et al. (2009), research shows that teachers most often pose questions that require students to recall facts, rules and procedures as opposed to questions that require explanations or justifications. When students have opportunities to explain their own mathematical ideas and descriptions of how they solve problems, including why and how, teachers gain insight into student thinking and can use that thinking to make instructional decisions. This relates to a community of learning in that student thinking is a centerpiece in a community of learning. In mathematical learning environments, teachers' questions can position student thinking in relation to the mathematics in ways that support student understanding (Franke et al., 2009), which correlates with the principle of problematizing content. These researchers found several benefits to teachers posing probing questions in the first and second grade classrooms in which they conducted their study. According to their study, asking probing questions contributed to student sense making in the classroom in three ways: (1) teachers, armed with a deeper understanding of student thinking, were able to make informed instructional decisions in the classroom; (2) in responding to teachers' probing questions, students were able to clarify, solidify, and correct their own thinking and (3) when students responded to the teacher's probing questions, this gave opportunities for other students in the class to connect their own thinking to what was being said and correct misunderstandings. Supporting students' explanations is an important component of providing opportunities for students to problematize content. It requires teachers to not only provide sufficient time and appropriate tasks for students, but also requires teachers to press students for justification and explanation (Silver & Smith, 1996). CGI studies show that students in elementary mathematics classrooms can problematize content and make intellectual contributions to the classroom learning community, as opposed to just participating by means of assimilating facts and procedures (Carpenter, et al., 1996; Franke, et al., 2009; Warfield, 2001).

The studies described in this section all involved specific interventions. Lampert's (1990) intervention involved developing moral qualities in her students and reversing the roles of teacher and students, Boaler and Greeno's (2002) intervention involved engaging students in discussion-based mathematical problem solving, and the work of Carpenter et al. (1996) and Franke et al. (2009) involved developing teachers' abilities to use student thinking for guiding instruction in CGI classrooms by helping teachers learn how to ask probing questions and to use student thinking in instructional decisions. These studies provide examples of classrooms in which students have opportunities to engage in problematizing content. Investigations of the teaching and learning of mathematics in the lower grades, however, should include research conducted in the naturalistic setting. Specifically, research regarding opportunities that exist for students to elicit curiosities and engage in sense making skills should also incorporate studies that do not include an intervention, conducted in the natural setting of the elementary school mathematics classroom.

Giving Students Authority. The second principle for fostering productive disciplinary engagement in a community of learning classroom as devised by Engle and Conant (2002) is described as giving students authority. For this to be enacted, students need to have an active role in defining, addressing, and resolving problems in a mathematics classroom. For students to have authority, all members of the learning community, teacher and students, need to position students as stakeholders in their claims, approaches, explanations, designs and other responses to problems. “By giving students authority, we mean that the tasks, teachers and other members of the learning community generally encourage students to be authors and producers of knowledge, with ownership over it” (Engle & Conant, 2002, p. 404).

The term authority relates to the notion of expertise (Brown et al., 1993), the principle of authority (Engle & Conant, 2002), and the construct of agency. Brown et al. (1993) associated the term expertise with mutual distribution of expertise in the classroom, also referred to as shared cognition, in a collaborative learning environment. Brown and colleagues describe interdependence as a defining feature of a community of learning and distributed expertise is essential for interdependence. According to Brown et al. (1993), “ideas and concepts migrate throughout the community via mutual appropriation and negotiation" (p. 224). Brown (1997) refers to agency as the consistent ways in which members of the learning community actively engage in learning by searching for understanding, putting forth effort and finding meaning. Several researchers have focused on the agency aspect of a classroom learning community (Boaler, 2003; Boaler & Greeno, 2000; Lampert, 1990; Schoenfeld, 1992, 2016).

The construct of agency and principle of authority relate to the work of Schoenfeld (1992) involving studying problem solving, metacognition and sense making in mathematics. Schoenfeld described a problem-solving class he was teaching, in which he employed a method of coaching with his college students while they worked in groups. He described his job as a "roving consultant," peppering the students with questions such as, "What are you doing? Why are you doing it? How can it help you?" According to Schoenfeld (1992), after one semester, students were more capable of answering his questions and comfortable with problem solving. He states, "by virtue of good self-regulation, the students gave themselves the opportunity to solve the problem" (p. 357). The idea of the teacher as a roving consultant asking probing questions positioned the students as stakeholders in their learning. The teacher, in this action, shared responsibility for learning with the students, which gave students agency. The teacher did not tell students what the answer was, or give explicit instructions for solving the problem. Rather the teacher shared authority with their students in letting the students solve the problem in whichever way they wanted. The students had agency in that when the teacher assumed the role of roving consultant, the students were placed in positions to become authors and producers of knowledge.

The principle of authority relates to students producing knowledge and having ownership over it. By participating as a roving consultant, Schoenfeld altered his teaching role, which in turn gave students more authority. Lampert (1990) did this as well in her fifth-grade classroom by abandoning what she referred to as the traditional role of teacher in favor of serving as a facilitator in her classroom. She stated that by altering the role of teacher and students, students held authority in determining mathematical truths. Yackel and Cobb (1996) invoked a similar instructional strategy with teachers in their study, which gave students agency in second and third grade classrooms. The teachers in their project classrooms facilitated whole class discussions around a problem, then provided opportunities for students to work collaboratively together, before engaging in a whole class discussion during which students were expected to provide explanations, justifications, and argumentation in line with established classroom norms. The teacher's role during these discussions were to ask questions such as "What are different ways of solving?" and "Why?" Students were the authority on determining which strategy or solution paths they favored and were expected to justify their thinking. The notion of giving students agency or authority in the classroom, and distributing expertise, permeates all grade levels as evidenced by these studies. A tie that binds these studies is that they involved interventions, which may provide an opportunity for additional studies related to the principle of authority to be conducted in naturalistic settings.

More recently, Schoenfeld (2016) developed the Teaching for Robust Understanding (TRU) framework, which includes five dimensions for characterizing powerful mathematics classrooms (see Appendix B for the descriptions of the dimensions). Schoenfeld developed the TRU framework and these five dimensions after conducting a series of literature reviews that identified hundreds of aspects of powerful K-16 mathematics classrooms and examining videos of classrooms which did not include any interventions. These videos involved classrooms that were and were not considered examples of excellent teaching (Schoenfeld, 2019). The dimension of agency, ownership, and identity (AOI) is particularly relevant to authority in terms of a community of learning. Schoenfeld (2016) describes the dimension of agency, ownership and identity as follows:

> The extent to which students are provided opportunities to “walk the walk and talk the talk”—to contribute to conversations about disciplinary ideas, to build on others’ ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners. (p. 9)

AOI encapsulates the ideas of Brown (1997) and Engle and Conant (2002) regarding shared expertise and authority in a community of learning. Specifically, AOI focuses on opportunities students have for generating and sharing their own ideas, and acknowledges that student ideas need to be encouraged, recognized, and supported in the classroom. AOI focuses on building upon student ideas in the collaborative construction of collective understanding in the classroom. AOI incorporates Brown’s (1997) notion of mutual distribution of expertise in a collaborative learning environment and the positioning of students as equal stakeholders in the production of knowledge. As Engle and Conant (2002) describe, when students are positioned in active roles of defining, addressing, and resolving problems, when authority is shared equally in the classroom community, then students have opportunities to become productively engaged in teaching and learning and in the production of knowledge.

In the study conducted by Lampert (1990) described previously, students had opportunities to be productive engaged in teaching and learning and in the production of knowledge. Lampert’s goal was to create a culture of shared responsibility among her students as they engaged in meaningful learning. Lampert intended to create a classroom community where the students and the teacher were equally responsible for seeking to understand the mathematical assumptions behind the problems they were attempting to solve. For Lampert, the role of the teacher as a member of the community, as a shared stakeholder, was important to the development of a meaningful mathematical learning community. This is similar to Brown's (1997) ideas of agency in a community of learning. When the teacher participates as a shared stakeholder in the learning, it does not give sole authority to any one person in the classroom community, rather, it supports the notion of shared authority which depends upon all participants being equal stakeholders in the teaching and learning. This participation also resonates with the dimension of AOI according to Schoenfeld (2019). Students in Lampert's classroom developed agency by way of contributing to conversations of disciplinary ideas and building on one another's ideas. I mentioned the normative practices of discourse in Lampert's classroom, which contributed to student engagement with problematizing content, and those normative discourse practices also gave students authority in the classroom. Students drove conversations by presenting their own ideas. The students challenged and questioned one another. Brown's idea of agency in terms of shared expertise, and the consistent ways in which members of the learning community actively engage in learning by searching for understanding, putting forth effort and finding meaning, are evident in the principle of authority as well as AOI (Brown, 1997; Engle & Conant, 2002; Schoenfeld, 2016).

The notion of agency was also the focus of research conducted by Boaler (2003). Boaler (2003) investigated the development of agency by looking at learning through teaching practices employed in three high school classrooms. She was primarily concerned with comparing what she called a "traditional" approach to teaching and learning and a "reform" approach to teaching and learning. The former is described as an approach through which teachers demonstrate mathematical methods and students practice those methods individually after the demonstration. The latter is described as an approach through which students are given ample time to solve open ended problems without a demonstration of mathematical methods. The reform approach aligns with giving students authority and encourages students to produce knowledge. Students explore their ideas primarily in groups with limited teacher intervention, which resonates with the collaborative learning groups in the study conducted by Yackel and Cobb (1996). Interpreting the results of the study, Boaler (2003) characterized students in the traditional group as “passive knowers” and students in the reform group as “active knowers.” She describes how the environments in the reform classes required students to take on different roles than the traditional classes, namely that they were encouraged to propose ideas and theories, suggest mathematical directions, and pose questions to one another. Students were positioned as having agency and authority in terms of learning and producing knowledge in the reform classes. Her work calls upon researchers in mathematics education to devote more time to study “the process of *transformation* by capturing some of the practices of teaching and converting them into a set of carefully documented records of practice” (Boaler, 2003, p. 15). The goal of my research study would help answer that call, in that investigating opportunities for cultivating a community of learning in the natural classroom setting involves capturing classroom practices to understand mathematics teaching and learning. Moreover, it extends this research work to the early elementary grades. Extending research on understanding mathematical practices that influence teaching and learning in an elementary school classroom, in relation to a community of learning and the four guiding principles, my study would build upon the goal of incorporating diverse groups of students in this type of research.

According to Engle and Conant (2002), the principle of giving students authority in a community of learning may critically shape student engagement. Students who are positioned to have more active intellectual roles are likely to be more engaged in the classroom community. NCTM (2014) refers to teachers supporting students in developing a disposition to persevere in solving problems by giving students opportunities to engage in productive struggle and acknowledging the value of productive struggle in learning mathematics. This teaching action is associated with eliciting and using evidence of student thinking. When students develop a disposition to persevere in solving problems, and their intellectual work is valued, students can see themselves as learners and doers of mathematics, which relates to student engagement in the classroom. I connect the idea of students' developing this type of disposition with students having authority in the classroom, in terms of agency and expertise in their learning and in doing of mathematics. Lampert's (1990) findings from her study of fifth grade students demonstrate how a mathematical community of discourse allowed students to become stakeholders in their learning in an elementary school mathematics classroom. Boaler (2003) explored contrasting high school learning environments to describe how students developed agency in terms of the way they were positioned in learning and producing knowledge in classrooms that incorporated a reform approach to teaching. She described a relationship between students who developed a sense of agency with being positioned to engage with mathematical learning in a more meaningful way. This work correlates with the supposition of Engle and Conant (2002), that authority encourages students to become authors and producers of knowledge in a community of learning classroom. I provided evidence of several studies conducted in relation to the principle of authority and concept of agency in the classroom. However, missing from this research are naturalistic K-5 studies focusing on authority and agency in early elementary school mathematics classrooms, which I was unable to identify.

Holding Students Accountable. The third principle for fostering productive disciplinary engagement in a community of learning as devised by Engle and Conant (2002) is holding students accountable to others and to disciplinary norms. This principle refers to members of the learning community responding to others with justification and respecting classroom norms. Accountability is ensuring that students' "intellectual work is responsive to content and practices established by intellectual stakeholders" (Engle & Conant, 2002, p. 405). Several researchers have focused on social and sociomathematical norms within a classroom community (e.g., Cobb, Gravemeijer, Yackel, McClain & Whiteneck, 1997; Cobb, Yackel & Wood, 1993; Lampert, 1990; Yackel & Cobb, 1996). Social and sociomathematical norms in a classroom community relate to the principle of holding students accountable because by looking at the norms or practices that have been established in the classroom, and the normative practices in which students engage in the classroom, student accountability becomes evident. The ways students respond to one another and the ways students explain, justify, and argue about content, can be investigated in studies related to social and sociomathematical norms.

Researchers who have focused on social and sociomathematical norms and students' participation in classroom practices, as well as students' mathematical dispositions and development of intellectual autonomy position themselves from a cognitive constructivist and social constructivist theoretical orientation. Cobb and colleagues have studied learning in elementary mathematics classrooms and believe that students construct their own knowledge of mathematics based upon their activities, experiences, and participation in the classroom community (e.g., Cobb, Stephan, McClain & Gravemeijer, 2001; Cobb & Yackel, 1996; Cobb, et al., 1993). Cobb and Yackel (1996) share the views of others (Boaler & Greeno 2000; Kantowski 1977; Lampert 1990; Schoenfeld 1992) that mathematical learning is "an active problem-solving process" through which students develop specific mathematical beliefs and values and, consequently, develop certain mathematical dispositions. Cobb, Yackel and Wood (1993) reported on a study involving a teaching experiment in a second-grade classroom aimed at translating the theory of constructivism into the practice of doing mathematics. Student accountability was evident in this study through the students' collaborative construction of knowledge. The teacher played an important role in creating an environment in which students were held accountable to one another and to disciplinary norms. Cobb et al. (1993) describe that the teacher achieved this by capitalizing on students' constructivist activities to help students arrive at a consensus, rather than focusing on steering students toward a predetermined mathematical solution or answer. This resonates with the third and fourth principles of FCL, collaboration and culture. Creating an atmosphere of shared responsibility in the classroom community involves collaboration and negotiation to arrive at a consensus. This relates to what Brown (1997) refers to as the culture in a community of learners' classroom.

Yackel and Cobb (1996) illustrated in their research how classroom norms regulated mathematical argumentation and helped elementary students develop mathematical dispositions and intellectual autonomy. They define autonomy with respect to students' participation in the practices of the classroom community. They state that students who are intellectually autonomous in mathematics are aware of, and draw on their own intellectual capabilities when making mathematical decisions and as they participate in the classroom community, which is similar to the reflection principle of FCL, or student's insight into their own thinking (Brown, 1997). The development of mathematical dispositions among students is related to the shared values and beliefs established by the classroom community.

In their work, Yackel and Cobb (1996) collaborated closely with second and third grade mathematics teachers to “radically revise the way they teach mathematics” (p. 460). A major goal of their research was to investigate the establishment of sociomathematical norms in an inquiry-based classroom mathematics tradition. Interventions in this study were the instructional tasks and the instructional strategies that teachers used. Teachers presented an inquiry-based task to the whole class, then gave students opportunities to collaborate and solve in small-groups before following-up with a whole class discussion in which students were pressed to explain and justify their interpretations and solutions. The teachers in this study recognized that acceptable explanations and justifications of mathematical argumentation had to involve mathematical actions and interpretations rather than procedural instructions. This provides evidence of the principle of accountability in this study. Students were expected to explain, justify, and argue about mathematical actions and interpretations, which were the sociomathematical norms that were established in the classroom. Students’ specific beliefs and values helped them form judgements and develop mathematical explanations. Yackel and Cobb (1996) state, “students can take over some of the traditional teacher’s responsibilities only to the extent that they have constructed personal ways of judging and that enable them to know in action both when it is appropriate to make a mathematical contribution and what constitutes an acceptable mathematical contribution” (p. 473). In addition, teachers in this study were trained to elicit multiple solution strategies without placing value on one strategy over another, thus students were left to decide which solution path they favored. This is evidence of the principle of authority. The students decided which solution path they favored, based in mathematical reasoning, which gave students expertise in the classroom and agency in their learning. Students in this study provided explanations that were conceptual as opposed to calculational in nature and presented ideas for solving problems in meaningful ways because they felt an obligation in ensuring that other students could understand and make sense of their mathematical reasoning. This was based upon the sociomathematical norms of what was acceptable as mathematical explanation in the classroom. Furthermore, students made judgements about what constituted appropriate mathematical contributions and explanations based upon these norms. Students were held accountable for the teaching and learning of mathematics in this classroom, and students had authority.

In Lampert's (1990) classroom of fifth-grade students, the norms and practices that governed classroom interactions paralleled standards for argument in the mathematical community. In establishing classroom norms and practices for making their thinking a public and collaborative endeavor, students became responsible for justifying, explaining, and revising their beliefs. Students were responsible for making their mathematical reasoning explicit, responding to one another, and posing questions to one another before legitimatizing their beliefs to determine collective mathematical truths. Lampert was able to achieve this in her classroom by intentionally constructing an environment in which traditional roles of student and teacher were reversed, and in which students developed the traits of vulnerability, modesty, and courage. Students learned what was an appropriate contribution to the activity of learning mathematics and practiced being responsive participants in the classroom community. In these ways, students were held accountable to others and to disciplinary norms within the classroom.

Another study which addresses the notion of student accountability in the classroom focused on promoting conceptual thinking in upper-elementary mathematics classrooms, specifically grades four and five (Kazemi & Stipek, 2001). The teachers participating in this study had been trained in mathematics reform curricula. In this study, Kazemi and Stipek (2001) found that when classroom practices focused on students developing deeper understandings of mathematical ideas and concepts, engagement and accountability was evident. When students collaborated with their peers to construct mathematical understandings, they became skilled in describing and defending their differing mathematical interpretations and solutions. Students were held accountable in their own thinking about mathematical content and were responsible for responding to the thinking of others in the form of mathematical argumentation to reach consensus. Engle and Conant (2002) specifically refer to this principle as “an internal accountability, in which students' influence within their learning environment is affected by how well they account for how what they are doing is responsive to what others have done and to community norms for good practice” (p. 405). This is what Brown (1997) conceptualized in terms of culture and collaboration regarding FCL. A culture of learning, negotiating, sharing, and producing work in an atmosphere of joint responsibility, mutual respect, and a sense of personal and group identity, are the backbone of a community of learning.

Schoenfeld (1992) addresses the concept of accountability in the mathematics classroom by discussing students’ social participation in the classroom and enculturation with the discipline of mathematics. As mentioned previously, Schoenfeld (1992) presented the term “enculturation” as another word for socialization, which is described as entering and picking up the values of a culture, crediting Resnick (1988) for the term. Schoenfeld identified a problem with how students see themselves fitting into the mathematical enterprise because of their enculturation with mathematics. Building upon the conceptions of social learning theory as presented by Vygotsky (1978), Schoenfeld contends that we need to help learners develop the habits and skills of interpretation and develop the ability to construct meaning in their own mathematical learning in order to help them think mathematically, or see the world the way mathematicians do. He identifies the enculturation into mathematical learning as a problem. In his words:

> … if we are to understand how people develop their mathematical perspectives, we must look at the issue in terms of the mathematical communities in which students live and the practices that underlie those communities. The role of interactions with others will be central in understanding learning, whether it be understanding how individuals come to grips with the specifics of the domain or more broad issues about developing perspectives and values. (Schoenfeld, 1992, p. 31).

I associate Schoenfeld's (1992) statement above with the way Brown and Campione (2002) talk about the collaborative learning environments in their FCL project classrooms. They argue that with repeated experience explaining, arguing, questioning, and justifying claims with evidence, students adopted these skills as part of their personal repertoire as ways of knowing. A main underlying idea of a community of learning is that students actively participate in the collective construction of knowledge (Brown, 1992). The way learners interact with others in pursuit of this collective construction of knowledge relates to how they identify with learning. A community of learning creates a culture of learning, where students learn to think deeply and help others think deeply (Brown & Campione, 2002). If students are enculturated into the learning of mathematics in this way, their perspectives, the way they learn mathematics, and the value they place on that learning could be impacted.

Schoenfeld (1992) advocates for an approach to teaching and learning mathematics which requires students to seek solutions, explore patterns, and formulate conjectures on their own to make their learning of mathematics meaningful. This approach to teaching and learning mathematics requires students to be held accountable to one another and to disciplinary norms in mathematical learning communities. It requires that all members of the classroom community be actively engaged with disciplinary content (Schoenfeld, 2016). When students engage in these practices, they approach the learning of mathematics in ways which silhouettes what it means to think mathematically (Schoenfeld, 1992). As Schoenfeld (1992) asserts “membership in a community of mathematical practice is part of what constitutes mathematical thinking and knowing” (p. 12). Schoenfeld’s statement essentially describes the notion of a community of learning within mathematics. These assertions made by Schoenfeld (1992) are applicable to all classrooms of mathematical learning. His conceptions of how to approach the teaching and learning of mathematics first appeared in a handbook for teaching and learning mathematics, designed to assist teachers at every level, K-12. Students begin to engage with school mathematical practices when they start learning mathematics in school. Therefore, the assertions of how students should engage with mathematical practices to develop their abilities to think mathematically are relevant across grade levels. Students’ social participation in terms of mathematical learning are not reserved for the upper grades, as evident in the research discussed in this section (Brown, 1992; Lampert, 1990; Yackel & Cobb, 1996). This principle refers to members of the learning community making sure they respond to one another and follow practices established by the community, and this is applicable across grade levels.

Providing Relevant Resources. Finally, the fourth principle, providing relevant resources, refers to providing whatever is necessary to support student engagement, such as the tasks with which students engage, giving students sufficient time to pursue a task, having access to sources of information relevant to a task, or any other materials to supplement student engagement or help students make sense of the task being investigated. Relevant resources include sources of information or tools that are necessary to support student engagement and learning. The curriculum is one resource that plays an important role in supporting students' learning in relation to the other four principles (Engle & Conant, 2002)

Brown (1997), as well as Engle and Conant (2002), did not use a prescribed textbook in their work with fostering communities of learning. They both describe that their curricula involved a combination of artifacts, hands-on experiments, books, videos, newspapers, and periodicals that students used in the classroom. Brown (1997) describes that this sort of curriculum relied heavily on disciplinary content and provided students with sufficient rigor, providing students with challenge, room to explore, and opportunities to delve more deeply into content. On the other hand, most mathematics classrooms in the United States incorporate the use of a set curriculum to guide teaching and learning (Reys, Reys & Rubenstein, 2010). Remillard and Heck (2014) define intended curriculum as a plan for the experiences learners will encounter, and enacted curriculum as the actual experiences students encounter to help them reach specified objectives. In recent years, curricula have changed to emphasize mathematical thinking and reasoning, conceptual understanding, and problem-solving. These changes to curricula require teachers to play a substantially different role in the mathematics classroom than has been typical among teachers in the United States in the past (Remillard, 2005). Some changes to curricula include increasing the level of rigor and providing students with more opportunities to engage more deeply with content. For example, Pearson enVision 2.0 (2016) has incorporated productive struggle, problem-based learning and personalization into their K-8 curriculum rendered in 2016. They specifically describe their K-8 mathematics curriculum as offering "superior focus, coherence, and rigor to empower every teacher and student."[2] Changes such as those that Pearson enVision 2.0 (2016) describe correlates with the sort of curriculum

[2] Pearson enVision Math 2.0 Program for Grades K-8. Retrieved March 28, 2020: https://www.pearsonschool.com/index.cfm?locator=PS2xIm.

Brown (1997) created for her students through the use of multiple information sources, involving sufficient rigor, challenge, and opportunities for exploration with deep disciplinary content. However, one notable difference is that Brown developed her own curriculum while the Pearson curriculum is typically provided to teachers in schools.

Using mathematics curricula is a complex and dynamic process and is mediated by a teacher's knowledge, beliefs, and dispositions (Remillard, 2005). Use of a curriculum involves a teacher interacting with curricular materials. This brings to focus the idea of intended curriculum versus enacted curriculum. The *intended curriculum*, or *written curriculum*, refers to the specific learning goals for students set by the official curriculum, the curricular aims, and objectives as presented in published teaching guides and curricular materials used in the classroom (Stein, Remillard, & Smith, 2007). The *enacted curriculum* refers to what actually takes place in the classroom related to the curriculum including the teacher's implementation of the official curriculum and the ways in which learners experience the official curriculum (Gehrke, Knapp & Sirotnik, 1992; Remillard & Heck, 2014). Given the importance of curriculum according to the fourth principle of relevant resources described by Engle and Conant (2002), it is important to differentiate intended versus enacted curriculum and define curriculum in the context of mathematics education. This is important because intended and enacted curriculum are two different things. The former is the formal curriculum, or what is written on paper while the latter is what happens in the classroom and how students experience the curriculum. These things could be very similar or very different, and they both relate to relevant resources because for a community of learning, the curriculum can be interpreted to support (or not) the other principles.

Beyond the curriculum, teachers of mathematics need to understand how to use tasks in their teaching that can help cultivate a community of learning. One of the eight effective teaching practices for strengthening the teaching and learning of mathematics (NCTM, 2014) involves the implementation of tasks that promote reasoning and problem solving. NCTM (2014) describes that students who have opportunities to engage actively in reasoning, sense making, and problem solving, can develop a deeper understanding of mathematics. In many instances, tasks that are incorporated into the classroom come from the intended curriculum. Therefore, tasks that are included in the curriculum (curricular tasks) should be designed to provide opportunities for reasoning, sense making and problem solving. Curricula introduced in recent years in mathematics education have addressed this by increasing rigor and providing opportunities for students to reason, discuss and make sense of problems (e.g., Pearson enVision 2.0, 2016). Henningsen and Stein (1997) provide evidence from research they conducted involving the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project which investigated the relationship between reform features of instruction with student understanding of mathematics in four middle schools. Their study involved an intervention, reform-based instruction, and studied the effects of that intervention on student understanding. The authors identified a discernible set of factors which influenced students' engagement in high levels of thinking and reasoning in the classroom. These factors related to the appropriateness of the task and supportive actions by teachers such as consistently pressing students to provide meaningful explanations or make meaningful connections. Their work illustrates the importance of instructional tasks and how student thinking and reasoning can be supported in the classroom.

Henningsen and Stein (1997) coined the term *cognitive demand* in their work with the QUASAR project to refer to the kinds of thinking processes with which students engaged. Stein, Smith, Henningsen, and Silver (2009) build on this work by defining cognitive demand as "the kind and level of thinking required of students to successfully engage with and solve the task" (p.1). According to these researchers, instructional tasks form the basis of students' opportunities to learn mathematics. As such, it is important to match tasks with goals for student learning. For example, if the goal is to increase students' ability to think, reason and solve problems, then students should be presented with tasks that are cognitively challenging and have the potential to engage students in complex thinking. To help teachers set up appropriate level tasks with learning goals, Stein and Smith (1998) created a Task Analysis Guide (Figure 3). This Task Analysis Guide (TAG) lists characteristics of tasks at four levels of cognitive demand: *memorization*, *procedures without connections*, *procedures with connections* and *doing mathematics*. Tasks classified as memorization or procedures without connections are grouped together in lower levels of cognitive demand. Tasks that are classified as involving procedures with connections or doing mathematics are grouped together in higher levels of cognitive demand. These thinking processes can range from memorization of procedures and algorithms (with or without attention to concepts, understanding, or meaning) to complex thinking and reasoning strategies (e.g., conjecturing, justifying or interpreting).

Lower-Level Demands	Higher-Level Demands
Memorization • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	Procedures With Connections • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focuses solely on describing the procedure that was used.	Doing Mathematics • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Figure 3. The Task Analysis Guide (Stein, Smith, Henningsen, & Silver, 2009, p. 6).

The TAG is important because it can be helpful to teachers in selecting, classifying, and even modifying tasks for their classrooms. Cognitive demand of tasks and the TAG relate to the principle of relevant resources in that the TAG offers guidance in terms of the different levels of cognitive demand a task requires. The descriptions of each of these types of tasks relate to the other principles. For example, a task associated with doing mathematics requires students to explore and understand the nature of mathematical concepts, processes, or relationships, which is problematizing content. Another characteristic of a Doing Mathematics task is that it demands student self-monitoring or self-regulation of one's own cognitive processes, which relates to authority. The type of task presented to students is important in terms of the type of thinking it invokes in students. The TAG is a relevant resource in terms of relating tasks to the kinds of thinking with which students engage in the classroom.

One of the main goals in fostering a community of learning, according to Brown (1992) was to create an environment in which students engaged in deep thinking about disciplinary content. In mathematics classrooms, resources such as the intended curriculum, enacted curriculum, as well as tasks and implementation of tasks are important elements related to how students engage in deep thinking about disciplinary content. These types of resources are relevant and important in fostering productive disciplinary engagement in a community of learning classroom.

Conclusion

Social learning theories have led practitioners and researchers to conceptualize communities of learning in the classroom, especially in mathematics education (Lampert, 1990; Schoenfeld, 2016). Researchers have focused on investigating how mathematics teaching and learning promotes the construction of mathematical knowledge that is meaningful by embedding their research in the social complexities of the classroom and investigating the identities that students develop as they engage in this learning (Schoenfeld, 1992; von Glasersfeld, 1991). As stated earlier, the classroom is the living social context of education and it is ripe with complexities. According to Brown (1992) the classroom culture is the where and the how of learning. In a community of learning, students are given significant opportunity to collaborate and take charge of their own learning. In the discipline of mathematics, learners should engage in mathematical practices in the classroom that allow them make sense of mathematics and engage with mathematics as a dynamic process of discovery (NCTM, 2014; Schoenfeld, 1992). As Lampert (1990) pointed out, intellectual authority is central to how mathematics is known in the discipline and should be central to how mathematics is known in school.

The research cited herein regarding aspects and conceptualizations of a community of learning involve data derived from studies conducted in classrooms across all grade levels, including elementary, middle, and high school mathematics classrooms. The current study involves studying the teaching and learning in elementary mathematics classrooms in relation to a community of learning. However, to unpack the current understanding of communities of learning in elementary mathematics classrooms, it became necessary for me to incorporate studies involving middle and high school mathematics classrooms in this literature review due to the lack of studies related to aspects of a community of learning in elementary schools, especially the earliest grades (K-2). Moreover, most studies described in this chapter included an intervention of some sort. Researchers wanted to see the effect of an intervention such as reform-based curriculum, a certain type of professional development program for teachers, or a certain type of classroom design. There are many studies that involve studying teaching and learning with the incorporation of an intervention. However, there are very few studies aimed at studying the teaching and learning in a classroom in the naturalistic setting with no interventions. My research differs from the studies I described in this literature review in that I situated my study in its natural setting, the elementary mathematics classroom, without any interventions.

In addition, my experiences as a teacher influenced my interest in conducting research in an early elementary mathematics classroom, rather than upper elementary. The notion of inquiry and curiosity in young children, and my personal experiences of the dwindling inquiry and curiosity in learning mathematics with first, second and third graders, fueled my desire to study the teaching and learning of mathematics when students begin school. My rationale is based in part on Schoenfeld’s (1992) emphasis on students’ enculturation with mathematics. Specifically, Schoenfeld identifies that there is a problem with how students see themselves fitting into the mathematical enterprise because of their enculturation with mathematics.

The mathematics education community has also developed frameworks related to what should be happening in mathematics classrooms, across all grade levels, and the type of teaching and learning that should take place to achieve effective mathematics teaching and learning (e.g., *Principles to Actions* and TRU). For example, NCTM (2014) has recommended specific experiences for learners of mathematics, across grade levels, and essential teaching practices for teachers of mathematics, across grade levels. These recommendations are steeped in research. Mathematics classrooms can and should support the production of intellectually generative knowledge in a social learning context. Research supports that students should engage in problematizing content, be accountable for their own thinking and reasoning while evaluating and responding the thinking and reasoning of others, have authority or agency in their learning, and be provided with resources that help them reach deeper levels of understanding (e.g. Boaler, 2003; Lampert, 1990; Schoenfeld, 2016; Yackel & Cobb, 1996). These are all tenets of a community of learning. Elementary school classrooms, even as early as kindergarten, involve students who often like sharing their own ideas about problem solving (Warfield, 2001), which can create opportunities for teachers to elicit and use student thinking in the classroom. Given the focus on helping students reach deeper levels of understanding, it makes sense that students (their thinking, their learning, their ideas, their actions, and their questions) should be a centerpiece of the classroom.

The goals of the current research study are to investigate the natural opportunities for the development of a community of learning in a first-grade elementary school mathematics classroom using the four guiding principles for fostering productive disciplinary engagement

(Engle & Conant, 2002). The principles, based upon Brown's (1992) conceptualization of a community of learning, include problematizing content, giving students authority, holding students accountable, and relevant resources. It is important to investigate the teaching and learning in an elementary school mathematics classroom in the natural setting, as I have not found any current studies which investigate this experience. To fill this gap in the research, I sought to answer the following question: What opportunities are there for cultivating a community of learning in an elementary school mathematics classroom based upon the principles for productive disciplinary engagement?

CHAPTER 3: METHODS AND METHODOLOGY

This chapter describes the design of the study, research site, participants, data sources, collection, and analysis. The data were collected over two consecutive months in the middle of the 2019-2020 academic year in a first-grade elementary school mathematics classroom. The goal of this study is to investigate opportunities for cultivating a community of learning in an elementary school mathematics classroom based upon the principles for productive disciplinary engagement.

Design of Study

I designed this research study using a qualitative approach. Because this study is built upon the tenets of social constructivism and communities of learning, I used a qualitative research approach, which has roots in the fields of anthropology, sociology and psychology, and draws upon the philosophies of constructivism. A qualitative approach allowed me, the researcher, to seek to understand and interpret complex social phenomena, an elementary school classroom, in terms of its participants (Merriam & Tisdell, 2016). Qualitative research emerged from "researchers interested in understanding how people interpret their experiences, how they construct their worlds and what meanings they attribute to their experiences" (Merriam & Tisdell, 2016, p. 6).

Qualitative case study methodology is appropriate for this study given that a case study can be described as "an in-depth description and analysis of a bounded system" (Merriam & Tisdell, 2016, p. 37). A classroom of learners, which includes the teacher as a learner, is a bounded system since it is single entity. The boundaries of this study are that this study has a specified focus within a specified classroom involving participants who regularly participate in this classroom. The goal of this study is to investigate opportunities for cultivating a community of learning in an elementary school mathematics classroom. The nature of this study required a method which would allow me to observe students in their natural setting within the bounded system of their classroom. As case studies are characterized by the unit of analysis, not the topic of investigation, the unit of analysis for this study is the classroom of learners.

More specifically, I used an exploratory case study methodology (Yin, 2018). Exploratory case studies look at any phenomenon in the data which serves as a point of interest to the researcher. Exploratory case studies are used when there is no predetermined outcome and are appropriate when asking "how" and "what" (Yin, 2018). The research question for this study is "what opportunities are there for cultivating a community of learning in an elementary school mathematics classroom based upon the principles for productive disciplinary engagement?" There is no predetermined outcome and the goal of the study is to explore phenomena in the data relative to these opportunities.

In case study methodology, triangulation of data (collecting data from multiple sources for analysis) is important. Case study methodology seeks to offer an intensive and holistic description of a single bounded system and therefore, "conveying an understanding of the case is the paramount consideration in analyzing the data" (Merriam & Tisdell, 2016, p. 233). To maintain the integrity of conveying an honest and true understanding of the case, a researcher must draw upon multiple sources of data such as observations, interviews, artifacts, and documents. Merriam and Tisdell (2016) point out that while qualitative researchers can never capture an objective reality, triangulating data (comparing and cross-checking using multiple sources of data) increases credibility or the internal validity of the research. The design of this study addresses issues of validity by focusing on triangulation of data, incorporating multiple sources of data for collection and analysis. Specifically, I collected data from three sources:

observations, interviews, and artifacts/documents. I will describe these sources in greater detail in the data collection section of this chapter.

Context

The school that is the context for my study is a public, suburban, elementary school, serving kindergarten through fifth grade in northern New Jersey. Since it was important for my research to be conducted in an early elementary classroom, I conducted my research during mathematics instruction in a first-grade classroom. In particular, the studies I identified regarding aspects and conceptualizations of a community of learning in classrooms were intervention-based and situated in elementary, middle, or high school mathematics classrooms. Situating this study in a first-grade classroom, in the natural setting, makes this study unique. The goal of this study is to investigate what is happening in the classroom relative to opportunities for cultivating a community of learning when students initially engage with learning mathematics in school. This goal is motivated by four factors: (1) Schoenfeld's (1992) description of the enculturation of students with learning and doing mathematics in school, (2) the natural inquisitive and curious nature of young students who have not yet been muddled by learning in school (Dewey, 1906), (3) the gap in research regarding naturalistic studies which investigate the teaching and learning in early elementary classrooms without an intervention as illustrated in the literature review, and (4) my personal experiences.

In designing the study, I considered both kindergarten and first grade mathematics classrooms. First grade emerged as a more fitting setting for my research for several reasons. By first grade, most students have experienced at least one year of school. They have experienced some academic, emotional, and social aspects of school, and have learned a bit about how to read, speak and write within the school context. Also, first grade mathematics curricula incorporate a greater array of concepts than kindergarten. In first grade curricula, I see greater opportunities for the cultivation of a community of learning. In kindergarten, students spend most of their time learning how to represent and compare whole numbers and describe shapes and spaces (National Governors Association Center for Best Practices, 2010). In first grade, students are supposed to continue the work of kindergarten and focus on four new areas: (1) understanding addition and subtraction within 20, (2) developing an understanding of whole number relationships including place value (tens and ones), (3) developing an understanding of linear measurement and (4) reasoning about attributes of, and composing and decomposing geometric shapes.[3] The additional concepts covered in first grade coupled with my perception that first grade students have had some experiences with reading, speaking and writing, led me to believe there could be greater opportunities for investigating a community of learning in a first grade classroom.

I deliberately chose to situate this study in an elementary school in which I had no prior experience. The elementary school for this study was chosen mostly out of convenience. I wanted the school to be somewhat close to my home and still be a place in which I did not know any of the students or teachers. I thought that if I was collecting data in a school in which I had no relationships, no preconceived beliefs, and no knowledge of the students, the trustworthiness of my study and my objective integrity as a researcher would be maintained. As Merriam and Tisdell (2016) assert, credibility hinges partially on the integrity of the researcher.

I identified the school through an acquaintance who knew someone that worked at the school. That person put me in touch with the principal. I emailed the principal and inquired if he would be willing to let me conduct research for this dissertation at his school. He invited me in

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[3] This information was retrieved from the Common Core State Standards for Mathematical Practice (2010), by grade level, accessible at http://www.corestandards.org/Math/Practice/.

for a meeting. During that meeting I provided the principal with a verbal description of the research goals described herein (that I was seeking to investigate opportunities for cultivating a community of learning in an elementary mathematics classroom). I specifically stated that I was interested in collecting data in a first-grade classroom. The principal suggested a first-grade teacher, Miss X.[4]. Later, I asked him why he chose Miss X and he stated that she had the most experience of all the first-grade teachers. There were three first-grade teachers at the school and the teacher whom he chose was in her third-year teaching. Among the other two first-grade teachers, he stated, one was a first-year teacher and the other was a maternity leave replacement teacher.

Participants. There were 18 students in the class. The students in the class were described by the teacher as being "very sweet and helpful." This class is representative of a typical first grade suburban public-school mathematics classroom in a middle-class community (NCES, 2012). There were students of mixed levels in the class, including at least one student requiring one-on-one support, and the size of 18 students in the class is typical in the United States. The average number of students in an elementary school classroom in the United States is 21, and in New Jersey, that average is 19 students.

The teacher, Miss X, was in her third-year teaching and had spent all three of her teaching years in the same first grade classroom. She held a dual undergraduate degree in the areas of Early Childhood Education and Psychology. After field placements across various grades and districts during her undergraduate education, she decided to refine her major concentration from elementary to early childhood education. She stated that she always knew she

[4] I use pseudonyms for teachers and students throughout this manuscript.

wanted to be a teacher and that she “always loved working with children, particularly in learning environments.”

Daily schedule. The teacher had a daily schedule posted in the front of the classroom. The daily schedule included a list of times and activities for the day. Math was scheduled for 10:45-11:30 AM each day but was listed as “Daily 3” to refer to mathematics instruction. Students moved between three activities: a journal question, a worksheet called “Math by Myself”, and a game to be played with a peer. The students worked within one of three groups for each activity. As the students moved through these activities, the teacher would meet with one small group of students to work through a specific lesson from the textbook (enVision Math 2.0, Pearson, 2016). The idea was that while the teacher worked with one of the three small groups, the rest of the class was working on another aspect of the “Daily 3.”

The students were well versed in the “Daily 3” and transitioned easily. Around 10:45 AM, the teacher would kick off the “Daily 3” with a brief whole class mini lesson (launch). The teacher would begin the whole class mini lesson by asking all the students to sit on the rectangular carpet in the front of the classroom. She stood in front of the students, positioned in front of the interactive whiteboard and would teach the students a new concept, review a previous concept, and/or teach the students a new game. The teacher always followed the mini lesson with directions for the “Daily 3” activities. These directions usually included directives for what learning materials the students would need (e.g., a ten frame or number line), a description of the worksheet “Math by Myself,” a review of their journal problem, and identification of the game (sometimes the game was “free choice”). The mini lesson ended when the teacher called a small group to the back table to work with her on a worksheet from the textbook. Other students would be instructed to get their worksheets for “Math by Myself.” An aspect of the "Daily 3" was that the students had to work through the tasks in the order in which they were listed. This meant they always started with "Math by Myself," then completed the journal entry, and then played the game. As the students worked through these tasks, the teacher would call one group at a time to rotate through small group instruction with her in the back of the classroom (see Figure 4 for the "Daily 3" activities).

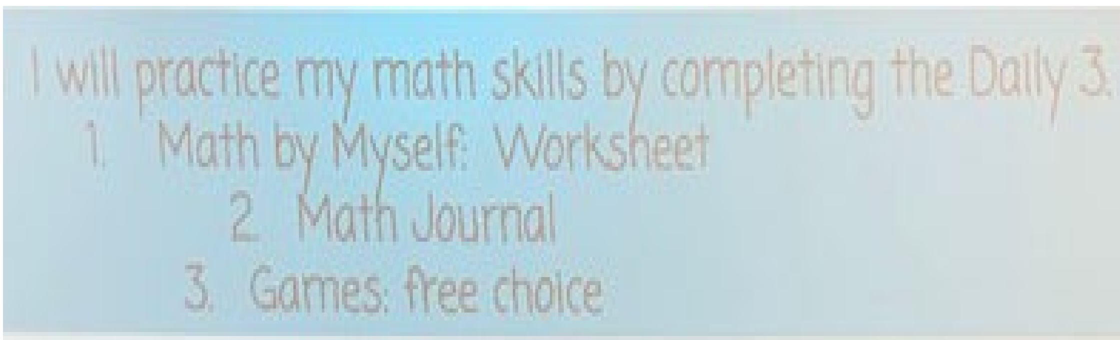
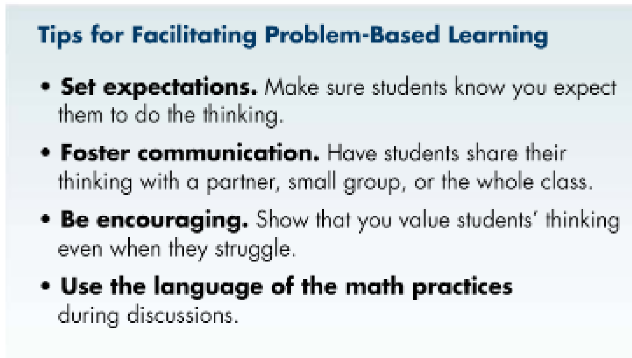


Figure 4. The "Daily 3."

Curriculum. The curriculum used by the school and the teacher was Pearson's enVision Math 2.0 (2016). The developers of this reform-based curriculum state that they "embraced the research and classroom-proven curriculum and instruction principles shown to promote the depth of mathematical understanding needed for student success and higher achievement" (Pearson enVision Math 2.0, 2016, p. 24). The curriculum aligns with the newly adopted standards for mathematics education (National Governors Association Center for Best Practices, 2010). The curriculum was developed based on research for highly effective learning and teaching practices and emphasizes teaching mathematics with understanding. Pearson enVision 2.0 (2016) provides students with opportunities to achieve deeper understanding of concepts by encouraging students to explain their thinking and giving students sufficient time to solve rich problems. This is made explicit in the Teacher's Guide (Pearson enVision 2.0, 2016). These tenets align with cultivating a community of learning, emphasizing student thinking as essential for building understanding of deep disciplinary content (Engle & Conant, 2002).

The curriculum is a problem-based learning curriculum that includes many tools to help students develop deep understanding of concepts through solving rich problems. One of these tools and a feature of the curriculum is the introduction of concepts through a problem-solving experience called the "Solve and Share." The "Solve and Share" was designed based on research that students develop conceptual understanding when new mathematical concepts are introduced "in the context of solving a real problem in which ideas related to the new content are embedded" (Pearson enVision 2.0, 2016, p. 26). The "Solve and Share" requires students to make connections to prior knowledge, develop new knowledge and persevere in solving a real problem. Another tool of the Pearson enVision 2.0 (2016) curriculum to help students develop deep understanding is visual learning. The curriculum describes this visual learning as a bridge, or a way of connecting students' thinking from the "Solve and Share" activity with new concepts. This bridge encourages using a series of visual illustrations to enhance understanding. Some of these visual appear in the way of comments and conversation-starter questions in student learning materials.

Focus, coherence, and rigor is outlined for each lesson, which is another tool for the teacher. The focus connects the lesson to mathematical standards and practices for effective teaching and learning. The focus provides an objective and essential understanding for each lesson. The coherence of a lesson refers to looking back and looking ahead, relating concepts of a new lesson with prior concepts to help teachers engage learners in connecting prior knowledge to new ideas. Rigor reflects the balance of conceptual understanding, procedural skill and fluency, and application. Rigor is supported with tools in the curriculum aimed at fostering high-level, question-driven classroom conversations (Pearson enVision 2.0, 2016). These tools include teacher action items, tips for facilitating problem-based learning and conversation starter questions. See Figure 5 for an image of one of these tools, "Tips for Facilitating Problem-Based Learning" which makes clear the importance of student thinking and sharing students' thinking.


Tips for Facilitating Problem-Based Learning

- **Set expectations.** Make sure students know you expect them to do the thinking.
- **Foster communication.** Have students share their thinking with a partner, small group, or the whole class.
- **Be encouraging.** Show that you value students' thinking even when they struggle.
- **Use the language of the math practices** during discussions.

Figure 5. Tips for Facilitating Problem-Based Learning (Pearson enVision 2.0, 2016, p. 45).

The curriculum provided a suggested pacing guide and proposed sequence of lessons based on clusters of topics related to the Common Core State Standards (National Governors Association Center for Best Practices, 2010). The first grade textbook is organized around a large cluster of topics related to understanding addition and subtraction. In November and December, while I was in the classroom, the students were mostly working through the end of Topic 4: Subtraction Facts to 20: Use Strategies and all of Topic 5: Work with Addition and Subtraction Equations. Prior to my observations in the classroom, the students had already moved through the first three topics of the curriculum, which included Topic 1: Solving Addition and Subtraction Problems to 10, Topic 2: Fluently Add and Subtract Within 10 and Topic 3: Addition Facts to 20: Use Strategies. To assist with the presentation of results in this chapter, I created a table which lists each of the topics covered during my observations and the content of each topic and lesson (Table 1). I also included the dates and times of five interviews in this table.

Topic 4: Subtraction Facts to 20 (Use Strategies)		
11/18	Observation #1 & Interview	Topic 4, Lesson 5 - Use Addition to Subtract
11/20	Observation #2	Topic 4, Lesson 7 - Explaining Subtraction Strategies

11/22	Observation #3 & Interview	Topic 4, Lesson 9 - Subtraction Reasoning, Math Practices and Problem Solving
11/25	Observation #4	Topic 4 Reteaching
Topic 5: Work with Addition and Subtraction Equations		
12/3	Observation #5	Topic 5, Lesson 2 - True or False Equations
12/4	Observation #6	Topic 5, Lesson 3 - Make True Equations
12/6	Observation #7	Topic 5, Lesson 5 - Add Three Numbers
12/9	Observation #8 & Interview	Topic 5, Lesson 4 - Word Problems with Three Addends
12/11	Observation #9 & Interview	Topic 5, Lesson 6 - Solve Addition and Subtraction Word Problems
12/13	Observation #10 & Interview	Topic 5 Assessment

Table 1. Dates of observations, informal interviews and corresponding curricular lessons and topics (Pearson enVision Math 2.0, 2016).

Student groups. The students were assigned to one of three small groups based on the teacher's assessment of their academic ability from a beginning of the year assessment. These groups are referred to as the silver, gold and copper group.[5] The silver group was the lowest group, the gold group was the highest group, and the copper group was in the middle. In my ten observations, the teacher worked with the silver group first, all but one time. She said that she typically started with the silver group because they needed the most help. I noticed that in all my observations, she was only able to work with two groups during the scheduled time between 10:45 and 11:30 AM. She mentioned this to me in an interview, stating that she would often meet with the gold group, or the highest group, at the end of the day, when they had extra time. In one of my observations she started the "Daily 3" fifteen minutes early, at 10:30 instead of 10:45 AM, because she specifically wanted to get to all three groups. She asked me to come in

[5] The names of the groups have been changed to protect the privacy of the students.

15 minutes early that day and she was able to meet with all three groups between 10:30 and 11:30 AM.

Physical space and resources. The physical space of the classroom was conducive to a community of learning. According to Brown (1997), collaboration is necessary for a community of learners, and the tables and chairs were designed to promote collaboration by all facing one another with multiple seats together. The teacher shared with me that in October she had the students' individual desks and chairs removed and adopted a flexible seating classroom (Figure 6). The flexible seating was as follows: four rectangular standing tables on wheels that could accommodate between two and four students each; one round table located very close to the floor (I refer to this table as the floor table in my observations), which could accommodate approximately four to six students; one rectangular high top table with six high back stools; one round table that was standard height off the floor that included soft benches around it which could accommodate four to six students; and a kidney table in the back of the classroom with the teacher at the center and flexible stools for students to sit around that could accommodate six to eight students comfortably. The students kept their personal materials (e.g., pencil cases, workbooks, journals) in cubbies in the back of the classroom and the majority of tools and resources in the classroom were shared (number lines, white boards, dry erase markers, game pieces, etc.) and stored in accessible cubbies on top of the radiator and on bookshelves. There was an abundance of resources and the classroom was neat and organized.

Figure 6. Flexible seating classroom layout.

General classroom norms. There were three classroom rules that the teacher and students established at the beginning of the school year (Figure 7) and these rules were posted on the closet door. These rules included (1) take care of your body, (2) take care of your friends and teachers, and (3) take care of Room 9. In all my observations, the students demonstrated behavior based on the norms of the school. Specifically, the students did not shout, they were not disruptive, they were not physical with each other, and they listened to the teacher. The teacher told me that this was a great class. She recalled previous years with students who were disruptive and did not listen. The teacher utilized a class reward system. When all the students worked exceptionally hard, she added a piece to a “Mr. Potato Head” doll (such as an arm, leg, or mouth). When he was fully assembled the class earned a free choice of an activity, such as time for games, extra outdoor recess, or another sort of reward of their choosing.

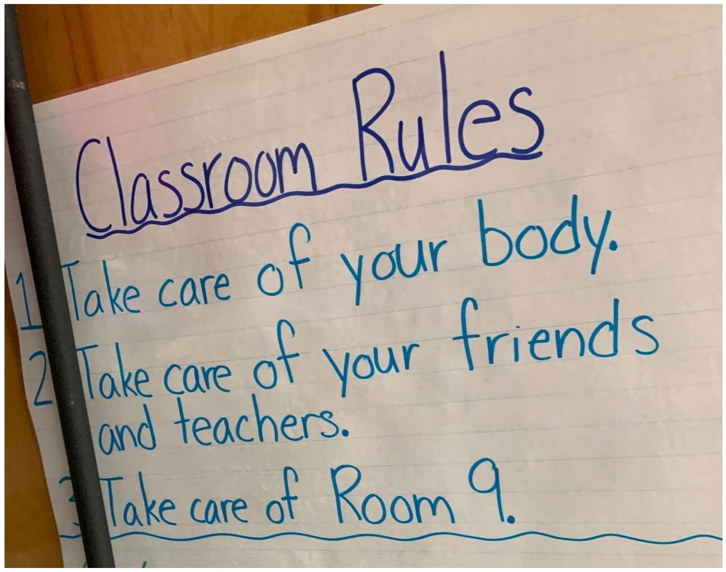


Figure 7. Classroom rules.

Analytic Framework

While exploratory case study is the methodology of choice for my research, I use the four principles for fostering productive disciplinary engagement in a community of learners' classroom by Engle and Conant (2002) as my analytic framework. To reiterate, Engle and Conant's principles for a community of learning are (1) problematizing content, (2) giving students authority, (3) holding students accountable to others and to disciplinary norms and (4) relevant resources. Case study methodology coupled with this analytic framework helped me to investigate a community of learning in greater depth with a specific lens. Case studies often enable the illustration of principles from instances in a bounded system (Cohen, Manion and Morrison, 2000). Specifically, instances in Miss X's first grade classroom of learners illustrated some of Engle and Conant's principles, which helped me to identify opportunities for possibly cultivating a community of learning in this classroom. Case studies provide "a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply by presenting them with abstract theories or principles" (Cohen et al., 2000, p. 253).

Data Collection

I collected data over the course of six weeks in the 2019-2020 academic year during mathematics instruction. My three data sources included observations, interviews, and artifacts/documents. Observations are a major means of collecting data in a qualitative case study about a classroom because they are richly descriptive. When combined with interviewing and artifact/document analysis, observations allow for a holistic interpretation of the phenomenon being investigated which includes a description of the context, participants and activities of interest (Merriam & Tisdell, 2016). During data collection, I assumed the role of observer. My observer activities were known to the group, but I did not participate in the group (Merriam & Tisdell, 2016). My goal was to gather information.

The triangulation of data, collecting and analyzing data through these three sources, enhances the validity of my findings. According to Stake (1995), it is the use of multiple data sources in collection and analysis that forms successful case study research. As described previously, the integrity of a case study depends upon the holistic and realistic representation of the bounded system (in this case, the classroom of learners), therefore, it requires the triangulation of data in both collection and analysis. Merriam and Tisdell (2016) assert that the use of multiple sources of data to confirm findings gives a study validity and is imperative when studying how people experience phenomena. The collection and analysis of data from these multiple sources enabled me to provide a realistic representation of this classroom of learners. I was committed to generating rich, thick descriptions in my data collection and when writing up elements of the study.

Observations. I observed a total of ten mathematics lessons over two months, alternating between two and three observations per week. The teacher allotted 40 minutes for math each day between 10:45 AM and 11:25 AM. One lesson (the ninth lesson I observed), the teacher allotted an extra fifteen minutes and began at 10:30 AM instead of 10:45 AM. This was due to the content of the lesson (multi-step word problems).

Stake (1995) emphasizes that when a researcher conducts an observation, they must keep "a good record of events to provide a relatively incontestable description for further analysis and ultimate reporting" (p. 62). While an observation requires using all five senses to observe a physical setting, participants, activities, interactions, conversations and behaviors, it is the written documentation of those observations that become the raw data which inform the study's findings (Creswell & Poth, 2018). Field notes are one source for written documentation of a researcher's observations and must be approached with care (Merriam & Tisdell, 2016). I referred to Merriam and Tisdell (2016) for tips on being a careful observer and taking detailed field notes. These tips included paying attention, using a narrow angle during observations (focusing on a small group, a specific person, or an interaction as it occurred), looking for key words in conversations, concentrating on the first and last remarks in conversations, mentally playing back scenes during breaks in talking, and limiting time in between observing and reviewing field notes.

I was especially committed to limiting the time between observing and reviewing my field notes. Since I usually observed on a Monday, Wednesday, and/or Friday (see Table 1), there were days in between my observations that allowed me to review and organize my notes. On the days in between observations, I re-read my field notes, as well as edited and added more details to limit the possibility of missing any important events that could have occurred. I wanted to ensure that my field notes were as accurate and honest as possible, that they provided a rich description of the observation and that they would make sense when I revisited them. Having these days to review data in between my observation days, also proved helpful during my ongoing data analysis, which is described in the next section.

I added details to my field notes by embedding pictures of the classroom, resources, and the white board. Merriam and Tisdell (2016) assert that the content of field notes should be highly descriptive and written in a way that makes the reader feel as if they were there seeing what the observer saw. I was committed to this in my work and believe that adding pictures helped me to be more descriptive. I also was intentional in being specific in my language in order to paint a more accurate rendering of what was happening in the classroom. For example, instead of saying "students were working in different places around the room," I would write specifically, "Four students were seated at the high back stools table; four students were at the standing tables; three students were sitting on the floor by the floor table; one student was at the round table with benches; and six were seated around the teacher's table."

While field notes themselves primarily focus on rich factual descriptions of what is happening during observations, observers' comments are where the observer has an opportunity to include reactions, initial interpretations, feelings and speculations. I included a wide margin in my field notes to add my own observer comments. I also used a different color font for my observer comments, so they would stand out and be easily identified. Merriam and Tisdell (2016) share an example of a researcher's field notes in which the field notes include the name of the researcher, the date and time of the observation, the location, purpose of the observation and provide space for recording the physical layout/map of the space and observation notes. I created a field notes template that followed this format which I used during each observation (see Appendix A).

A limitation of this study is that I was not allowed to audio or video record. I reached out to the teacher via email to ask her permission to audio and video record and she responded that she would not allow me to do so. She cited three reasons: she was not comfortable with it; she wanted to protect the privacy of herself and her students; and she had not achieved tenure yet and did not want anything to interfere with that achievement. Given the complexity of the classroom environment as described by Brown (1992) and Bruner (1996), the ability to audio and video record would have enhanced my study in several ways. First, it would have allowed me to investigate all groups around the room at the same time. Since I am one person, I had to stick with one group and could not gather data related to what all groups of students around the room were doing simultaneously. Second, my data is based on typed observation notes, artifacts and interviews, and while I tried to very carefully transcribe interview data and exact statements of students and the teacher during observations, there is the possibility that I may have missed statements or misheard something that was said. Lastly, if I had been able to audio and video record, I could have revisited classroom activities and interactions multiple times to confirm what I said, heard or was reporting on, and to better understand an interaction.

This constraint made it even more important for me to be as thorough and detailed as possible in my field notes. Due to this limitation of having to choose one place to be in the classroom at a time, I made the deliberate decision during my second observation to stick with whatever group involved the teacher. This meant I either stayed with the whole class, when the whole class was interacting with the teacher together (such as the launch of the lesson), or I stayed with a small group when that group was working with the teacher. During my first observation, I initially tried to follow different students and groups as they worked around the room. It became clear that when students were not involved in an interaction involving the teacher, they were often not discussing mathematics. Several students, when they were completing the journal entry, the worksheet “Math by Myself,” or playing the daily game, were engaged in conversations unrelated to mathematics, such as talking about their favorite toys or soccer practice. Focused on my goal of investigating opportunities for a community of learning, I decided to stay with any group that involved the teacher. I was more confident that groups involving the teacher would include conversations about mathematics.

Interviews. The second source of data I collected and analyzed for this study was teacher interviews. Interviews can range from highly structured interviews in which order and questions are set in advance, to totally unstructured interviews with nothing predetermined (Merriam & Tisdell, 2016). I incorporated the use of informal interviews with the teacher before, during or after each lesson. Informal interviews are described by Merriam and Tisdell (2016) as being open-ended, flexible, and exploratory, and occur like a conversation. The informal interviews were an appropriate tool for seeking to make sense of the experiences in the classroom in the identification of evidence of a community of learning as they helped me address the unplanned questions that arose during data collection. In addition, interviewing the teacher provided me the teacher’s perspective on what I was observing. For example, in my first observation, the teacher kept referring to a potato piece. She would say to the students “This might be potato piece worthy!” I had no frame of reference for this statement. Post observation, I conducted an informal interview with the teacher and asked her what “potato piece worthy” meant, to which she informed me of the class reward system described above.

I conducted five informal interviews just with the teacher, Miss X, not with students. My general feeling was that the teacher wanted as little interruption in the students’ learning as possible and by conducting interviews with only the teacher, I was purely an observer in the classroom, which also maintained the integrity of the natural setting since I was not affecting anything the students were doing. I did not want to pull students away from doing mathematics to interview them. I also did not want to disrupt their natural environment by imposing questions on them during teaching and learning. I kept a log of my informal interviews with Miss X and typed the dialogue word for word into my log as we engaged in conversation. I attempted to transcribe our conversation in-the-moment, exactly as it occurred, since I had no audio or video recording device. Interviews and conversations are important in case study research as they offer insight regarding the phenomenon of interest. Merriam and Tisdell (2016) state that informal interviews are most often used in exploratory case study methodology, which is true for this study.

As I began to collect data, it became clear that Miss X was not eager to participate in interviews. I was also only able to engage Miss X in an informal interview half of the time (five out of ten days I was in the classroom). This was based on her actions and behaviors in the busy environment of the classroom with little time to spare. Miss X would answer my questions, amidst children asking for things and while she was answering my questions, she was dealing with students walking into chairs, forgetting materials, lining up, getting lunches, etc. For my first few observations, I arrived at the school 10 minutes early with hopes to set up early and have a chance to ask the teacher questions during the transition to math. Miss X did not allow me in the classroom until the exact time my observation was to begin. In addition, at the conclusion of my observations, the students and teacher had lunch. She would walk out of the classroom with the students and say goodbye to me as she shut off the lights. These behaviors made it clear that she was not interested in spending any extra time answering interview questions. I did manage to gather informal interview data during five observations, and although these interviews are short, they are helpful. The information gleaned from these interviews was important to this study, even though the amount of interview data I collected in the end did not match my initial expectations.

Artifacts/documents. The third piece of data I collected and analyzed were artifacts and documents. Artifacts occur naturally in the research setting. Artifacts are defined as "three-dimensional physical 'things' or objects in the environment that represent some form of communication that is meaningful to participants and/or the setting" (Merriam & Tisdell, 2016, p. 162). Documents are the "wide range of written, visual, digital, and physical material relevant to the study" (Merriam & Tisdell, 2016, p. 162). Given the exploratory nature of this case study, allowing for the collection of artifacts and documents that address the research question was important. Creswell and Poth (2018) note the role that artifacts and documents play in supplementing interviews and observations; they also emphasize that important contextual information can be generated from the review and collection of individual and organizational artifacts and documents. An information-rich case study seeks to build an in-depth picture of the case, and as such, it is important to include physical artifacts and documents in data collection and analysis. Examples of artifacts and documents that I collected in this study include student work, curricular documents, and emails from the teacher. As I moved through this study, it became clearer how important the artifacts and documents were in telling the story of this classroom.

Data Analysis

The purpose of my study is to investigate opportunities for cultivating a community of learning based upon the four principles for fostering productive disciplinary engagement (Engle & Conant, 2002). According to Engle and Conant (2002), productive disciplinary engagement depends upon the combined interactions of each of the four principles. In my study, although I investigated opportunities for cultivating a community of learning based on all four principles, problematizing content and relevant resources were most salient. To analyze my data, I used a qualitative research method in which I incorporated all three sources of data on a regular and ongoing basis. This included reviewing and re-reading my field notes on days in between observations. I also reviewed and re-read my informal interview log in between observations. During these reviews and re-reads, I created personal notes about the data as I immersed myself in the data and attempted to make sense of it using the four principles.

The preferred way to analyze data in a qualitative study is to do it simultaneously with data collection, so that the final product is shaped by the data. Without ongoing analysis, the data can be unfocused, repetitive, and overwhelming (Merriam & Tisdell, 2016). For my analysis, I applied the research method outlined in the Data Analysis Spiral (Figure 8) rendered by Creswell and Poth (2018). According to Creswell and Poth (2018), a researcher begins with managing and organizing data or looking at the big picture, reading and re-reading data, writing down emergent ideas and/or creating memos, then describing and classifying codes and themes, and moving around in this spiral cycle narrowing in on developing an interpretation of the data and finally representing the data to account for one's findings. This spiral pattern repeats during data analysis and collection. In the following paragraphs, I describe how I moved through each of these steps, beginning with managing and organizing the data.

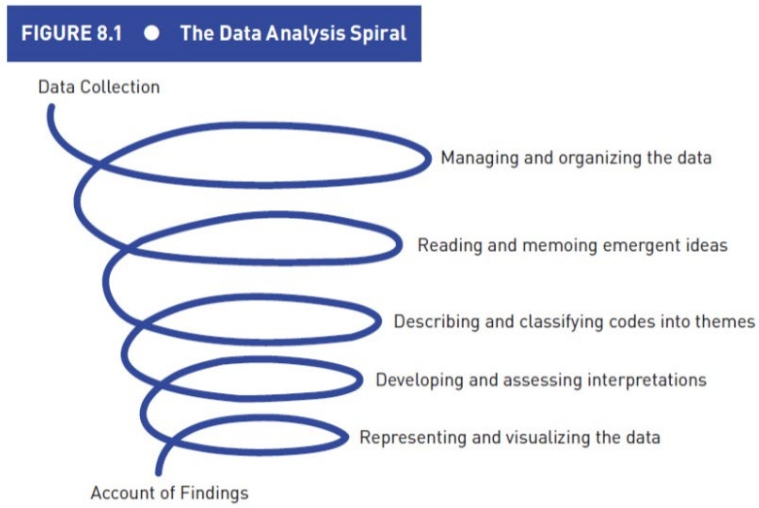


Figure 8. Data Analysis Spiral (Creswell & Poth, 2018, p. 186).

Managing and organizing the data. Anticipating a large volume of data, I was careful to keep the data organized throughout my collection and analysis. I created a folder for my observation field notes, photos from each observation, and photos of any artifacts/documents which included the number and the date of the observation. I used the same template for all my field notes that included a reminder of the purpose of my observations at the top: *To record moments in time where one (or more) of the four principles are, or could have been, evident in the classroom.* I used this reminder as a managing and organizational tool for the data, as the reminder helped me divide the field notes into "instances" that became my units of analysis. After my initial recording of field notes, I re-read and reviewed my field notes to make sure I did not miss anything. For example, I embedded pictures from the previous day's lesson into the field notes and added any additional details that occurred to me during the review. Along with field notes and photos, I maintained a single document, an "Informal Interview Log," where I would type, word for word, as the teacher and I spoke, our informal interviews as they occurred. I organized interview data in this log in chronological order. I used the days in between observations to re-read and review the interview data.

As stated above, my unit of analysis was an instance. I relied on the four principles for fostering productive disciplinary engagement (Engle & Conant, 2002), problematizing content (PC), authority (AUTH), norms (NORMS) and resources (RR), to help me identify instances within the data. An instance refers to a classroom event in which I identified an opportunity for one or more of the principles of a community of learning to be cultivated. I considered an instance to end when the classroom event shifted, meaning the focus of talk or nature of an interaction changed. These shifts were often at the conclusion of a conversation, end of giving directions, or completion in solving a problem. This process was similar to the way Engle and Conant (2002) relied on interactions to illustrate how these four principles were embodied in the classroom in their original study. I identified a total of 144 initial instances. The process of identifying instances involved a back and forth between looking at each of the principles and the data, including opportunities that were acted upon and those that were not. Prior to data collection, I created a one-page document which described each of the principles and gave possible examples for each (see Appendix A). It was not an exhaustive list but offered me guidance as I moved through the data analysis process. While identifying instances, I created a data log in a spreadsheet to help manage the data. I recorded each instance in my data log, in chronological order. I added important contextual information next to each instance (including the date and the focus of the lesson). The data log is where I conducted the reading and memoing of emergent ideas, which is the next phase of the data analysis spiral.

Reading and memoing emergent ideas. This step in the data analysis process involved engaging in reflective thinking about the data and formulating emergent ideas. In this step, the goal was to immerse myself in the details to try and get a sense of the whole and all its parts, before coding and applying themes. It was a built-in step for exploring and thinking about the data. This part of the process also enabled me to track the development of my ideas.

During this step in the process, I created a summary, using my own notes, of each instance during my re-reading and reviewing of the data. These occurred every couple of days between observations. For example, I summarized instances after the first two observations, then summarized again after the next two observations. I conducted ten observations over the course of two months and engaged in this summarizing process four times between observations. As I reviewed the instances with my summaries, I wrote emergent ideas and interpretations for each instance. To organize the reading and memoing of emergent ideas, I included an additional column in my data log for these interpretations. Over time and as a result of my second and third reads of the instances, the emergent ideas changed. A few examples of the emergent ideas in the early stages of analysis include the following: (1) teacher reads problem to students but does not read it word for word; (2) before the teacher reads a problem to the students, she tells them what to do (draw a part-part-whole mat); and (3) teacher presents a problem to the class and then asks the students to give an equation to solve the problem. These emergent themes became the basis for the next stage of my analysis, describing and classifying codes into themes.

Describing and classifying codes into themes. In this stage of the data analysis spiral, I began the process of defining and creating codes, followed by describing and classifying those codes into themes. It is important to note that this activity happened in concert with the memoing of emergent ideas. Specifically, I moved through this analytic cycle many times at various times throughout my data collection. I began this process by reading through the instances in the first two observations. After reading through the instances, I coded instances based on the principles. Specifically, I applied one or more principle codes (PC, AUTH, NORMS, and/or RR) to each instance. I then added more descriptive codes that incorporated my summaries and the principle codes. After initially coding two observations, I went through this process a second time, re-reading the instances and adding new notes about the instances and coding the next three observations. The codes that emerged after these activities included the following: no student talk, all teacher talk, lack of PC, over scaffolding, manipulation of resources, some student talk, teacher is expert/authority. I then classified these codes into themes that I used as I analyzed instances from new observations. This back and forth between reading and memoing the instances followed by describing and classifying codes into themes allowed me to redefine, reorganize and reclassify codes into themes several times. The cyclical nature of the data analysis provided me with the opportunity to re-read my codes and themes to see if they made sense, which allowed new ideas to emerge.

This ability to go back and forth is possible based upon the design of the cycles of analysis in the data analysis spiral. After coding all ten of my observations, I had coded the instances and classified them according to the three themes of resources, authority, and student talk. However, after reviewing the entire data corpus through the lens of these three themes, I felt that most of the instances identified in one theme were also related to authority (108 out of 144), that authority became an underlying characteristic of the instances rather than an independent theme. In addition, I recognized that my initial theme of resources only referred to the curriculum, both the intended and enacted. Since I was only able to gather data from groups that involved interactions with the teacher (whole class and small group) I did not encounter many other resources such as games or tools for problem solving. I changed the code of resources to curriculum and removed the code of authority. I eventually arrived at two themes, curriculum, and discourse, which I discuss in the next chapter and I used these themes to develop and assess interpretations of the data.

Developing and assessing interpretations. The next step in the process of data analysis is to develop and assess interpretations. According to Creswell and Poth (2018) this step involves “abstracting out beyond the codes and themes to the larger meaning of the data” (p. 195). This is the step where I interpreted the big picture of my data and related that big picture to conceptions of a community of learning which already exist in educational research literature. I considered my data through the lens of opportunities for cultivating a community of learning in the natural setting of this first-grade classroom. I then reflected on how my data related to the larger body of literature. In particular, I thought about how the data from this study related to studies that involved interventions, general philosophies of teaching and learning, and research describing what effective mathematics teaching and learning should look like in the classroom. To record these interpretations, I created a new document I called “Discussion/Results.” In this document I began to connect all the pieces: the principles, the themes, the greater literature, and personal views. I share the results of this process and my interpretations of the data in chapter five.

Representing and visualizing the data. This last step of the data analysis spiral is to represent and or create a visual image of the data. I developed hypotheses regarding relationships among themes and the principles according to my data and created a visual image to capture those relationships. This image moved through several iterations before I was satisfied with how it visually represented the data, which I share in chapter five. In the next chapter, I present the results of my study and discuss the four themes I identified during data analysis.

CHAPTER 4: RESULTS

The focus of this study was to investigate opportunities for cultivating a community of learning in an elementary school mathematics classroom based on the four guiding principles for productive disciplinary engagement (Engle & Conant, 2002). My investigation of the datum involved moving in a circular pattern through specific steps of analytic activities outlined in the data analysis spiral by Creswell and Poth (2018). I used the four guiding principles for productive disciplinary engagement to help me identify instances in the data and organize the data and created a data log of these instances. Using this data log, I re-read, summarized, and wrote notes about emergent ideas for each instance, which helped me track the development of ideas. I used the data log to also look for similarities and differences across instances, which led to the creation of categories and themes. Upon reflection, I noticed overlap among thematic ideas, thus I conducted another round of analysis and focused on specific elements of each instance within categories. Using the elements of each instance, I identified two overarching themes, curriculum and discourse, and use these themes to present the results of my investigation.

In the sections to follow, I first describe how the classroom setup provided opportunities for cultivating a community of learning. I then provide a description of the themes, summarize the number of instances associated with each theme and corresponding categories, and present the results of the data using these themes. My research question was around opportunities for cultivating a community of learning. I saw those opportunities through the classroom setup, the daily routines, and the intended curriculum. However, throughout my observations I did not see the teacher taking advantage of these opportunities. Therefore, as I analyzed my data, I identified missed opportunities for cultivating a community of learning through two themes related to the way the teacher enacted the curriculum and facilitated discourse in the classroom.

Classroom Setup

Several aspects of the classroom setup related to Brown's (1992) vision of a community of learning and the four guiding principles (Engle & Conant, 2002). In the paragraphs to follow, I discuss the way the physical space and seating in the classroom were arranged, the daily classroom routine, and the intended curriculum, as aspects of the classroom setup that align with ideas underlying a community of learning and the four guiding principles. Prior to observing any interactions between students and/or teachers in the classroom, these aspects of the classroom setup show that there were opportunities for the cultivation of a community of learning. In addition, in considering the general classroom norms I observed that the students seemed to respect each other, their teacher, and their classroom. This was evident in the way they assisted each other and responded to each other and their teacher.

Physical Space and Seating

The physical space and seating of the classroom were arranged in a way to promote collaboration. Collaboration is necessary in a community of learning (Brown, 1992). One way the community can establish joint expertise is through collaborative learning. The classroom in this study was a flexible seating classroom, ripe with spaces designed to promote group learning. These spaces included round tables with chairs and benches facing each other, rectangular tables with stools or standing tables that positioned students facing each other, and a large kidney shaped table to work with the teacher, as well as a large carpet space. Students were free to move about the classroom when not engaged in the whole class lesson launch or in small group instruction with the teacher. The positioning of students facing one another and the flexibility of students to move about connects with the third principle of Engle and Conant (2002), for students to be held accountable to others and to disciplinary norms.

According to third principle of accountability, for students to be held accountable, they must consult others in constructing understanding. The flexibility and arrangement of the physical space and seating in this classroom supported the notion of students consulting one another to construct mathematical understandings. This type of classroom setup could reinforce normative practices associated with student accountability when cultivating a community of learning. A normative practice of accountability is that students are responsible for sharing their thinking and responding to the thinking of others. Studies by Lampert (1990) and Yackel and Cobb (1996) involved students participating in normative practices that included presenting their ideas to others because they felt an obligation for sharing their thinking. A classroom environment in which all the desks/chairs were facing one direction, or in which students had an assigned seat, may be less conducive to this type of group learning. According to the layout of this classroom, students were not restricted to one specific seat or space, rather the classroom space was designed to be flexible with spaces arranged in ways to promote students learning together. The spaces involved students facing one another which would make it easier for students to share their thinking and attend to the thinking of others. When I asked the teacher about the flexible seating arrangement, she told me that they had just adopted this new flexible seating arrangement in September. The principal had offered this option to any teacher that was interested. She told the principal that she was interested, stating that she did not like the students' old desks and preferred for her students to have the flexibility to move around and work in groups.

Classroom Routine: The Daily 3

In addition to the way the physical space and seating were arranged in the classroom, the daily classroom routine (the "Daily 3") included aspects which could support the cultivation of a community of learning in the classroom. Brown and Campione (1994) describe a community of learning as having a ritual, familiar participant structure. The daily routine involved ritualistic activities and offered a familiar structure for students to participate. Every day, students engaged in the Daily 3, which included an independent journal question, worksheet or "Math by Myself", and a game to play with peers. In addition to these ritualistic activities that were one of the Daily 3, other parts of the daily routine included small group work with the teacher and a whole class lesson launch. As mentioned previously, the teacher began each lesson with a whole class lesson launch. After that, she called one small group of students at a time to work with her at a kidney shaped table in the back of the classroom. During the time that the teacher worked with small groups, the rest of the class was to be working on an aspect of the Daily 3. This structure, the Daily 3, exhibits opportunities for cultivating a community of learning in terms of the first and second principles of Engle and Conant (2002), problematizing content and authority.

Problematizing content involves students questioning, challenging, proposing, and making intellectual contributions in constructing mathematical understanding. Engagement with mathematical content in any of the Daily 3 activities just described could support opportunities for students to question, challenge, propose ideas and make intellectual contributions. Small group work and playing a game with peers could be spaces in which students talk with one another and discuss ideas collaboratively. Active participation in the lesson launch could provide an opportunity for students to question, challenge, propose ideas and make intellectual contributions by making their thinking and ideas public and engaging in collaborative sense making with others. An example of this sort of participation in the classroom is what Lampert (1990) described as students challenging one another's thinking in respectful ways in group settings.

"Math by Myself" offered a chance for students to implement their mathematical ideas independently and the journal activity provided a space for students to elicit curiosities and make meaning through solving a task independently in their journal. Math journals have been found to have a positive influence on elementary school students' communication of their own mathematical thinking (Kostos & Shin, 2010). Kostos and Shin (2010) conducted their research on the use of math journals with second graders and assert that even with minimal writing skills, younger students could also illustrate mathematical thinking by using pictorial representations in math journals. Communicating mathematical thinking is essential for a community of learning. Schoenfeld (2016) describes in the TRU framework dimension of agency, ownership, and identity, which I associate with the principle of authority, that students need to have ownership over content. The Daily 3 could provide opportunities for students to have ownership over content, especially through students engaging in these activities independently, such as the math journal, Math by Myself, and the game. The elements of the Daily 3 all support opportunities for students to problematize content.

The Daily 3 relates to notions of students having authority in the classroom. Moving through the Daily 3 activities requires self-regulation which creates an opportunity for students to have authority in the classroom. When I contacted the teacher to ask about her classroom set up and routine prior to my observations, she told me in an email that during the Daily 3 students had to self-regulate as they moved through tasks. Authority according to Engle and Conant (2002) involves students having shared responsibility in their learning. In the Daily 3, students had opportunities to engage in a game with peers, to complete "Math by Myself," and to complete a task in their math journal independently. Moving through these tasks could provide students with authority in that it involves students having responsibility in their learning. They self-regulate as they move through tasks and participate in the learning community as stakeholders. The daily routine, in terms of the elements of its design, demonstrated opportunities for students to engage with mathematical tasks and problematizing content, as well as monitor their own thinking and have authority.

Intended Curriculum

The curriculum also exhibited characteristics that could provide opportunities for cultivating a community of learning in this classroom. The curriculum Pearson enVision 2.0 (2016) was designed with the goal of deepening students' abilities to think mathematically through problem-based learning. The curriculum is described as a learner-centered curriculum, with components and tiered activities to accommodate all learners and maintain rigor (Pearson enVision 2.0, 2016). For example, an aspect of the curriculum called the "Solve and Share," which was the intended launch of the lesson, included discussion provoking tasks often consisting of word problems that required students to make meaning, strategize and connect concepts in order to solve. The curriculum described the "Solve and Share" as being a way for students to engage in productive struggle, to use any strategy for solving that they wished, and to engage in discussion with one another regarding mathematical ideas. According to the Teacher's Edition, organization of lessons and curricular tasks were designed to "keep students on a path to higher levels of cognitive demand" (Pearson enVision 2.0, 2016, p. 40). The curriculum aligns with the goals of a community of learning, to enable students to think deeply about disciplinary content.

The Pearson enVision 2.0 (2016) curriculum was developed in response to the changes in mathematical standards and align with recommended mathematical practices for effective teaching and learning set forth in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, 2010) and NCTM (2014). This curriculum emphasizes mathematical thinking and reasoning, conceptual understanding and problem solving. I mentioned previously how each lesson began with a "Solve and Share," an opportunity for students to discuss their own ideas for solving. In addition, the tasks presented in the curriculum required sense making to solve. There is a direct link made explicit between tasks in the curriculum and mathematical practices for effective teaching and learning. I include an example of a task from a lesson on explaining subtraction strategies (Topic 4 Lesson 7). I chose to share this example because it includes a reference to the mathematical practice of making sense (MP.1). According to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, 2010), the first standard of mathematical practice that teachers should develop in their students involves making sense of problems and persevering in solving them. These connections are made explicit in the curriculum. Moreover, there is an image of a character posing questions to students (Figure 9). Next to the problem, a speech bubble presents the questions, "What's your plan for solving the problem? What else can you try if you get stuck?" These questions center on students' thinking, like a community of learning, and are aimed at provoking discussion and explanation, which relate to the principles of problematizing content and accountability. These questions also align with effective mathematics teaching practice of facilitating meaningful mathematical discourse in the classroom (NCTM, 2014). These types of questions, which appear throughout the curriculum, support opportunities for facilitating meaningful mathematical discourse and appear next to tasks that require students to make sense and engage in deep thinking. I described in the previous chapter how the nature of a task plays an important role in engaging students in deep thinking. The nature of tasks in this curriculum, provide opportunities for students to think deeply and make sense of mathematical ideas. The curriculum, based on its design and types of tasks included, demonstrates opportunities for cultivating a community of learning.

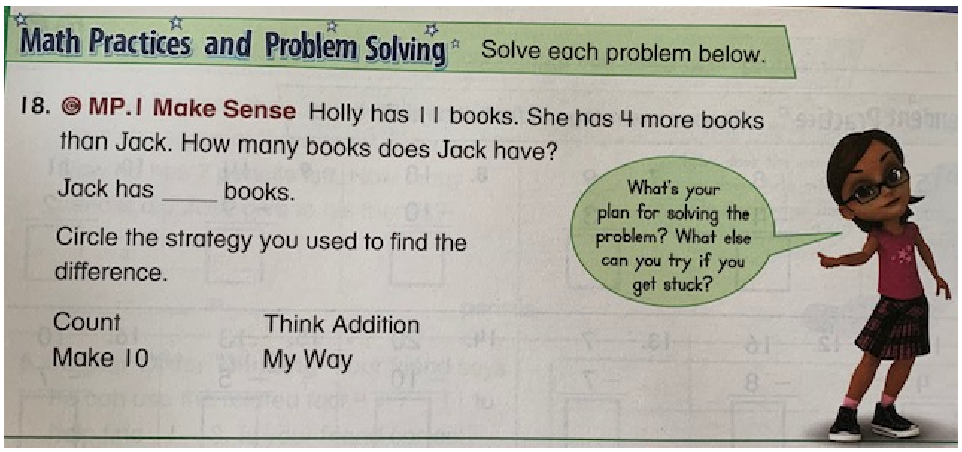


Figure 9. Curricular task related to mathematical practices.

The ways in which the classroom was set up, the arrangement of the physical space and flexible seating, the daily routine of the classroom, and the curriculum, supported opportunities for cultivating a community of learning in this classroom. This was clear from the information gathered about the class prior to my observations. In the next section, I describe the two themes I identified during data analysis and present the results of this study using these two themes.

Description of Themes

I classified my codes into the following two themes to share the results of the data: curriculum and discourse. Every instance I identified in the data relating to opportunities for cultivating a community of learning involved either changes to the intended curriculum and/or discourse that occurred in the classroom. Within each of these themes I established two categories. Instances associated with the theme of curriculum were grouped as either premeditated changes to the curriculum by the teacher or in-the-moment changes to the curriculum by the teacher. As described above, because of the way the curriculum was designed, it presented opportunities for cultivating a community of learning through presenting problem-solving based tasks to students to invoke discussion and allow them to make connections to prior knowledge. Therefore, in this theme of curriculum when I identified instances, they were about how the teacher enacted the curriculum. Instances associated with the theme of discourse were grouped as either IRE (I-initiation, R-reply, and E-evaluation, described in detail below) or as a type of discourse characterized by the teacher giving direct instructions. Because of the way that the classroom was set up and the nature of the Daily 3, there were opportunities for cultivating a community of learning through the teacher's facilitation of discourse. The instances I identified related to the facilitation of discourse involved the teacher only facilitating discourse in two ways, in the form of IRE or direct instruction, which is why these forms of discourse are the only ones associated with this theme. See Figure 10 for a visual image of these themes and corresponding categories.

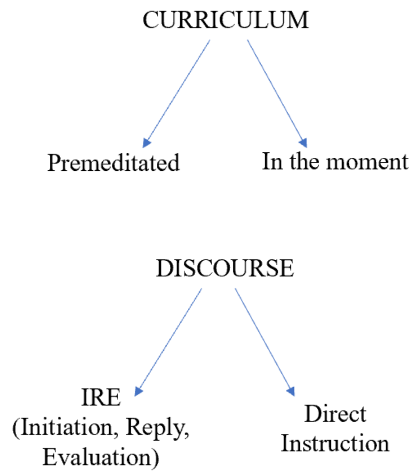


Figure 10. Themes and corresponding categories identified during data analysis.

As described previously, the goal of this study was to investigate opportunities for cultivating a community of learning in an elementary school classroom based on the four principles of Engle and Conant (2002). I used these four principles: problematizing content (PC), authority (AUTH), norms (NORMS) and resources (RR) to help me manage and organize the data. I identified instances within the data related to a community of learning based on these four principles. Using the four principles to analyze the data, I identified 129 instances. I identified 70 instances that related to the theme of curriculum and 122 instances that related to the theme of discourse. (Refer to table below.)

Theme 1: Curriculum	
Number of instances involving premeditated curricular changes:	36
Number of instances involving in-the-moment curricular changes:	34
Total number of curricular instances:	70
Theme 2: Discourse	
Number of instances involving IRE patterns of discourse:	75
Number of instances involving direct instruction type of discourse:	47
Total number of discourse instances:	122
Total number of instances:	129

Table 2. Numbers of instances according to each theme and category.

As seen in Table 2, there was an overwhelming number of instances related to the theme of discourse that fall into the IRE category, 75 instances in all. There were 47 instances associated with the teacher providing direct instruction according to the theme of discourse. In addition, there were a somewhat equal number of instances related to both categories of curriculum: 36 instances associated with premeditated curricular changes, and 34 instances associated with in-the-moment curricular changes.[6] It is important to point out that some instances are associated with both themes, which is evident from the table totals.

The first theme, curriculum, relates entirely to the teacher's use and enactment of the curriculum. In terms of curriculum, there is the intended curriculum, or the way the curriculum appeared in the textbook, and the enacted curriculum, or the way the teacher engaged with the curriculum during implementation. The teacher in this study often made changes to the intended curriculum. The changes that the teacher made to the curriculum impacted opportunities for cultivating a community of learning in the classroom. The teacher made these types of changes so frequently that they became an overarching theme according to the data. Based on the numbers above, there were 70 instances I identified in which the teacher modified the intended curriculum. Sometimes these changes occurred before a lesson. I grouped instances associated with the teacher making changes to the curriculum prior to a lesson as premeditated. There were 36 instances of premeditated changes to the intended curriculum. Examples of premeditated changes that the teacher made to the intended curriculum involved changing the launch task entirely, following a different sequence of lessons from the order in which they appeared in the curriculum, or modifying problems on worksheets and the assessment. Also, the teacher made

[6] Although I named these instances in-the-moment, because I was unable to interview the teacher after instruction, I do not know if the teacher had some prior experience to inform these alterations or if she thought of them at that time. Therefore, I use this term to distinguish these from the pre-made tasks and worksheets.

changes to the curriculum during a lesson or in-the-moment. There were 34 instances of in-the-moment changes to the intended curriculum. Examples of in-the-moment changes include the teacher changing the language of a problem while reading it aloud and changing the type of problem being presented.

The second theme, discourse, refers to the types of talk I identified within instances in the data. There were instances that involved teacher-student-teacher talk, and instances in which the teacher was the only one who spoke. In all 122 instances associated with the theme of discourse, the teacher initiated the talk. Whether it was instances involving teacher-student-teacher talk, or instances involving teacher talk only, the teacher was always the first, and sometimes only one, to speak. Discourse became an overarching theme based on the ways these types of discourse impacted opportunities for cultivating a community of learning in this classroom. All 75 instances of teacher-student-teacher talk that I identified followed a pattern of talk referred to as IRE. This pattern of talk is characterized by the teacher initiating (I) a question, a student responding (R) to that question, and is followed by an evaluation (E) of the student's response (Mehan, 1979; Smith & Stein, 2011). In this type of discourse, the teacher arbitrated student responses, which impacted opportunities for cultivating a community of learning. The 47 instances associated with teacher talk only, typically involved the teacher giving direct instructions of some sort. Either the teacher told students what materials to gather and gave instructions regarding tasks, or the teacher told students what to draw or write on their papers. Sometimes the teacher told a student how to solve a problem without eliciting any student input whatsoever. These instances were categorized as direct instruction within the theme of discourse and the nature of this talk impacted opportunities for cultivating a community of learning.

As stated above, there were instances that I coded in more than one theme. For example, one instance I identified involved Miss X working with a small group of students and is associated with both themes: curriculum and discourse. During this instance, Miss X read a problem from the textbook to a small group of students. As she read the problem aloud, in-the-moment, she changed words in the problem which changed the nature of the problem. In addition, she told students not to worry about the second part of the task, which was selecting the strategy they used to solve the problem. Because this instance involved a change the teacher made to the curriculum during enactment, this instance is associated with the theme of curriculum. This instance is also associated with the theme of discourse and the corresponding category of IRE. During the exchange of talk between the teacher and students during implementation, the teacher initiated a question, "How many books did she give to Jack?" and students responded, "7." Then the teacher evaluated that response, "Okay, good," and indicated that the students were finished completing the problem by saying "We aren't going to worry about the rest of the words there." This instance can be associated with both themes. It includes an in-the-moment modification to the intended curricular problem and discourse related to IRE. In the next two sections, I discuss each theme and present the results of the data.

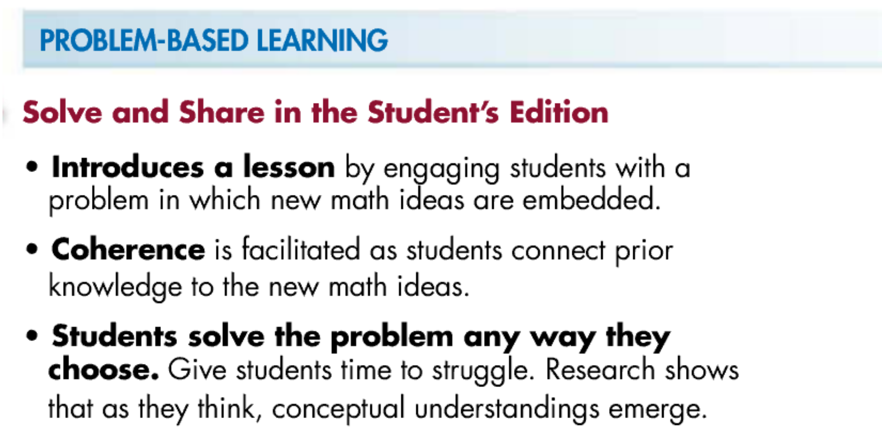
Curriculum

The modifications to the curriculum, both prior to a lesson and in-the-moment, impacted opportunities for cultivating a community of learning in this classroom. These modifications altered content with which students engaged, the types of tasks and problems they encountered, and what opportunities students had to engage in deep thinking through the altered content. These modifications relate to accountability because many of the modifications the teacher made emphasized correct answers rather than holding students accountable for justifying or sharing their thinking about how they solved a problem. These modifications relate to authority in that by changing the tasks, the teacher held authority over student learning by dictating solution paths or use of resources. For example, one way of changing tasks was that the teacher included scaffolding techniques, such as adding blank diagrams next to certain problems on pre-printed materials or hurrying students through a strategy when solving in a problem in a group setting, and this took authority away from students in the sense that students did not have the liberty to solve a problem any way they chose, but were limited to the teacher's strategy or approach. Modifications to the curriculum also relate to the fourth principle of relevant resources. Changing the nature of the tasks in the ways I just described, altered the resource of the curriculum in such a way that it no longer supported the other principles.

As mentioned, evidence of this theme includes the teacher's use and enactment of the curriculum premeditated (prior to a lesson) and in-the-moment (during a lesson). I identified 36 instances of premeditated modifications to the curriculum, and these instances involved the teacher creating her own version of aspects of the lesson (e.g., "solve and share" or launch), modifying a worksheet (e.g., "reteaching worksheet" and assessment), or changing the order of lessons within a topic. I identified 34 instances of the teacher modifying the curriculum by either choosing to skip problems during a lesson, changing the language in a word problem when reading the problem aloud, or changing the problem itself during the lesson.

Premeditated. Examples of premeditated modifications to the curriculum include instances in which the teacher modified aspects of the curriculum prior to teaching a lesson. The teacher made premeditated modifications to the "Solve and Share," the Topic 4 "Reteaching" worksheet, the Topic 5 "Assessment," and by changing the order of lessons within Topic 5.

Solve and Share. One aspect of the curriculum that the teacher modified was the "Solve and Share." As mentioned previously, Pearson's enVision 2.0 (2016) is a problem-based learning curriculum, with a focus on giving students time to solve problems with information at hand and connect prior knowledge to new mathematical ideas. The curriculum makes it clear that students should be given the opportunity to grapple with mathematical ideas, have sufficient time to engage in productive struggle, be permitted to solve problems any way they choose, and have opportunities to connect prior knowledge with new mathematical ideas when engaging in the "Solve and Share." (Figure 11). These ideas align with community of learning practices.


PROBLEM-BASED LEARNING

Solve and Share in the Student's Edition

- **Introduces a lesson** by engaging students with a problem in which new math ideas are embedded.
- **Coherence** is facilitated as students connect prior knowledge to the new math ideas.
- **Students solve the problem any way they choose.** Give students time to struggle. Research shows that as they think, conceptual understandings emerge.

Figure 11. Description of "Solve and Share" (Pearson enVision Math 2.0, 2016, p. 45).

In my observations, the teacher, Miss X, incorporated her own version of a "Solve and Share" during the whole class launch of a lesson, which I refer to as the teacher's launch. Miss X created problems for the teacher's launch, and often her problems looked quite different from the problems presented in the curricular "Solve and Share." The way Miss X presented the problems during her launch when I was observing did not give students time to struggle or the choice to solve the problem in any way they chose. Miss X often presented problems in her launch that were algorithmic in nature, and the operations and procedures were made explicit to students. The types of tasks that Miss X presented in her launch did not require explanations or create opportunities for students to share new mathematical ideas or engage in discussion. Miss X's launch was heavily scaffolded as she provided step-by-step guidance for students in solving problems. In Figure 12, I provide an example of an original "Solve and Share" activity alongside Miss X's modification for a teacher's launch (for a list of all modifications of the "Solve and Share," see Appendix C.)

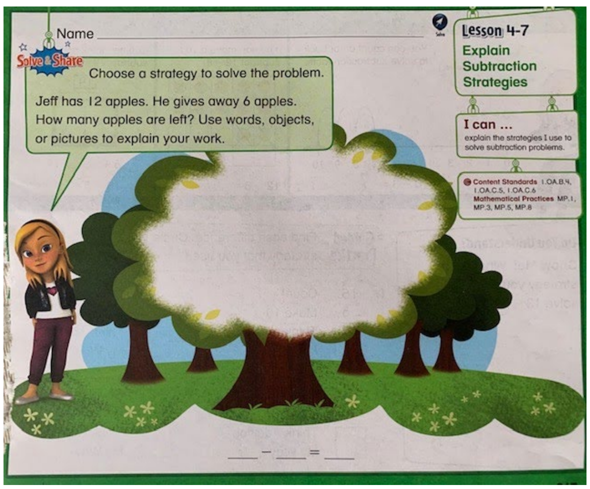
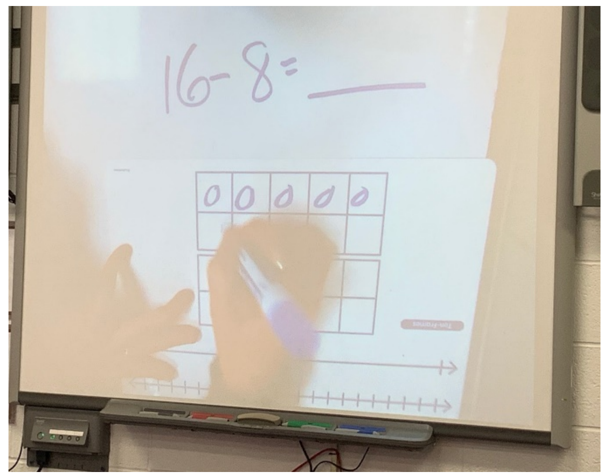
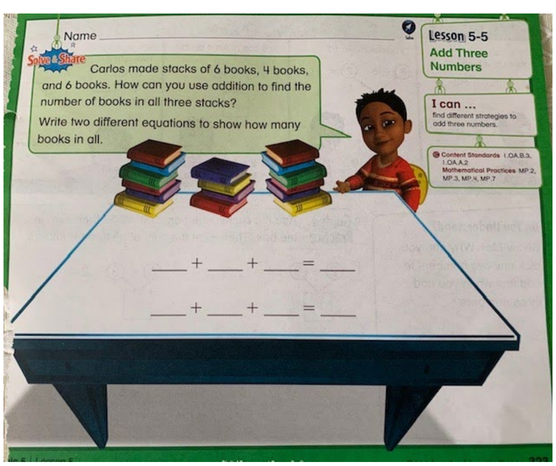
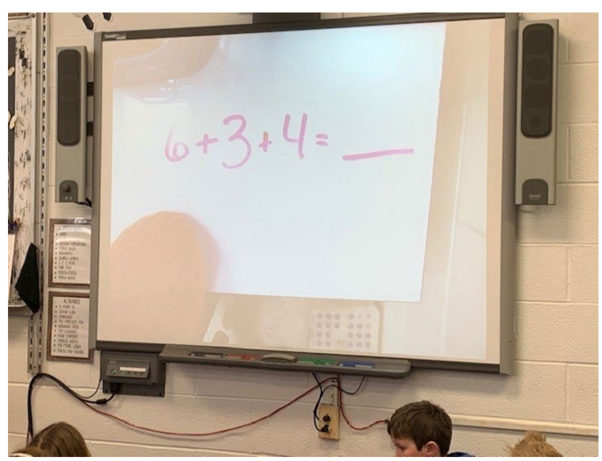
CURRICULUM SOLVE & SHARE	TEACHER'S LAUNCH
 Observation #2: Solve & Share Lesson 4-7 (Explain Subtraction Strategies)	 Observation #2: Teacher's Launch

Figure 12. Curricular "Solve and Share" activity compared to the teacher's launch.

The example illustrated in Figure 12 comes from my second observation where the focus of the "Solve and Share," according to the curriculum, was for students to explain their subtraction strategies (Topic 4 Lesson 7). The "Solve and Share" problem in the curriculum was a word problem, which did not give students an equation. This problem stated: "Jeff has 12 apples. He gives away 6 apples. How many apples are left? Use words, objects or pictures to explain your work." In contrast to this problem, Miss X presented a decontextualized subtraction problem in her launch. Miss X presented the problem "*16-8=_*" and she presented the problem alongside a ten frame. The premeditated changes to the problem were that Miss X presented the problem as a number sentence and she did not give students an opportunity to try to solve the problem in whatever way they chose. More specifically, while both problems involved separating with an unknown result (starting with a whole and separating a part), the teacher presented her version of the problem as a decontextualized subtraction number sentence. Additionally, Miss X presented her problem alongside a ten frame. By showing the problem alongside a ten frame, Miss X emphasized one strategy to use when performing the operation, before eliciting student ideas or promoting student discussion regarding strategies.

In eight of my observations, Miss X conducted a teacher's launch, and in each of these eight observations, she made similar premeditated modifications to the "Solve and Share" aspect of the curriculum. A second example of such an instance, during my seventh observation, related to finding different strategies to add three numbers (Topic 5 Lesson 5). The curricular "Solve and Share" included the following word problem: "Carlos made stacks of 6 books, 4 books, and 6 books. How can you use addition to find the number of books in all three stacks? Write two different equations to show how many books in all." In contrast, Miss X presented the problem "*6+3+4=_*" in her launch (Figure 13).

<table>
<tr><th>CURRICULUM
SOLVE & SHARE</th><th>TEACHER'S
LAUNCH</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Observation #7: Curricular Solve & Share Lesson 5-5 (Add Three Numbers)	Observation #7: Teacher's Launch

Figure 13. Curricular "Solve and Share" activity compared to the teacher's launch.

Like the previous example, the problem that Miss X presented in her launch was in the form of a number sentence, "*6+3+4=_*." While both problems involved joining (or adding) three numbers with an unknown result, Miss X's version of the joining problem was as a decontextualized addition number sentence. The curricular version required students to construct their own two number sentences based upon the contextual information provided in the problem. The changes that Miss X made removed any ambiguity from the task and removed any connection to the underlying concept of the commutative property of real numbers.

Reteaching worksheet. Miss X made premeditated modifications to the curriculum outside of the "Solve and Share." Some of these modifications were made on the "Reteaching" worksheet for Topic 4 (Strategies for subtraction facts to 20). According to Pearson enVision 2.0 (2016), the "Reteaching" worksheet is an intervention tool designed to assist teachers in providing students with additional guided practice to build their understanding. In the Topic 4 "Reteaching" worksheet, Miss X specifically altered four out of the original ten problems (see original and modified problems in Figure 14).

CURRICULUM RETEACHING PROBLEMS	TEACHER'S MODIFIED RETEACHING PROBLEMS

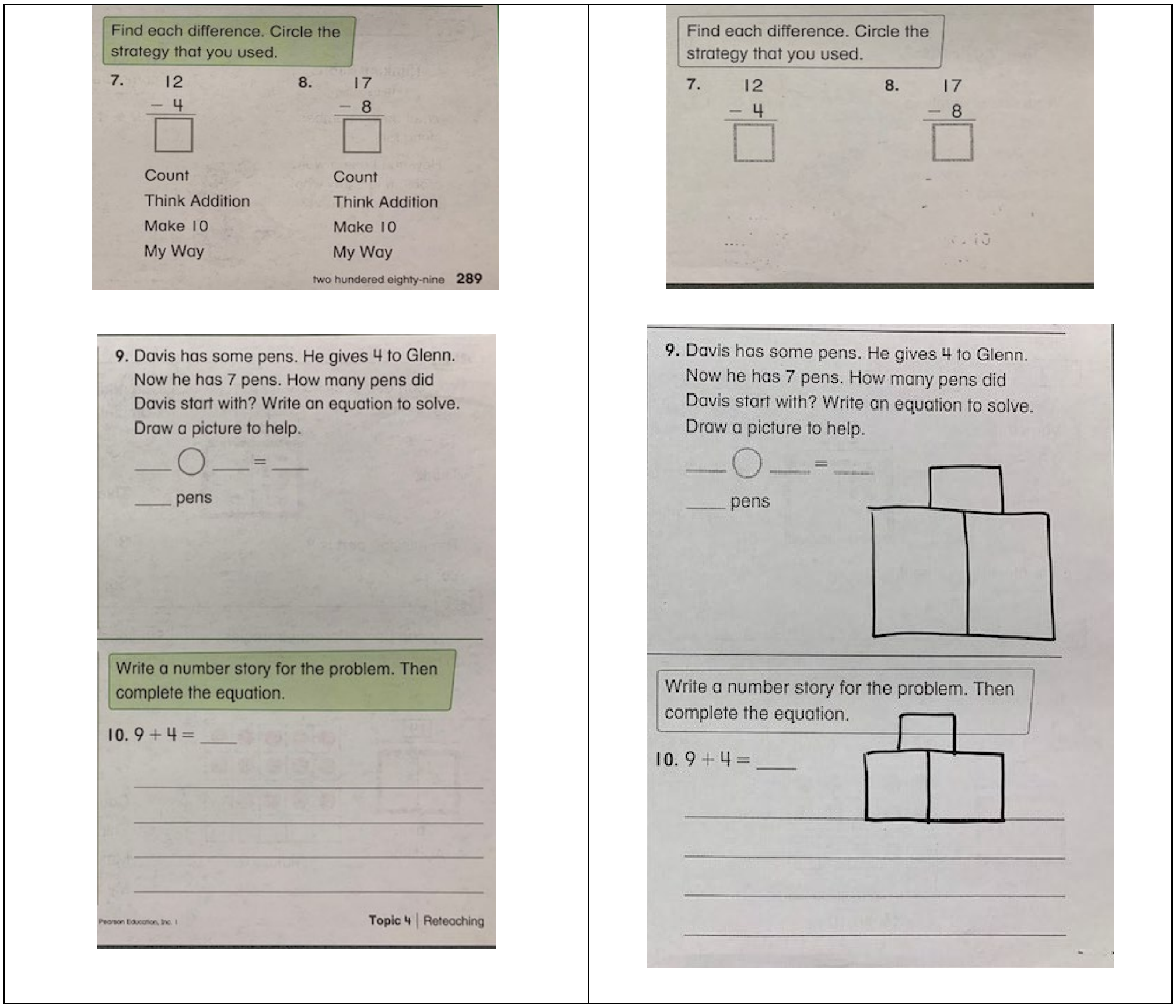


Figure 14. Reteaching worksheet for Topic 4 original (left) and modified (right).

The first two problems that were modified (number seven and number eight) included the following directions: “Find each difference. Circle the strategy that you used.” As seen in Figure 14, both subtraction problems were presented in the form of an equation. Beneath the equations were four choices of subtraction strategies: count, think addition, make 10, and my way. These four choices represented subtraction strategies referenced in previous topics in the curriculum, as well as in Topic 4. My observations began in November with Topic 4 Lesson 5. The strategies listed under problem number seven above would have appeared in previous lessons prior to my observations. The strategy of counting back to subtract was the focus of Topic 2 Lesson 6; the strategy of think addition was the focus of Topic 2 Lesson 7; the strategy of using a number line to subtract was the focus of Topic 4 Lesson 1; and the strategy of making ten to subtract was the focus of Topic 4 Lessons 2 and 3. In my observations, I observed Miss X using the strategies of counting back, using a number line, and using a ten frame to subtract.

The modification that Miss X made to problems number seven and eight on the reteaching worksheet was removing the choice of strategies beneath each subtraction equation, requiring students to only provide an answer to the equation without selecting the strategy that they used. I do not have information regarding Miss X's decision to remove the choice of strategies from this problem due to limited opportunities for interviewing her. However, the premeditated modification shows a focus on the answer as opposed to a focus on process or strategy. Students sharing their thinking is an important aspect of a community of learning and sharing the strategy they used to solve a problem is one way that students share their thinking in the classroom (Engle & Conant, 2002; Franke & Kazemi, 2001). The principles of problematizing content and accountability focus on student thinking regarding the problem and understanding through collaborative learning.

For problems number nine and ten, the teacher added the image of a diagram intended to scaffold student thinking. This diagram is a part-part-whole diagram. Part-part-whole diagrams stem from the model method, which is a method of using diagrams or pictures to represent or model certain relationships. The model method is often incorporated into elementary school mathematics classrooms as a tool to help students visualize and represent relationships (Ng & Lee, 2009). The model method involves students learning through textual, pictorial, and symbolic modes of representation. In the lower grades, students use objects, pictures, and symbols to model combinations of numbers (Ng & Lee, 2009). The part-part-whole diagram is a visual model to help students see quantitative relationships. In this instance, by adding a blank part-part-whole diagram next to problem number nine on her modified version of the "Reteaching" worksheet, Miss X was representing an arithmetic situation of combining two parts to make a whole. By adding this diagram, Miss X's students were directed toward a specific pictorial way of thinking about the textual problem. Similarly, Miss X added a blank part-part-whole diagram in her modification for the next problem. This problem, displayed in the curriculum as an equation "*9+4=_*", included these directions: "Write a number story for the problem. Then complete the equation." In addition to including the diagram for problem number ten, Miss X altered the directions to the problem by not requiring students to write a number story for the number sentence. Instead, she instructed the students to solve the problem and fill in their part-part-whole diagram. The activity of solving an equation and writing a number story to represent an equation requires different kinds of thinking. Solving an equation is focused on producing a correct answer while writing a number story is focused on making connections with conceptual ideas that underlie a procedure (Stein et al., 2009). In her modification, Miss X left little abmiguity regarding how to solve the problem. By not asking students to contextualize the equation by writing a number story to go with it, students did not have an opportunity show their understanding of addition as a procedure for joining two quantities in the conext of a real-world problem. In terms of cultivating a community of learning, students must have opportunities to engage in complex thinking.

Assessment. Miss X also made premeditated modifications to the "Assessment" for Topic 5 (Work with addition and subtraction equations). Miss X's version of the assessment, which she called "Show What You Know," included different problems than the assessment presented in the curriculum. I describe three examples of these differences here. For a list of all the modifications Miss X made to the "Assessment", see Appendix D.

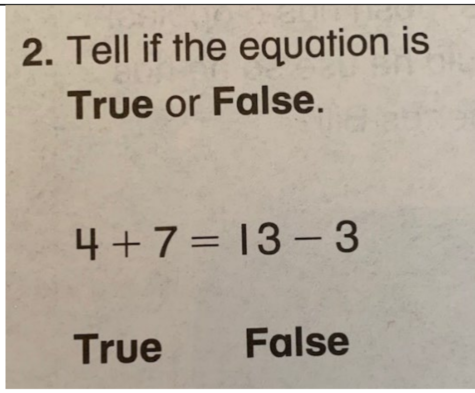
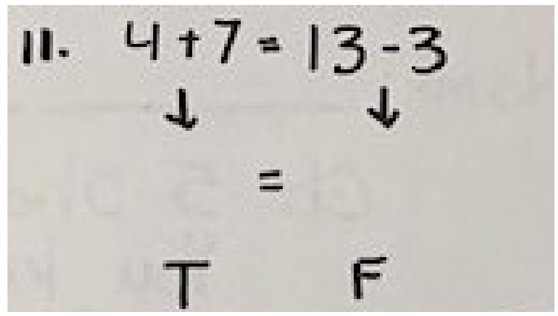
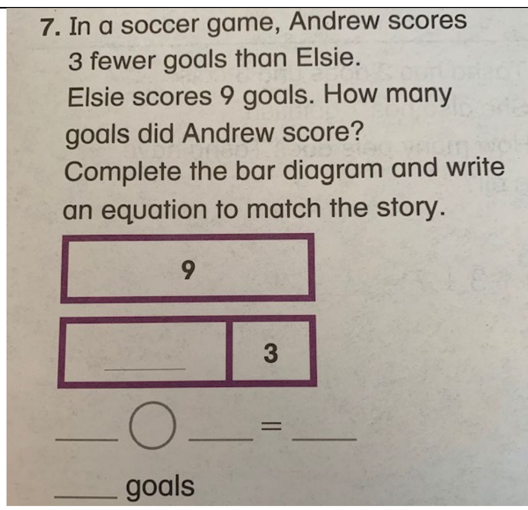
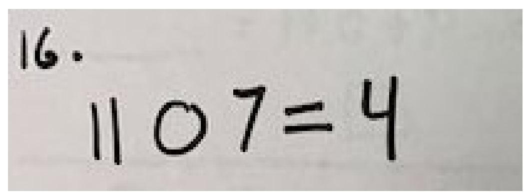
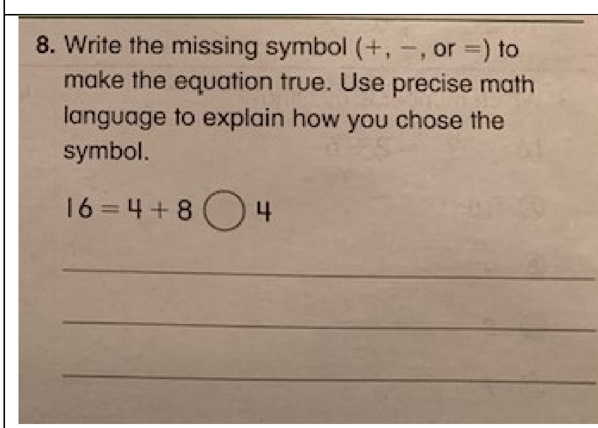
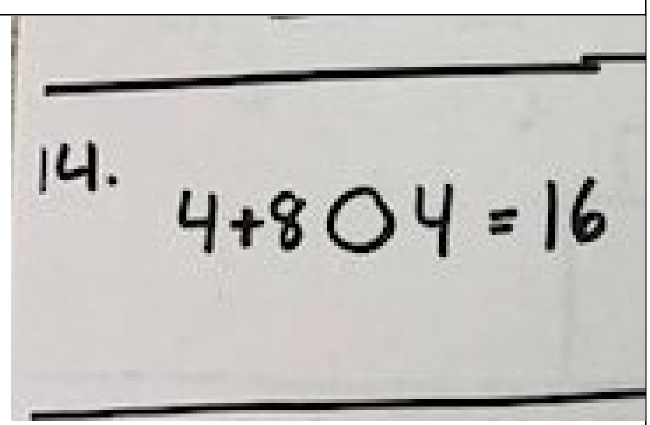
CURRICULUM TOPIC 5 ASSESSMENT	MISS X's TOPIC 5 ASSESSMENT
 2. Tell if the equation is **True** or **False**. 4 + 7 = 13 − 3 **True** **False**	 11. 4 + 7 = 13 - 3 ↓ ↓ = T F
 7. In a soccer game, Andrew scores 3 fewer goals than Elsie. Elsie scores 9 goals. How many goals did Andrew score? Complete the bar diagram and write an equation to match the story. 9 ___ 3 ___ ◯ ___ = ___ ___ goals	 16. 11 ◯ 7 = 4
 8. Write the missing symbol (+, −, or =) to make the equation true. Use precise math language to explain how you chose the symbol. 16 = 4 + 8 ◯ 4	 14. 4 + 8 ◯ 4 = 16

Figure 15. Examples of teacher's modifications to problems in the assessment presented in the curriculum.

There were several differences between the assessment presented in the curriculum and Miss X's assessment (Figure 15). First, the assessment presented in the curriculum included eight problems, four of which were in the form of a word problem. Miss X's "Show What You Know" included more problems, eighteen in total, of which only one was in the form of a word problem taken from the assessment presented in the curriculum. For some modifications, Miss X did not change the problem entirely, rather provided some sort of scaffolding for students. For example, Miss X's version of problem number two shown above, "*4+7=13-3*," included the addition of arrows. Beneath the statements on each side of the equals sign, Miss X included arrows to presumably guide students in solving a problem side. The arrows signaled to the students that they were to add four and seven then write the sum beneath the arrow, then subtract three from thirteen and write the difference beneath the arrow. After solving each side and following the arrows to write the sum or difference, the students were to indicate if the statement was true or false. This was a procedure that the students had practiced in class and I believe that that by adding arrows, Miss X focused students on the procedure for solving. This modification could have had the effect of limiting students' thinking about underlying concepts when engaging in tasks involving the various properties of equalities and properties of addition and subtraction. I do not know about the teacher's intention in adding arrows to these types of problems, but I observed that she often modified problems in the intended curriculum by including arrows like this and diagrams. I interpreted these changes as having the effect of limiting students' conceptual thinking based on research regarding procedural knowledge and conceptual knowledge. Procedural knowledge is developed when students engage in procedural tasks, which are described as familiar tasks that involve problem types students have seen before and know how to solve (Rittle-Johnson, 2015). Procedural tasks often involve transfer problems, that require recognition of a known procedure rather than understanding of an underlying concept. However, procedural approaches do not always support growth in conceptual knowledge (Rittle-Johnson & Schneider, 2015). A major goal of the intended curriculum was to provide students with opportunities to build conceptual knowledge (Pearson enVision 2.0, 2016). It is my interpretation that Miss X used arrows as a way to focus students on the procedure for solving these types of problems, and prompted their thinking in terms of that procedure as opposed to providing opportunities for students to build on their conceptual knowledge of addition and equality. In doing this, Miss X enacted the curriculum in a way in which it was not intended.

Problem number seven in the assessment presented in the curriculum asked students to write an equation to match this story: "In a soccer game, Andrew scores 3 fewer goals than Elsie. Elsie scores 9 goals. How many goals did Andrew score?" Miss X did not include any problems like this in her assessment, but rather included problems that involved a missing operation. The curricular problem demanded a different level of thinking in that students had to make sense of the story to construct their own equation. The problems related to separation in Miss X's assessment provided students with all three numbers given and the students were required to fill in the operation. Specifically, Miss X's modification focused on assessing the students' ability to identify which operation was needed to complete the equation. Miss X did not ask students to construct their own number sentence based on their conceptual understanding of an operation in relation to contextual information provided in the problem. While this section is focused on presenting the premeditated modifications Miss X made to the curriculum, it is important to point out that when Miss X implemented this assessment, she provided a high-level of scaffolding. For example, for problem number sixteen on the "Show What You Know," when the students were working on this problem, Miss X said “So for number sixteen, we have an equals sign so you decide, is it plus or is it minus? 11 blank 7 equals 4. Is it plus or minus?” I addressed this level of scaffolding with Miss X in an informal interview after this observation. I told Miss X that I noticed she provided a lot of guidance to the students in solving problems when administering the assessment, she responded by saying, “If I just pull back from guided instruction and just say do it, who knows what they would do with it.” I interpret this comment as evidence of Miss X’s belief that her students were either not capable or developmentally ready to engage with tasks without a high-level of support from her. I was not able to ask any follow up questions, as our interview time was limited and after this question, the students and teacher left the classroom.

Problem number eight on the assessment presented in the curriculum required students to write the missing symbol to make this equation true, “*16=4+8_4*” and “use precise math language” to explain how they chose the symbol that would make the equation true. Miss X changed the equation by writing her problem as “*4+8_4=16”* and additionally, she did not ask students to explain their thinking. The problem presented by Miss X in her enactment of the curriculum followed a traditional format with the operations on the left side and result on the right side of the equal sign. In contrast, the problem presented in the intended curriculum followed a nontraditional format, with the result on the left and the operations on the right side of the equal sign. There is research-based evidence showing that students can develop a greater understanding of the concept of mathematical equivalence when they are presented with problems in the nontraditional format. This research attributes students’ understanding of the concept of equivalence when learned through practice with nontraditional problems with students’ abilities to generalize beyond arithmetic or operational patterns, and in turn, consider mathematical equivalence in terms of the relationship between quantities (McNeil, Fyfe, Petersen, Dunwiddie & Brlectic-Shipley, 2011). Although Miss X seemingly made a minor change, her change may have narrowed students' experiences which may limit their abilities to develop a deeper conceptual understanding of mathematical equivalence. I interpret this change as a lowering of cognitive demand. As presented in the curriculum, the problem in the nontraditional format would have focused students on using a procedure to develop a deeper understanding of a mathematical concept, which is a "Procedures with Connections Task" according to TAG (Stein et al., 2000). Miss X's change to present the problem in the traditional format lowered the cognitive demand by allowing students to perform the operation with limited understanding of the concept of equivalence. In addition, Miss X did not require students to explain their thinking or reasoning. Stein and Smith (1998) associate tasks that do not require students to explain their thinking or reasoning with lower levels of cognitive demand (Stein & Smith, 1998). In fact, Miss X's version of the assessment, the "Show What You Know" did not include any problems that required students to explain or describe their mathematical thinking or reasoning. In addition, Miss X included scaffolding in the form of arrows, part-part-whole diagrams, and magic boxes[7] to a much greater extent than the assessment presented in the curriculum and she changed the format of problems, which may have impacted students' development of conceptual understanding.

Based on my analysis of the data, it is my conclusion that the problems Miss X included in her assessment were largely algorithmic in nature and focused students on producing answers rather than giving students an opportunity to explain their thinking or express their strategies and

[7] A magic box was a strategy Miss X used to show the students how to add a problem that included three addends. She instructed students to add two numbers and write the sum of those two numbers in a magic box, then add that number in the magic box to the remaining addend.

processes for solving. A similar focus on having students provide answers without explaining their thinking was presented in the examples shared above relating to premeditated modifications to the “Solve and Share” and “Reteaching” worksheet. The modifications that Miss X made to the “Solve and Share,” the “Reteaching” worksheet and the assessment presented in the curriculum limited students’ opportunities to share their thinking, which I surmise impacted opportunities for cultivating a community of learning in this classroom. These examples connect to another way that Miss X made premeditated modifications to the curriculum, changing the order of lessons.

Changing the order of lessons. During my seventh observation (Topic 5), Miss X presented Lesson 5 (adding three numbers) before presenting Lesson 4 (word problems with three addends). I discussed this decision with Miss X during our interview that day. During the interview, I said, “I thought you would do 5.4 today.” Miss X replied:

> We have to do 5.5. first. They introduce you to three addends in equations in 5.5. So, we are going backwards. I don’t know why they do it this way (rolls eyes toward teacher’s guide). It makes more sense to do 5.5, equations, before 5.4, which is word problems. You can’t do word problems until you learn the equations.

I interpreted Miss X’s decision to change the order of lessons to be based on her belief that word problems were supposed to come after equations. This could be a result of Miss X’s perception that word problems to be more difficult for students than equations, or symbolic problems. This change to the order of the lessons impacted how the mathematical ideas were presented to students, namely that the students encountered three addends in the form of an equation before encountering three addends in the form of a word problem. The order in which the curriculum presented the lessons, word problems with three addends before equations, aligned with the tenets of the curriculum, a problem solving based curriculum supporting students in productive struggle, with making connections to prior knowledge and in promoting higher order thinking (Pearson enVision 2.0, 2016). Word problems give students real-world context for adding three numbers, it gives meaning to the operation, before students engage in adding three numbers in the form of a decontextualized number sentence.

In a later interview during a different observation (still within Topic 5), Miss X confirmed this perception that she regarded word problems as being difficult for students. During my ninth observation involving Topic 5 Lesson 6 (solving addition and subtraction word problems), after changing the language of the curricular word problems when reading the problems aloud to one group of students, I asked Miss X if she thought the word problems were difficult for the students. Her response was, “Totally. I am going to make up problems for the next group, as you will see. I won't even attempt these. These are too hard.” Her statements provide insight regarding her belief that the students in her classroom were either not capable of engaging with these types of problems or that they were not developmentally ready to engage with the problems presented in the curriculum.

In this section, I present data illustrating the ways in which Miss X made decisions prior to instruction that modified the intended curriculum. These included Miss X creating her own version of aspects of the lesson (such as the “Solve and Share”), modifying worksheets (the “Reteaching” worksheet and assessment), and changing the order of lessons within a topic. Beyond these premeditated modifications, Miss X also modified the curriculum during her teaching. I call these modifications “in-the-moment” and discuss them below.

In-the-moment. Other instances that involved the teacher making modifications to the curriculum that impacted opportunities for cultivating a community of learning occurred during instruction. Examples of these “in-the-moment” modifications represent changes the teacher made to the curriculum in her enactment of the curriculum during a lesson. In the following paragraphs, I revisit the examples shared in the previous section involving the “Solve and Share” problems and discuss the ways that Miss X continued to make modifications to the curriculum by presenting the problems differently than what was intended by the developers. In addition to these examples, I discuss how the teacher modified the curriculum in-the-moment as she read word problems aloud to students. Reading word problems aloud to students is common in first grade, especially because students are on different reading levels (Powell & Nelson, 2017). Miss X would read problems aloud to students and routinely use different language or change the problem entirely in her verbal presentation.

Solve and Share. As discussed in the previous section, during one of Miss X’s launches, she posed the subtraction problem “*16-8=_*” to her students with the picture of a ten frame. The intended curriculum, according to the teachers’ guide (Pearson enVision 2.0, 2016), was to give students time to struggle, allow students to solve the problem in any way they chose, and create opportunities for students to connect prior knowledge with new mathematical ideas. Instead, immediately after posing the problem, Miss X began providing students with step-by-step procedures for solving the problem and elicited student input in the form of one or only a few words. The exchange of talk during this launch was as follows:

Teacher: If I wanted to use this ten frame to solve the problem, sixteen minus eight, what would I do?

Student 1 (after raising their hand): Fill in the ten frame.

Teacher (Fills in the ten frame with sixteen circles.): Now what do I do?

Student 2: Cross them off.

Teacher: Why would I cross them off?

Student 2: Because it is subtraction.

The teacher then asked the students where to start crossing off and a student told her to start at the bottom. The teacher crossed out circles and students counted along until she had crossed out eight circles.

Teacher: So, sixteen minus eight equals?

Student 3: Seven.

After this exchange, Miss X asked the students to count again to double check how many circles remained. It is important to point out that when Miss X did not directly respond to the contribution from Student 3, she communicated an implicit evaluation to the students, connecting this instance to the theme of discourse and IRE as well. After this exchange, the students counted eight circles remaining. The teacher said, "So sixteen minus eight equals eight" and moved on to a different problem. In this instance, the enacted curriculum did not follow the intent of this aspect of the lesson. Specifically, in her enactment of the problem, Miss X did not give students time to struggle or share their mathematical ideas. Moreover, Miss X neglected to incorporate Student 3's response into the lesson, which may be considered as not reflecting the practices of a community of learning. One of the tenets of a community of learning is that participants, students and teachers alike, respond to and discuss the ideas of others. Miss X moved into procedures for solving the problem and neglected Student 3's response. The limited opportunity for students to respond to and discuss ideas of others is also seen in how the students participated in this launch by providing short responses to the teacher's questions (i.e., What do I do first? How many circles do I cross off?). The teacher chose and carried out the strategy for solving and did not provide the students an opportunity to choose the strategy, which also reduced the complexity of the task. In addition, the teacher took over the thinking and reasoning, or the problematizing of content, by walking the students through each step of solving, rather than letting students explain their reasoning themselves.

A similar example related to another "Solve and Share" I described in the previous section involved this addition equation with three addends, "*6+3+4=_*." In her enactment of this task, Miss X again did not give students an opportunity to think about, discuss, or share ideas regarding how to solve the problem. After showing the students the problem, she moved right into guiding students through step-by-step procedures to solve, eliciting student input in the form of only one- or two-word responses to her questions. The launch occurred as follows:

Teacher: Who wants to read this to me? [Student 1][8], go ahead.

Student 1: 6 plus 3 plus 4.

Teacher: We are used to adding part-part-whole. Today we are doing part-part-part whole. There is magic involved when you add three numbers. Here is what the magic looks like, you have to start by adding just two numbers. So, somebody pick just two numbers. Two of these numbers.

Student 2: 4 and 6.

Teacher: (Teacher crosses out 6 and 4). We can magically change that into? 7…8…9…10. (Teachers counts on using her fingers). We can magically change that into?

Students: 10!

Teacher: Now we take 10 and magically add it to 3.

[8] The naming of students does not correlate across instances. For example, Student 1 from other instances is not the same student as Student 1 in this instance. I named students based on the order in which they spoke in each instance.

Students: 13!

In this instance, Miss X routinized the task by pressing students to move through specific procedures to solve, namely, to choose two numbers to add first before adding the third number. Moreover, as in the previous example, she did not provide students with opportunities to share their thinking or engage in discussion about ideas for solving. Student 2's idea to add four and six may have provided an opportunity for a rich classroom discussion about a mathematical idea (the commutative property of addition). The student selected two addends out of order, and possibly strategically to create a partial sum of 10. 10 is easier to add with other addends. The number 10 is considered a benchmark number because we use a base-10 number system. Students would have been introduced to the strategy of making 10 to add in Topic 3 Lessons 6 and 7, "Making 10 to Add" and "Continuing to Make 10 to Add" (Pearson enVision 2.0, 2016). Miss X did not address this or elicit the student's thinking regarding their selection of which two numbers to add first. In this moment, the principle of accountability could have been cultivated. Miss X could have asked the student to explain their thinking or press the student to justify their selection of which two numbers to add first.

Changing words to problems. Other examples of in-the-moment modifications to the curriculum that Miss X often made was in changing problems while reading them aloud. An example of this type of in-the-moment modification to the curriculum occurred during my ninth observation as Miss X worked with a small group of students. She said, "Sandi makes 11 fruit cups. She gives five away to Jack. How many are left?" This verbiage did not match the way the problem appeared in the curriculum. In the curriculum, the task was stated as follows: "Jack makes 5 fewer fruit cups than Sandi. Sandi makes 11 fruit cups. How many fruit cups did Jack make?" (Figure 16).

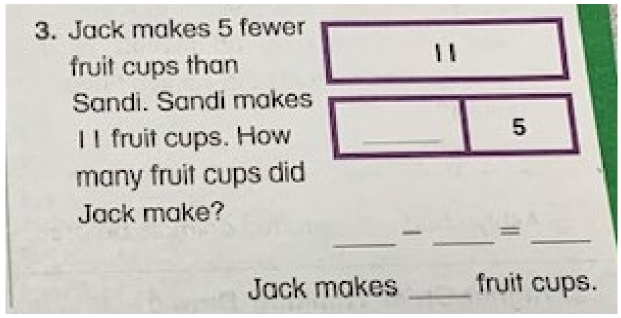


Figure 16. Task as it appeared in the curriculum.

There are several differences between these two presentations of this problem. First, in the curricular presentation, Sandi did not give five fruit cups away to Jack. The problem stated that "Jack makes 5 fewer fruit cups than Sandi." Second, the question was not asking about how many fruit cups are left. In the curriculum, the problem was asking how many fruit cups Jack made. The way Miss X stated the problem presented it in the form of a separation type problem (starts with a whole and gives some away). The way the problem was worded in the curriculum presented it as a part-part-whole type problem (starts with a given part and the whole).

The types of problems presented, the way they are presented in the curriculum and how the teacher presented them, are an important aspect of the data. Carpenter et al. (2014) make distinctions among different types of problems in regard to CGI and discuss how the wording of problems impact the way students think about and solve them. In first grade, students are working with addition and subtraction problems. These can be grouped into four main problem types: join, separate, part-part-whole and compare. Join problems involve a direct action in which a set is increased by a certain amount. Such as "Jane has four dolls. She gets two more dolls. How many dolls does she have now?" Separate problems are similar, but an initial quantity is decreased. An example is "Will has four fish. He gives two fish to Jack. How many fish does Will have left?" There are more variations with problem types regarding the result being unknown, the change being unknown, or the start being unknown. In part-part-whole problems, there is a static relationship among a set, with two disjoint subsets. Part-part-whole problems do not imply an action. Either two parts are given and the whole needs to be found, or one of the parts and the whole is given, and the other part needs to be found. An example is “4 boys and 3 girls were playing baseball. How many children were playing baseball?” In this problem, there is no implication for joining or separating. Compare problems involve relationships between quantities rather than implying joining or separating, like part-part-whole problems. In compare problems, however, two distinct, disjoint sets need to be compared, a referent set and a compared set, in order to find the difference between sets. For example, a compare problem might be worded as “John has four footballs. Ryan has eight footballs. How many more footballs does Ryan have?” In this example, the four footballs John has is the referent set, the eight footballs Ryan has is the compare set, and the difference, or solution to this problem, is the amount between those sets.

According to Carpenter et al. (2014) “variations in the wording of the problems and situations they depict can make a problem more or less difficult for children to solve” (p. 10). The authors describe how the wording of problems are easier for children to solve when the wording of a problem relates to the action sequence for solving. In the instance I just described, the teacher changed the trajectory of the problem by changing the words of the problem. She changed it from a separate problem to a part-part-whole problem. The modification she made “Sandi starts with 11 fruit cups and gives 5 away to Jack” makes this problem easier for students to think about and solve because the action to solve the problem is clearly described in the sequence. The teacher started with a whole and described giving some away. The curricular problem reads “Jack makes 5 fewer fruit cups than Sandi. Sandi makes 11 fruit cups. How many fruit cups did Jack make?” This original problem provides “a more rigorous test of whether children are carefully analyzing the problem or just mechanically operating on the numbers given in the problem" (Carpenter et al., 2014, p. 11). In making this modification, the teacher made the problem easier for students to solve which impacted opportunities for cultivating a community of learning, and the opportunities for students to engage in complex thinking.

During my ninth observation, the teacher changed another problem while reading it aloud as well. In both this instance and the previous one, the students had the original curricular problem in front of them, but the teacher said something entirely different. The teacher expressed the problem as, "Harry has 5 buttons and Tina has 7 buttons. How many buttons do they have altogether?" The problem in the book stated, "Harry has 5 fewer buttons than Tina. Harry has 7 buttons. How many buttons does Tina have?" (Figure 17).

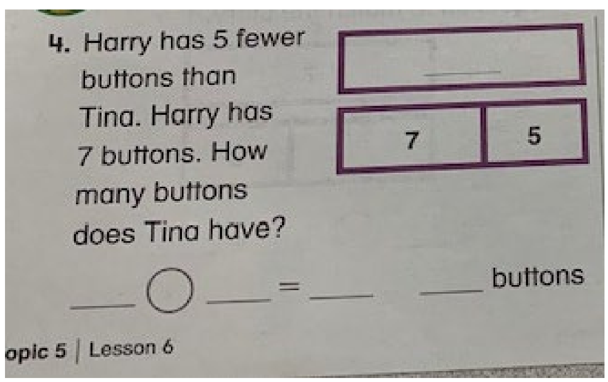


Figure 17. Task as it was presented in the curriculum.

The curricular problem is asking about how many buttons Tina has after giving the information that (1) Harry has five fewer than Tina, and (2) Harry has seven buttons. The way the teacher read the question aloud presented an entirely different problem, requiring students to think about how many buttons they had all together as opposed to how many buttons Tina had in relation to Harry. The problem presented in the curriculum was a compare problem, which involves comparing quantities rather than joining or separating quantities. In the problem presented in the curriculum, the compared quantity, the number of buttons that Tina has (the larger amount), is the unknown (Carpenter et al., 2014). The teacher presented the problem as a part part whole problem which involves a static relationship between a particular set and its two disjoint subsets. It is important to note that the intended curriculum included a diagram next to this problem which showed seven and five as the two parts with the larger quantity unknown. Based on the contextual information in the problem presented in the curriculum, the seven in the diagram represents the amount of buttons Harry has, or the referent, and the five in the diagram represents how many buttons Harry has compared to Tina, or the difference. The number of buttons that Tina has, or the compared quantity, is unknown and is represented in the diagram by the blank line. One must understand the contextual information provided in the problem for the model to make sense. One challenge in using a model such as the diagram presented in this problem is that if the textual information presented in the problem is not interpreted correctly, it can lead to unsuccessful problem solving and limit conceptual understanding (Ng & Lee, 2009). I do not know why Miss X changed the words to the problem during her enactment of the curriculum or how the diagram impacted her modification. However, my interpretation of this modification is that by changing the words of the problem Miss X presented a different type of problem to students. Moreover, the problem she presented can be associated with a lower level of cognitive demand because part part whole problems are considered easier types of problems to solve (Carpenter et al., 2014) and the compare problem presented in the curriculum would require more complex thinking.

This problem is like many of the “Solve and Share” and teacher’s launch examples, in that Miss X presented a problem that did not allow students to grapple with perplexing ideas. The perplexing idea in this instance was the comparison of two amounts to determine a larger unknown, as opposed to the less of looking at two parts to determine a whole. As described previously, Carpenter et al. (2014) describe compare problems as involving relationships between quantities which require the comparison of two distinct, disjoint sets. The compare problem demands a higher level of cognition. Unfortunately, I was not able to ask the teacher about why she modified the problem in this way, or how the diagram may have impacted her modification. I am only able to report on how she modified the problem and my interpretation of that modification based on the data I was able to collect.

Another instance of the teacher modifying the curriculum was when the teacher and students were working on Topic 4 Lesson 7 (explaining subtraction strategies). I present this example to highlight the principles of problematizing content, accountability, and authority in relation to the teacher's modifications and implementation of this task. The curricular task was presented as follows: "Holly has 11 books. She has 4 more books than Jack. How many books does Jack have?" The problem was categorized as a "make sense" problem and included the choice of four strategies from which students could choose to indicate their strategy for solving. In addition, there was a picture of a character next to the problem, with a speech bubble posing the questions "What's your plan for solving the problem? What else can you try if you get stuck?" (Figure 18). The curricular problem was presented as a comparison problem. According to the information provided in the problem, students were to compare the number of books that Holly had (11), with the difference between how many books Holly had in relation to how many Jack had (Holly had four more than Jack). In addition to solving the problem, the curricular task required students to indicate which strategy they used to solve. The intended curricular problem is presented in a way that would give students joint responsibility in the production of knowledge. The questions, "What's your plan for solving the problem? What else can you try if you get stuck?" could distribute authority among the students and serve as a way to hold students accountable for explaining their thinking. Students were instructed to indicate which strategy they used to solve the problem, which gave them expertise in terms of deciding upon a strategy and relates to the principle of authority. In addition, the two questions about their plan for solving and what to do if they were stuck, solicited an explanation from students regarding how they made sense of the problem which relates to the principle of accountability. When students have opportunities to explain their thinking and discuss strategies for solving a problem in the classroom, they develop agency and ownership over content and are held accountable in a community of learning. (Engle & Conant, 2002; Schoenfeld, 2016).

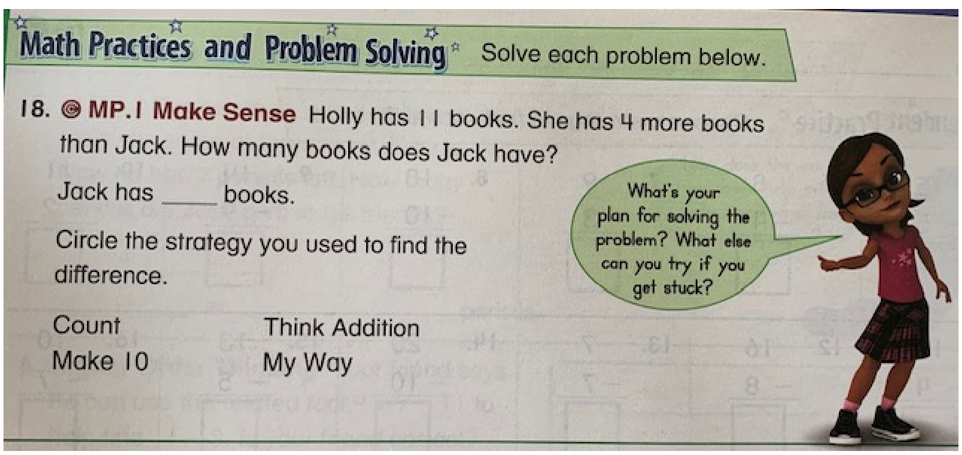


Figure 18. Curricular task prior to teacher modifications (cognitive demand and authority).

During implementation, while the students had the problem as written in front of them on their worksheet, the teacher told the students what to write on their papers as she read the problem to them, which limited students' authority. The interaction occurred as follows:

Teacher: Holly has 11 books. Holly has 11. Draw a part-part-whole model and put 11 on top. She keeps four. She gives these to Jack (teacher points to the empty "part" box). Figure out how many she gives to Jack. (Pause). How many books does she give to Jack?

Student 1: Seven.

Student 2: Seven.

Teacher: Anyone get anything that is not seven? (Students do not respond). Where it says Jack has blank books, fill in seven. We are not going to worry about the rest of the words there (referring to the speech bubbles and the strategy choices).

Miss X not only changed the type of problem presented, but she also changed the nature of student participation. The curricular problem was presented as a comparison problem with the smaller part unknown. The teacher changed it to a separate problem, Holly has 11 books, keeps four and gives some away, with the result unknown. As described previously, comparison problems require a different level of thinking than separate, result unknown problems (Carpenter et al., 2014). In her modification of the task and implementation, she impacted opportunities for cultivating a community of learning, especially in terms of problematizing content, as her modifications to the problem reduced the level of cognitive demand.

Skipping problems. Another way Miss X modified the curriculum during enactment was when she skipped problems during a lesson. Such an instance occurred during my first observation, when Miss X was working with a small group of students on a curricular worksheet during Topic 4 Lesson 5 (Using addition to subtract). She told the students to skip two problems, numbers six and seven (Figure 19).

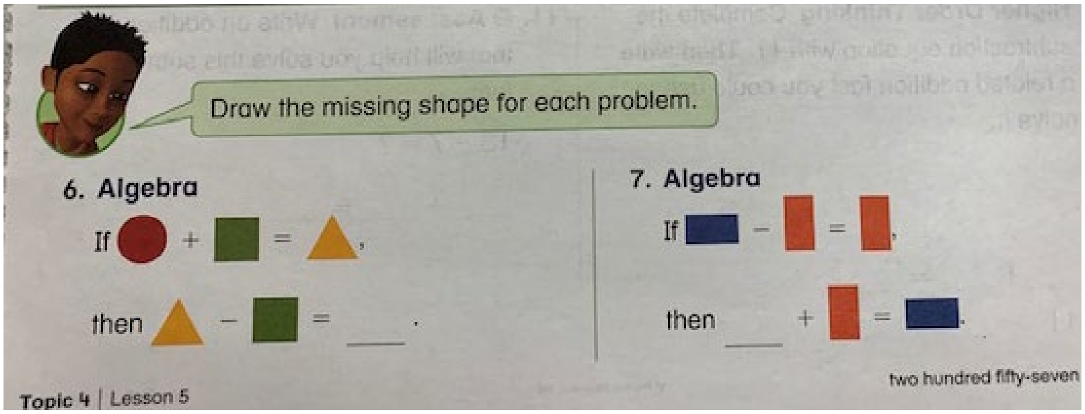


Figure 19. Problems the teacher skipped (#6 and #7).

Miss X decided what conceptual ideas were relevant to students and her enactment of the curriculum reflected this decision. I asked Miss X why she skipped these problems in a brief interview after the lesson. Her response was “This was not relevant to them, the algebra. Number six would have been frustration for them.” Algebraic concepts are associated with complexity in cognitive demand, as are word problems (Fuchs et al., 2016). These statements provided by Miss X offer insight regarding the complexity of problems and her perception about her students’ cognitive abilities. These statements also demonstrate how she did not promote productive struggle. I would have liked to probe further, but our opportunities for informal interviews were limited.

The modifications to the curriculum that I described in this section provide examples of the teacher doing the following: changing problems in her verbal presentation of the problem; not giving students time to grapple with tasks; and not invoking student thinking or reasoning. These actions of both premeditated and in-the-moment modifications to the curriculum relate to lowering the cognitive demand in the classroom and impacted opportunities for cultivating a community of learning. The teacher’s premeditated and in-the-moment modifications to the curriculum were an obstacle to the intended curricular goals as well as an obstacle to cultivating a community of learning. In the next chapter, I discuss the teacher’s modifications to the curriculum within the context of a community of learning in relation to effective teaching and learning.

Discourse

As mentioned previously, all the instances of classroom talk that I identified in my data were teacher initiated. To reiterate, the focus of my observations was on interactions that included the teacher and students as participants (e.g., small group work or whole class learning). As a reminder, I did not follow students who were working on other aspects of the Daily 3. My experience during my first observation was that students who were not working with the teacher were not discussing mathematics and I did not have the ability to audio or video record during observations. I was limited in the sense that I could not follow all groups at the same time, thus I made the decision to focus on interactions that involved the teacher. In order for me to investigate opportunities for cultivating a community of learning, I needed to look at interactions that involved talk about mathematics. What I found was that interactions including the teacher were more likely to include talk about mathematics. The concept of a community of learning is built around students thinking deeply about disciplinary content, so it was necessary for me to focus on interactions that included mathematical content as part of the conversation.

Through whole class and small group instruction, I identified instances of talk that occurred between teacher and students (teacher-student-teacher talk), which I categorized as IRE. I identified other instances involving teacher talk only, which I categorized as direct instruction. Over the course of ten observations, I identified 122 instances related to discourse. 75 of those instances were grouped into the category of IRE, and 47 of those instances were grouped into the category of direct instruction. Every instance of talk I identified fell into the category of IRE or direct instruction. I provide examples of instances among both categories and describe how these types of talk impacted opportunities for cultivating a community of learning.

Teacher-student-teacher talk following an IRE pattern. After careful analysis of the instances involving teacher-student-teacher talk, it became clear that every instance involved a pattern of discourse commonly referred to as IRE: initiation, reply, evaluation (Mehan, 1979). In these classroom interactions, the teacher initiates the discourse typically by asking a question. A student (or students) respond, and the teacher evaluates that response. IRE is a historical, commonly used interaction pattern in which the student's participation consists of responding to teacher questioning with their response being evaluated by the teacher (Smith & Stein, 2011). The purpose of the interaction is for the teacher to check whether the student knows the answer, and often, students' answers are in the form of one or only a few words (Wood, Williams & McNeal, 2006). A constraint of this type of questioning is that it does not deepen students' comprehension or build on student thinking (Smith & Stein, 2011). Instances associated with the theme of IRE occurred both during the teacher's launch as well as during small group instruction and included both verbal and nonverbal input from both the teacher and the students.

Launch. One of these instances I coded as an IRE pattern of interaction occurred during the teacher's launch. In this instance, Miss X was guiding students through steps for determining if an equation was true or false and helping students understand the meaning of the equal sign. Students were looking at the number sentence "*7+1=2+6*" to determine if it was a true or false statement. The following exchange between Miss X and the students occurred:

Teacher: Let's start with seven plus one. What is that?

Students: Eight.

Teacher: Now two plus six. It is easy to say six and count two in my head. So, start with six and count on two.

Students: Six… seven… eight…

Teacher: (Writes eight equals eight under the original equation.) Now, is this true or false?

Students: True.

Teacher: Yes, true.

The instance above relates to the IRE pattern of discourse. In particular, the teacher posed questions requiring one- or two-word answers. The students recalled information or provided an answer after each question, and the teacher evaluated their answers. The I, or initiation, in this interaction is Miss X asking, "Seven plus one. What is that?" The R, or response, is the student responding with "Eight." The E, or evaluation, is implicit when Miss X says, "Now two plus six." By moving on to the next step, Miss X implies that the student is correct (this is her evaluation). Then the IRE pattern repeats, "Now two plus six…" is the initiation from the teacher, and "Six, seven, eight" is the response from the students. The E, or evaluation, is when the teacher writes the eight under the original equation. Then the IRE pattern repeats, again with Miss X initiating by saying, "Now is this true or false?" and the students reply with "True." Miss X implicitly evaluates this statement by moving on to another problem. IRE interactions are not conducive to cultivating a community of learning since they do not promote discussion about deep disciplinary content among students. Additionally, these types of interactions do not give students authority or shared expertise. These types of interactions do not encourage discussion among students in determining mathematical truths. These types of interactions merely solicit one- or two-word answers from students and do not elicit student thinking as a centerpiece of discussion.

The previous example illustrates an IRE pattern in which the teacher's evaluation was nonverbal or implicit in her teaching moves. During my first observation, I observed an IRE pattern of interaction that involved both verbal and nonverbal student responses. I share this example to further demonstrate how some of the instances I identified involved nonverbal interactions in the classroom. In this instance, Miss X drew a part-part-whole diagram on the white board with twenty as the whole and sixteen as one of the parts. The class was trying to figure out the missing part. The following exchange between teacher and students occurred:

Teacher: I want to know how to find the answer. What do I need to do to figure out the answer?

Student 1: Count on.

Teacher: [Student 1], count on from where? And where do I stop?

Student 1: Sixteen… twenty.

Teacher: (Draws circles on the board in the part-part-whole diagram, starting with seventeen, following the student's directions for solving, until she had four circles drawn in the second part of the diagram and reached the number twenty in her counting (Figure 20)). Silent thumbs up if you agree.

Students: (Give silent thumbs up.)

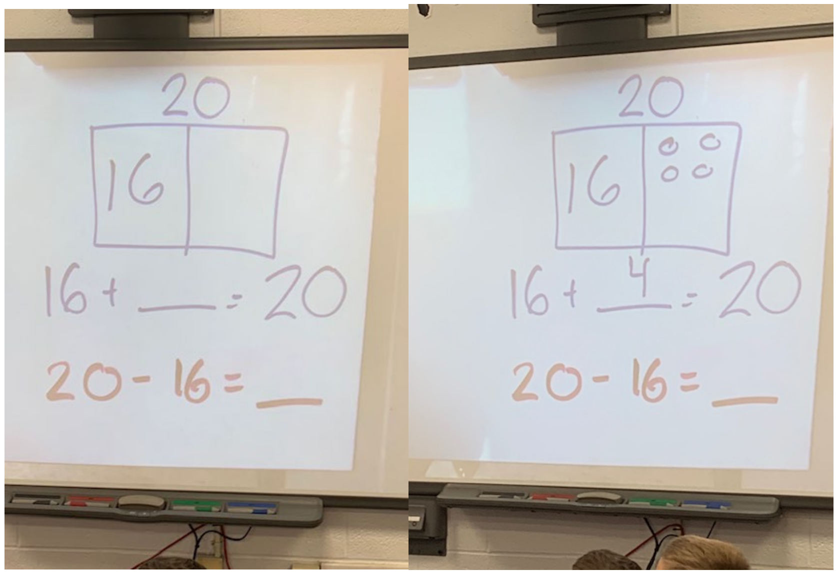


Figure 20. Teacher models solving using student response.

In the example above, Miss X initiated a question when asking her students how to find an answer, “What do I need to do to figure out the answer?” A student responded with “Count on.” Miss X then implicitly evaluated that student’s idea by asking a follow up question, “Count on from where? And where do I stop?” The IRE pattern continued. Miss X’s follow up question initiates another student response. The student responds with “16” and then “20” and Miss X again nonverbally evaluates these answers in the way of modeling the proposed strategy for solving in front of the whole class beginning with 16 and counting on to 20. Miss X then asks students for a silent thumbs up if they agreed with the strategy and solution (nonverbal input). While this pattern of talk still resembles IRE, the types of question that Miss X asked in this instance were different than previous instance. The type of question she asked in this instance allowed the student to propose a strategy for solving a problem. The question did not elicit an explanation, but the question the teacher asked required more than just recalling information in the problem or stating a fact. The question elicited an idea for solving. This pattern of talk still poses constraints on student contributions, but the type of question Miss X asked in this instance may have been an attempt to give the student more authority, although that authority was constrained by the teacher's arbitration of student responses. There is research that shows how the types of questions a teacher asks impacts student learning (Boaler & Brodie, 2004), which I discuss in the next chapter.

Another example of an IRE pattern of interaction involving verbal and nonverbal student responses during the teacher's launch occurred in my fifth observation. In this instance, the goal of the lesson was to help students understand that the equal sign is a symbol which means both sides have the same value as one another. Students were practicing this understanding by determining if an equation was true or false. The interaction occurred as follows:

Teacher: (Writes the equation "*10+2=16-3*" on the white board.) Who would like to read this whole thing to me?

Student 1: Ten plus two… (hesitates).

Teacher: (Assists student in reading the rest of the statement) …equals sixteen minus three. Who can help me figure this out? [Student 2]? What is ten plus two?

Student 2: Twelve.

Teacher: Thumbs up if you agree.

Students: (Give silent thumbs up.)

Teacher: (Writes twelve on the white board.) Alright.

In this instance, Miss X initiated student talk by asking questions related to the problem that required students to read the problem, provide a one-word answer, and give a thumbs up if they agreed with the answer. Miss X evaluated the students' responses by asking students for a thumbs up if they agreed, and by saying "Alright" and writing the answer on the board. One important thing I noted was that there was not one instance during my observations in which I observed the students being asked to give a thumbs up if they agreed with an answer that was not correct. This resulted in the same response to this question in all my observations. Specifically, whenever the teacher asked students to give a thumbs up if they agreed, they gave a thumbs up. My interpretation of this type of exchange is that this was a way Miss X evaluated student responses and was another way in which Miss X arbitrated student participation. This is another example of how opportunities for cultivating a community of learning were impacted in this classroom. The nature of discourse in this classroom did not encourage student participation that was more than surface level participation. The culture of this classroom did not encourage students to share their thinking, to pose questions, to disagree, or be pressed to justify their own thinking or ask others to justify their thinking. Student thinking and student discussions were absent in the discourse that occurred in this classroom during interactions that involved the teacher.

While the above examples describe instances in which both verbal and nonverbal exchanges took place, there were other instances that involved verbal input only. An instance representative of this type of teacher-student-teacher interaction that occurred during the teacher's launch took place during my seventh observation. In this instance, which I described in the previous section, Miss X was introducing students to adding more than two numbers at a time. She began by writing an equation on the white board. The interaction that followed involved similar questioning as in the first examples in this section. That is, Miss X asked questions to gather information from the students and led students through a particular solution strategy. (See Figure 21 for the way the white board represented this interaction):

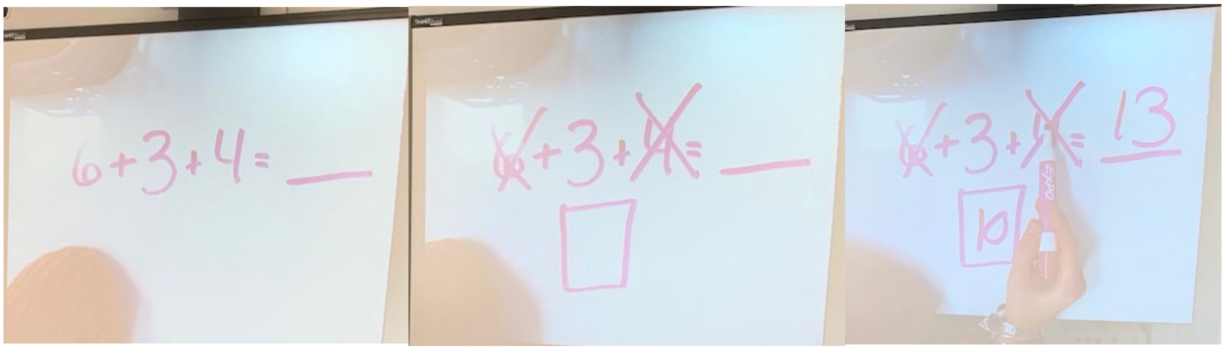


Figure 21. Teacher models solving using student response.

In this instance, the teacher initiated the interaction by asking the students if a problem was easy or hard. She asked the students what direction to take when solving the problem, stating "Pick any two numbers." Miss X could have initiated this interaction, and all the other interactions, very differently based upon the types of questions she posed. For example, in this problem, the teacher could have said "What is the problem asking us?" or "What do you see here?" or, "Who wants to talk about this problem?" or "How do you think we should solve this problem and why?" These types of questions could have encouraged the students to explore the mathematical relationships within the problem or orient their thinking to the underlying mathematical ideas, such as the use of the commutative property of addition. Instead, she told students to pick any two numbers to add. The student chose two numbers and Miss X followed that student's direction, which was her implicit evaluation and acceptance of the student's response. She did not probe the student's thinking to find out why the student chose the numbers that they did, which could have led to a rich discussion of the student's mathematical thinking. Interestingly, the student chose two numbers that when added together made the sum of 10, which I discussed previously. The teacher did not give students time to think about the problem before moving them through steps for solving and did not probe their thinking.

As I pointed out above, Miss X routinely evaluated student's correct responses through nonverbal interactions such as moving on or asking the students for a thumbs up. However, there were other instances demonstrating the IRE pattern of questioning in which the teacher evaluated a student who provided an incorrect answer. This instance occurred during the teacher's launch in my eighth observation as Miss X was focused on teaching a new concept, word problems with three addends. The following exchange was observed between teacher and students:

Teacher: Paul has eight cookies. Robert has two cookies. Richard has five cookies. How many cookies do the boys have all together? Listen, Paul has eight, Robert has two and Richard has five. Now I need to write an equation. Tell me a number that I can write.

Student 1: Seven.

Teacher: No! A number that I used in the problem.

Student 1: Eight.

Teacher: I need to add two numbers first. Which two numbers should I add?

Student 2: Eight and five.

Teacher: (Initiates counting on five from eight.) Nine… ten… eleven… twelve... thirteen. What number is left?

Student 3: Two.

Teacher: Okay, so let's count on. (Initiates counting on two more from thirteen.) Fourteen… fifteen.

During this interaction, it is difficult to infer what was Student 1 was thinking. It is possible the student mentally added 5 and 2, before sharing their response of seven. But Miss X did not ask questions to probe that student's thinking, rather she evaluated their response. Miss X initiated student talk by stating a problem and asking the students to tell her what to write. Miss X responded by saying “No!” when a student provided a response the teacher seemingly did not want to elicit. My interpretation from this exchange is that the teacher wanted students to provide numbers in the problem, and when this did not happen, rather than inquire what the student was thinking, the teacher evaluated the response as incorrect. Following this evaluation, the teacher evaluated what she deemed as a correct response, “two” by responding, “Okay, so let’s count on.” Miss X’s initiation in these instances, “Tell me a number I can write.” And “What numbers are left?” follow the IRE pattern of discourse, through which students are prompted to recall known information and present facts, which the teacher then evaluates. This type of interaction has consequences for the knowledge that children display in the classroom and the cultivation of a community of learning. When this pattern of questioning is used, the teacher imposes constraints on what students share in the classroom (Mehan, 1979). These constraints are not what Brown (1992) envisioned in her conception of a community of learning. Rather, Brown (1992) envisioned an environment in which students could become independent, self-motivated, critical thinkers with responsibility for life-long learning. Brown specifically describes that in a community of learning, the teacher does not know the answer to a question, rather the teacher defers to the community for evaluation. In addition, with experience in justifying their claims with evidence and explaining their thinking, in a community of learning, students adopt critical thinking strategies (Brown & Campione, 2002). The constraints imposed on students in the classroom based upon an IRE questioning pattern impacts opportunities for cultivating a community of learning because the teacher arbitrates student responses.

Another interesting way in which I noticed the IRE pattern of discourse appeared through interactions involving the teacher initiating a question and the students providing a response in unison in the form of a whole class choral response. Beyond the nonverbal thumbs up request, there were instances in which the students collectively provided verbal responses to the teacher's questions. The instance below that illustrates this type of discourse occurred during a lesson on fact families. This lesson involved representing the relationship between numbers in a fact family using addition and subtraction equations. The intended goal, according to the curriculum (Topic 4 Lesson 5), was to use addition facts to subtract. The interaction occurred as follows:

Teacher: (Writes the equation "*2+3=5*" on the white board). What do we think?

Students: (In unison) Yes.

Teacher: (Writes the equation "*3+2=5*" on the white board). Yes?

Students: (Nod in unison).

Teacher: Someone give me a subtraction equation.

Student 1: Five minus two equals three.

Teacher: And the last one?

Student 2: Five minus three equals two.

Teacher: (Writes an incorrect equation, "*3-2=5*" on the white board). Can I write this?

Students: (In unison) NO!

Teacher: But it has all the numbers. (Teacher points to the numbers). Because when we do subtraction, we always start with the…

Students: (In unison) BIGGEST NUMBER!

The teacher initiated by asking questions that required students to provide facts regarding a family of numbers. The questions she asked required immediate answers and her implicit evaluation was in her moving on. For example, when she asks for a subtraction equation and Student 1 says, "Five minus two equals three," Miss X responds with "And the last one?" The final question she asked, “Can I write this?” after writing an incorrect equation required students to respond with a rehearsed classroom rule about subtraction, that it always starts with the biggest number, which is not necessarily true. In fact, when students reach higher grades and encounter integers this does not hold true, and by using ideas that do not generalize could result in misconceptions or misuses by students (Karp, Bush, & Dougherty, 2014). The focus of this analysis, however, is the pattern of discourse in terms of a community of learning. In this example, the IRE pattern of interaction is repeated until the students had provided all possible relationships between the numbers in the family (two, three and five). This interaction impacts opportunities for a community of learning in that students are practicing rehearsed facts and procedures rather than collaborative learning and discussion of disciplinary content. When the teacher presented the equation, 3-2=5 on the board, there were opportunities for rich discussion about subtraction. The teacher neglected these opportunities in steering students towards recalling a classroom rule for subtraction rather than eliciting their ideas and their thinking about the statement, fact families, and the connection between addition and subtraction.

Small group instruction. As mentioned, the IRE pattern of interaction was also evident in small group instruction. These instances were like the whole group instruction instances in that they include the teacher arbitrating student responses. They are different in that Miss X was working with only five or six students at a time in these instances, rather than with the entire class. While a small group setting might present greater opportunities for the teacher to engage students in discussion and elicit student thinking, the data presented here shows that the teacher continued the IRE pattern of discourse much like in the launch of the lessons.

During my fifth observation, Miss X was working with a small group of students after a launch I described above. In the launch I described above, Miss X was showing students how to answer a question about the truth of a number sentence and engaged in an IRE pattern of discourse. Miss X continued this IRE pattern of discourse while working with a small group of students on the same concept. The problem the small group lesson was focused on involved determining if "*5+5=6+4*" was a true or false statement. The exchange between teacher and students occurred in an almost identical way as the launch.

Teacher: Under five plus five draw an arrow and under six plus four draw an arrow. It is easy peasy. What is five plus five?

Students: Ten.

Teacher: Yes, write your ten. Now we need to do six plus four. (Shows students how to make four hops on the number line starting with six and landing on ten). This answer is ten. Write your ten. Fill the equals sign in (Figure 22).

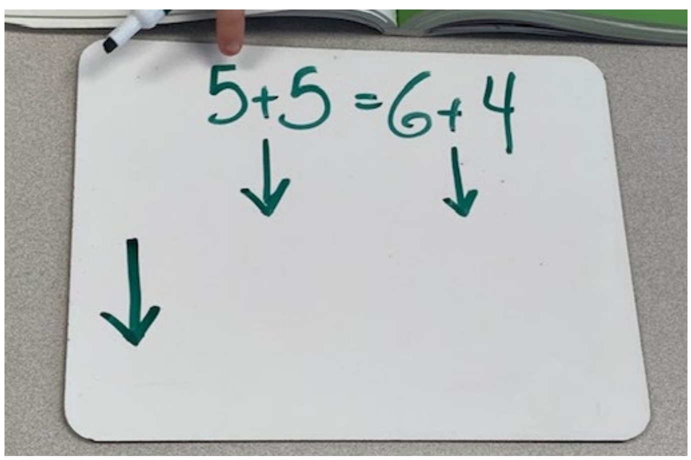


Figure 22. Teacher's white board during small group instruction.

In this instance, the teacher initiated the interaction when she asked, "What is 5 plus 5?" The students responded, in unison, with a one-word answer, to which the teacher responded "Yes." The teacher's response provided an evaluation of the students' answer, thus making this interaction representative of an IRE pattern of teacher-student-teacher talk. Students were constrained in sharing their thinking by the nature of the teacher's questions and the purpose of the teacher's questions were to see if the students produced correct answers.

Another example of this type of teacher-student-teacher talk following an IRE pattern during small group instruction took place during my eighth observation. In this instance, the following exchange took place:

Teacher: Alright, number three. Listen. As I say the number, I want you to circle the number in pencil. So, pick up your pencil. Here we go. (She reads) "Pat has his favorite cards. He has eight baseball cards. Circle the number eight."

Student 1: Where?

Teacher: (Points to the number eight in the problem.) Pat has cards with his favorite athletes. Circle eight. Everyone should have eight circled by now. Second time I am saying it. He also has two football cards and three basketball cards. So, what are the numbers we need to add in our equation?

Student 2: Eight.

Teacher: Yes, what is another one?

Student 3: Two.

Student 4: Three.

Teacher: Okay, draw a magic box for yourself after you write your three numbers down.

In this instance, Miss X first tells students to circle numbers in the problem as she reads them. In this way, she is emphasizing the numbers in the problem and possibly leading students towards answers for the questions she is about to pose. Focusing on key words and emphasizing numbers is commonly referred to as the key word approach to solving word problems, or the direct translation approach (Hegarty, Mayer & Monk, 1995). This approach often does not involve building an understanding of the situation described in the problem. Hegarty et al. (1995) sought to specify differences in the way successful and unsuccessful problem solvers treated numbers and key words in word problems and found that unsuccessful problem solvers most often used a key word or direct translation approach, and successful problem solvers incorporated a meaning-based approach to solving word problems. They refer to this emphasis on numbers and relational terms as a "short-cut" approach and recommend that teachers provide instruction to students that would help students develop a meaning-based approach to word problems. The researchers discourage emphasizing numbers and relational terms when solving word problems. In this instance, Miss X emphasized the numbers in the problem. She initiated student talk in this instance by asking the question, "What are the numbers we need to add to our equation?" This question involves students recalling information from the problem and giving an answer, which related to the numbers Miss X had already instructed them to circle. Miss X evaluated the students' responses by saying, "Yes," and "Okay." Beginning this interaction by telling students what to circle in the problem, Miss X did not give students an opportunity to problematize content, nor did she give students an opportunity to discuss strategy or meaning. Instead, she implores students to circle numbers before even making sense of the problem or reading the problem in its entirety. This type of discourse limits the opportunities for a community of learning to be cultivated in the classroom. In particular, the principles of problematizing content and accountability are not cultivated. Miss X does not elicit student thinking. She does not ask students about how they would approach solving the problem or allow students to engage in a discussion about mathematical content with one another, which are the principles of problematizing content and accountability. Miss X did not allow student thinking to be a central component in the teaching and learning that took place and Miss X continued this practice of arbitrating student contributions which limited opportunities for cultivating a community of learning.

I relate other instances involving the IRE pattern of interaction to the principle of relevant resources. During my second observation, Miss X was working with a small group on Topic 4 Lesson 7, explaining subtraction strategies. I share this instance to show an example of how the tool, or resource, of a ten frame was used during the interaction. The problem that Miss X was asking the students to solve originated from the curricular worksheet but was being enacted by Miss X as a number sentence and was presented to the students alongside a ten frame. This instance was discussed in the Curriculum section as an in-the-moment modification to the curriculum. As described above, Miss X took a word problem and replaced it with a decontextualized number sentence, 16 – 8 = ___. Miss X asked students, "What's the first thing I should do in my ten frame? What should I do with sixteen?" After which a student suggested, "Draw sixteen." The purpose of the teacher's initial question was to enable students to rehearse a known procedure (e.g., subtraction means to start with a number and take some away), and since the problem was presented alongside a ten frame, the known procedure was to fill in the ten frame with the starting number. After a student responded, the teacher evaluated the student's response by drawing sixteen circles in her ten frame. It is important to recognize that Miss X's questions were leading the students through a procedure using a ten frame. The questions gathered information rather than providing students with the opportunity to discuss possible strategies, connect mathematical ideas with different representations, or articulate their thinking. To provide these opportunities, Miss X could have asked the students questions such as, "What can I do with this ten frame?" "What other ways can I solve this problem using the ten frame?" Or "How can this ten frame help me solve this problem?" These types of questions would have allowed students to discuss strategies for solving and make sense of the tool of a ten frame in solving a subtraction problem, which would provide opportunities for her to cultivate a community of learning through the principles of problematizing content and authority.

Journal entries. Outside of the instances described so far, there was one other type of interaction that occurred on a regular basis which followed the IRE pattern and involved a nonverbal evaluation from the teacher. These interactions did not occur during the launch or small group instruction, rather, these interactions had to do with the teacher responding in writing to students' journal entries. Specifically, students completed a daily journal entry, which was a problem they worked on independently while the teacher rotated through small group instruction. Every journal question I observed was an equation of some sort where the teacher was seeking an answer, as opposed to prompts for students to write about their thinking or discuss an idea. After the students wrote their answers in their journals, Miss X would evaluate their work by either drawing a happy face if their work was correct, or a sad face if their work was incorrect on their journal page (Figure 23). Every so often, the students who received sad faces would be asked to correct their work independently with no discussion. I considered this exchange to follow the IRE pattern since the teacher initiated a question, the students responded, and the teacher evaluated their response, even though it was a nonverbal exchange. This interaction is important to point out within this theme, as the journal activity could have provided opportunities for students to problematize content and develop mathematical ideas. However, the fact that their journal entries never garnered discussion made it appear that the journal served as an assessment of some sort. I asked Miss X if she checked the students' journals during an informal interview. She responded by saying, "Yes, I check the journals every day. Sometimes I make a smiley face if they got it, or a sad face if they don't. Sometimes I will circle it or just write ‘Check!’” I would have liked to ask Miss X more about the intent and purpose of the journal, but I was not able ask these questions given time constraints for the interviews.

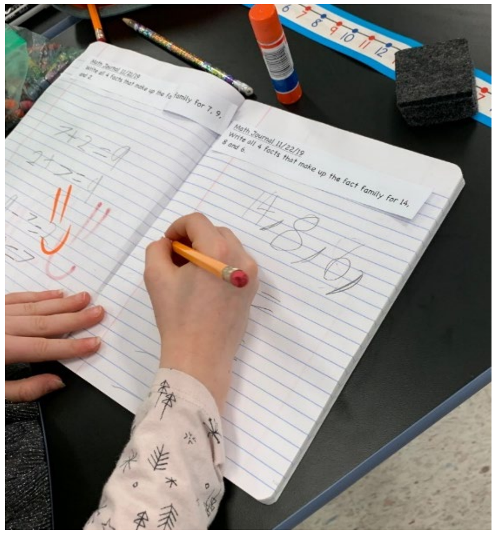


Figure 23. Teacher’s evaluation of student entries in their daily journal.

The instances detailed in this section provide evidence of teacher-student-teacher exchanges following the IRE pattern of discourse. I shared examples of verbal and nonverbal input in the form of students providing information for solving a problem, suggesting a strategy, or students giving a silent thumbs up in both the teacher’s launch and during small group instruction. All these responses were initiated and evaluated by the teacher. The third guiding principle of Engle and Conant (2002) is accountability. This principle refers to students being held accountable to one another and to disciplinary norms. This principle refers to internal accountability, in which students are expected to explain their own thinking and their own ideas, as well as be responsive and critical to the work of others. When the teacher engages in an IRE pattern of discourse, as Miss X routinely did in this classroom, students are not provided with opportunities to hold each another accountable. They are not able to pose questions to one another, to engage in discussion, and to share new ideas. IRE patterns of interaction do not align with a community of learning; in a community of learning, student talk and student thinking are a centerpiece. The point of a community of learning is for students to construct shared knowledge with all members of the community, students and teacher alike, being equal stakeholders, in the collaborative construction of knowledge. In an IRE pattern of discourse, the teacher arbitrates student responses and thus holds more authority. In the next chapter, I further discuss how this type of teacher-student-teacher talk relates to the principles for fostering productive disciplinary engagement in a community of learning, and relate other aspects of teaching and learning to my interpretation of these principles in this classroom.

Direct Instruction. The second category of discourse I identified in my data relates to instances in which the teacher was the only one who spoke during an instance. These were coded as instances because based on the way the classroom was setup and the nature of the intended curriculum, the teacher did have opportunities to cultivate a community of learning, rather, she engaged in direct instruction. I identified 47 instances in which the teacher was the only one who spoke. I describe several examples of teacher only talk that occurred during the teacher's launch and small group instruction. I examined the nature of these instances carefully and found that these examples often involved the teacher giving directions or telling students what to draw or write on their papers, sometimes even before reading a problem aloud. These instances also involved the teacher guiding students through specific step-by-step procedures for solving a problem during which the teacher was the only one who spoke. Finally, some of these instances involve the teacher giving instructions such as what materials to gather and the order of activities for the day. This type of direct instruction was a normal part of the classroom environment.

Instances of direct instruction in which the teacher was the only one to talk often involved Miss X giving directions toward the end of the teacher's launch. This is when Miss X would provide direction instructions to students regarding the "Daily 3" activities. These directions included telling students what materials (e.g., number lines, ten frames) to bring to their small group instruction for that day, what tools to use for completing the journal entry or worksheet, what materials students needed for the game, or even how to solve problems that the students would encounter in their independent activities. While teacher directions are a component of any classroom, the teacher's directions in this data are important because of the way these directions relate to a community of learning. Specifically, in many of these instances the teacher picked the relevant resource or learning tool for students rather than the students having a choice. Moreover, the way the teacher stated directions related to the ideas of positionality, placing the teacher in a position of authority and taking authority away from students, which I discuss in the next chapter.

One example of direct instruction during the teacher's launch that involved the teacher giving directions occurred during my seventh observation. I share this instance because it involves the teacher doing multiple things described above, both telling students what tool to use and telling them how to solve a problem. These actions relate to the principles of problematizing content and authority. In this instance, Miss X said the following to the students:

> When you come see me today you are going to be doing some math magic. So, you will find your page and you will need a number line. 'Math by Myself' looks like this (teacher held up a worksheet). You have equations. They all have equal signs, but they have no plus or no minus, you have to decide if it is a plus or minus. Let's try this one, one blank three equals four. Let's try plus. One plus three equals four. Does that work? It works. So, it must be plus. If it didn't work, you would try minus. Math Journal. Is this true or false? (Pointing to an equation). Draw your arrows to show.

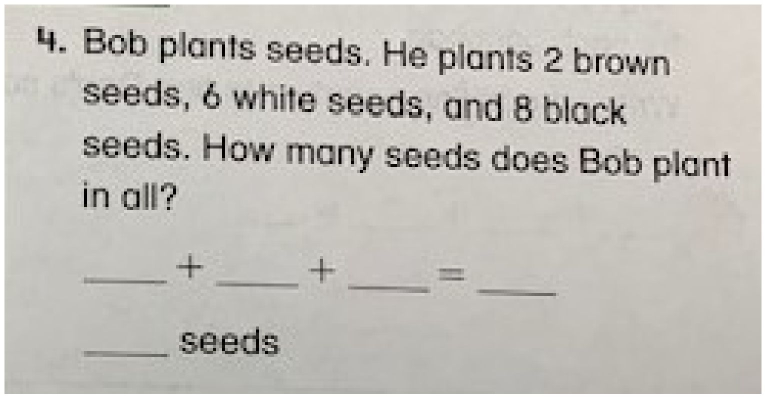
At the conclusion of the teacher giving these directions, the students moved into groups. Important elements related to a community of learning highlighted through this instance include that in her directions, Miss X told the students what tool to select and was very specific in telling students how to solve the problem in their math journal. By telling the students what tool to select, by not giving students a choice of tools, the teacher held authority. Although I am not able to infer the teacher's thinking in this instance, it is clear that she wanted the students to use a number line. By telling students how to solve the problem in their math journal, the teacher dictated mathematical thinking for her students. She did not give them the opportunity to figure out how to solve the problem on their own, based on their own ideas or prior knowledge. She provided students with a strategy and steps for solving. The mathematics became hers rather than theirs. An important aspect of this instance is that the teacher left little choice to the students and in this way, limited opportunities for cultivating a community of learning. The journal could have provided a great opportunity for cultivating a community of learning. If Miss X had used the journal in a different way, such as allowing students to think about a problem and attempt to solve it in their own way, and then following up with a discussion about their thinking students would have engaged in problematizing content and been held accountable for sharing their thinking.

A similar instance of this sort of direct instruction took place during my fourth observation. As with the previous example, the teacher talk occurred in the form of the teacher giving directions at the conclusion of the teacher's launch. This example shows how the teacher held expertise in the classroom and relates to the principle of authority. Miss X gave the following instruction to the students:

> So, you are going to do all the math and then come see me. I just want to double check and that way if you make a little oops-open your eyes, it's so rude, just so rude (talking to a student who was closing their eyes)-if you make an oops. So, fourteen minus seven, there is no way it is 440,000. I want to check and make sure there are no oops. When you are done with math today, you will start coloring these, in those nice fall colors. You can do them all yellow or orange, or two yellow or two orange, but I should not see pink and purple and blue because those are not fall turkey colors. When you are done with that, with coloring all of them then you can cut. I have a feeling we are going to get to number two today. Would blue be a good color? No. Maybe I would pick like green, brown, black or red. Not yellow, yellow you can't see. Cool? As soon as I give it to you put your name on the back. Okay, my first row of friends.

In this instance, Miss X was the only one who spoke, and she gave specific directions including directions regarding reasonability of responses and directions regarding how students should color their turkey craft. She told the students to get their work checked by her before moving on to the next activity. She was very specific in her directions about how the students could color their turkey craft, which is not related to mathematics, but adds to the story of how the teacher ran the classroom and her arbitration of student contributions. An element of this instance that I will discuss in the next chapter and relates to a community of learning is how the teacher's directions positioned her as authority/expert.

I encountered a similar instance during my eighth observation. During this instance, the nature of talk in the classroom involved Miss X telling students exactly what to write and draw on their papers. In this instance, Miss X was working with a small group and she instructed students what to write on their papers as she read a problem aloud (Figure 24). Miss X said, "Listen to number four. Circle the numbers as I read it. Bob plants seeds. Two brown seeds, six white seeds and eight black seeds. Circle the numbers, two, six, and eight. Write an equation and draw your magic box."


4. Bob plants seeds. He plants 2 brown seeds, 6 white seeds, and 8 black seeds. How many seeds does Bob plant in all?

____ + ____ + ____ = ____

____ seeds

Figure 24. Problem the teacher used when telling students what to write/draw.

As the students worked, they circled the numbers, wrote an equation, and drew their magic box. Because the students wrote exactly what Miss X told them to write, it was not clear to me if the numbers they wrote were meaningful to them in any way, or if they made any sense of the problem. Generally, telling the students what numbers to circle in the problem draws their attention toward following a procedure rather than allowing them to make their own connections between mathematical ideas while making sense of the problem (Stein et al., 2009).

In this section, I provided evidence of instances categorized as direct instruction, instances in which the teacher was the only one who spoke as she was interacting with students. These instances are distinctly different than the IRE pattern of discourse because the students were not offering up input for evaluation. In these instances, the teacher was the only one who spoke and even when the students did not speak, the teacher was the arbitrator of how students engaged in activities. In terms of the first principle, problematizing content, Miss X offered up her own thinking for consumption, which may have impacted opportunities for students to come up with their own ideas or share their own mathematical thinking. In terms of my research question, opportunities for cultivating a community of learning in this way were compromised. These instances relate to the teacher holding authority, the teacher problematizing content and students not being held accountable. Classroom discourse is critical in a community of learning and the four principles are cultivated through student discourse.

Conclusion

In this chapter, I described the teaching and learning that occurred within a first-grade classroom. Through my use of the data analysis spiral I was able to represent the data using the themes of teacher-student-teacher talk following an IRE pattern and direct instruction. Representing the data through these themes enabled me to provide an account of my findings. The way instances presented themselves within these themes related to broader notions in the teaching and learning of mathematics. These relationships demonstrate how fostering a community of learning may be a worthwhile endeavor in the elementary mathematics classroom.

In the next chapter, I connect these results with the concept of a community of learning, and other important ideas related to the teaching and learning of mathematics. Many instances I described in this chapter show distinct relationships to larger ideas in mathematics education, such as cognitive demand, eliciting student thinking, and authority in the classroom. Student talk is central in a community of learning and the results of this study make it clear that the teacher's practices play an important role in facilitating such a community. While there was some evidence of opportunities for cultivating a community of learning in this study according to the four principles, overwhelmingly, the results of this study point to missed opportunities for cultivating a community of learning in this first-grade elementary school mathematics classroom.

CHAPTER 5: DISCUSSION AND IMPLICATIONS

The goal of this study was to investigate opportunities for cultivating a community of learning in a first-grade elementary school mathematics classroom based on the four guiding principles for fostering productive disciplinary engagement. My results show that opportunities for cultivating a community of learning in this classroom were presented, however these opportunities were overwhelmingly missed. As a reminder, the four guiding principles that served as my analytic framework include problematizing content, accountability, authority, and relevant resources (Engle & Conant, 2002). In the previous chapter, I presented the results related to missed opportunities using two themes I identified during analysis: curriculum and discourse. In this chapter, I discuss limitations, interpretation of the findings including a visualization, and areas for future research. The processes of interpreting, representing, and visualizing the data to provide an account of my findings follow the cycles of the data analysis spiral (Creswell & Poth, 2018). Since this was a case study, the results relate to the particular classroom of learners in which I conducted this study (the bounded system), however the findings and my interpretation of the findings can be generalized to theoretical propositions (Merriam & Tisdell, 2016). The findings from this study add to a limited body of research within the field of mathematics education involving naturalistic studies aimed at investigating aspects of a community of learning in early elementary school mathematics classrooms.

Limitations

Before I discuss my interpretation of the findings and the implications of this study, it is important to point out limitations. I mentioned these limitations in previous chapters but reiterate them here to discuss how they impacted this study. The first limitation was that I was not able to audio or video record in this classroom. Because of this limitation I committed my observations to interactions that involved the teacher. The launch of the lesson always included all the students and the teacher, so those findings are not affected by this limitation. However, my findings do not include data gleaned from interactions that occurred during other aspects of the Daily 3, when students were working by themselves or in groups that did not involve the teacher. My focus on observing small group activity that involved the teacher limited my understanding of what was happening in the rest of the classroom during that time. Specifically, when students were engaged in other aspects of the Daily 3 such as completing the math journal, playing the game, or doing the "Math by Myself" worksheet, I was not able to see or hear what was happening. I made the decision to focus my observations on interactions that involved the teacher because during my initial observation, I tried to follow groups of students that were not interacting with the teacher and I found that they were not discussing mathematics. Therefore, in order to understand opportunities to cultivate a community of learning around mathematics with the limitation of no audio or video recordings, I restricted my observations to those which I perceived would allow for me to best capture classroom activities related to mathematics. Despite this limitation, there is an important story to tell about the opportunities for cultivating a community of learning in this classroom based on the findings from the data that included interactions which involved the teacher, which I was able to collect and analyze.

Another limitation, which I was not aware of going into this study, was the limited opportunities I had for interviewing and talking with the teacher. In my study design, I had planned on conducting interviews with the teacher at the conclusion of each observation. While the teacher agreed to this before my observations began, in practice she did not leave room for these discussions. I can only hypothesize why this was the case. Miss X mentioned at one point that she was not comfortable with me recording classroom activities because she was a non-tenured teacher. Perhaps she was uncomfortable engaging in interviews for the same reason and worried that she may say something that would affect her tenure. Another possible explanation could be the nature of the elementary school day, which is busy and does not provide teachers with many breaks or time away from the students. I recall when I was teaching, my lunch and prep periods were precious. I used every minute of them, often for things like preparing for future lessons, emailing parents, or reviewing student work. So, while I do not know if this is true for Miss X, her classroom did appear to be busy and immediately following my observation time, Miss X as well as the students had their lunch period. She often hurried out of the classroom with the students at this time, signaling that my time had ended. Perhaps Miss X did not want to give up time during her lunch period to answer my questions.

Regardless of the reasons why Miss X was not open to interviews, the impact on the study is that it lacks the teacher's voice. For example, when the teacher commented that she thought things were too hard for students, I was not able to follow up with her to probe why she thought this. When the teacher modified a problem, I was unaware of her intentions and the reasons she decided to make those modifications. I was not able to learn how she interpreted diagrams, arrows, or other models with problems and during instruction. Such insights could have helped me understand choices she made regarding the enactment of the curriculum and facilitation of discourse, especially in relation to cognitive demand. Additionally, I was not able to ask the teacher about her choices in terms of the ways she enacted the curriculum. For example, what influenced her decision to change certain problems? Why did she dictate tools students should use for particular tasks? Knowing the answers to these questions from Miss X's perspective could have provided me with additional evidence for why she did not take advantage of opportunities presented for cultivating a community of learning. Knowing more about Miss X's perspective in her own voice could be beneficial to understanding this case more entirely, which could have helped me learn more about what focused support teachers could benefit from to build on opportunities for cultivating a community of learning in the classroom. Nevertheless, despite these limitations, there is an important story to tell regarding opportunities for cultivating a community of learning in this classroom based on the data I was able to collect and analyze.

Discussion

As mentioned, the goal of this study was to investigate opportunities for cultivating a community of learning in a first-grade elementary school mathematics classroom based on the four guiding principles of disciplinary engagement (Engle & Conant, 2002). According to Engle and Conant (2002), productive disciplinary engagement in a community of learning is only possible when there is synergy between all four principles. This means that the principles are codependent upon one another, and it is only through their combined interactions that a community of learning can be cultivated (Engle & Conant, 2002). The results of this study provide additional evidence of the necessary interaction between all four principles in cultivating a community of learning. In this study, I found missed opportunities for cultivating a community of learning through the combined interactions of the four principles based on the teacher's enactment of the curriculum and facilitation of discourse. I reported on the existing opportunities for cultivating a community of learning in this classroom in the way the classroom was set up, in terms of the daily routine, and as presented in the intended curriculum. However, I did not observe the teacher taking advantage of these opportunities. In particular, through her enactment of the curriculum and facilitation of discourse, I perceive that the teacher reduced cognitive demand in the classroom and did not elicit student thinking. Because of the lack of teacher voice in the study, I do not know why she enacted the curriculum or facilitated discourse the way she did. However, through my carefully constructed observations and analysis, I conclude that these actions were barriers to taking advantage of the opportunities to cultivate a community of learning for this classroom. These findings show that the teacher played an important role in cultivating a community of learning in the classroom and reinforce that cognitive demand and student thinking are important elements in cultivating a community of learning (Brown, 1997; Engle & Conant, 2002; Franke et al., 2009; Stein et al., 1996).

An implication for this study is my visualization of the results. I incorporated the elements of cognitive demand and student thinking with the four guiding principles (Figure 25). This visualization represents the ways in which the principles and elements may interact within a community of learning. There is both overlap and back and forth activity between cognitive demand, student thinking and the principle of authority. In the center of the image, at the intersection of these elements, is the principle of problematizing content. I believe when high levels of cognitive demand are maintained and student thinking is used and elicited, students have authority, are held accountable to one another and to disciplinary norms, and use relevant resources to support the principles and elements. These all merge so that students can engage in problematizing content and a community of learning can be cultivated.

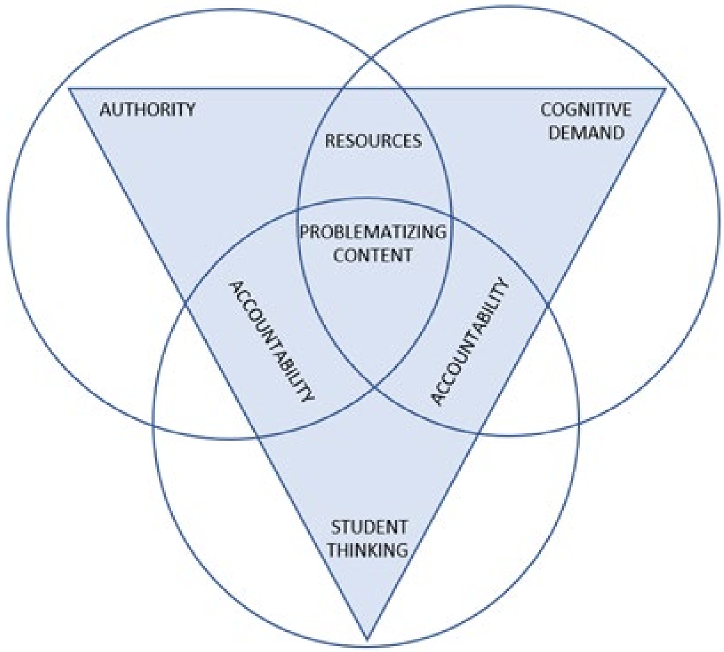


Figure 25. Visualization of the interplay between principles and elements for cultivating a community of learning.

The principle of accountability, accountability to others and to disciplinary norms, is evident in the interplay between student thinking and authority, as well as student thinking and cognitive demand. Accountability relates to student thinking, as it is based upon students sharing, justifying, and explaining their own thinking as well as responding to the thinking of others. When students share their thinking, they become accountable to others and to disciplinary norms, which relates to students having authority in the classroom. Cognitive demand relates to student thinking in that a characteristic of classrooms associated with higher levels of demand is that students are held responsible for explaining, justifying, and describing their thinking (Stein, Smith, Henningsen & Silver, 2009). Students, by sharing their thinking in these ways, are held accountable to others and to disciplinary norms. These actions related to accountability can either be supported or hindered by a teacher's actions during instruction through how they facilitate discourse, particularly the ways in which a teacher elicits and builds on student thinking.

The principle of providing relevant resources is especially evident in the interplay between authority and cognitive demand. According to the results of this study, when the teacher enacted the curriculum, students were presented with tasks requiring lower levels of cognitive demand. Instances in which the level of cognitive demand was reduced for students relate to an unequal distribution of authority (i.e., the teacher holding most of the authority). Students need to be provided with appropriate resources to allow them to engage in high levels of cognitive demand and maintain shared authority in the classroom. While the principle of relevant resources refers to anything that supports student engagement and learning across all principles and elements, it relates to student authority and cognitive demand in particular in that providing students with an opportunity to engage with highly demanding tasks becomes relevant when students also have the authority to engage in such tasks.

My interpretation of the findings of this study are that the teacher's enactment of the curriculum reduced the level of cognitive demand in the classroom and contributed to missed opportunities for cultivating a community of learning. I also conclude that in her facilitation of discourse, which occurred only in the form of IRE or direct instruction, the teacher did not elicit student thinking thus also contributing to missed opportunities for cultivating a community of learning. I contend that these two actions, reducing the level of cognitive demand and not eliciting student thinking, were barriers to cultivating a community of learning. The teacher in this study, by not giving students authority, not eliciting student thinking, and not maintaining levels of cognitive demand, also impacted the principles of accountability, relevant resources, and problematizing content. Students were not held accountable, resources were not used in a relevant way and students did not have opportunities to problematize content. While Miss X did have opportunities to cultivate a community of learning during mathematics instruction, I argue that she did not take advantage of these opportunities, thus resulting in a lack of synergy between the four principles and inability to cultivate a community of learning in her classroom.

Opportunities for Cultivating a Community of Learning

I described in the previous chapter how the classroom set up, the Daily 3 and the intended curriculum all presented opportunities for cultivating a community of learning in this classroom. Recall that I did not intentionally choose this classroom based upon the opportunities it demonstrated for cultivating a community of learning, but rather I chose this site out of convenience and because it was unfamiliar to me. Given that this was a naturalistic study without any interventions, seeing opportunities for cultivating a community of learning based upon the classroom set up, daily routine, and intended curriculum prior to conducting any observations created an expectation for me that I could find a teacher taking advantage of some of these opportunities.

To understand the meaning and impact of these opportunities for cultivating a community of learning, it is important to highlight how these opportunities are related to the principles of disciplinary engagement. The classroom set up in terms of physical space was arranged in a way that could promote discussions and productive exchanges. There were tables with seats or stools facing one another, to support collaborative learning groups. There were flexible seating options, including standing tables, which appeared to provide students with opportunities to interact with one another and move about. The teacher’s table was a kidney shaped table with six chairs, with student seats and the teacher’s seat all facing each other, the teacher positioned in the middle. Based on this set up, there was an opportunity for the teacher to engage students in discussions about mathematics at this table. Such conversations could have been used to support a community of learning through the principles of authority, accountability, and problematizing content. Specifically, the set up supported the distribution of authority among students, in that by facing one another, all students could participate equally in the learning. Students were positioned in a way that could have supported them in listening to and responding to one another. In this way, students could have had opportunities to explain their thinking, ask each other questions, propose ideas, and challenge each other, which could have supported students being held accountable to responding to the work of others.

The design of the Daily 3 also provided opportunities for cultivating a community of learning in that during this daily routine, students seemingly would have authority in the classroom as they moved through three different activities, being responsible for self-regulating their activities and learning. The daily journal, math game and "Math By Myself," components of the Daily 3, are relevant resources that could have provided opportunities for students to have authority by providing students with the ability to have an active role in defining, addressing, and resolving mathematical problems. Moreover, these components could have allowed students to engage in high levels of thinking and share their thinking, which would have supported them in problematizing content. Finally, the Daily 3 may have also been used to ensure the students' intellectual work was responsive to content. This accountability was not present in the classroom, as students did not seem to utilize their time away from the teacher to engage in mathematical thinking and learning.

The intended curriculum was a problem-solving based curriculum designed to promote deep thinking and engage students in sharing their thinking and problem-solving strategies. The intended curriculum included opportunities for students to make meaning and connections among mathematical concepts and to engage in productive struggle through problem-solving types of tasks. This relates to the principles of problematizing content and the element of cognitive demand in that engaging in deep thinking about mathematical content presented in challenging tasks is a shared characteristic of both the principle and the element. The intended curriculum incorporated probing questions to spark discussions, and it was made explicit in the teacher's guide that students should be given time to engage in productive struggle and share their thinking (Pearson enVision 2.0, 2016). These aspects of the intended curriculum could encourage discussion, which relates to the element of eliciting student thinking and the principle of accountability; by sharing their thinking and responding to the thinking of others in the form of discussion, students are held accountable. In these ways, the intended curriculum could have been leveraged to cultivate a community of learning.

Another aspect of the classroom set up, which provided opportunities for cultivating a community of learning in this study, involved the availability and access to relevant resources. Tools such as number lines, ten frames, and dice were available around the room in baskets and cubbies. These relevant resources could have been used to cultivate a community of learning specifically around students' problematizing content. The use of mathematical tools can help students making sense of and reasoning about mathematics, which relates to the principle of problematizing content. Tools can also help students communicate their mathematical thinking, another important element in a community of learning. According to NCTM (2014), "An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (p. 78). Building on this idea as it relates to the principles, I recognize that opportunities to choose tools to guide their own mathematical thinking and reasoning distributes authority to students and allows students to problematize content based on their own thinking.

Despite the presence of these opportunities for cultivating a community of learning based on the four principles of disciplinary engagement, the teacher's enactment of the curriculum and facilitation of discourse in the classroom resulted in missed opportunities. According to Engle and Conant (2002) interaction of the four principles creates a classroom environment in which students are passionately engaged, develop arguments, use evidence, generate questions, and negotiate meaning through productive discourse. Despite the way the classroom was set up, the daily routine, the intended curriculum, and availability of resources, I did not observe these characteristics in the classroom.

One interpretation of these findings could be that classroom norms for students to engage in learning in the ways described by Engle and Conant (2002) were not established in this classroom. However, I have little data about the classroom norms beyond the general norms. Some unknowns around the sociomathematical norms that are salient for this study include why did I not see students engage in discussions about mathematics with each other or the teacher and why did I not observe students providing evidence of their thinking or negotiate meaning through productive discourse. I do not know why I did not observe students engaging in mathematics lessons in these ways, but conjecture that the ways in which the teacher enacted the curriculum and facilitated discourse may have been a result of established classroom norms related to mathematical learning. Moreover, despite the availability of resources in the classroom, the students in my study were not given authority to select tools related to their own mathematical thinking when solving problems. The norm for the classroom seemed to be that students would wait to be told by the teacher which tools to use and how to use them. By arbitrating the resources that students were able to use in their learning, the teacher missed an opportunity to cultivate a community of learning in the classroom through the principles of authority and relevant resources.

I acknowledge that I did not have the opportunity to discuss classroom norms with the teacher due to limited interview time, and therefore have only been able to interpret the norms around mathematics learning based on my observations. The norms I observed seemed to be that students saw their role as providing solutions to problems by following the teacher's directions. They did not seem to have a shared belief that asking questions or engaging each other in mathematical conversation was an appropriate form of communication. Rather, their shared belief seemed to be that they should wait to be asked a question or be prompted by the teacher, provide a short response, and receive evaluation from the teacher. There was one instance in which a student, unsolicited, stated that two problems were different. The student made this statement without being prompted or asked a question, and the teacher did not build on or acknowledge his contribution. This small action could have served to reinforce the communication norms in the classroom. In an intervention study in a second-grade classroom Cobb et al. (1993) reported that second grade students were able to engage in the collaborative construction of knowledge, a tenet of a community of learning, because of the norms that were established in the classroom. Cobb et al. (1993) concluded that the teacher cultivated a learning environment that built on students' constructivist activities in order to arrive at a consensus, rather than steering students toward mathematical solutions or answers. According to their study, which involved an intervention, the teacher held students in the classroom accountable to norms and gave students authority or agency in their learning, allowing students to have joint responsibility in their learning. In my study, I did not observe the collaborative construction of knowledge, and the teacher did not hold students accountable to norms in which students could illustrate authority or agency in their learning.

Overall, my findings show that despite the characteristics of the classroom set up, the daily routine, the intended curriculum and availability of resources, there was no evidence of students having authority; student thinking was not elicited; cognitive demand was reduced; students were not held accountable; students did not engage in problematizing content; and relevant resources were arbitrated by the teacher. The implication of this is that even though the teacher created or was presented with opportunities for cultivating a community of learning, she did not take advantage of these opportunities. The principal supported the teacher's decision to have a flexible and collaborative seating arrangement in her classroom. The school utilized a curriculum which supported opportunities for cultivating a community of learning. The daily routine the teacher implemented for mathematics instruction provided opportunities to cultivate a community of learning, and in her classroom, student learning tools were accessible and available. Nevertheless, the teacher did not take advantage of these opportunities to cultivate a community of learning. This makes me wonder about the teacher's intention for her decisions and how she understood the different aspects of the impact of the classroom set up or daily routine. In terms of the classroom set up, the teacher had said she was offered the option of flexible seating and she said she liked it, but did she realize its purpose and potential? How could the seating arrangement rise above aesthetics in the classroom? I wonder about the professional learning opportunities the teacher had to learn how to best leverage flexible seating and implement the curriculum in ways that kept aspects of it intact (e.g., cognitively demanding tasks, openness to multiple strategies). These outstanding questions suggest a need for further investigations as to how teacher educators may provide support for teachers in cultivating a community of learning in their classrooms when presented with opportunities.

Barriers: Cognitive Demand and Eliciting Student Thinking

In the previous section I describe the opportunities for cultivating a community of learning for the teacher in this study and how these opportunities were missed. In this section, I discuss the themes of curriculum and discourse and relate them to what I perceived to be the main barriers to leveraging opportunities for cultivating a community of learning: cognitive demand and eliciting student thinking.

When I looked at instances associated with the theme of curriculum, I observed that student participation was arbitrated by the teacher; her enactment of the curriculum determined what tasks the students engaged with and her set up and implementation of tasks changed the intended student participation. When I looked at instances associated with the theme of discourse, I observed that when classroom discourse followed the IRE pattern of teacher-student-teacher talk and student participation was in the way of providing answers to questions, recalling facts, or giving solutions. When discourse followed direct instruction, I observed no student participation and in these instances the teacher either gave directions or solved a problem without eliciting student input.

In the following sections, I discuss further how the teacher's enactment of the curriculum and facilitation of discourse in the classroom took authority away from students, impacted students' accountability and opportunities to engage in problematizing content, and did not provide students with relevant resources for cultivating a community of learning. My first interpretation of the findings is that the teacher's enactment of the curriculum led to a reduction of cognitive demand in the classroom. Literature regarding effective mathematics teaching and learning emphasizes the importance of giving students opportunities to engage in higher levels of cognition (NCTM, 2014). Brown (1992) describes how the conception of a community of learning grew from wanting students to engage in thinking deeply about disciplinary content. Therefore, for a community of learning to be cultivated, there must be opportunities for students to engage in high level thinking. In my study, the way the teacher set up tasks, either premeditated or in-the-moment, and the way she implemented those tasks, lowered the cognitive demand in the classroom. Since the design of a task correlates with student learning based upon the kind of thinking it requires of students (Henningsen & Stein, 1997), it is important that teachers present students with higher-level tasks. I argue that the teacher's modifications to the intended curricular tasks and the ways in which she set up these tasks routinely lowered the cognitive demand and the ways she implemented the tasks additionally highlighted the lack of eliciting student thinking during discussions.

Figure 26 illustrates the relationships among task-related variables and student learning outcomes (Henningsen & Stein, 1997). In this figure, the rectangles show the relationship between the intended mathematical task, the task as set up by the teacher, and the implementation of the task by students in the classroom to student learning outcomes. The circles refer to the factors which influence task to task set up, and task set up to task implementation.

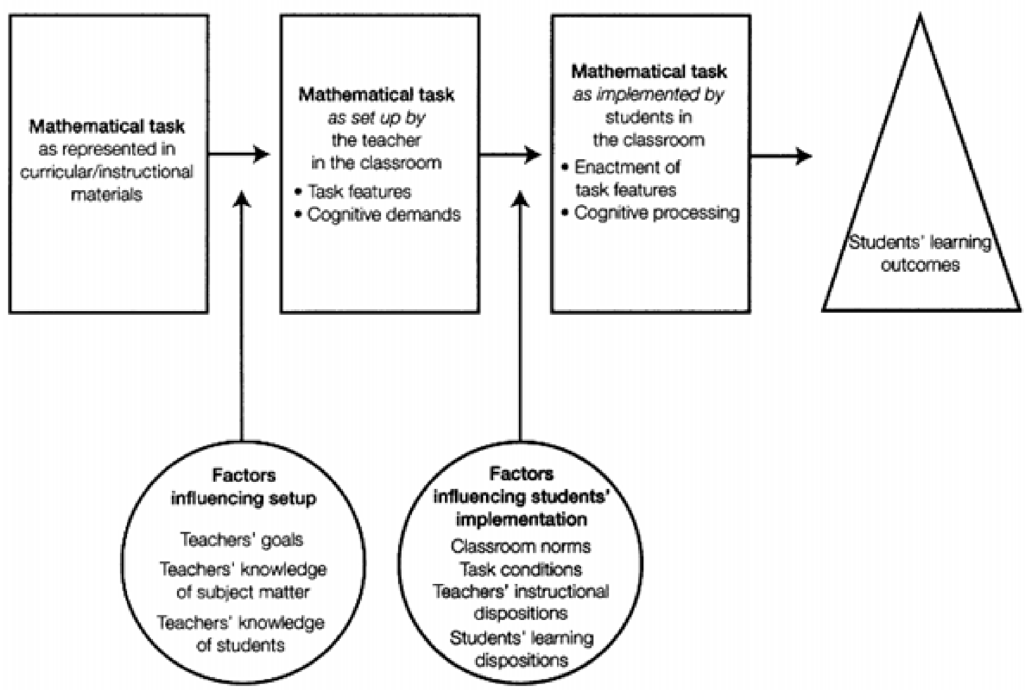


Figure 26. Relationships among various task-related variables and students' learning outcomes (Henningsen & Stein, 1997, p. 528).

In the sections to follow, I first discuss how the tasks presented in the intended curriculum are higher-level cognitive demand tasks and how in her enactment of the curriculum, the teacher set up these tasks in ways that lowered the cognitive demand. I then discuss how this trend continued through implementation and how Miss X did not effectively elicit student thinking in this stage, which I argue is most salient in both her questioning strategies and use of scaffolding.

Task Set Up

As the results demonstrate, there were differences between the tasks presented in the intended curriculum and the tasks the teacher presented in her enactment of the curriculum in terms of cognitive demand. Tasks presented in the intended curriculum, including the "Solve and Share," "Reteaching" worksheet, and "Assessment," could be categorized as having higher levels of cognitive demand. I described the cognitive demand of tasks in the literature review and presented Stein and Smith's (1998) Task Analysis Guide (TAG). The TAG classifies the characteristics of tasks at four levels of cognitive demand (Stein & Smith, 1998). According to the TAG, tasks requiring lower levels of cognitive demand include memorization tasks or procedures without connection tasks, and tasks requiring higher levels of cognitive demand include procedures with connections and doing mathematics tasks.

The intended curriculum presented tasks in the "Solve and Share" that were discussion provoking tasks, often consisting of word problems which required students to make meaning, strategize, and construct their own equations to solve. These tasks often included directions for students that in addition to solving the problem, students were required to describe, explain, or represent their thinking about the task. The "Solve and Share" was described as a way for students to engage in tasks that promote productive struggle and would allow students to solve in any way they wanted (Pearson enVision 2.0, 2016). These tasks were open-ended and did not include explicit pathways for solving. The nature of these tasks was that they were set up to spark student discussions around mathematical ideas (Pearson enVision 2.0, 2016). Based on these characteristics, the "Solve and Share" tasks in the intended curriculum can be classified as higher-level tasks (Stein & Smith, 1998), and thus presented opportunities for the teacher to cultivate a community of learning. In particular, these tasks were presented as a relevant resource that would hold students accountable for their thinking. Relevant resources in the form of curricular tasks support student engagement and help students make sense of the tasks being investigated, thus allowing the students to engage in intellectual work that is responsive to the content (Engle and Conant, 2002). Moreover, the intended tasks have the necessary characteristics to give students opportunities to problematize content, and their directions would help position students as stakeholders in their production of knowledge, thus giving students authority in their learning.

Tasks included in the intended curriculum on the “Reteaching” worksheets and in assessments, shared some similar characteristics to the “Solve and Share” tasks. The “Reteaching” worksheets were slightly different, in that this tool was designed as an intervention to help build students’ understanding of concepts through guided practice (Pearson enVision 2.0, 2016). Because these worksheets were an intervention tool, the tasks included more explicit pathways for solving problems. For example, many of the problems presented in the “Reteaching” worksheets included a choice of strategies for solving. However, even with these suggestions, students needed to be able to make sense of these tasks. The worksheets also focused on process rather than solutions, which is a classification of tasks requiring higher levels of cognitive demand (Stein & Smith, 1998). Similarly, tasks included in the assessments in the intended curriculum focused on process rather than solution. However, because these were used as assessment tools, these pathways for solving were not suggested to students. Students were required to come up with their own pathways for solving. Tasks included in the assessment presented in the curriculum required students to explain their strategies for solving using precise mathematical language, to contextualize equations by writing word problems to go along with them, and to make sense of word problems while constructing equations in order to solve them. Thus, these tasks can be classified as tasks requiring higher-levels of cognitive demand as they required complex thinking related to understanding the nature of mathematical relationships, and they required students to consider solution strategies and the affordances of various solution strategies based on the task presented. In these ways, the tasks were presented in ways that would elicit student thinking, hold students accountable for their intellectual work, and provide students with an opportunity to problematize content. In short, the intended curriculum presented opportunities for cultivating a community of learning based on the principles and these tasks presented opportunities for higher levels of cognitive demand in the classroom.

In Miss X's enactment of the curriculum, I argue that she presented tasks to students which required lower levels of cognitive demand and this impacted her ability to take advantage of opportunities presented for cultivating a community of learning. There were similarities among the types of tasks she presented in her teacher's launch, in her version of the "Reteaching" worksheet, and in her version of the assessment. I described in the previous chapter how Miss X's premeditated or in-the-moment modifications to tasks resulted in her presenting tasks to students which did not hold them accountable for sharing their thinking. For example, the teacher set up tasks as decontextualized number sentences as opposed to word problems and did not prompt students to share their reasoning. A common feature of the higher-demand tasks presented in the intended curriculum was that they required students to contextualize equations by constructing word problems to go along with them. The activity of solving an equation and writing a number story to represent that equation requires complex thinking. Solving an equation is focused on producing a correct answer while writing a number story focuses on making connections with conceptual ideas that underlie procedures (Stein et al., 2009). Miss X did not set up the tasks in ways that required students to contextualize equations by writing a word problem to go along with them. In this way, I believe the types of tasks that Miss X presented in her enactment of the curriculum included mostly procedures without connections tasks, which were algorithmic in nature and did not allow students to make connections between procedures and their underlying meanings. Therefore, through task set up, Miss X limited students' abilities to problematize content or be held accountable for their thinking.

Miss X frequently set up tasks in her enactment of the curriculum that included diagrams. The use of diagrams to represent or model certain relationships is a way to scaffold student thinking and stems from the model method. The model method is often incorporated into elementary school mathematics classrooms as a tool to help students visualize and represent relationships (Ng & Lee, 2009). The model method involves students learning through textual, pictorial, and symbolic modes of representation. In the lower grades, students are often presented with objects, pictures, and symbols to help them model combinations of numbers. While the model method can be helpful in guiding students' understanding of relationships between quantities, the model method is also described as being a way for primary students to "circumvent the cognitive demands that are inherent in solving algebraic equations" (Ng & Lee, 2009, p. 293), including abstractions and the use of symbolic representations that are not appropriate for their level. I contend that in some ways, Miss X's use of diagrams during task set up went beyond these affordances and reduced cognitive demand. In particular, instead of students making sense of the problem, it appeared that the use of diagrams focused students on filling in parts of the diagram, thus removing any ambiguity about what needed to be done. In this way, the use of diagrams may have limited students' abilities to make meaning of the relationships between quantities. By doing this, Miss X missed an opportunity to cultivate a community of learning by taking away students' authority to own their approach to the problems. She also limited students' ability to problematize content by focusing on the procedures and answers to problems rather than encouraging students to ask questions and propose their own solution strategies.

While the reduction in demand during the task set up above relates to Miss X's premeditated changes to the curriculum, I also observed in-the-moment modifications that had similar results. One of the ways Miss X set up tasks during her enactment of the curriculum, was in reading problems aloud to students. Even though students had the problems in front of them, Miss X would use different words when reading problems aloud, changing the type of problem presented to students. These changes in problem type typically reduced the cognitive demand of the task. Carpenter et al. (2014) make distinctions among different types of problems in connection with CGI and discuss how the wording of problems impact the way students think about and solve them. In first grade, students are working with addition and subtraction problems. These can be grouped into four main problem types: join, separate, part-part-whole and compare. Most often, Miss X changed the type of problem from a compare problem, which requires students to make sense of quantities in relation to one another and involves complex thinking, to a join or separate problem with the result unknown. Result unknown problems provide the starting and change quantities, and the result quantity is unknown. These are known to be the easiest types of problems to solve. Specifically, the wording of problems is easier for children to solve when the wording of a problem relates to the action sequence for solving (Carpenter et al., 2014). Therefore, Miss X set up tasks for her students in ways that made them easier for students to think about and solve because she clearly described the action sequence to solve the problem when reading it aloud.

Miss X also sometimes changed compare problems to part-part-whole problems. In making these changes, she did not allow students to grapple with perplexing ideas. The perplexing idea in a compare problem is the comparison of two amounts to determine an unknown, as opposed to the less demanding cognitive task of putting two parts together to identify a whole. The tasks presented in the intended curriculum often involved compare problems, which requires the comparison of two distinct, disjoint sets rather than simply looking at the relationship between a set and its subsets. Carpenter et al. (2014) describe part-part-whole problems as involving a static relationship among a set and its two disjoint subsets. In contrast, they describe compare problems as involving relationships between quantities. By changing the words of the problem and thus changing the problem type from a compare problem to a part-part-whole problem, Miss X presented tasks requiring a lower level of cognition to solve. I classify the types of problems Miss X set up when reading problems aloud as procedures without connections tasks according to the TAG. In making the action sequence clear, while reading the problem aloud, and changing the type of problem from compare problems to either part-part-whole or join or separate result unknown problems, Miss X took away perplexing ideas and made the procedure for solving the problem explicit, which resulted in her presenting problems that required lower levels of cognitive demand to solve.

In this section I argue that in her enactment of the curriculum Miss X lowered the cognitive demand of tasks, which led to missed opportunities for cultivating a community of learning. While the intended curriculum provided opportunities for this cultivation, Miss X's task set up did not hold students accountable for providing their mathematical reasoning. The tasks as set up by Miss X required lower levels of cognitive demand in that they lacked ambiguity, focused students on producing correct answers, and included explicitly stated procedures for solving. These actions limited the power of the intended curriculum as a relevant resource that could support student engagement and learning, which subsequently restricted students' ability to problematize content. Miss X's set up of tasks in these ways lowered the cognitive demand in the classroom and became a barrier to her ability to leverage opportunities presented for cultivating a community of learning in this classroom.

Task Implementation

The second relationship among task-related variables and student learning outcomes (Henningsen & Stein, 1997) seen in Figure 26 focuses on the transition from the mathematical task as set up by the teacher to mathematical task as implementation by students. The implementation by students is impacted by classroom norms, task conditions, and both teacher and student dispositions. While limitations of my study make it difficult for me to assess the impact of classroom norms and teacher and student dispositions, through my results I have been able to draw conclusions about how after set up, the teacher enacted the curriculum in ways that maintained a lower-level cognitive demands while not eliciting student thinking.

Tasks that are set up as low-level tasks are likely to result in low level implementation (Stein et al., 2009). Task implementation “is defined by the manner in which students actually work on the task” (Stein et al., 1996, p. 460), which is mediated by teacher actions. During implementation, Miss X often took over mathematical thinking and reasoning in the classroom by telling students how to solve problems and asking students to recall information or provide answers as she solved problems in front of them. Miss X often hurried students through tasks, moving quickly to the next step in solving. To better understand the implementation of tasks, Stein et al. (2009) identified factors associated with the decline of high-level cognitive demands and the maintenance of high-level of cognitive demands (Figure 27). The results of my study show factors associated with the decline of cognitive demands based on the ways the teacher implemented tasks in her enactment of the curriculum.

Factors Associated with the Decline of High-Level Cognitive Demands	Factors Associated with the Maintenance of High-Level Cognitive Demands
1. Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells students how to do the problem). 2. The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer. 3. Not enough time is provided to wrestle with the demanding aspects of the task or too much time is allowed and students drift into off-task behavior. 4. Classroom management problems prevent sustained engagement in high-level cognitive activities. 5. Inappropriateness of tasks for a given group of students (e.g., students do not engage in high-level cognitive activities due to lack of interest, motivation or prior knowledge needed to perform; task expectations not clear enough to put students in the right cognitive space). 6. Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade).	1. Scaffolding of students thinking and reasoning. 2. Students are provided with means of monitoring their own progress. 3. Teacher or capable students model high-level performance. 4. Sustained press for justifications, explanation, and/or meaning through teacher questioning, comments, and/or feedback. 5. Tasks build on students' prior knowledge. 6. Teacher draws frequent conceptual connections. 7. Sufficient time to explore (not too little, not too much).

Figure 27. Factors associated with the maintenance and decline of high-level demands. (Stein, Smith, Henningsen & Silver, 2009, p. 16).

Related to the implementation of tasks is the element of eliciting student thinking. Factors related to maintaining high levels of cognitive demand include scaffolding students' thinking and reasoning and providing sustained press for justifications, explanations and/or meaning through teacher questioning. One of my main findings in this study is that student thinking was not elicited despite opportunities for this to be possible through the classroom set up, daily routine, and intended curriculum. These aspects of the classroom could have empowered students or given students authority to share mathematical ideas with other students. Specifically, the layout of the classroom and flexible seating provides students with the opportunity to collaborate and interact more naturally. Further, the daily routine, in theory, could have given students opportunities to problematize content by questioning and challenging each other through math games or have authority in producing their own knowledge through their math journals. Moreover, in the teacher's enactment of the curriculum and in her facilitation of discourse, students were not prompted to share their thinking. Students sharing their thinking is an important aspect of a community of learning and explaining their strategies for solving problems is one way that students can share their thinking in the classroom (Engle & Conant, 2002; Franke & Kazemi, 2001). The principles of problematizing content and accountability focus on student thinking, as these principles relate to how students make sense of a problem in discussing their understanding of a problem and explaining their ideas for solving. Facilitating discourse which elicits student thinking is another way that teachers provide students with authority to problematize content in the classroom (Engle & Conant, 2002).

In the sections below, I describe how teacher questioning and scaffolding in this study did not elicit student thinking and thus contributed to the decline of cognitive demand during task implementation and resulted in missed opportunities for cultivating a community of learning. That is to say, the enacted curriculum lowered cognitive demand and the facilitation of discourse (IRE and direct instruction) limited Miss X's ability to elicit student thinking, which I argue is important because these barriers restricted her ability to cultivate a community of learning based on the guiding principles for productive disciplinary engagement.

Questioning. According to my findings, there is no evidence of Miss X pressing students to explain or justify their thinking. The ways in which Miss X facilitated discourse in this classroom, as described in the previous chapter, were in the form of IRE or direct instruction in which student thinking was not elicited. The results of this study show that Miss X asked questions that required students to provide one- or two-word answers or recall information. Questions that are concerned with seeking answers, rehearsing known facts, recalling information, and using correct mathematical terminology are associated with a decline of cognitive demand and are typical within the IRE pattern of discourse (Mehan, 1979: Stein et al., 2009). These types of questions do not elicit student thinking and do not foster students' development as active thinkers, constructors, and evaluators of knowledge (Boaler & Brodie, 2004; Schoenfeld, 2016). Because of this, I perceive these types of questions to inhibit opportunities for cultivating a community of learning.

According to research, asking good questions that elicit student thinking can ensure active thinking and participation among students in the classroom (Smith and Stein, 2011). In addition, the types of questions a teacher asks can promote deeper levels of thinking, or higher levels of cognitive demand (NCTM, 2014). Posing questions that require students to articulate, explain or justify their thinking holds students accountable for their intellectual work (Engle & Conant, 2002). When teachers pose questions related to how students are making sense of or thinking about content, they help students problematize that content. By asking students questions that elicit their thinking in the form of explaining, justifying, reasoning or reflecting on intellectual content, students activate their sense-making skills. According to NCTM (2014), questions should involve "the teacher attending to what the students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates" (p. 37), which relates to all of the principles of disciplinary engagement. I perceive that the ways Miss X facilitated discourse in this study, in addition to lowering the cognitive demands in the classroom, created a barrier in leveraging opportunities that were presented to cultivate a community of learning.

Miss X could have taken advantage of the opportunities presented in the intended curriculum to facilitate discourse to elicit student thinking. I mentioned in the previous chapter how the intended curriculum included suggested questions in the teacher's guide. These questions sprinkled throughout each lesson and topics are designed to elicit student thinking in the form of explanations and provoke discussion. The purpose of these questions was to help students make sense of mathematics and engage in deep thinking about mathematical concepts (Pearson enVision 2.0, 2016). As described in the previous paragraph, since the questions center on eliciting student thinking, they relate to the principles of problematizing content and accountability. These questions also align with the effective mathematics teaching practice of facilitating meaningful mathematical discourse in the classroom (NCTM, 2014). These types of questions were included throughout the intended curriculum and provided an opportunity for Miss X to facilitate meaningful mathematical discourse, help students to make sense of mathematical ideas and engage students in deep thinking. Despite this opportunity, Miss X's facilitation of discourse in the form of the IRE pattern of questioning and in the form of direct instruction during task implementation limited her ability to leverage this opportunity.

Boaler and Brodie (2004) studied teacher questioning in the context of the classroom and constructed the following table to describe the types of questions teachers ask and provide examples of each question type (Figure 28). Question types 3, 4 and 5 are associated with eliciting student thinking, higher levels of cognitive demand and the guiding principles (Engle & Conant, 2002; Stein et al., 2009). Smith and Stein (2011) highlighted these types of questions as being important in that they go beyond simply gathering information or leading students through a procedure, important aspects for problematizing content. Questions of these types allow students to think deeply about mathematical ideas, which can result in the maintenance of higher levels of cognitive demand and become relevant resources to support student engagement. The classroom set up and daily routine also presented opportunities to leverage questions types 3, 4 and 5. The type of seating could have promoted collaboration and the daily routine could have encouraged students to articulate, elaborate or clarify ideas through journaling or in small groups, all of which relate to the principles of authority and accountability.

Question type	Description	Examples
1. Gathering information, leading students through a method	Requires immediate answer Rehearses known facts/procedures Enables students to state facts/procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3. Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations	Where is this x on the diagram? What does probability mean?
4. Probing, getting students to explain their thinking	Asks student to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5. Generating Discussion	Solicits contributions from other members of class.	Is there another opinion about this? What did you say, Justin?
6. Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life	In what other situations could you apply this? Where else have we used this?
7. Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used	Would this work with other numbers?
8. Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics	What is the lottery? How old do you have to be to play the lottery?

Figure 28. Types of teacher questions (Boaler & Brodie, 2004, p. 777).

Engle and Conant (2002) state that the idea underlying the principle of problematizing content is that teachers should "encourage students' questions, proposals, challenges and other intellectual contributions, rather than expecting that they should assimilate facts, procedures and other answers" (p. 404). When teachers ask questions that elicit student thinking, students have opportunities to explore, explain and discuss mathematical ideas and construct meaning. In this study, students were not provided with those opportunities. The questions the teacher asked were mostly question type 1, gathering information and leading students through a method. According to the results of this study, while we know about the types of questions teachers should ask to elicit student thinking in practice, the teacher in this study did not ask those types of questions. Therefore, implications of these findings for the field of mathematics education can be related to the ways in which teachers can be supported in their practice to elicit student thinking. When students are pressed to articulate, explain, and justify their thinking and ideas, they are held accountable for their intellectual work (Engle & Conant, 2002). Smith and Stein (2011) assert that good questions "force students to articulate their thinking so that it is understandable to another human being; this articulation, in and of itself, is often a catalyst to learning" (p. 62). By asking good questions that elicit student thinking, teachers facilitate student discourse which can lead to increased mathematical knowledge and understanding in the classroom (Franke et al., 2009). Even though we have known these things for over a decade, this study shows how teachers continue to struggle with asking good questions in practice.

Miss X's facilitation of discourse, which did not elicit student thinking, not only reduced cognitive demand in the classroom but also created a barrier to leveraging opportunities presented for cultivating a community of learning in this classroom. As described previously, the classroom set up, the daily routine, and the intended curriculum presented opportunities for student thinking to be elicited, however, Miss X did not take advantage of these opportunities. Another way that Miss X reduced the cognitive demand in the classroom and failed to elicit student thinking was in her use of scaffolding during task implementation.

Scaffolding. Scaffolding is an instructional tool that teachers use to guide students toward understanding mathematical concepts and in solving problems. The term “scaffolding” connects to the work of Vygotsky and the “zone of proximal development” (ZPD). Psychologists who have studied the ZPD in different educational contexts have found that encouraging students to tackle the most difficult tasks within their ZPD leads to the most learning (Glassman, 2001). Henningsen and Stein (1997) state that “scaffolding occurs when a student cannot work through a task on his or her own, and a teacher or more capable peer provides assistance that enables the student to complete the task alone, but does not reduce the overall complexity or cognitive demand of the task” (p. 527). The key to successful scaffolding is relying on prior knowledge that a student possesses to provide a bridge for the student to construct a more complete conceptual knowledge. Scaffolding in this way, in terms of students’ conceptual knowledge, improves students’ problem solving (Rittle-Johnson & Koeniger, 2005).

One way that teachers can support students’ learning is by using scaffolding to focus on students’ mathematical thinking and use their understanding of students’ mathematical thinking to guide instruction (Franke et al., 2009). When teachers focus on students’ mathematical thinking, with the goal of organizing their own knowledge and understanding, teachers are able to incorporate that thinking into their instructional decisions (Franke et al., 2009). Using student thinking to guide instruction is not a new concept. This was the basis of many CGI studies in which teachers were explicitly trained how to use and analyze students’ thinking to guide instruction (Carpenter et al., 2000). CGI teachers were trained to encourage students to solve problems in their own ways and engage students in conversations to elicit their thinking for the purpose of understanding and analyzing it. CGI teachers then made instructional decisions based on their knowledge of what they had analyzed. Carpenter et al. (2000) reported that students of teachers who knew more about their students' thinking had higher levels of achievement in problem solving than students of teachers who had less knowledge of their students' thinking.

In this study, scaffolding could have been used in ways which would have supported opportunities to cultivate a community of learning. This could have included Miss X focusing on eliciting student thinking in her use of scaffolding, to bridge their learning with prior knowledge, and could have included her presenting tasks in ways that maintained the level of demand of tasks within the intended curriculum. In these ways, Miss X could have given students the authority to complete a task on their own, with her guidance, rather than her holding the authority in taking over the thinking and reasoning for students. However, I observed scaffolding being used in other ways in this classroom, described below, and thus Miss X’s use of scaffolding in this study became a barrier in terms of opportunities presented for cultivating a community of learning.

In the previous chapter, I described instances in which the teacher provided step-by-step instructions when solving problems with students. Based upon the relationship of these instances with the principles of problematizing content, authority and accountability, I coded these instances as *overscaffolding* in my initial reading and memoing of emergent ideas. In these instances, I observed the teacher engaging in discourse that involved IRE or direct instruction in which she was doing most of the talking about a problem. She provided specific step-by-step instructions for what to do next when solving the problem and told students what to write or draw on their papers. The ways that Miss X moved through procedures and provided explicit instructions for solving, while neglecting to elicit and use student thinking, is evidence of how Miss X's use of scaffolding actually limited the students' abilities to tap into their higher-order thinking and engage in problematizing content. It also provides evidence of her holding authority in the classroom as the mathematics was not generated by the class as a whole and the students were unable to be authors and producers of their own knowledge (Engle & Conant, 2002).

Vygotsky presented his work on the zone of proximal development in 1978, and since then, teachers have incorporated scaffolding techniques. It is reasonable to assume that most teachers are aware of the tool of scaffolding and how to effectively incorporate scaffolding techniques in their teaching. However, in this study, scaffolding in ways that supported the cultivation of a community of learning were not observed. This is interesting because the teacher did have opportunities to use scaffolding in the classroom in ways that would support student learning. Miss X could have taken advantage of the collaborative classroom set up, situating students with more knowledgeable peers, to work on tasks together. During small group instruction, Miss X could have been intentional in eliciting and using student thinking to support their learning and build on their thinking through questioning and prompts. These actions would have supported the principles of giving students authority and problematizing content, as well as holding students accountable for sharing their thinking and using relevant resources.

Scaffolding connects to cognitive demand and student thinking through the principle of accountability. Stein et al. (2009) associate the maintenance of high-level cognitive demands with appropriate teacher scaffolding. Appropriate teacher scaffolding is described as guiding student thinking by asking thought-provoking questions that preserve the complexity of the task. When students share their thinking and respond to the thinking of others, they are held accountable to others and to disciplinary norms. When students have opportunities to engage in complex thinking and tasks associated with high levels of cognitive demand, they are also held accountable as their intellectual work can then be responsive to the content (Engle & Conant, 2002). The results of this study show how the teacher's enactment of the curriculum and facilitation of discourse in terms of scaffolding led to the decline of cognitive demand and impacted opportunities for students to be held accountable in their learning. Scaffolding is not a new concept in teaching, as Vygotsky presented his work on the zone of proximal development, which is synonymous with scaffolding in 1978. However, in this study, scaffolding in ways that supported the cultivation of a community of learning were not present.

In this section, I described how Miss X lowered the cognitive demand in her set up of tasks and then maintained that low level of demand during implementation. I described how during task implementation, the types of questions teachers ask should serve to deepen students' thinking and understanding of disciplinary content and hold students accountable by invoking explanations and pushing students to provide evidence to support their claims (Boaler & Brodie, 2004). I described the importance of eliciting student thinking in the classroom during scaffolding to inform teachers' instruction and to give students authority, hold students accountable, and allow students to engage in problematizing content. Student thinking is fundamental in a community of learning. When students participate in classroom discourse and share their thinking, they become accountable to other members of the classroom as they publicly investigate their ideas. And lastly, when students have an opportunity to explain their thinking, share ideas and ask questions, they have expertise and authority in the classroom. These things did not happen in the classroom in my study, and as such, opportunities presented for cultivating a community of learning through the classroom set up, daily routines, and intended curriculum were missed.

Conclusion

In this study, I found the teacher missed opportunities to cultivate a community of learning. Through her enactment of the curriculum and facilitation of classroom discourse, Miss X reduced the level of cognitive demand and failed to elicit student thinking. This reduction of cognitive demand and failure to elicit student thinking were barriers in taking advantage of the opportunities presented for cultivating a community of learning through her classroom set up, daily mathematics routine, and the intended curriculum. These barriers are not new to the field of mathematics education. Over the past several decades, researchers have called for attention to the benefits of enacting high-level tasks and facilitating meaningful mathematics discourse (e.g., Schoenfeld, 2016; Smith & Stein, 2011). There is also evidence that teachers need to be supported in their practice to employ these elements effectively (Franke et al., NCTM, 2014; Stein et al., 1996). This study reinforces this need as we see that in a naturalistic setting, this first-grade teacher, while having the opportunity to implement curriculum and facilitate discussions in the classroom in ways that could cultivate a community of learning, seemed to need support in how to effectively do this.

There are serious difficulties in enacting a curriculum as intended (Remillard, 2005). Researchers describe that "curriculum enactment is a complex and multifaceted process, involving different actors and operating at multiple levels" and have identified a need for teachers to be supported in implementing curricula (Remillard & Heck, 2014 p. 716). In addition to the need to support teachers in their enactment of a curriculum, it is clear based on the results of my study that teachers also need to be supported in cultivating a community of learning while enacting a curriculum. This is significant because, as my study demonstrates, taking advantage of opportunities to cultivate a community of learning while enacting a curriculum can create new challenges for teachers and require different types of support. One of the findings of my study is that even though the teacher was presented with a curriculum that demonstrated characteristics which appeared to support the cultivation of a community of learning, and that the classroom set up and daily routine demonstrated characteristics that could support a community of learning, she could have benefitted from being supported in understanding how to take advantage of these opportunities. Whether in the form of professional development, mentoring, coaching or dedicated planning, teachers need to be supported in how to enact curricula, set up norms and facilitate discourse in ways that can cultivate a community of learning. A contribution from my work is the template I used in my observations (Appendix A), which could be used as a self- or peer-reflection tool for teachers to connect the principles for productive disciplinary engagement to their teaching actions. This is a one-page document, which lists what students should be doing in the classroom in relation to cultivating a community of learning according to each of the four principles and could be used in mentoring or coaching meetings to help teachers identify aspects of their practice for improvement. For example, if Miss X had become aware of how she was modifying tasks as she read them to her students, she may have been able to change this behavior to maintain the level of demand of those tasks. By maintaining demand, she would have given herself the opportunity to support students in problematizing content. I imagine a follow up study involving teacher interviews which might be aimed at investigating the impact of this tool on teacher practice and the cultivation of a community of learning.

Another follow up study might be aimed at investigating the relationship between the teacher's actions and her beliefs regarding cultivating a community of learning. Miss X seemed to position her own mathematical thinking as the centerpiece of the classroom, as opposed to the students' mathematical thinking, which prevented students from being held accountable for their thinking and responding to the thinking of others. Miss X did not give her students opportunities to engage in productive struggle, which relates to them problematizing content and having authority in the classroom. In my one exchange with Miss X where I pressed her on this, she could only point to a perception that the material was too hard for her students. Why did she believe this? And what other beliefs led to her actions? Investigating the teacher's actions and beliefs as they relate to a community of learning could help researchers to better support teachers in taking advantage of opportunities for cultivating a community of learning in their classrooms.

Research regarding effective teaching and learning in mathematics classrooms, including the types of questions teachers should ask to facilitate productive student discourse in the classroom and ways to establish and maintain levels of cognitive demand in the classroom are made clear in mathematics education literature (Boaler & Brodie, 2004; NCTM, 2014; Stein et al., 2009; Schoenfeld, 2016). According to the results of my study, this research is not always translating into practice. Teachers continue to struggle with critical aspects of effective teaching, making the findings of this study important. My findings indicate that we have not yet handled this problematic aspect of teacher professional learning and supporting teachers in their practice to achieve effective teaching and learning of mathematics in their classrooms.

What I found in this study is that this first-grade teacher missed opportunities for cultivating a community of learning in her classroom. This highlights the fact that teachers need to be supported in understanding how to cultivate a community of learning and how to do that using the four guiding principles. This is significant because there is research from the past few decades that a community of learning can support students' learning in meaningful ways, and there are many intervention studies that have been conducted which demonstrate aspects of a community of learning being realized in mathematics classrooms. However, based on the findings of this naturalistic study, it is clear that understanding how to cultivate a community of learning and how to take advantage of opportunities for cultivating a community of learning is necessary. Moreover, knowing how to elicit student thinking and maintaining the level of cognitive demand in mathematics classrooms continues to be a struggle for some teachers. Therefore, we need to do something different to support in-service teachers in their preparation, practice, and professional development as it relates to these important teaching practices. Using my tool, which outlines what students should be doing in a community of learning classroom in relation to cultivating a community of learning, could support teachers in relating their practices with each of the four guiding principles for fostering productive disciplinary engagement in a community of learning. If we can support our early elementary teachers in these ways, we can better support our youngest learners in powerful and meaningful ways.

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APPENDIX A

Four Principles for Fostering Productive Disciplinary Engagement

in a Community of Learning

Problematizing content (PC): Students question, propose, challenge and contribute intellectually to the content; teacher encourages this (opposite of simply assimilating facts or procedures); claims are examined, evidence is sought for explanations and procedures are accompanied with an explanation. Evidence of PC may include:

• Student asking question • Student proposing idea/strategy/solution • Student challenging another student/teacher idea or a strategy/procedure • Student explaining their thinking	• Student seeking evidence for claim • Student engaging in discussion about mathematical ideas • Student seeking credibility in sources

Giving students authority (AUTH): Students have an active role (agency) in defining, addressing, and resolving problems. Students are positioned as stakeholders in publicly identifying claims, approaches, explanations, designs and other responses to problems. Evidence of AUTH may include:

• Student ideas solicited • Student challenge welcomed • Student is not told if they are right or wrong • Student plays an active role in solving a problem • Student contributes to a collaborative project	• Student referred to as expert/knower • Student produces knowledge • Student defines/debates idea • Student convinces others to change thinking

Holding students accountable to others and to disciplinary norms (NORMS): Students are responsible for ensuring that their intellectual work is responsive to content and practices established by other members of the learning community. Evidence of NORMS may include:

• Students consult others in constructing understanding/making meaning • Students attend to/respond to the work/thoughts of others	• Student demonstrates that others in the community share the same level of authority • Students negotiate evidence to support a claim

Providing relevant resources (RR): Resources needed to support these principles are evident. Evidence of RR may include:

• Students have sufficient time • Students are provided with scaffolding	• Students have access to resources (manipulatives, media) • Students have opportunities to discuss

Engle, R. A., & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners' classroom. Cognition and Instruction, 20(4), 399-483.

APPENDIX B

Description of the Teaching for Robust Understanding (TRU) Framework

The TRU framework is a way of characterizing powerful learning environments in crisp and actionable ways. Central to TRU are the 5 dimensions of classroom activity. Classrooms that do well on these 5 dimensions produce students who are powerful thinkers.

The Five Dimensions of Powerful Classrooms

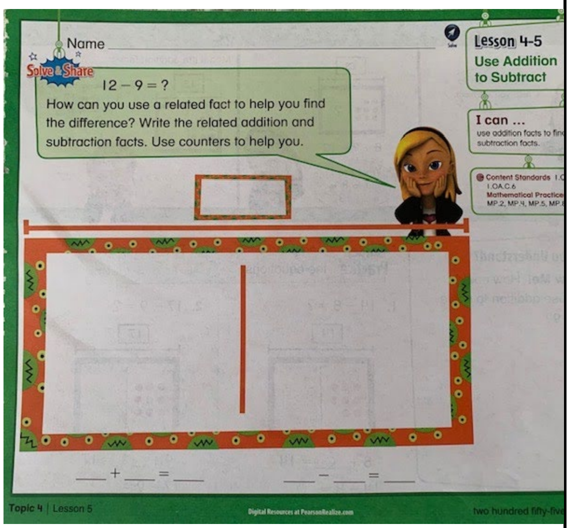
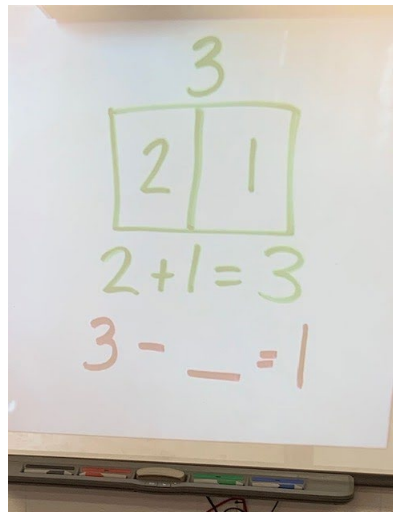
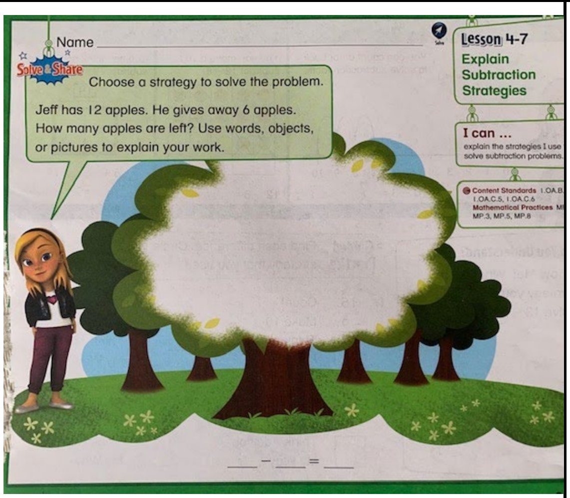
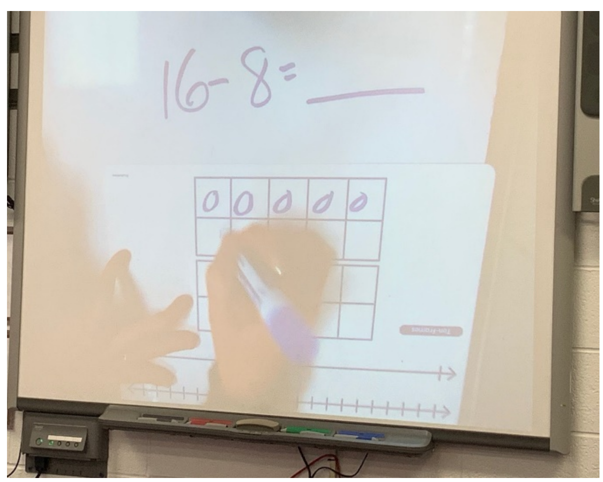
The Content	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.	*The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

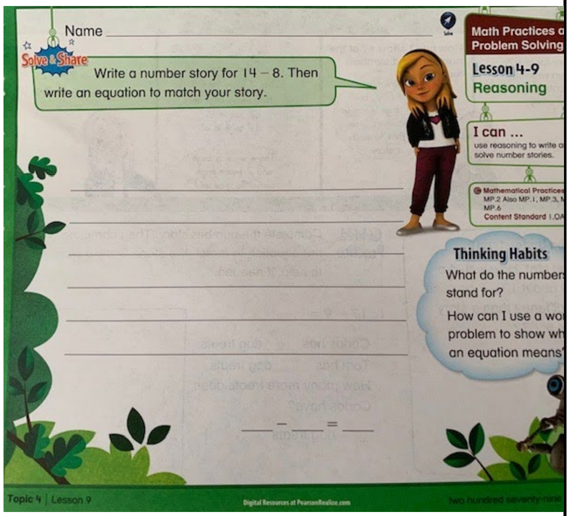
Schoenfeld, A. H., and the Teaching for Robust Understanding Project. (2016). The Five Dimensions of Powerful Classrooms [Online Image]. Retrieved February 22, 2020 from https://www.map.mathshell.org/trumath.php.

APPENDIX C

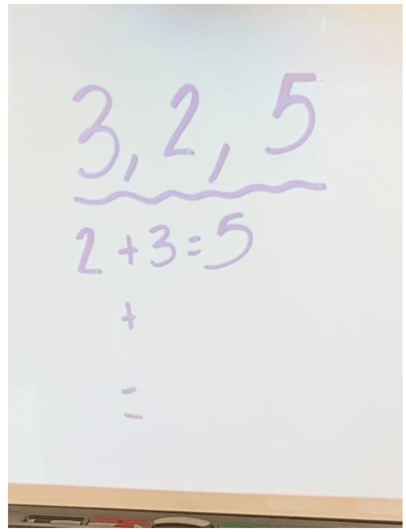
Pearson enVision Math 2.0 (2016) "Solve and Share"

and Teacher's Launch

CURRICULUM SOLVE & SHARE	TEACHER LAUNCH
 Observation #1: Solve & Share Lesson 4-5 (Using Addition to Subtract)	 Observation #1: Teacher's Launch
 Observation #2: Solve & Share Lesson 4-7 (Explain Subtraction Strategies)	 Observation #2: Teacher's Launch

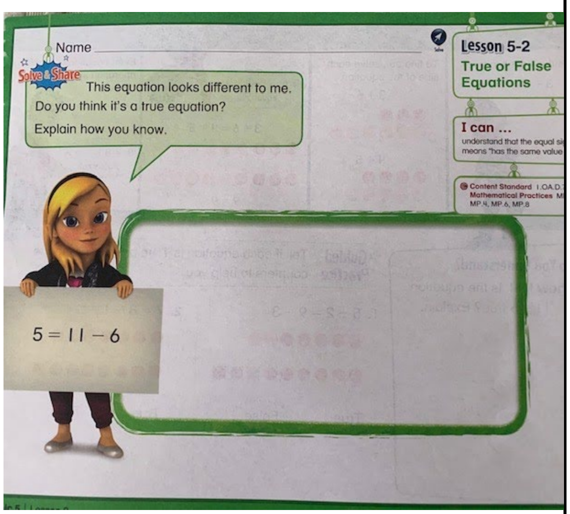


Observation #3: Solve & Share
Lesson 4-9 (Reasoning - Subtraction)

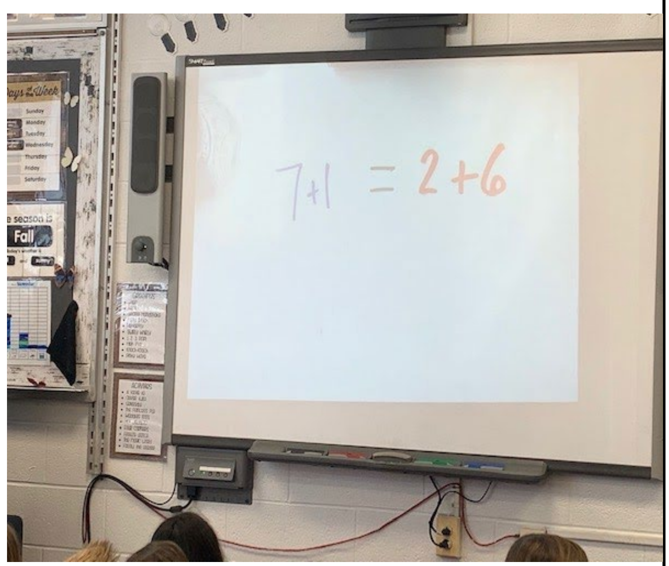


Observation #3: Teacher's Launch

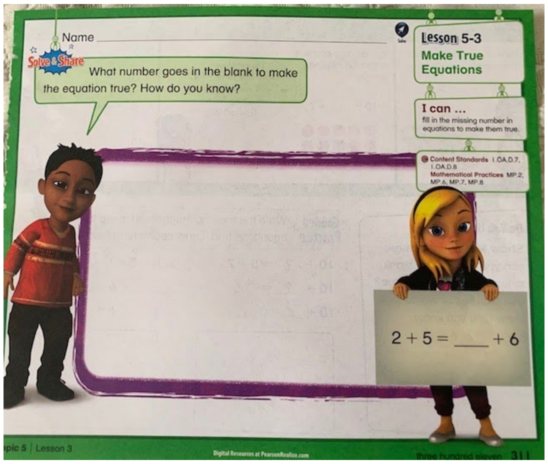
Observation #4: Reteaching Topic 4
(Teacher did not do a launch)



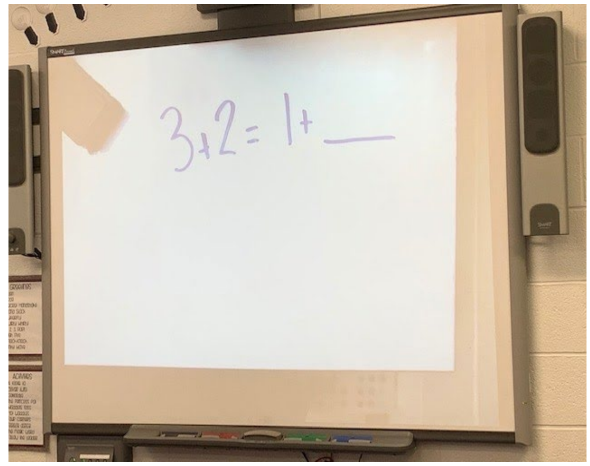
Observation #5: Book Solve & Share
Lesson 5-2 (True or False Equations)



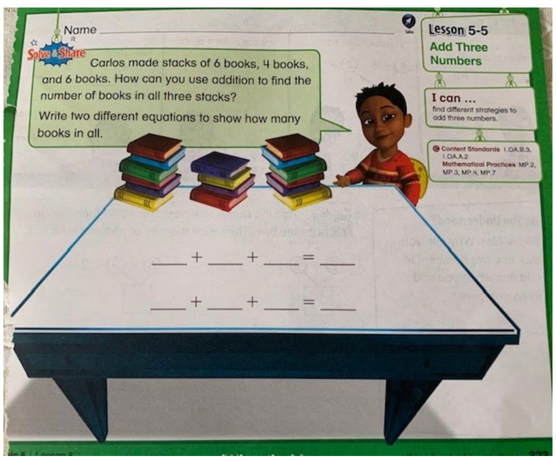
Observation #5: Teacher's Launch



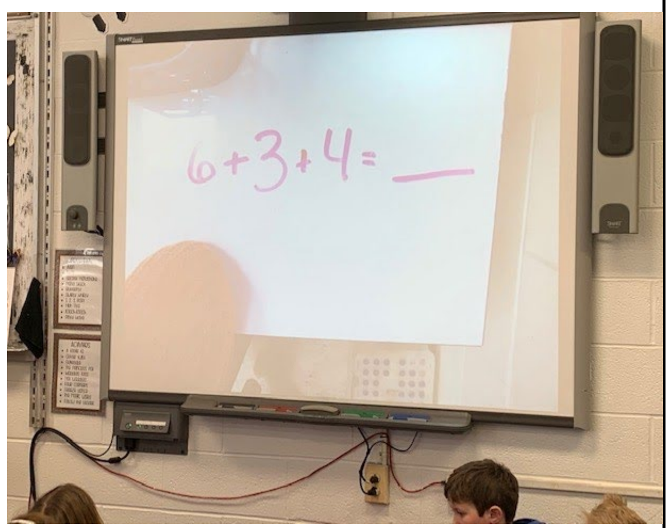
Observation #6: Book Solve & Share Lesson 5-3 (Making True Equations)



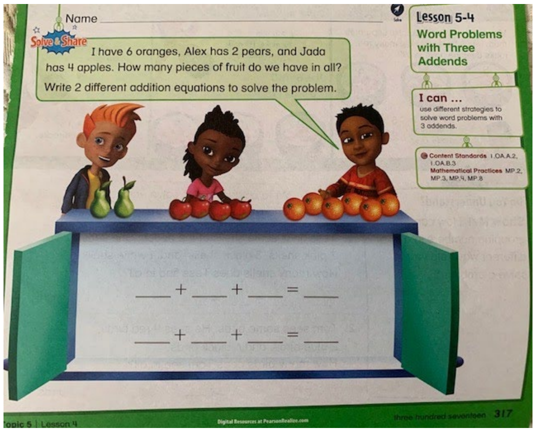
Observation #6: Teacher's Launch



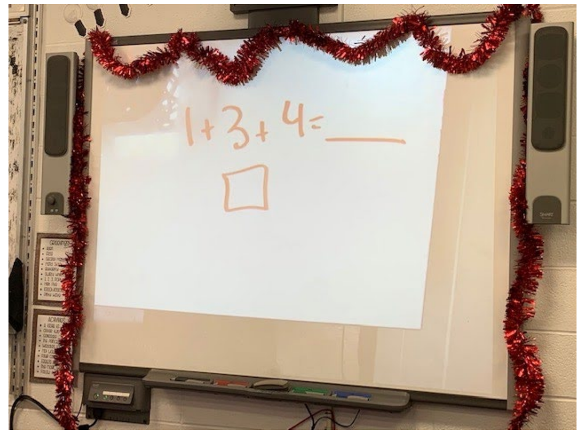
Observation #7: Book Solve & Share Lesson 5-5 (Add Three Numbers)



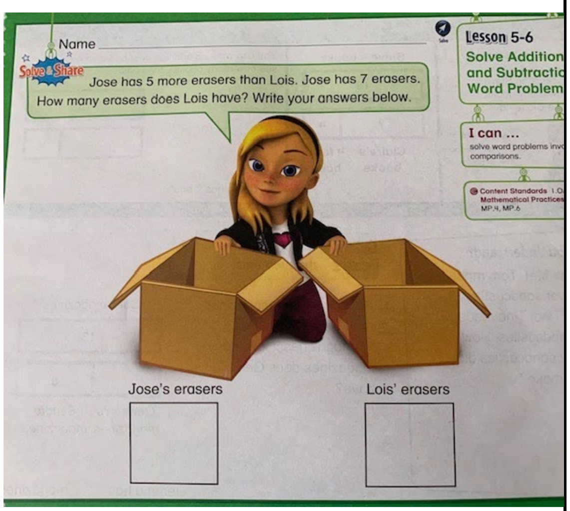
Observation #7: Teacher's Launch



Observation #8: Book Solve & Share Lesson 5-4 (Word Problems Three Addends)



Observation #8: Teacher's Launch



Observation #9: Book Solve & Share Lesson 5-6 (Solve Addition and Subtraction Word Problems)

"Miss X has 9 cookies. Heather has 4 less. Heather has 4 less cookies than Miss X. What would be the equation?"

Observation #9: Teacher's Launch

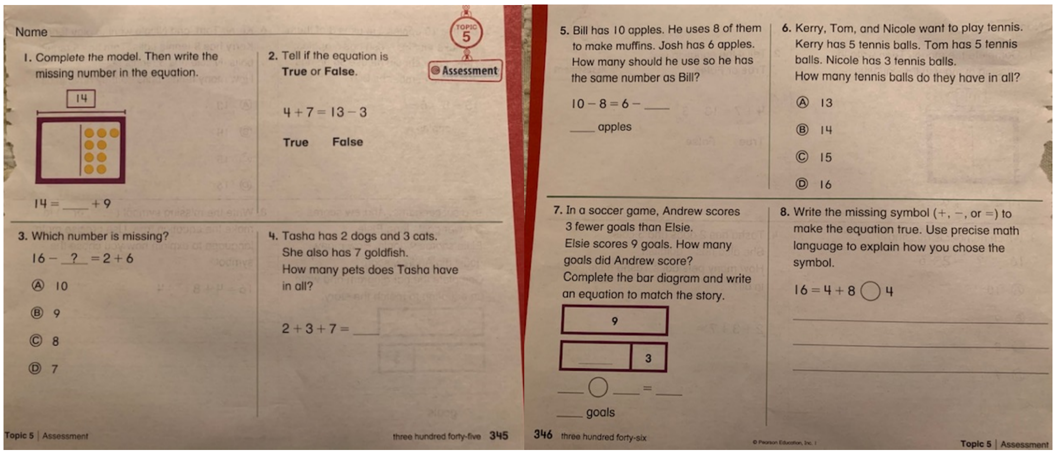
Observation #4: Reteaching Topic 4
(Teacher did not do a launch)

Charles, R. I., Bay-Williams, J. M., Berry, R. Q., Caldwell, J., Champagne, Z., Copley, J., Crown, W., Fennell, F., Karp, K., Murphy, S., Schielack, J., Suh, J., & Wray, J. (2016). EnVision Math 2.0. Glenview, IL: Scott Foresman, Pearson Education.

APPENDIX D

Pearson enVision Math 2.0 (2016) "Topic 5 Assessment"

and Teacher's Topic 5 "Show What You Know"



Name

Topic 5 Assessment

1. Complete the model. Then write the missing number in the equation.

14

14 = ___ + 9

2. Tell if the equation is True or False.

4 + 7 = 13 − 3

True False

3. Which number is missing?

16 − ? = 2 + 6

Ⓐ 10
Ⓑ 9
Ⓒ 8
Ⓓ 7

4. Tasha has 2 dogs and 3 cats. She also has 7 goldfish. How many pets does Tasha have in all?

2 + 3 + 7 = ___

Topic 5 | Assessment three hundred forty-five 345

5. Bill has 10 apples. He uses 8 of them to make muffins. Josh has 6 apples. How many should he use so he has the same number as Bill?

10 − 8 = 6 − ___

___ apples

6. Kerry, Tom, and Nicole want to play tennis. Kerry has 5 tennis balls. Tom has 5 tennis balls. Nicole has 3 tennis balls. How many tennis balls do they have in all?

Ⓐ 13
Ⓑ 14
Ⓒ 15
Ⓓ 16

7. In a soccer game, Andrew scores 3 fewer goals than Elsie. Elsie scores 9 goals. How many goals did Andrew score? Complete the bar diagram and write an equation to match the story.

9

3

___ ◯ ___ = ___

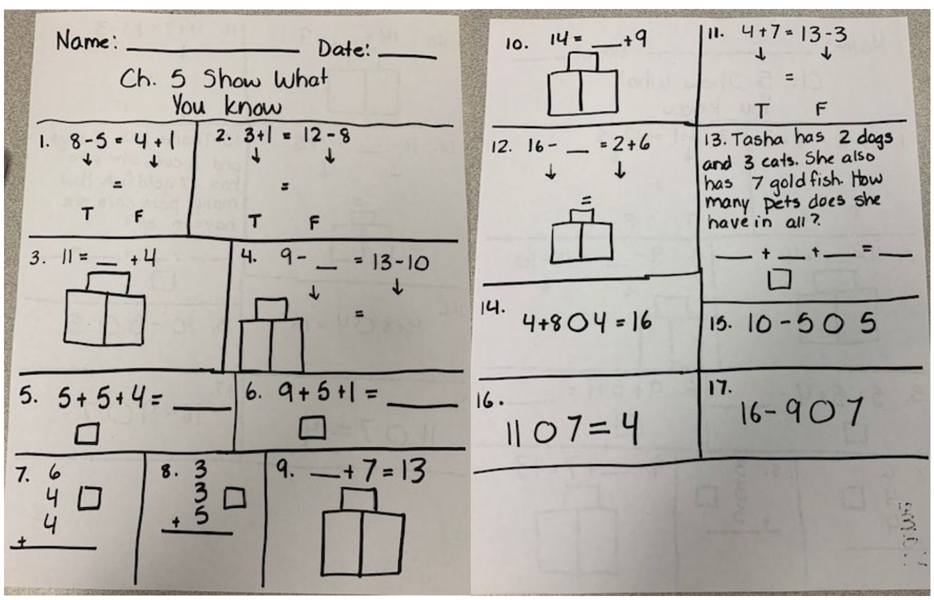
___ goals

8. Write the missing symbol (+, −, or =) to make the equation true. Use precise math language to explain how you chose the symbol.

16 = 4 + 8 ◯ 4

346 three hundred forty-six Topic 5 | Assessment

Book "Topic 5 Assessment"



Name: ___ Date: ___

Ch. 5 Show What You know

1. 8−5 = 4+1 T F
2. 3+1 = 12−8 T F
3. 11 = ___ +4
4. 9− ___ = 13−10
5. 5+5+4= ___
6. 9+5+1 = ___
7. 6 4 4 +
8. 3 3 + 5
9. ___ +7=13
10. 14 = ___ +9
11. 4+7 = 13−3 T F
12. 16− ___ = 2+6
13. Tasha has 2 dogs and 3 cats. She also has 7 goldfish. How many pets does she have in all? ___ + ___ + ___ = ___
14. 4+8 ◯ 4 = 16
15. 10−5 ◯ 5
16. 11 ◯ 7 = 4
17. 16−9 ◯ 7

Teacher's Topic 5 "Show What You Know"

Charles, R. I., Bay-Williams, J. M., Berry, R. Q., Caldwell, J., Champagne, Z., Copley, J., Crown, W., Fennell, F., Karp, K., Murphy, S., Schielack, J., Suh, J., & Wray, J. (2016). EnVision Math 2.0. Glenview, IL: Scott Foresman, Pearson Education.