1-2021

Fostering Mathematical Creativity Among Middle School Mathematics Teachers

Ceire H. Monahan
Montclair State University

Fostering Mathematical Creativity Among Middle School Mathematics Teachers

A DISSERTATION

Submitted to the Faculty of

Montclair State University in partial fulfillment

of the requirements

for the degree of Doctor of Philosophy

by

Ceire H. Monahan

Montclair State University

Montclair, New Jersey

January 2021

Dissertation Chair: Dr. Mika Munakata

MONTCLAIR STATE UNIVERSITY

THE GRADUATE SCHOOL

DISSERTATION APPROVAL

We hereby approve the Dissertation

Fostering Mathematical Creativity Among Middle School Mathematics Teachers

of

Ceire H. Monahan

Candidate for the Degree:

Doctor of Philosophy

Graduate Program:
Mathematics Education

Certified by:

[redacted]

Dr. M. Scott Herness
Vice Provost for Research and
Dean of the Graduate School

Jan. 26, 2021
Date

Dissertation Committee:

[redacted]

Dr. Mika Munakata
Dissertation Chair

[redacted]

Dr. Erin Krupa

[redacted]

Dr. Pablo Tinio

[redacted]

Dr. Ashwin Vaidya

Abstract

Fostering Mathematical Creativity Among Middle School Mathematics Teachers

by Ceire H. Monahan

The purpose of this research was to understand in-service teachers' experiences with and ideas about mathematical creativity after participating in a targeted professional development program. The professional development program encouraged participants to think creatively and foster students' creativity. In this study I present the results from the main unit of analysis, 12 participants in a professional development program, along with a deep analysis of three exemplar teachers from each of the identified groups, *adherence to traditional teaching practices (traditional), appreciation for teaching for creativity (creative but hesitant),* and *teaching for creativity (creative)*. The findings of this study highlight the combination of participants' personal characteristics, lesson plans, problem-solving processes, and the environment that allowed teachers to foster creativity among middle school mathematics students. It also describes challenges participants faced when addressing mathematical creativity and how professional development programs can further support teachers as they implement elements of mathematical creativity into their teaching practice.

Keywords: Professional development, creativity, mathematical creativity, K-12 education, teaching and learning

Acknowledgements

I would like to express my appreciation to my chair, Dr. Mika Munakata, who believed in me from my first day as a doctoral student. Thank you for your expertise, guidance, and patience. You helped me to become a better researcher, writer, and thinker, guided me through every stage of this journey, and helped me find my voice. I am forever grateful for you.

I would also like to thank Dr. Krupa who became a mentor to me from my first class as a doctoral student. Thank you for always believing in me, pushing me to achieve my dreams, and encouraging me to be the best that I could be. Drs. Tinio, and Vaidya, thank you for your continued support and feedback that helped shape this research.

Thank you to Megan, Tory, and Madhavi who provided inspiration, encouragement, and hope that this was possible, regardless of the challenges. To my writing group, Adam, Katie, and Laura who were constantly there for support, and Jess who lived this experience alongside me.

A special thank you to my family. To my husband, Tommy, for your patience and thoughtfulness on this journey, and for the countless weekend adventures with Gerard and Tessa so I could work. To our son Gerard and daughter Tessa, thank you for being filled with so much hope and wonder, demonstrating that every day is an opportunity for a new beginning. To my Mom and Dad for the unending support, instilling in me the importance of education, and seemingly endless extra hours of time you allowed for me to focus while you entertained and loved Gerard and Tessa. To Caitlin, Ryan, Jane, and Audrey who somehow from afar continued to be a main source of steadiness, focus, and logic, as always. To the friends who have become family, thank you for providing encouragement and laughs when I needed it most.

And lastly, my gratitude to the 12 teacher-participants in this study who demonstrate daily that despite the challenges, amazing things are happening in middle school mathematics teaching.

Dedication

I dedicate this dissertation to the person who sacrificed alongside me throughout this process. Tommy, your endless support, selflessness, and patience are admirable and very much appreciated. Thank you for believing in me and helping me achieve this goal and for being a wonderful husband and father.

Contents

List of Tables

List of Figures

CHAPTER 1: INTRODUCTION

> All mathematics today is the creative product of human imagination; invented as a means to communicate, to solve practical problems, to entertain, to search for understanding of how the systems in our world connect and function, and to explore ideas we find beautiful and elegant. (Mann, 2020)

Most inventions arise out of an inherent need. Mathematics is no different as it is rooted in counting, measuring, and quantifying for survival. Humans' creative abilities allowed for survival and also helped understand our world and the universe. The expansive view of creativity as a means to understanding the world leads one to consider creativity as being central to mathematics. Although creativity may be thought to be central to mathematics, it is most often recognized in art, music, and poetry.

Although traditionally recognized in the arts, researchers have also explored the importance of creativity in mathematics (Mann, 2005; Shriki, 2010; Sriraman, 2005). Researchers debate whether creativity is general or specific to a particular domain (Baer & Kaufman, 2005; Plucker & Beghetto, 2004) and if there is a place for creativity in school (Kaufman & Beghetto, 2013). However, others argue that mathematics is inherently creative and classroom teaching, which often discourages creativity (Geist & Hohn, 2009), should instead reflect the creative nature of mathematics (Pólya, 1962). If there is ever a place for creativity to be emphasized, it might be in mathematics.

This study explores how a professional development program focused on elements of mathematical creativity impacts in-service middle school mathematics teachers' conceptions of mathematics and creativity, and how these ideas are implemented in practice. Because creativity is difficult to define, researchers have defined it in many different ways. For the purposes of this study, I developed a theoretical framework based on Rhodes' (1961) categorization of creativity as four strands: person, process, product, place, the spectrum of creativity (Beghetto & Kaufman, 2007; Csikszentmihalyi, 1996, 1998), and triangular theory of creativity (Sternberg, 2018). This study allowed me to consider all three theories together and elucidate how they are related to one another. I developed this framework by combining the three theories within the context of mathematics, highlighting how these theories relate to each other and to middle school mathematics teaching and learning.

Rationale

Traditional teaching practices celebrate speed and accuracy as mastery in mathematics, producing students who are, "prodigious calculators—frequently quite uneducated [people]—who can very rapidly make very complicated numerical calculations…such talent is, in reality, distinct from mathematical ability" (Hadamard, 1945, p. 58). Sternberg (1996) echoed this idea when he referred to comments from mathematicians who suggested that, "It is creative mathematical thinking that is the most important" (p. 313). Similarly, Boaler (2016) suggested the importance of creativity in mathematics when she noted that, "Powerful thinkers are those who make connections, think logically, and use space, data, and numbers creatively" (p. 31). Mathematics requires more than mastering computational skills. Instead, the essence of mathematics is to develop new ideas and make connections among seemingly unrelated concepts. However, teachers often emphasize the importance of accuracy, speed, and algorithms in mathematics classrooms. This can be discouraging and result in students who are able to reproduce procedures but lack the skills to apply knowledge to new situations. A shift in traditional teaching practices is needed to foster an appreciation for and development of students' creative ideas in mathematics. In order to promote change in teaching practices, it is necessary to understand teachers' beliefs for the purposes of shifting instruction away from traditional practices and toward strategies that foster creativity in mathematics, and better understand characteristics of developing mathematical creativity (Mann, 2006, 2009).

Although creativity can improve students' achievement in mathematics (Bahar & Maker, 2011; Karwowski et al., 2020), the majority of research on students' creativity is among gifted students (Hong & Aqui, 2004; Zenasni et al., 2016) and limited research on general students' creativity. These researchers have studied mathematics Olympiads and highlighted the creativity of gifted students. Giftedness is often assessed based on speed and accuracy rather than on students' ability to apply ideas in different ways (Kim et al., 2004; Mann, 2009). Restricting mathematical creativity as possible only for gifted students ignores general students' creativity (Mann, 2009) and the benefits creativity might have for all mathematics learners.

There is also limited research on how to prepare in-service teachers to foster creativity among students in mathematics. While some researchers have focused on preservice elementary mathematics teachers' awareness of creativity in mathematics (Levenson, 2013a; Shriki, 2010), they do not identify how in-service teachers might foster these ideas among students. With this in mind, Levenson (2013b) stressed a need to better understand teachers' beliefs and attitudes toward creativity in mathematics.

Professional Development

One way to support teachers in understanding and implementing mathematical creativity in classrooms is to design professional development programs that encourage participants to think about and implement these ideas in practice. Although a large amount of research on general and content-specific professional development exists, little research has been done to investigate how professional development influences teachers' practices to foster creativity in mathematics, understand how teachers can encourage students' mathematical creativity in practice, or explore how professional development programs can prepare teachers to do so. Therefore, there is a need to design and research professional development programs that introduce teachers to mathematical creativity and encourage them to reconsider their teaching practices.

Although some research has addressed how teachers' conceptions of creativity impact their instruction in mathematics and the types of teaching environments that are effective in fostering creativity (e.g. Chamberlin & Moon, 2005; Shriki, 2010), further research should explore professional development programs focused on domain-specific applications of creativity. The focus of this research continues the investigation of innovative problem solving through collaboration and creativity, and identifies how teachers understand and utilize creativity specific to mathematics to foster creativity among students.

Significance

This research attempts to address gaps in literature related to encouraging mathematical creativity, particularly as it pertains to teacher professional development. The model of professional development under study has the potential to inform teaching and learning and other professional development programs. Developing a better understanding of teachers' implementation of creativity in mathematics can contribute to our understanding of what creativity in mathematics looks like as teachers prepare lessons and students work on mathematical tasks. It may also help students to develop deeper understanding and lasting knowledge of the subject in conjunction with mastering computational skills.

This study is situated within the context of a larger project funded by the National Science Foundation (NSF) Improving Undergraduate STEM Education grant (project #1611876).

The Creativity in Mathematics and Science project (CMS) focused on mathematical creativity at the undergraduate level by revising existing courses based on research of best practices for fostering mathematical creativity among undergraduate students. Through this project, researchers designed, piloted, and refined learning materials to engage students in thinking creatively about mathematics, evaluated effective instructional strategies and students' perceptions of STEM learning, and partnered with institutions to foster lasting change in undergraduate STEM education. The study presented here, implementing a professional development program focused on creativity in K-12 education, was motivated by this NSF-funded research at the undergraduate level and by the call for research to address student and teacher implementation of mathematical creativity.

The purpose of this research was to understand in-service teachers' experiences with and ideas about mathematical creativity after participating in a targeted professional development program. This professional development program encouraged participants to think creatively and foster students' creativity. To better understand teachers' ideas about mathematics and mathematical creativity and how a professional development program focused on mathematical creativity influences participants' implementation of these ideas, I was interested in answering the following research questions:

1. How do in-service mathematics teachers' conceptions of mathematics and mathematical creativity evolve as they participate in a professional development program focused on mathematical creativity?
2. What are in-service mathematics teachers' experiences infusing creativity into their mathematics lesson(s) as they participate in a professional development program?

a. How do the ideas of fostering mathematical creativity explored in a professional development program transfer to in-service teachers' practice?

b. How do teachers describe the development and implementation of lessons that require creativity and encourage creativity among students?

In the following chapters, I first review the literature on creativity, teachers' and students' beliefs about mathematics, and best practices for fostering student creativity. Second, I describe the study design and data analysis process. This study utilized an embedded, single-case study design to approach the main unit of analysis involving all 12 participants in the professional development program, and further examined the different ways in which three exemplar teachers approached creativity in their teaching and learning. Lastly, I discuss the significance of the study in the context of the literature, suggest its contributions to research on professional development, and outline limitations and avenues for future research.

CHAPTER 2: REVIEW OF THE LITERATURE

In this literature review I explore research on creativity, mathematical creativity, and effective professional development programs. I first present theories of creativity in order to define the term and identify traits of creativity and their connections to mathematics (DeHaan, 2009; Yaftian, 2015). Second, I explore students' and teachers' views about and understanding of mathematics and creativity (Bolden et al., 2010; Levenson, 2013a; Shriki, 2010). Lastly, I present research on effective professional development programs (Borko, 2004; Guskey, 2000; Loucks-Horsley et al., 2009) and investigate strategies and classroom environments that promote creative thinking (Ball, 1992; Hong & Kang, 2010). Finally, I identify gaps in the literature and outline areas for future research.

Teaching Creatively Versus Teaching for Creativity

The National Advisory Committee on Creative and Cultural Education (NACCCE) (1999) distinguish between teaching creatively and teaching for creativity. Teaching creatively is defined as, "teachers using imaginative approaches to make learning more interesting, exciting, and effective" (p. 102). Teaching for creativity differs from teaching creatively because it attends to encouraging, identifying, and fostering creativity among students. Similar to NACCCE's (1999) distinction between teaching for creativity and teaching creatively, Bolden et al. (2010) categorized teachers' conceptions of creativity as creative teaching and creative learning, noting that creative teaching involves the resources used to teach mathematics, whereas creative learning includes students' participation in activities and investigations to develop mathematical flexibility. Although different, teaching creatively and teaching for creativity are both pertinent to mathematics education. To highlight this distinction, I first draw from the general creativity literature to define the term and explore this idea within the context of mathematics.

Defining Creativity

Creativity is defined differently within the literature. Many researchers have offered behaviors and characteristics that creative people tend to display. However, creativity itself is difficult to define because of its complex and multidimensional nature (Cropley, 2000; Runco, 2007; Said-Metwaly et al., 2017). Instead of one decided-upon definition for creativity, researchers have focused on different dimensions of creativity. While some researchers identify characteristics of a creative person (Neumann, 2007; Sternberg, 2006; Sternberg & Lubart, 1995), others highlight the contrast between convergent and divergent thinking (Chamberlin & Moon, 2005; Guilford, 1968), and many explore the connection between intelligence (or giftedness) and creativity.

Still others explore how to categorize creativity and whether the product (what is created), person (the traits and characteristics of someone deemed creative), process (the thinking and process behind creativity), or place (the environment in which creativity occurs) should be the creative component under study (Rhodes, 1961; Runco, 2004). Although there are many areas of research on creativity, almost all definitions of creativity have some component of one's ability to connect ideas, produce something novel and useful (Kaufman & Sternberg, 2010), and be innovative. For example, Koestler (1967) suggested that creativity is a *bisociative process* where a person is able to connect two previously unrelated ideas in order to generate a novel insight, which is an important thinking strategy for both teachers and students.

Product, Person, Process, and Place

As noted in the previous section, one question in defining creativity is whether the product, person, process, or place (4P's) is creative (Rhodes, 1961). Sternberg and Lubart (2000) define creativity as the "ability to produce work that is both novel (i.e., original, unexpected) and appropriate (i.e., useful, adaptive concerning task constraints)" (p. 3). Plucker et al. (2004) see creativity as "the interaction among aptitude, process, and environment by which an individual or group produces a perceptible product that is both novel and useful as defined within a social context" (p. 90). While Sternberg and Lubart (2000) focus on the work that is produced by a person, Plucker et al. (2004) focus on the process and environment in which creativity takes place. Although the product is ultimately what is considered creative in these definitions, the attributes of the person producing the product and the environment in which they work are also integral to these creative products. Each of the four categories offers a lens with which to analyze creativity but a strong overlap in categories also exists.

Product

Most often, a creative product is associated with something novel, innovative, and useful (Sternberg & Lubart, 2000). A product may be in the form of publications, works of art, inventions, or ideas. For a teacher, the product may be a lesson plan or project guidelines whereas for a student the product may be multiple responses to a question or the development of knowledge previously unknown to that community of students. Of the four categories, *product* is viewed as the most objective because what is produced can often be viewed and judged by others. However, looking only at the product removes any sense of the person behind it or the process it took to create the work. Another issue with viewing the product alone is that a product, although arguably the most objective category of the four, may be judged by one person to be creative but another may see it as less so, or not creative at all. Although it is possible to see other categories as also being subjective, the consideration and judgement of a product is more prominent because something is actually produced and critiqued.

Person

A creative product is developed by a person or group of people. Creativity might be considered as part of a teacher's skill set as an educator and a student's repertoire as a learner. Traits and characteristics of a creative person pertain both to a teacher being creative and fostering students' creativity. Research defines characteristics of a creative person as someone who is persistent, innovative, and able to develop something novel. Sternberg and Williams (1998) identified traits of creativity including the ability to connect ideas, see similarities, have flexibility and aesthetic taste, be inquisitive, and question norms. Creativity also calls on people to dismiss "conventional thinking" by seeking new perspectives and approaches, make decisions about which approaches are worthy of pursuing, and convince others of the values of their ideas (Sternberg, 2006).

Amabile (1983) extended research on the traits of creativity to include a person's motivation and willingness to try new things, and the social context that impacts these motivations. Although various traits and characteristics of a creative person exist, some traits are fixed (e.g. persistence) whereas others are malleable (e.g. connecting ideas, questioning norms). This is an important distinction for educators who should consider the characteristics students already possess and those which might be encouraged and developed through the proper structuring of a creative learning environment.

Persistence, a main personality trait of people identified as creative, is most notable in Thomas Edison's recounting of his development of the light bulb. The great inventor commented, "I have not failed 10,000 times…I have succeeded in proving that those 10,000 ways will not work. When I have eliminated the ways that will not work, I will find the way that will work" (Furr, 2011). The idea of persistence is present in many fields. For example, Dr.

Stanley Prusiner spent his career, as many researchers do, working on a single question. Through his persistence in looking for a cause to understand degenerative, deadly neurological diseases, Dr. Prusiner discovered prions, “abnormal, pathogenic agents that are transmissible and are able to induce abnormal folding of specific normal cellular proteins…that are found most abundantly in the brain” (CDC, n.d.). He was recognized in 1997 as a Nobel Prize recipient for his work because he was steadfast in his exploration and ultimately uncovered a completely new area of medical research. He notes that the discovery of prions and accolades for it were luck. However, he possessed several traits of creativity including persistence, questioning norms, seeking new approaches, and convincing others of his ideas, which are more likely the reason for his success and recognition.

Similarly, traits of creativity are also exemplified in other seemingly unrelated fields. For example, the Brazilian football phenom, Edson Arantes do Nascimento, better known as Pelé, demonstrated skill and creativity on the football pitch for over 20 years. His innovation and flexibility as he developed and executed novel plays in each game demonstrated many of the characteristics identified in research as creative. Furthermore, being persuasive, questioning and challenging norms, having flexibility, and connecting ideas were all apparent in Pele’s successes and alteration of the game worldwide. His creative attributes and notable persistence allowed him to innovate amazing outcomes on the field. These examples are provided to give a tangible understanding of several personal characteristics explored in creativity research.

Although these characteristics sufficiently describe a creative outcome, they still describe a creative person or event rather than define the term. While a person might be considered creative, the process in which one undertakes to develop a product can also be recognized as creative. These qualities are attributed to the person who is creating, as opposed to the previous category, product, which focuses on the outcome.

Process

In developing a creative product, a person undertakes a series of mental mechanisms and components of creative thought (Mumford et al., 1997), and stages of processing (Mace & Ward, 2002; Simonton, 1984) which are considered part of the creative process. Process can be defined as what occurs when a person is engaged in creative thinking. More specifically theories that fall under the process category are concerned with stages of the mind's processing including preparation, incubation, illumination, and verification (Wallas, 1926).

Although theories to evaluate the process exist, understanding the creative process is more difficult to carry out than research in other categories because of the latent nature of mental processes and the fact that processes are influenced by several things, namely the constraints and environment in which one creates. However, the type of thinking one demonstrates when being creative has often been discussed and categorized as either convergent or divergent (Guilford, 1950). While convergent thinking is represented in tasks that result in one, correct, conventional answer, divergent thinking is needed for tasks that have multiple responses or can be solved in a variety of ways. Divergent thinking was specifically defined by Guilford (1950) as generating novel solutions, meaning the exploration of several solutions to a problem. The creative process happens within the context of an environment which might also be described as creative.

Place

Researchers describe a creative place as the environment or conditions under which a person participates in creative activity. Place focuses on interactions between a person and his or her environment. According to literature, some environments are more conducive to fostering creativity than others. For example, Amabile (1990) noted that environments that allow people the opportunity to explore, and those that support and value originality are preferable over those that favor prescriptive, predetermined outcomes for creativity to flourish.

Another thing to consider is the social aspect of creativity. The social aspect of creativity explores how people interact in the creative process and negotiate with one another to formulate ideas. While some researchers emphasize the individual as creative (Lubart & Sternberg, 1998; Sternberg, 2018), others explore the importance of the social realm of creativity (Amabile, 1983; Plucker et al., 2004). Plucker et al. (2004) note that the social context in which creativity occurs is important for the identification of something as creative because “a context is requisite for determining whether (and how) a person, action, or product will be defined or judged as creative” (p. 92). The social context in which creativity occurs is an important focus for research.

Using cognitive, personality, and social frameworks, Amabile (1983) argued that “task motivation can be seen…as the most important determinant of the difference between what a person can do and what he or she will do” (p. 366). The potential of what a person can do is determined by one’s domain-relevant (subject-specific) and creativity-relevant (attributes displayed by creative people) skills whereas what one will do is determined by these same to two categories along with, “an intrinsically motivated state.” Amabile (1983) argued that personality traits alone do not fully explain one’s level of creativity. Therefore, the addition of focusing also on the influence of internal and external factors (i.e. cognitive skills, work habits, social-environmental variables, and personality traits) would improve creativity research (p. 373).

These four categories are often addressed separately. However, they are intertwined and inseparable. For example, a person may be creative in their process of creating a product, making these categories interlaced. It could be argued that these four categories are actually more connected because the person and process of creativity which happen under given conditions of the place, are integral to the product that is generated. In thinking about categorizing research on creativity, it is not only important to understand the 4P's but also the level of creativity on a spectrum and for whom the work impacts.

Creativity on a Spectrum of Little-c to Big-C

Research on creativity can also be thought of as a scale of little-c to Big-C creativity (Beghetto & Kaufman, 2007b; Csikszentmihalyi, 1996, 1998). Initially, Maslow (1967) distinguished between two types of creativity, primary (personal creativity one uses for his or her own purposes) and secondary (scholarly creativity that leads to an achievement that impacts a field). Csikszentmihalyi (1996, 1998) extended this distinction, defining creativity on a spectrum of little-c (everyday) to Big-C (eminent, discipline altering). Little-c refers to the creativity depicted in everyday life (Richards, 2007) but activities which do not make a major impact on a field. For example, little-c creativity is visible in a person's decision to explore different routes to get to work in an effort to decrease commute time or developing a recipe to enjoy. Big-C refers to domain-altering creative expression, exemplified in Freud's contributions to psychology or Beethoven's composition of music or Monet's contributions to art.

To further specify creativity in people's every day and professional lives, Beghetto and Kaufman (2007b) argued for two additional categories to be added to Csikszentmihalyi's existing model further distinguishing between subjective and objective forms of little-c creativity, and to clarify the distinction between little-c and Big-C creativity. The little-c and Big-C classification was expanded to include: *mini-c* (personal) and *pro-c* (professional). Mini-c creativity is exemplified in purposefully being creative as in writing poetry or solving a mathematics problem in an innovative way, and having those around you recognize your work. This is different from little-c creativity which is personal and not necessarily apparent to those around you. Having your work recognized is what distinguishes mini-c from little-c creativity. People who exemplify pro-c creativity make a living by their creative contributions, but their work does not necessarily alter the field in any way. This differs from Big-C creativity which changes the direction of a field. In distinguishing among little-c and Big-C creativity with these two additional categories, Beghetto and Kaufman (2007b) further extended Csikszentmihalyi's (1996, 1998) original model which they argued inadequately categorized people's creativity and missed important nuances of the discipline.

Theories of Creativity

To build on the connections among product, person, process, and place, and the spectrum of little-c to Big-C creativity, I delve into theories of creativity namely, the three-facet model of creativity, the investment theory of creativity, and Sternberg's (2018) most recent expansion upon these theories, the triangular theory of creativity. In an attempt to define creativity and further understand how creativity is understood in the literature, I explore how researchers have theorized about creativity, and bring us closer to a working definition of the term. In doing so, I draw from research on creativity to develop an operational definition for the purposes of this study.

Three-Facet Model of Creativity

In the three-facet model of creativity, Sternberg (1988) explored intelligence (the ability to create novel and useful ideas), cognitive style (how one prefers to use his or her abilities), and personality or motivation (willingness to take risks, possession of intrinsic motivation) in relation to creativity. This model of creativity grew out of the triarchic model of intelligence (Sternberg, 1985) and attempts to understand intelligence in terms of, "its relation to the internal world of the individual, its relation to experience, and its relation to the external world of the individual" (p. 132). Although the three-facet model of creativity originates from the triarchic theory of intelligence, intelligence is just one component of the model. Other important aspects of this model are personality attributes and the ways in which intelligence is enacted, both which impact a person's level of creativity.

In order to determine how these three facets were correlated, Sternberg (1988) sent a questionnaire to art, business, philosophy, and physics professors asking them to rate the prominence of each facet in their field on a scale of one to nine (uncharacteristic to extremely characteristic). Findings suggest that across disciplines, intelligence and wisdom were perceived as most closely related, intelligence and creativity were next most closely related, and wisdom and creativity were least related. Sternberg's three-facet model of creativity asserts that, "Creativity involves not just a set of skills, but also a set of stylistic, personality, attitudinal, and motivational constructs as well" (Sternberg, 2018, p. 51), further highlighting the connection between a creative person and the creative process. This model is an attempt to understand the internal attributes associated with creativity. However, Sternberg (1988) noted that the model is incomplete without also attending to the environment and personal variables that impact a person's creativity and therefore a theory was not developed from these ideas.

Investment Theory of Creativity

Building upon the three-facet model of creativity, Sternberg and Lubart (1996) attempted to define creativity by comparing it to a financial investment and argued that creativity is a choice to "defy the crowd." According to this theory, an investor will "buy [ideas] low and sell high" (p. 229), meaning that the creator is tasked with convincing people in their intellectual community of their creative innovations. Although success might come more easily in some cases, there is still a need to sell one's ideas to the community. This requires persistence despite the discouragement one may feel in going against the norm. To buy low, one, "generate[s] and promote[s] ideas that are novel and even strange and out of fashion" (p. 2). These ideas are not readily accepted but with persistence one may eventually, "sell high" meaning the idea becomes recognized and appreciated among the community (Sternberg & Lubart, 1995, 1996).

Several people (e.g. Johann Sebastian Bach, Vincent Van Gogh) now appreciated for their domain-altering, creative work, were not esteemed during their lifetime. Instead, their ideas were not sold high until well after their deaths. For example, Van Gogh, now considered a prominent Impressionist painter who is revered as having revolutionized art, was shunned and ostracized throughout his life. Although his creative work has had a lasting and immeasurable impact on the direction of Impressionist art, he was not recognized for these accomplishments until after his death. Although he was "buying low" during his life by studying and producing work in unexplored avenues of art, he was unable to "sell high" during his lifetime and instead was ridiculed and shunned. If he were able to convince others of his ideas and "sell high," he would probably have experienced more success while alive. This is an example of the difficulty one can face by introducing a creative idea to "sell high" to an intellectual community.

Six attributes are required in the investment theory of creativity including intelligence, knowledge, thinking style, attitude, motivation, and environment (see Table 1). Sternberg and Lubart (1996) argued that each of the six elements must be present in order for creativity to occur. However, they also noted that creativity is not something a person is born with and instead something that can be developed. In order to buy low and sell high, one must identify and be willing to take risks on unknown or unpopular ideas. Sternberg and Lubart argued that these attributes allow one to take these risks and succeed in creative endeavors. The investment theory of creativity, where ideas are bought low and sold high, is a way of thinking and approaching problems through these six categories.

Table 1

Investment Theory Attributes of Creativity

Attribute	Description of Attribute
Intelligence	Redefining a problem: generating ideas
	Evaluating the quality of ideas
	Promoting and refining ideas
Knowledge	Domain specific
	Informal
Thinking Style	Legislative
	Flexible
Personality	Perseverance
	Risk taking
Motivation	Intrinsic
	Extrinsic
Environment	Encourages, supports, and rewards creative thinking

Triangular Theory of Creativity

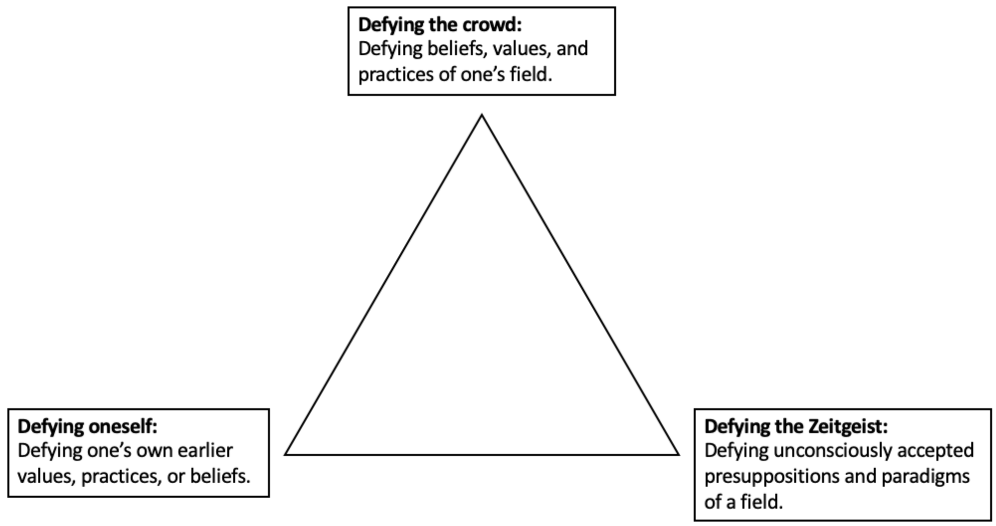
Recently Sternberg (2018) introduced an expansion of the three-facet model (Sternberg, 1988), investment theory of creativity (Sternberg & Lubart, 1995, 1996), and propulsion model (Sternberg, 1999) to include a triarchic model of defiance (Sternberg, 2018). Defiance in this context means, “1) to confront with assured power of resistance, 2) to resist attempts at, 3) to challenge to do something considered impossible” (*Defy*, n.d.). In the triangular theory of creativity, Sternberg (2018) defines defiance as, “resistance to something and to effective[ly] challenge that something” (p. 51). In investment theory, that “something” is the crowd. The triangular theory expands on the idea of defying the crowd to also include defiance of oneself and the Zeitgeist.

While the three-facet model focuses on intelligence, personality, and motivation, the investment theory of creativity highlights defying the crowd. The propulsion model of creativity describes creative behavior specific to domain-altering activities, moving a field forward or changing it in a significant way. The propulsion model incorporates the features of the three-facet model including intelligence, cognitive style, and personality/motivation needed for field altering creativity, and extends investment theory by focusing on the ways in which creative contributions affect a field or discipline. This is also aligned with Big-C creativity in that it describes how domains are altered through creative activity.

The triangular theory of creativity expands upon the investment theory of creativity which includes one category of defiance, the crowd, to three types of defiance adding defying oneself and the Zeitgeist (Sternberg, 2018). This theory also includes intelligence (creating novel and useful ideas), cognitive style (using one's abilities), and personality or motivation (willingness to take risks) from the three-facet model, and the intellectual abilities outlined in the investment theory of creativity. However, identifying these facets is not enough. Instead, defiance of three types is a necessary component of this theory in the process of defending ideas to oneself, the crowd, and existing, often subconsciously accepted beliefs of an intellectual community (see ***Figure 1***). These three types of defiance are aligned with the spectrum of creativity (Sternberg, 2018) described earlier, which highlights the different ways one can demonstrate creativity, from personal to domain altering contributions.

Figure 1

Types of Defiance in Sternberg's (2018) Triangular Theory of Creativity

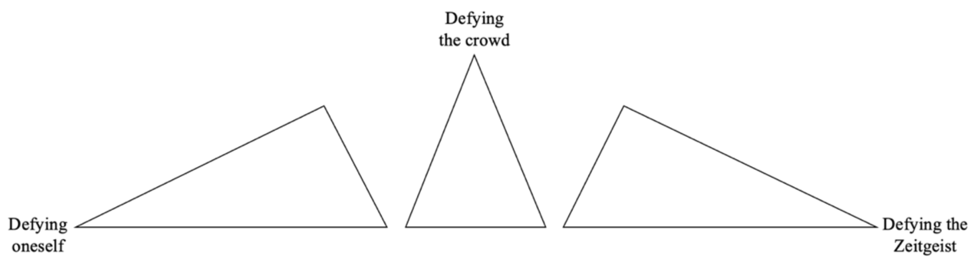


While

Figure 1 depicts an equal emphasis on each of the vertices, this triangle can be manipulated and morphed into differing types of triangles beyond equilateral to highlight the focus on one of the vertices. For example, in order to generate an idea, you may first need to more heavily focus on defying oneself, moving beyond existing personal ideas and assumptions. In other instances, it may be more appropriate to focus on convincing the crowd and selling an idea to the community or the Zeitgeist. The vertex that represents the stronger focus of defiance becomes more acute to visually represent the attention to that defiance (Figure 2).

Figure 2

Focus on each of the three vertices



Defying the Crowd. Defying the crowd means that one's ideas are unconventional to one or several people who are unfamiliar with your idea and "the crowd" may take issue with or need to be convinced of your innovations. Sternberg and Lubart (1995) assert that:

> People with a creative frame of mind do not join the crowd in its beliefs simply because there is safety in numbers, or because if everyone believes something it must be true. They think for themselves and resist tendencies toward groupthink and blind conformity. (p. 83)

Creative people understand that in the long term, defying the crowd may lead to reshaping the field. However, Sternberg (2018) recognized that defying the crowd is difficult and, "ideas that challenge the existing order often are very hard to get accepted" (p. 53). For example, the 1997 Nobel Prize winner, Dr. Stanley Prusiner, mentioned earlier, felt pressure and disagreement from the crowd as he published findings that went against accepted beliefs of the field. Throughout his career, many of his contemporaries were unhappy with his work and as he described, he felt "the long arm of the scientific community pressing down on me" which his friend more boldly described as Dr. Prusiner being, "attacked viciously…ridiculed, vilified, denounced" (Benderly, June, 2014). Dr. Prusiner noted, "While it is quite reasonable for scientists to be skeptical of new ideas that do not fit within the accepted realm of scientific knowledge, the best science often emerges from situations where results carefully obtained do not fit within the accepted paradigms" (Prusiner, 1997), highlighting that pressure and disagreement tend to be part of scientific advancement.

While the investment theory of creativity emphasizes defying the crowd, the triangular theory identifies that, "optimal levels of creativity result not just from defying the crowd…but also, from defying oneself and one's own beliefs as well as defying the usually unrecognized and perhaps unconscious field-based presuppositions" (p. 53). Sternberg (2018) recognized that defying the crowd alone was not enough to be deemed creative. Instead, one also must defy oneself and the Zeitgeist in an effort to generate new, creative ideas.

Defying Oneself. Although defying the crowd is challenging, it can be even more difficult to identify one's own beliefs and defy oneself. Often people get stuck and are unable to move beyond one idea or way of thinking resulting in "one idea people" (Sternberg, 2018, p. 54). Sternberg (2018) argues that people become entrenched, seeing others as barriers to creativity instead of themselves. In reality, one can be one's own worst enemy when attempting to formulate and defend new ideas. It is necessary to defy oneself in order to be creative because in doing so, one moves beyond held assumptions and opens a space for different ways of thinking. For example, Sternberg admitted the irony of his attachment to the number three. Through exploring intelligence and models of creativity, he drew upon three facets of the overarching constructs. Growing out of these ideas, he developed the triangular theory of creativity, and in his personal life he and his wife are parents of triplets, most all of his creations revolve around or include the number three in some way. Although still part of his most recent theory and difficult to break out of the comfort in the number, he has recognized and attempted to move away from thinking only in triples.

Defying the Zeitgeist. Perhaps the most abstract concept of the three included in the triangular theory of creativity is defying the Zeitgeist. This means identifying and defying norms of the existing culture and society. As part of a society, we hold subconscious assumptions and beliefs about a field. One who defies the Zeitgeist goes against the ways in which a culture has established norms resulting in vast changes to the development and sometimes direction of a discipline. For example, in the 1940's, schizophrenia, manic depression, and bipolar disorder were once seen as hysteria and treated with drastic measures (e.g. electric shock, lobotomy). However, a cultural shift beginning in the mid-1950's has begun to deescalate the hysteria and stigma around mental illness, recognizing them as diseases and treating them in more humane ways. Because of the complexities of these categories of defiance, further research is needed to better understand these types of defiance and their relationships with one another.

Operational Definition of Creativity

Prior to exploring mathematical creativity, I first compared and contrasted definitions of creativity noted above to describe my definition. I used the traits and characteristics of a creative person, definitions of creativity, and the theories of creativity, to formulate my own working definition of creativity. While traits of creativity including being innovative, identifying similarities and differences, and exploring multiple solutions to solving problems are part of what it means to be creative, creativity is also about producing something novel, defending these discoveries, inventions, and ideas to others, and redirecting the community forward in some way. My working definition of creativity is a person or group's ability to produce novel, compelling ideas and defend these ideas to themselves, the community, and the existing, subconsciously-accepted norms of that community.

The aforementioned theories of creativity are relevant to mathematics because they suggest a different way of thinking about and approaching problems. One must defy themselves and use their own knowledge, instead of relying on the crowd or previously held beliefs of a group. In mathematics, one generates new ideas through risk taking and drawing on motivation to persevere in solving difficult problems. This is pertinent to school mathematics where students should be encouraged to rely on their own knowledge and abilities to use their creativity to investigate and find solutions instead of relying solely on the teacher for knowledge. In the following sections I discuss creativity specific to mathematics and explore research on student and teachers' conceptions of creativity in mathematics.

Creativity and Mathematics

Much like creativity in general, one specific, clear definition of mathematical creativity does not exist (Haylock, 1987; Mann, 2005; Sriraman, 2005). However, for several years, researchers have explored the idea of mathematical creativity (Poincaré, 1948). Poincaré (1948), a prominent mathematician of the twentieth century, argued that mathematics is creative and in order to be considered creative, mathematics has to be novel and useful, combining familiar ideas in new and unfamiliar ways. According to Neumann (2007), a good scientist possesses qualities such as, "rigorous intellect, the ability to get the job done and the ability to have creative ideas" (p. 203). Boaler (2016) identified mathematical creativity as a flexible mental construct and highlighted the importance of including creativity in the teaching and learning of mathematics. Research suggests that one way in which creativity is exemplified in any discipline is when alternate solutions to a problem are identified. However, creativity in mathematics goes beyond that to involve the ability to demonstrate non-algorithmic decision making (Ervynck, 1991), and generate novel solutions (Chamberlin & Moon, 2005; DeHaan, 2009; Shriki, 2010).

Although researchers have identified the importance of creativity in mathematics, there is also still debate about when creativity in education is appropriate (Kaufman & Beghetto, 2013). For example, a certain level of knowledge is needed for any demonstration of creativity to exist. While there is a need for students' basic procedural knowledge, there is also a place for creativity alongside these procedures in mathematics.

Sheffield (2008) described mathematically creative students as having similar characteristics to those identified in literature on general creativity (e.g. originality, innovation, flexibility), adding mathematical aspects of creativity including: (a) striving for mathematical elegance and clarity, (b) explaining reasoning, (c) demonstrating curiosity about mathematical connections, and (d) continuing to explore after a problem has been solved. Boaler (2016) argued that, "The new evidence from brain research tells us that everyone, with the right teaching and messages, can be successful in math" (p. 4) and highlights the importance of introducing creativity in mathematics to help develop students' mathematical mindsets.

To further specify creativity in mathematics, I recall Guilford's (1950) definition of divergent thinking as generating novel solutions. Building on the idea of divergent thinking as the ability to identify multiple solutions or think about a task from multiple angles, Bolden et al. (2010) noted that one can be creative by separating from, "established mind-sets" (p. 144) such as generating an idea that does not fit existing beliefs of a discipline. Established or static mind-sets are defined as a firmly established way of thinking. These established mind-sets represent norms of a field or a classroom, and it is creative to go beyond what others accept to be true, as is evident in defying the crowd in the investment theory of creativity (Sternberg & Lubart, 1996) and the triangular theory of creativity (Sternberg, 2018). These notions of creativity are particularly important in mathematics where there is often more than one viable approach to solving a problem. This differs greatly from a static mindset that requires and accepts only one solution or way to solve a problem. Imbedded within these ideas of the structure of divergent thinking and separating from established mindsets, are varying levels of mathematical creativity.

Levels of Mathematical Creativity

Similar to the spectrum of little-c to Big-C creativity, varying levels of mathematical creativity range from students to professional mathematicians. Researchers agree that both novice learners and professional mathematicians can be creative in their exploration of mathematics. For example, Levenson (2013b) asserts that a new idea or discovery could be one that is new to the person (little-c creativity) or one that is a novel discovery to the mathematics community (Big-C creativity), both of which are equally creative. It is creative to come to understand something one previously did not know, whether at the elementary school level or as a professional mathematician. The meaning of a new idea may differ based on a person's level of experience with mathematics, but one level is no less creative than another. Similarly, Hadamard (1945) viewed students' problem solving in geometry or algebra and the work of an inventor as having similar qualities asserting, "there is only a difference of degree, a difference of level, and both works being of similar nature" (p. 104).

Sriraman (2005) also thought about levels of mathematical creativity, questioning whether students' mathematical ideas, novel to them, are a creative act of doing mathematics. Although one can be creative at various levels of mathematics, the type of mathematical creativity differs. Sriraman developed two definitions of mathematical creativity. He defined mathematical creativity in grades K-12 as developing insightful solutions to a problem or formulating new questions for other students to consider. At the professional level mathematical creativity is defined as focusing on the production of original work and the development of questions for other mathematicians. Although both definitions attend to questioning and developing avenues for further questioning, the level of expertise and the extent of these ideas differ.

In thinking about levels of mathematical creativity, school-aged children have varying levels of mathematical creativity and abilities namely, students who struggle, are deemed average, and are gifted. A large amount of research exists on developing creativity among gifted students (Aiken, 1973; Sriraman, 2005; Zenasni et al., 2016). However, very few studies explore how to nurture creativity among the general population of students. The literature that does exist on students' mathematical creativity is limited to gifted students and curriculum geared toward this population. However, it might be beneficial instead to seek and help foster creativity among all students, identifying ways to draw general education students, particularly those who struggle, into mathematics using creativity by helping them explore their own potential within the context of the subject (Mann, 2020; Runco, 2014).

Focusing on creative ability only at top levels of a discipline excludes all other mathematics learners. Silver (1997) argued that although creativity is often seen as something attainable only for gifted or exceptional students, "it can be productive for mathematics educators to view creativity instead as an orientation or disposition toward mathematical activity that can be fostered broadly in the general school population (p. 75). Therefore, it is important for educators, policy makers, teachers and students to focus on the little-c and mini-c creativity exemplified by people in everyday life, to help students develop as mathematics learners and creators. It is not appropriate to rule out students' creativity in mathematics simply because they lack domain specific knowledge to make Big-C discoveries or are presumed to be gifted in mathematics. Lacking the professional and robust knowledge that mathematicians possess does not rule out the creative endeavors of teachers and students. Instead of excluding students from exemplifying little-c creativity by drawing on their abilities and knowledge, and discovering mathematics for themselves, teachers should shift their focus to foster this creative ability in mathematics.

Assessment of Mathematical Creativity

Another issue in research on mathematical creativity is the way in which creativity is assessed. Although general assessments are often used in studies that attempt to measure students' levels of creativity (TTCT, GAU), specific assessments of mathematical creativity also exist. Researchers have developed and tested two major assessments of mathematical creativity, namely the Creativity Abilities in Mathematics Test (CAMT), considered a seminal instrument in the field of mathematical creativity (Balka, 1974), and the Mathematical Creativity Scale (MCS) (Akgül & Kahveci, 2016). Although these assessments of mathematical creativity exist, Akgül and Kahveci (2016) argued that not enough valid, reliable scales of creativity in science and mathematics exist, nor have these assessments of mathematical creativity been widely used.

The development of the CAMT (Balka, 1974) included input from several content experts (i.e. mathematicians, mathematics professors, classroom teachers). This instrument consists of five items, focused on aspects of creativity including fluency, flexibility, and originality. In each of the five items, participants are asked to create problems when given mathematical situations. For example, one item describes a situation containing information including numbers, and participants are tasked with writing as many mathematical problems as they can (see Balka, 1974 for detailed items and coding schemes). This instrument has been used to assess school children's mathematical creativity. However, it is difficult and time consuming to score these assessments and therefore somewhat impractical for classroom use.

In seeking a simpler way to identify creative potential, particularly so assessments could be administered more frequently in school settings, Mann (2005) investigated whether a measure, or combination of measures, could be used to determine student performance on CAMT and whether a measure of students' mathematics achievement predicted performance on CAMT. The author also investigated whether gender, mathematical ability, student perception of creativity and attitude toward mathematics, and teacher perception of students' mathematical talent and creative ability, could predict student performance on the CAMT. Findings suggest that mathematical achievement, student attitude toward mathematics, and student self-perception of creative ability were positively correlated with CAMT scores. For every one-point increase on the mathematics assessment, CAMT scores increased 0.17 points. For every one-point increase on the student attitude scale, CAMT scores increased, on average, 0.15 points. Similarly, for every one-point increase on student self-perception of creative ability, CAMT scores increased, on average, 0.15 points. Using CAMT, it was determined that students' mathematics achievement, attitude toward mathematics, perceptions of their own creativity were significantly correlated with the mathematical creativity score on CAMT.

The other known assessment of mathematical creativity, MCS, was developed by Akgül and Kahveci (2016). The MCS, a 5-item assessment, for middle school students, grades 5-8 focusing on students' divergent thinking. To develop the MCS for middle school students (grades 5-8), researchers drew on Haylock's (1987) framework for assessing children's mathematical creativity and Balka's (1974) CAMT. Participants included 296 Turkish students and their answers to the items were used to determine the correlation of divergent thinking variables (fluency, flexibility, and originality). Similar to the CAMT, items on MCS focus on fluency, flexibility, and originality. However, types of thinking (e.g. analytical, logical, spatial) were also included in the development of the MCS. Fluency, flexibility, and originality were found to be highly correlated (.78-.96) with one another. Findings suggest that because flexibility and originality were each highly correlated with creativity, these constructs could be used alone to determine a students' level of mathematical creativity. Akgül and Kahveci (2016) noted that other factors beyond divergent thinking may also impact students' creative ability in mathematics including their interest, motivation and attitude toward mathematics, and personality traits. Because of this, further research is needed to explore the MCS in relation to these hidden constructs.

Beliefs and Conceptions About Mathematics

The aforementioned issues in research on creativity and mathematical creativity, including difficulty defining general and domain specific creativity and issues with the assessment of mathematical creativity, are important areas of research to further explore. However, in thinking about creativity in mathematics, it is also important to attend to teachers and students' beliefs and perceptions about the subject and how creativity might have an impact on these beliefs. Research suggests that students develop negative perceptions of the mathematics as they progress through formal schooling (LaRocque, 2008; Ma & Kishor, 1997; Wilkens & Ma, 2003) and although teachers identify creativity as an important part of mathematics, they have difficulty implementing it practice (Leikin et al., 2013; Shriki, 2010). Because of this, it is necessary to understand teachers' and students' views about mathematics and mathematical creativity, and how these ideas might be improved by the exploration of creativity.

K-12 Students

Research suggests that as students progress through formal schooling, their positive attitudes toward mathematics decreases (Wilkens & Ma, 2003). Several studies suggest that students in lower elementary grades enter school with positive attitudes toward mathematics but that these attitudes become less positive as they progress through school, often becoming negative during high school years (LaRocque, 2008; Ma & Kishor, 1997; Wilkens & Ma, 2003). Middle and high school students often describe mathematics as a series of procedures, steps to memorize, and a subject where success is measured by the speed in which one can complete a problem (Brown et al., 1988; DÍaz-Obando et al., 2003; Frank, 1988; Garofalo, 1989). However, teachers play an important role in task choice and implementation (e.g. whether a challenging task is reduced to a procedural one, a good task is implemented effectively, mathematics is presented as static and boring or challenging and meaningful) which impacts students' beliefs about the subject (Hershkovitz et al., 2009; Wilkens & Ma, 2003). In a meta-study on students' beliefs about mathematics, Muis (2004) identified that students across grade levels saw mathematics as a series of procedures and did not see the usefulness of mathematics outside of school.

For example, Kloosterman and Cougan (1994) explored K-6 students' views on school and mathematics. Three to four students from each grade level were chosen as participants in the study. Researchers administered assessments consisting of five belief categories and seven mathematics problems, and conducted interviews to explore students' beliefs about the subject. The majority of participants reported that they liked both school and mathematics. Using standardized tests and the process problems from the mathematics test, researchers categorized students as high, medium, or low mathematics achievers. Using responses to interview questions and the beliefs questions from the assessment, students were ranked as high, medium or low in five affective categories: (a) liking school, (b) liking mathematics, (c) parental support for school in general, (d) parental support for mathematics, and (e) self-confidence in mathematics. Findings suggest that consistency between liking school and liking mathematics exists. Higher levels of self-confidence in mathematics was associated with students' success in the subject. The majority of students in this study believed anyone could learn mathematics and effort was needed in order to be successful in the subject. Although these findings are encouraging for elementary school mathematics, they do not hold true in research with older student populations and little is understood about the nuances among K-6 grade levels.

For example, Wilkens and Ma (2003) conducted a quantitative study following 3,116 seventh-grade students for six years to understand the change in students' attitudes toward and beliefs about mathematics. Students' attitudes were measured based on their responses to questions on the social importance of mathematics and the nature of the subject. Questions also attended to students' enjoyment of mathematics, feelings of success and understanding of the subject, and if mathematics made students feel nervous. Findings suggest that attitudes toward and beliefs about the social importance of mathematics became less positive as they progressed through school. However, teachers who expected students to do their best and demonstrated the importance of mathematics were found to positively influence students' affective domains. High school students who had taken more advanced mathematics courses (e.g. pre-calculus, calculus) had lower rates of decline in attitude toward mathematics. These findings highlight the importance of teacher influence and the need to implement classroom practices that encourage students to see the importance of mathematics and continue taking mathematics courses.

In scouring the literature on students' perceptions of creativity in mathematics at the undergraduate level, only one article was found. Munakata and Vaidya (2012) surveyed 218 college students and determined that participants attribute creativity to disciplines such as music, dance, and theater whereas science and mathematics disciplines were said to require less creativity. However, students' views on creativity in mathematics is an important area of study because research suggests that students who enter college as STEM majors are more likely to change majors. More specifically, 52% of mathematics majors leave the discipline before graduation (NCES, 2017). A significant number of students are enrolled in remedial or developmental mathematics courses and note having a dislike of the subject. Because students tend to have negative experiences in mathematics, and only one study focused on college students' beliefs about creativity in mathematics, it is important to further understand how exploring creativity in mathematics earlier in students' schooling might impact student learning and retention in STEM fields at the undergraduate level. By helping students see creativity in mathematics earlier in their schooling, we might help enhance their learning, beliefs about mathematics, and subsequently their mathematics achievement. However, this idea has not been considered much in current research.

Teachers' Beliefs

While Muis' (2004) meta-analysis focused on students' beliefs, other researchers have found preservice elementary teachers to have similar beliefs and reservations about mathematics (Shriki, 2010). Preservice elementary school teachers also tend to see mathematics as a set of rules and the manipulation of symbols. Because these beliefs shape their teaching (Muijs & Reynolds, 2015), teacher preparation programs should provide opportunities to encourage teachers to see mathematics differently and infuse creativity into the teaching and learning of the discipline. The studies included in this section focus on the type of mathematics instruction that is presented (e.g. teacher- or student-centered, focus on memorization, traditional versus reform instruction) which tends to differ by grade level. Studies that identified instructional methods in mathematics as exploratory and student centered tended to be implemented in kindergarten and first grade, while those aligned with memorization and replication were most often recognized in upper elementary and high school classrooms. Specifically exploring teachers' conceptions of creativity in mathematics classrooms may attend to these grade-level differences and highlight areas for future research.

For example, Leikin et al. (2013) were interested in teachers' perspectives on creativity and administered a survey to 1,089 mathematics teachers from various countries to assess the variation in their perspectives. Researchers developed a survey to understand high school mathematics teachers' views on creativity from a cultural perspective. The 100-item questionnaire was divided into four sections: (a) who is a creative student in mathematics, (b) who is a creative mathematics teacher, (c) how is creativity in mathematics related to culture, and (d) who is a creative person (p. 312). Findings suggest that teachers across the six countries included in the study similarly labeled students as creative if they demonstrate investigative abilities, mathematical flexibility, and problem-solving skills. Mathematics teachers were thought to be creative if they provided opportunities to develop students' creativity and were flexible in lesson planning. However, based on the information gained from this study, more attention should be paid to mathematical creativity in educational policy, instructional practices, and teacher education because although these ideas about mathematical creativity are encouraging, whether practices mirror these beliefs is unclear.

Shriki (2010) was also interested in middle and high school teachers' beliefs about mathematics and studied the impact a didactic learning environment had on 17 preservice teachers' beliefs. During a methods course, participants considered creativity and invented a new geometrical concept which they shared and discussed with their classmates. Participants reflected on their experience of developing a creative product. Instructors of this program attempted to encourage students to, "rethink their conceptions of what mathematics is, how it is learned, and how it should be taught" (p. 164) by structuring a learning environment to support preservice teachers' awareness of mathematical creativity. Findings suggest that preservice teachers often associated creativity in mathematics with genius or talent. However, participants saw creativity as domain specific and something that, with the proper guidance, could be developed. Proper development of course structure is needed because having learned in traditional environments themselves, teachers, often replicate this type of instruction in their classrooms (Shriki, 2010).

Similar to Shriki (2010), Bolden et al. (2010) focused on preservice elementary school teachers' beliefs about creativity in mathematics and identified a need to encourage and assess creativity in preservice teacher education programs. The purpose of the mathematics portion of the course was to develop preservice teachers' understanding of the subject knowledge applicable to teaching elementary-age students. Topics emphasized connections to students' mental representations of mathematics but did not include any direct instruction about mathematical creativity. Participants completed a questionnaire toward the beginning of the course, and were subsequently interviewed near the end of the course to gain an understanding of their conceptions of creativity in mathematics. The questionnaire, administered at the beginning of the course, included questions about creative mathematics lessons and teachers' conceptions about mathematical creativity. The interview, conducted at the end of the semester, gave researchers an opportunity to further explore teachers' ideas about mathematical creativity and ask for participants to provide examples of creative lessons.

Findings suggest that preservice elementary teachers' held narrow beliefs about mathematics, with teachers viewing the subject as a set discipline with little room for imagination (Bolden et al., 2010). Participants' views of creativity in mathematics were influenced by their views of mathematics and they did not describe mathematics as creative. However, during the interview, teachers' conceptions of mathematical creativity, although still limited, seemed to shift to see mathematics as having the potential for creativity. Participants noted that creativity in mathematics was important. However, they were unsure what this might look like in practice. This further highlights the importance of preparing teachers to encourage and foster creativity in mathematics on a practical level.

Similar to Bolden et al.'s (2010) distinction of teaching creatively and teaching for creativity, Lev-Zamir and Leikin (2011) studied how teachers conceive of creativity in mathematics, categorizing conceptions into teacher- and student-directed. Haylock's (1987) asserted that both mathematics and creativity must be included in the definition of mathematical creativity. Lev-Zamir and Leikin (2011) extended this definition to suggest that, "any definition of mathematical creativity in mathematics teaching must refer to mathematics teaching, learning, and creativity" (p. 19). This definition of mathematical creativity, as it pertains to teaching, is situated in literature on general creativity including fluency (use of basic knowledge), flexibility (approaching problems in various ways), originality (thinking in a unique way and developing unique products), and elaboration (the ability to generalize ideas). Researchers note that these categories are unique to teaching creatively because they are not integral to teaching in general.

Lev-Zamir and Leikin (2011) interviewed and observed 11 elementary and middle school mathematics teachers to understand their conceptions of creativity in mathematics teaching. Findings suggest that in-service elementary and middle school teachers categorize mathematical creativity as teacher-directed (teacher as creative) and student-directed (providing opportunities for students to develop creativity), and that teachers believed creative mathematics teaching involved going beyond the textbook lesson and being flexible in teaching strategies. Teacher-directed creativity was described by teachers as demonstrating flexibility to change mathematical tasks and their pedagogy. One example of teacher-directed creativity is when a participant transformed an addition-related task to a multiplication-related one to make it more interesting. Researchers noted that this example showed teachers' flexibility in mathematics teaching. Student-directed creativity was described as how teachers aid in the development of students' mathematical creativity. For example, the same participant mentioned previously incited students' creativity by stimulating students' ideas. The teacher in this example questioned students and offered other possibilities while simultaneously letting them explore and create their own ideas, allowing and encouraging students to provide varying responses.

This distinction between teacher- and student- directed creativity demonstrates a need to explore tasks and pedagogical choices teachers make when implementing mathematics lessons (Lev-Zamir & Leikin, 2011). Similar to this idea, Levenson (2013a) was interested in teachers' task-choice and studied teachers' thoughts regarding the features and cognitive demands associated with mathematical creativity. In this study, mathematical creativity was defined as non-algorithmic decision making (Ervynck, 1991), divergent thinking (Guilford, 1968), and generating unusual solutions to problems (Sriraman, 2009). In order to gain an understanding of tasks that were associated with mathematical creativity and what impacted their choice of tasks, Levenson (2013a) studied a group of 43 students, with varying levels of teaching experience (ranging from 0 to 35 years), working toward a Master's degree in *Mathematics, Science, and Technology Education*. Similar to findings of other researchers (Shriki, 2010; Sternberg, 2006), participants valued flexible thinking and the development of original ideas as creative (Levenson, 2013a). However, although not instructed to do so, teachers considered attitudes, beliefs, emotions, and values that may impact the use of creative mathematical tasks in classrooms, and chose tasks based on how they thought students might react (e.g., frustration, curiosity, satisfaction, motivation). Teachers were concerned with how students may feel when presented with a task pointing to issues of affect when deciding whether to choose a task.

Teachers chose tasks that were potentially frustrating for students but in some cases allowed for multiple approaches to incite creativity, curiosity, and interest. For example, one participant chose "Cutting Sidewalks", a multi-step task that prompted students to develop multiple approaches to the problem and generalize to other scenarios. Students were encouraged to draw one, two, and eventually three lines with chalk on a sidewalk to divide the squares into different amounts of sections. During the task implementation several questions are posed to students including: (1) Let's say we add another line, how many sections will we get? (2) If we draw the same amount of lines but in different ways, will we get the same number of sections? (3) If you draw three lines, how many ways are there to divide the rectangle or square? (4) If you draw three lines, what is the smallest possible number of sections that you would get? What is the largest possible number of sections that you would get? (5) What if you were to organize a neighborhood tournament where a square is divided by 10 lines and the winner is the one with the most sections? (Levenson 2013). Teachers also deemed tasks as creative if a problem resulted in more than one possible answer, required non-standard thinking, allowed for all students (e.g. of varying levels of mathematical ability) to participate.

Task choice is directly related to lesson planning and preparing to teach creatively and foster creativity among students. Panaoura and Panaoura (2014) were interested in preservice teachers' ability to recognize creativity in mathematics and how participants might incorporate these ideas into lesson planning. They investigated ten preservice teachers, in their third year of a teacher preparation program, attending a 13-week course on "specialization in mathematics education." Findings suggest that although 8 out of 10 preservice teachers included originality in their definitions of creativity, they had difficulty producing original activities and demonstrating an understanding of mathematical creativity in their lesson plans. These findings align with Bolden et al. (2010) and Shriki's (2010) assertions that programs geared toward teachers should aide in the development of mathematical creativity and implementation of classroom knowledge into practice.

As explored in existing research, students have negative perceptions of and low achievement in mathematics, and many teachers view mathematics as a series of procedures to follow. Therefore, exploring different ways to think about and teach mathematics, using creativity as a lens might be beneficial for teaching and learning the subject. However, studies that report on teachers' conceptions of mathematics as creative note issues with creating lessons and demonstrating preparedness to implement these ideas in practice. Very few studies on students' and teachers' conceptions of mathematical creativity exist and even fewer on possible interventions to help general education students excel in mathematics through the use of creative teaching and teaching for creativity. In a review of the literature, no studies have addressed in-service teachers' willingness and ability to aide students in mathematically creative endeavors.

Using creativity as a lens to explore student thinking and exploration of mathematics may help improve the teaching and learning of mathematics as well as influence students' and teachers' beliefs about the subject.

Current Mathematics Teaching Practices and Reform Efforts

The Common Core State Standards of Mathematics (CCSSM) recommends that teachers should provide opportunities for students to demonstrate perseverance in problem-solving, reason abstractly, critique others' reasoning, and help develop students' mathematical expertise through innovative teaching strategies (National Governors Association Center for Best Practices, 2010). However, evidence suggests that although teachers recognize and vocalize the importance of reform strategies such as inquiry- and problem-based learning, they often continue to teach in traditional ways (e.g. lecturing, focusing on memorization, emphasizing one solution to a problem) (Hong & Kang, 2010; Marshall et al., 2009; Purnomo, 2017). Current problems posed to students (via textbook or teacher-chosen samples) often have predetermined answers, going against the CCSSM assertions that students should explore, analyze, and defend ideas in their mathematics learning. Implementing practices that allow students to develop creativity in mathematics requires a shift in teachers' beliefs about mathematics education.

In an effort to understand teachers' beliefs about and implementation of inquiry-based teaching, as suggested by the CCSSM, Marshall et al. (2009) surveyed 1,222 K-12 mathematics and science teachers. Using a 58-item survey, information was gathered regarding teacher demographics, teachers' beliefs about inquiry instruction, and the frequency with which teachers implement inquiry-based instruction. Findings suggest that teachers' perceptions of the importance of inquiry and the actual time spent implementing this teaching strategy do not align. Teachers reported that the ideal amount of time to devote to inquiry-based teaching ($M=57.3\%$)

was higher than the actual amount of time devoted to this type of instruction (M=38.7%). However, because results from the survey were teacher-reported results may not reflect authentic percentages regarding in-class time spent as inquiry-based learning. Furthermore, teachers of lower grades valued the importance of inquiry-based learning more than those teaching upper grades, particularly in terms of practicality (e.g. less focus on standardized test scores). This is aligned with research the suggests teachers in younger grades are more open to exploratory learning (Wilkens & Ma, 2003). Lastly, although teachers reported the importance of inquiry-based instruction, the quality of this type of instruction was not addressed in this study.

In thinking about discrepancies between teachers' beliefs and practice, Purnomo (2017) found that elementary teachers' practices in mathematics classes do not necessarily reflect their beliefs. A sample of 325 elementary school teachers responded to two questionnaires regarding their beliefs about mathematics and teaching practices. The questionnaire about teachers' beliefs included three subscales regarding teachers' beliefs about the nature of mathematics, beliefs about teaching mathematics, and assessment of mathematics learning. Although teachers in this study tended to report the importance of constructivist-oriented teaching, they also noted that they used traditional practices in their classrooms. Using traditional assessments and focusing on students' learning through accountability provided insight that teachers were drawn to more traditional practices even though they cited constructivist teaching as good practice.

The aforementioned studies highlight a need for teachers to teach creatively and foster creativity among students invoking characteristics including seeking new methods to solve problems, making decisions about appropriate approaches, and convincing others of their ideas (Sternberg, 2006; 2018) which are important for mathematicians and scientists. Although these characteristics have been deemed important for the development of creativity, they are rarely focused on STEM learning (Donovan & Bransford, 2005). Instead, rote memorization is often the emphasis in many K-12 mathematics classrooms and undergraduate lectures which leads students to lose interest in the subject and drop out of STEM fields (Goldberg, 2008). Because rote memorization is the emphasis in many mathematics classes, students define mathematics as a series of rules to be followed (Underhill, 1988), with the goal being to find one correct answer (Spangler, 1992). Breaking from what is current practice and instead guiding students to be creative in their approach to mathematics, moves away from the idea of prescribed problems with predetermined solutions and toward the construction of knowledge. Several instructional practices may help in the implementation of creative teaching and development of students' mathematical creativity.

Infusing creativity into teaching and learning mathematics is different from traditional, teacher-centered instruction where teachers transfer facts to students. Instead, students may benefit from instructional practices that include introducing mathematics by encouraging creativity, viewing the teacher as a guide for student exploration, changing the way problems are posed, encouraging alternate solutions to solving problems, setting up a culture for learning that allows students to make mistakes and take risks (Ball, 1992; Nadjafikhah et al., 2012; Schoenfeld, 1992), and moving away from a series of rules to memorize (Goldberg, 2008), which will be further discussed in the upcoming sections.

Practices to Promote Creativity in Students

Many of the traits that mathematicians demonstrate are similar to the ones described for creative people. Sternberg and Williams (1998) proposed ways for teachers to promote creativity among students. The authors deemed six traits necessary for teachers to employ in order to address and encourage students' creativity including the ability to: (a) model creativity, (b) repeatedly encourage idea generation, (c) cross-fertilize ideas, (d) build self-efficacy, (e) question assumptions, and (f) imagine other viewpoints. In a search of the literature, these characteristics and examples of teachers portraying these behaviors to promote creativity among students has not yet been studied, particularly in mathematics.

Munakata and Vaidya (2013) argue that university mathematics courses would be enriched by:

> Cultivat[ing] a culture of creativity within all aspects of the program, adoption of a creative attitude by our students, and development of a desire in our students for the lifelong pursuit of creativity and the aspiration to progress to higher levels of creativity (p. 767).

This might also be true of K-12 mathematics teaching. With this in mind, it is necessary to explore how creativity might be demonstrated in classrooms. Sternberg (2006) suggested that creativity is, in part, students demonstrating their ability to dismiss conventional thinking by seeking new approaches, making decisions and convincing others of their ideas. This type of thinking cannot flourish in traditional classroom environments. Specific instructional strategies may support creative exploration in classrooms including classroom culture, types of problem posing and problem-solving techniques, and encouraging students to think as novice mathematicians.

Classroom Culture

The culture within a classroom, school, or university setting is a determining factor in the success and development of creativity among students (Sternberg, 2006; Sternberg & Lubart, 1996; Sternberg & Williams, 1998). Therefore, it is beneficial to understand the type of classroom culture that allows for creativity to emerge. In order to foster a creative mindset, researchers have identified a need for an environment that encourages members to make mistakes, take risks, and support one another in their explorations (Ball, 1992; Levenson, 2013b; Nadjafikhah et al., 2012; Neumann, 2007). This idea of teachers developing an environment that encourages risk taking, supports multiple approaches to problems, and celebrates students' originality is aligned with creativity research which highlights a creative environment as one that allows people to explore and values originality rather than predetermined outcomes.

In an effort to devise a model for developing a culture that supports creativity in science, Neumann (2007) found that creative thinking is enriched by a culture that is supportive of the creative process. Allowing people to feel comfortable to make mistakes by setting up an inviting, secure classroom culture is important for encouraging students to take risks (Ball, 1992; Nadjafikhah et al., 2012). If the goal is for students to be willing to take risks and make mistakes in order to develop their ability to be creative in mathematics, the culture of the classroom must be such that this type of behavior is celebrated, not criticized. These environments are largely reliant on the ability of the teacher to be inclusive of and open to this type of thinking from members of the classroom community, furthering the need to prepare teachers in this endeavor.

Teacher as Guide

Part of the culture of a classroom is how participants in the community view each member's role. Instead of having the teacher as the center of classroom instruction, the teacher should act as a guide, helping and empowering students to collectively create solutions alongside students. Munakata and Vaidya (2015) noted that an open-ended learning environment allowed students to move away from seeing the instructor as all-knowing, giver of knowledge and instead as a facilitator, collaborating and learning with students. Collaboration among students as well as with the instructor is important in a classroom that fosters creativity to promote mathematics as a social rather than individual process (Schoenfeld, 1992).

Viewing the teacher as a guide instead of the omniscient point-person, empowers students to rely on their own knowledge and the collective knowledge of the classroom community. In doing so, students must be creative in their mathematical explorations, connecting seemingly unrelated ideas (Koestler, 1967), demonstrating persistence and being inquisitive (R.J. Sternberg & W.M. Williams, 1998). In doing so, teachers employ CCSSM's call for students to persevere in problem solving, and analyze and defend mathematical ideas.

Mirroring the Work of Mathematicians

In fostering an inclusive culture for students to learn mathematics, teachers should celebrate students' ideas, highlight mistakes and successes as equally important, and encourage students to think like mathematicians. For teachers to foster creativity in students, they should, "provide plenty of opportunities for students in the math classes to think like a novice mathematician" (Yaftian, 2015, p. 2532). When teachers think of students as novice mathematicians, they are empowering them to create knowledge instead of passively receiving information and reproducing set answers. School mathematics should reflect the work of mathematicians (Papert, 1972) including solving difficult problems without knowing the answer and being able to work on open-ended problems for extended periods of time (Carlson & Bloom, 2005).

Mathematicians display creativity through cyclic problem-solving behaviors such as orientation, planning, execution, and checking (Carlson & Bloom, 2005), that students would benefit from emulating. However, preservice and in-service teachers need to be prepared to develop this type of thinking among students. Attending to the development of little-c, everyday creativity can encourage students to think and learn not solely for a grade but to experience and to gain an understanding of mathematics. In order to do so, mathematics instruction must shift to include meaningful problem solving and flexible thinking, much like mathematicians' experience.

Moving away from traditional instruction, it is important that students demonstrate the ability to apply previous knowledge to new situations instead of repeating and replicating facts told to them (Munakata & Vaidya, 2013; Schoenfeld, 1992). To promote students' creativity, teaching algorithms through lecture should be replaced by activities that promote cognitive flexibility and the ability to problem solve (DeHaan, 2009). Allowing students to work like mathematicians enables them to engage in problem-solving behaviors, analyzing and defending their ideas, and developing alternate strategies to difficult problems as the CCSSM suggest. In a TedTalk, Wolfram (2010) described four stages of real mathematics: (1) posing a question, (2) translating the real-world question into a mathematics model, (3) performing calculations, and (4) moving from the model to a real-world setting to see if question was answered. This is vastly different than how school mathematics is traditionally structured. Because these practices of fostering creativity in students is important for their mathematics learning, teachers must be prepared to identify and encourage these behaviors, and create an environment conducive to these creative stages.

Problem-Solving in Mathematics

As noted earlier, current problem-solving in mathematics often involves a question with a predetermined answer known by the teacher. Although research suggests that students benefit instead from problem solving that allows students to explore difficult problems that may be unknown to the teacher, Beghetto (2007) found that high school mathematics teachers tend to lack the confidence and in some cases mathematical content knowledge to attend to students' unique solutions. Instead of being seen as a distraction or negatively challenging the teacher, students should be encouraged to solve mathematics problems alongside instructors, promoting flexible thinking and problem-solving skills. In thinking about problem solving behaviors, Carlson and Bloom (2005) studied 12 mathematicians and found that participants employed four problem-solving phases to successfully approach difficult mathematical problems including orientation, planning, execution, and checking. These behaviors, accompanied with frustration and joy, allowed mathematicians to approach and solve problems that others would abandon. Students might also benefit from integrating these processes into their own problem-solving strategies. However, if students are tasked with solving problems in a sophisticated way without predetermined solutions, teachers need to be prepared with the necessary content and pedagogical knowledge for handling these multiple solutions and differing approaches.

Project-Based Learning (PBL)

One way to shift traditional problem-solving in mathematics is through PBL, defined as a learning strategy that begins with a problem that needs to be solved and proceeds with students and teacher exploring solutions to the problem (Krajcik & Blumenfeld, 2006). This process encourages mathematical thinking and creativity through a hands-on, collaborative learning approach. PBL may allow for students to develop skills that are deemed creative (i.e. persevere in problem solving, generate alternate solutions to problems, be inquisitive and flexible). Koestler (1967) suggested that creativity, in part, is the exploration and connection of differing disciplines or ideas to generate innovative solutions to problems. This definition of creativity lends itself to PBL because it involves drawing on various disciplines and differing expertise to find solutions to a problem.

Although researchers note PBL as a successful strategy for teaching mathematics, Hong and Kang (2010) found that teachers struggle to consistently implement these practices in classroom settings. They studied South Korean and United States science teachers' conceptions of creativity and the perceived constraints of teaching for creativity. In doing so, they were interested in understanding if there were differences between conceptions of teachers in these differing places. Using a questionnaire, participants shared their ideas about the use of PBL to foster creativity and the importance of fostering creativity in classrooms. Although teachers noted the importance of these ideas, actual classroom practices were not consistent with this type of instruction. Researchers cited several obstacles that prevent teachers from fostering creativity among students including time constraints, narrow views of creativity, and assessment, and curricula. They noted that these obstacles might be alleviated by restructuring teacher preparation programs to highlight various ways for teachers to incorporate creativity into their teaching.

Problem Posing and Developing Alternate Solutions for Solving Problems

Research also suggests that problem posing is a central part of mathematics and mathematical thinking (Silver, 1997). Problem posing is defined as generating new problems or reformulating existing problems (Silver, 1994). This is aligned with research on mathematical creativity that suggests a teacher can encourage students' creativity by having students generate new problems for others to consider as a way to foster a deeper understanding of the content. For students, this might be in the form of generating problems from their own mathematical noticings. However, in order to foster this among students, teachers need to be prepared to encourage this in practice.

In order to foster creativity, teachers should provide opportunities for students to develop alternate solutions and strategies to solve problems. Little-c creativity calls on people to think differently about an everyday situation or problem. This type of creativity is relevant in classrooms where students can develop multiple solutions to a problem or uncover various ways to approach a problem. This is different than traditional practices where the teacher presents an example and monitors students' understanding of one or two solution strategies for a problem, often dismissive of other problem-solving methods. Allowing students to generate novel, useful solutions to problems encourages them to develop their creative thinking abilities in mathematics and may be beneficial for a shift in students' attitudes toward mathematics. This type of exploration in mathematics requires teacher preparation as it strays from common practice.

Creative Teaching Framework

To measure teachers' creative teaching behaviors and its impact on student achievement, Schacter et al. (2006) designed the Creative Teaching Framework (CTF) consisting of 19 items. This framework measured the frequency and quality in which in-service teachers fostered students' creativity. The CTF identified five ways teachers can promote student creativity: (a) teaching creative thinking strategies, (b) providing opportunities for choice and discovery, (c) encouraging intrinsic motivation, (d) establishing a learning environment conducive to creativity, and (e) providing opportunities for imagination and fantasy (p. 48). The purpose of the study was to identify a relationship between creative teaching and student achievement. Researchers observed each of the 48 classroom teachers (grades 3 through 6) eight times over the course of nine months. In applying this framework, they found that creative teaching improves student achievement. However, there was little evidence of quality instances of creative teaching and the researchers argue that teachers are not equipped to foster students' creativity. Although teachers were observed for a total of 437 hours, there was little evidence that they taught for creativity and inferences cannot be made regarding participant preparation to teach creatively or foster students' creativity. Therefore, it is necessary to design professional development programs that illustrate the importance of creativity and support teachers as they implement these ideas in practice.

Professional Development and Creativity in Mathematics

If educators hope to promote a deeper understanding of mathematics in K-12 classrooms and implement the previously discussed strategies, it is important that research investigates ways to revise the ongoing preparation of mathematics teachers and support in-service teachers, particularly related to the connections between mathematics and creativity. Teaching strategies such as modeling creativity, encouraging idea generation, and imagining others viewpoints (Sternberg & Williams, 1998), and establishing an inclusive learning environment that encourages creativity are important to develop mathematical proficiency and support teachers' and students' mathematical creativity. However, because teachers' prior classroom experiences are often more traditional and their beliefs about mathematics tend to be focused on memorization, both of which influence their teaching practice (Bolden et al., 2010), teachers need to be prepared to move away from traditional instruction and toward teaching that fosters creativity.

One way to expose teachers to these ideas is through professional development programs. Teacher professional development is widely practiced in education because it is believed to improve teaching and learning. However, there is little agreement on how teachers learn from professional development and how these ideas transfer to practice (Kennedy, 2016). In order to measure a professional development program's effectiveness, some researchers focus on program design (Blank & de las Alas, 2009; Timperley et al., 2007), while others investigate the content of a program (Scher & O'Reilly, 2009). Although the focus of research on professional development programs may differ, many researchers agree that some features of professional development are more effective than others. These more effective features include focusing on teachers' content and pedagogical content knowledge (Borko, 2004), fostering a sense of community among teachers (Borko, 2004; DuFour, 2004) and developing ways for professional development to be ongoing instead of isolated (Guskey, 1991; Loucks-Horsley et al., 2009).

Similarly, the National Council of Teachers of Mathematics (2000) and National Research Council National Research Council (1996) recommend that professional development for mathematics teachers draw on teachers' existing knowledge, develop teachers as leaders, and align with and support system-based changes to promote student learning. Consistent with these ideas, Loucks-Horsley et al. (2009) identified principles to guide the design of effective professional development including that professional development: (i) is focused on employing teaching strategies that are accessible to all students, not just those deemed mathematically gifted, (ii) develops teachers pedagogical content knowledge, (iii) provides engaging, active learning, (iv) provides opportunities for collaboration, and (v) supports changes in student learning such as assessment, curriculum, and classroom culture (p. 11).

These principals guided Givvin and Santagata (2011) in their development of a middle-school mathematics professional development program and their exploration of obstacles they faced in the practical implementation of professional development using these principles. Findings suggest although the developers followed the NCTM, NRC recommendations and Loucks-Horsley et al. (2009) guidelines, they still encountered issues with implementation. Therefore, they note the importance of spending time to develop a shared image of desired practices, considering teachers' thinking in order to meet their individual needs, and providing opportunities that teach participants how to collaborate. It is important to understand and address teachers' beliefs and assumptions, and their "own images" (p. 448) of the professional development content by continuously checking for understanding and acquiring teacher buy-in which allows teachers to be part of the development of ideas throughout the professional development. Attending to existing pedagogical content knowledge and assisting teachers in collaboration during professional development, instead of assuming they can do so without guidance, is also a necessary component for effective professional development for mathematics teaching.

Because teachers play an integral role in promoting students' mathematical creativity, professional development is needed to identify and develop teachers' beliefs, classroom practices, and the evaluation of students' work that support mathematical creativity (Hershkovitz et al., 2009). Professional development programs should mirror creative behaviors and practices that researchers have deemed to be effective. Using NCTM (2000), NRC (1996), and Loucks-Horsley et al.'s (2009) recommendations for professional development, and the practical applications of these principles (Givvin & Santagata, 2011) is needed for future research and professional development programs for mathematics teaching that fosters creativity.

Hershkovitz et al. (2009) assert that teachers play a central role in students' mathematical thinking at the elementary level. They argue that teachers determine how a task is implemented and that certain practices can promote or extinguish students' creative thinking. Because of the integral role teachers play in students' development and awareness of mathematical creativity, teachers need to be prepared for this type of teaching as it is often not common practice in traditional mathematics classrooms. The likelihood that teachers will understand how to foster creativity among students without necessary preparation is further reduced because many teachers were not taught in this way themselves and teachers often teach as they were taught (Shriki, 2010).

Very few articles exist regarding professional development that support teachers in the development of students' mathematical creativity. Similarly, no articles were found that explore how to foster teachers' mathematical creativity and support them in creating an environment that encourages students to explore their own creativity. However, several researchers have identified a need for such professional development in order to successfully prepare teachers to alter teaching practices and promote creativity among students (Hershkovitz et al., 2009; Levenson, 2013b; Shriki, 2010). This supports a need to design, implement, and study professional development programs that encourage creativity among teachers and support them in the development of students' mathematical creativity.

Gaps in Literature

After reviewing existing literature on categories pertaining to creativity and mathematical creativity, several gaps have emerged, namely a need for improved mathematical instruction and aiding teachers in understanding and implementing these strategies in classrooms through professional development. Kaufman and Sternberg (2007) noted that in creativity research, "there are many obstacles left to tackle – from definitional specificity and agreement to better methods of assessment" (p. 58). In conjunction with specifying a definition for creativity and focusing on ways to assess this phenomenon as Kaufman and Sternberg (2007) suggest, research is lacking on teacher and student beliefs about mathematical creativity and instructional practices that occasion mathematical creativity. Gaps in the literature include issues with instruction and assessment of mathematical creativity particularly as they pertain to teacher professional development. Furthermore, issues with students' mathematical achievement may be improved with further research in these areas. In order to adapt instruction to be consistent with reform teaching practices that have been proposed, professional development programs need to be implemented and researched to determine how to best prepare teachers to foster creativity among students.

Instruction

Research suggests the need to better understand teachers' beliefs about creativity and its connections to STEM disciplines (Bolden et al., 2010; Levenson, 2013a, 2013b; Shriki, 2010). Although some research focuses on preservice elementary teachers' awareness of creativity in mathematics (Levenson, 2013a; Shriki, 2010), researchers also identify a need to better understand teachers' beliefs and attitudes toward creativity in mathematics. In order to promote change in classroom practices, it is necessary to understand and positively impact these beliefs for the purposes of shifting instruction away from traditional practices and toward strategies that include and foster creativity.

Although some research has been done to address how teachers' conceptions of creativity impact their instruction in mathematics and the types of teaching environments that are effective in fostering creative thinking, further research should explore domain specific applications of creativity in mathematics. This exploration would ultimately add to the body of research that continues the investigation of innovative problem solving through collaboration and creativity. Teaching in a way that encourages students' creativity is different from teaching creatively. Although one can teach creatively without promoting creativity in students, the focus of this research is how teachers understand and utilize creativity specific to mathematics to foster creativity among students.

Professional Development

Sustained professional development focused on creative teaching practices and a concentration on innovative ways to foster students' creativity in mathematics is important for teachers and students. Wayne et al. (2008) argued that although clear features of successful professional development programs may be agreed upon, research on the ways in which professional development programs attend to these ideas is lacking. More specifically, although an immense amount of research on general and content-specific professional development exists, little research has been done to investigate how professional development influences teachers' pedagogical practices to foster creativity in mathematics. Effective professional development focuses on teachers' existing expertise and knowledge (Darling-Hammond & McLaughlin, 1995). Research on creativity specifies personality traits, task types, and classroom environments that encourage and allow for students to be creative. However, little focus has been placed on understanding how teachers can develop these ideas among students or how professional development programs can prepare teachers to do so, highlighting a need for research to develop and implement a professional development program that exposes teachers to these ideas and encourages them to alter their teaching practice to include mathematical creativity.

CHAPTER 3: METHODS

The purpose of this study is to contribute to research on professional development for middle school mathematics teachers. This professional development program explored teachers' creativity and encouraged the promotion of students' mathematical creativity. The research focused on teachers' experiences as they participated in this professional development program and how they designed and implemented mathematics lessons focused on creativity. Through this research, I aimed to answer the following research questions using qualitative research methods. Specifically, it utilized an embedded, single case-study methodology (Yin, 2018).

1. How do in-service mathematics teachers' conceptions of mathematics and mathematical creativity evolve as they participate in a professional development program focused on mathematical creativity?
2. What are in-service mathematics teachers' experiences infusing creativity into their mathematics lesson(s) as they participate in a professional development program?
    a. How do the ideas of fostering mathematical creativity explored in a professional development program transfer to in-service teachers' practice?
    b. How do teachers describe the development and implementation of lessons that require creativity and encourage creativity among students?

This chapter presents the methods and methodology used to answer these research questions. Data were collected between January and June 2020 with 12 middle school (grades 5 through 8) mathematics teachers. In the following section, I describe the context of my study including participant selection and methods of data collection and analysis that supported me in addressing my research questions.

Research Design

This study is a qualitative research study (Merriam & Tisdell, 2015) specifically employing an embedded, single case-study research methodology (Yin, 2018). This is an appropriate methodology for this research because an embedded case study represents a suitable setting for studying a program and involves “units of analysis at more than one level” (Yin, 2018 p. 51). The focus of the research was on teachers’ experiences within a professional development program. The main unit of analysis is the professional development program and the sub units of analysis are the individual teachers participating in the professional development program. From these 12 sub units, three exemplars were chosen for further analysis, providing detailed descriptions that were representative of participants’ overall experiences with the professional development program. In the following section, I outline the data collection methods and analysis aligned with embedded, single-case study research.

Context of the Study

The study occurred in a suburb of a large metropolitan city in the northeastern United States with approximately 680,000 residents. The Tarnos[1] school district serves approximately 9,500 students, of whom 5% are English language learners and 15.4% are classified as students with disabilities. Fifty-seven percent of the population is classified as economically disadvantaged. Students’ ethnicities are 62% white, 24% Hispanic or Latino, 9% Black or African American, 5% Asian, 1% American Indian or Alaska Native, and less than 1% Hawaiian or other Pacific Islander (New Jersey Department of Education, 2019). Twenty middle school mathematics teachers across 11, K-8 schools service the 2,179 middle school students. The

[1] All names are pseudonyms.

mathematics department is managed by a K-12 mathematics supervisor with a background in mathematics education.

Participants

To recruit participants, I presented an in-person plea to all middle school teachers during two district-wide mathematics department meetings, one for grade 6-8 teachers, one for grade 5 teachers. I was interested in including mathematics teachers from grades 5 through 8 to provide an opportunity for teachers to collaborate and discuss how ideas of mathematical creativity might be applied in multiple grade levels. To participate in the study, teachers completed written consent on the day of the meeting and provided their email for future communication and scheduling. All teachers who volunteered were considered for the study and chosen based on their willingness to attend eight researcher-designed professional development sessions and agreement to a preprogram interview, a postprogram lesson observation, and a postlesson interview. Of the 17 teachers who expressed an initial interest in the study, four withdrew prior to the start of the study due to scheduling constraints. The researcher conducted individual preprogram interviews with 13 teachers prior to the beginning of the professional development program. One participant completed the preprogram interview and attended five of the eight professional development sessions but stopped attending sessions and did not respond to researcher communication after the first remote professional development session on April 8th. The remaining 12 participants attended all eight professional development sessions and participated in the post-lesson interview. **Error! Reference source not found.** details participants' years of teaching experience and current grade level taught.

Table 2

List of Participants and Their Teaching Experience.

Pseudonym	Years of Teaching Experience	Grade Level
Pamela	27	5
Jordan	2	5
Phyllis	1	5
Andrea	26	6
Dana	15	6
Audrey	2	6
Stephanie	18	7
Drew	10	Special education 7
Alex	8	7
Megan	9	8
Jane	15	7, 8
Rachel	9	7, 8

Professional Development Structure

Research suggests that mathematics professional development should build teachers' content knowledge and tie to teaching practices, aid teachers in noticing and responding to students' mathematical thinking, focus on teachers' attitudes about teaching and learning mathematics, and foster learning communities that will sustain ideas presented beyond the professional development experience (Doerr et al., 2010). Other researchers highlight the importance of teaching creatively and promoting creativity among students (Leikin, 2009; Levenson et al., 2018; Shriki, 2010). Although research suggests a need for content focused professional development (Ball & Cohen, 1999; Hawley & Valli, 1999; Krajcik, 1994) and highlights collaboration as an important feature (Briscoe & Peters, 1997; Gajda & Koliba, 2008), teachers' experience with professional development is often fragmented, isolated from classroom practices (Ball & Cohen, 1999; Collinson & Ono, 2001), and not content specific (Chval et al., 2008). The researcher-designed professional development program under study was aligned with research on effective professional development and the call for professional development focused on mathematical creativity.

The objectives of the researcher-designed professional development were to have participants recognize mathematics as a creative discipline, foster creativity among students, apply ideas regarding creativity into their teaching practice, and collaborate with colleagues on a district-wide level. The professional development sessions were structured for participants to develop an understanding of research on mathematical creativity, engage with challenging mathematics content, and discuss, implement, and reflect on mathematical creativity in middle school classrooms. Throughout the program, participants engaged in tasks to help them better understand research on mathematical creativity and were challenged to bring these ideas into their classrooms and reflect on the result by developing mathematically creative tasks. Participants were also asked to identify challenges they might face when implementing these ideas and offer possible solutions to increase the likelihood that they would continue to attempt to infuse mathematical creativity into their teaching practice.

The sessions were held twice per month, after school hours from 3:15-4:15pm beginning January, 2020. The first four sessions were held in person as planned. However, due to the COVID-19 pandemic, the remaining four sessions were held remotely on Zoom. Participants received professional development credit hours toward the overall state requirement and had the opportunity to apply for a stipend through a National Science Foundation, Improving Undergraduate STEM Education grant, Creativity in Mathematics and Science (CMS) scholarship program (#1611876).

Each of the eight professional development sessions presented a theme of creativity research (e.g. spectrum of creativity, characteristics of creative persons, classroom environment that fosters student creativity), challenging mathematics content, and opportunities for participants to make connections between middle school mathematics and themes of creativity. The schedule of activities including mathematical tasks, opportunities for collaboration, and questions to encourage discussion of teachers' beliefs about mathematics teaching and learning, is presented in Appendix A.

During sessions one through four, participants engaged with activities where the facilitator (teacher role) presented tasks that modeled creativity and fostered participant creativity, and participants engaged as students (student role). During these sessions, participants acted in the student role by engaging in facilitator-presented tasks, intended to make participants uncomfortable with the mathematics and required them to think creatively. In Session four, participants transitioned from the student role to the teacher role as they completed a task focused on fractions. They engaged with the task as students might, but also began to take on the role of teacher as they thought about how they might implement this task creatively and require creativity of their students. The mathematics presented in this task was more accessible to participants than tasks from previous sessions. Session five, the first meeting after the switch to online teaching and learning, was organized as a discussion about challenges and successes with the switch to online teaching as well as what creativity was required of teachers and their students in this new environment. In the final three sessions, participants acted in the teacher role by creating and revising tasks focused on mathematical creativity, restructuring traditional tasks into mathematically creative ones, and preparing and implementing their own mathematically creative lessons in practice.

Sample Task: Order of Magnitude

I presented unusual tasks to model the types of activities and thinking I hoped that teachers would adapt for their students. For example, I presented participants with order of magnitude problems (Appendix B) and asked them to determine reasonable estimates, within one order of magnitude, for these seemingly unanswerable questions. As participants discussed their ideas, I modeled how to facilitate a discussion that fostered creativity by welcoming all ideas, encouraging participants to reflect and refine their ideas, and to be inquisitive and curious. The order of magnitude problems required participants to think creatively by seeing multiple approaches to the problem, engage in nonalgorithmic decision making, and explain their reasoning. Table 3 outlines a detailed list of the elements of creativity this task required for facilitator (teacher role) and participants (student role). After completing tasks similar to the order of magnitude tasks, during the professional development sessions, it was my intention that participants would be able to bring these behaviors to promote student creativity into their classrooms, assuming the teacher role they witnessed during the professional development sessions.

Table 3

Creativity Associated with the Order of Magnitude Task

Task	Facilitator as creative (teacher role)	Participant as creative (student role)
Order of Magnitude problems	• Reinforce the creative process, not just the end result. • Support and encourage students' nonconformist and unusual ideas. • Is tolerant of ideas that do not lead to the correct answer. • Provide opportunity for participant to reflect on learning. • Develop an atmosphere focused on inquiry, curiosity, and exploration	• Engage in non-algorithmic decision making • Generate novel solutions to problems • Explain reasoning • See multiple ideas or solutions to a problem • Persevere

Switch to Online Sessions

As per state regulations, the Tarnot school district switched to online teaching and learning on March 20, 2020 due to the 2019 pandemic. In order to continue the professional development sessions, the meeting format changed from in-person to remote for the remaining four sessions. The fifth professional development session scheduled for March 25th was postponed to April 8th to allow teachers time to adjust to online teaching and for the researcher to restructure initial in-person plans for remote sessions. In doing this, there was a one-month gap between the fourth and fifth professional development sessions. The first remote session on April 8th was structured as a group discussion allowing teachers to reflect on the switch to online learning and make connections to their own and students' creativity. This was different from the other seven professional development sessions where teachers were presented with research on creativity, engaged in challenging content, and developed or reflected on mathematically creative tasks.

The switch to remote meetings for the remaining four professional development sessions offered some affordances and challenges. Meeting remotely provided ways for participants to collaborate easily in breakout rooms and for the researcher to engage with each group, simulating in-person small group facilitation. During the remote meetings, all participants contributed to one Google Slide, which made sharing with the whole group more meaningful. The audio of remote breakout room recordings was of better quality than in-person small group recordings. In the remote sessions, it was easier to hear all participants without the background noise of other groups.

The shift from in-person to online teaching at Tarnot School imposed constraints requiring participants to be creative as they experienced this drastic and challenging shift in teaching formats. During the remote professional development meetings, teachers shared information regarding effective practices for online teaching and learning (e.g. developing Screencastify videos, posing questions to foster student participation, sharing helpful applications). Screencastify is a program that allowed teachers to take narrated screen recordings to share with their students. Teachers were not permitted to meet synchronously with students (even online) during the remainder of the 2019-2020 school year. Thus, the Screencastify feature that allows the user to record their desktop while simultaneously recording audio was helpful for the participants.

Although these affordances existed, there were also challenges with holding remote professional development sessions. During the in-person meetings, many teachers would remain after the session ended to discuss things related and unrelated to mathematics, further establishing cross-district collaboration and community. However, when the remote sessions concluded, teachers immediately left the meeting as they were not able to naturally break off into side conversations in the online setting. Due to the nature of in-home meetings, there were outside interruptions and personal Wi-Fi connection issues that would have been less intrusive in-person. For example, Dana's was unintentionally signed out of the meetings due to connectivity issues, Drew joined Session five 20-minutes late due to technology issues, and other teachers had difficulty finding the GoogleDoc or other materials referenced during remote sessions. Meeting remotely inhibited the researcher from assisting to remedy these technical difficulties.

Data Collection

Because the goal of case study research is to understand a process and reveal the uniqueness of a phenomenon (Merriam, 1998), a variety of data sources such as observations, interviews, and artifacts (Cohen et al., 2011) are needed. In this section, I will outline the research timeline and describe the data collection methods. This research employed data collection techniques including preprogram surveys, preprogram and postlesson interviews, participant-developed lesson plans, audio recordings of professional development sessions, artifacts created by participants during the professional development and for their classrooms, and researcher reflections after each professional development session. Data were collected between January and July 2020. Table 4 provides a timeline of data collection events and the dates for which they were collected, followed by a detailed description of each data source.

Table 4

Timeline of Research Events

Research Question Addressed	Event	Date
1	Preprogram survey	January 2020
1	Preprogram interview	January 2020
1, 2, 2b	Professional development sessions (in-person meetings)	Ongoing – 4 sessions (twice/month) January 29, February 12, 26, March 11
2b	Interim program survey – focus on the switch to virtual learning	March 2020
1, 2, 2b	Professional development sessions (remote meetings)	Ongoing – 4 sessions (twice/month) April 8, 22, May 6, 20
2, 2a	Lesson plan collection	May-June 2020
2b	Postlesson interviews	June 2020 (11 teachers) July 2020 (1 teacher)

To answer my first research question, how do in-service mathematics teachers' conceptions of mathematics and mathematical creativity evolve as they participate in a professional development program focused on mathematical creativity? I conducted semi-structured individual interviews (Merriam & Tisdell, 2015), both before and after the program, and collected audio recordings from the first four professional development sessions, audio and video from the last four professional development sessions, and classroom artifacts. The preprogram interview helped me understand participants' perceived needs from a professional development program and their conceptions and definition of mathematical creativity prior to their participation in the program. The postlesson interview helped me gain insight into how teachers' ideas about mathematical creativity evolved.

The second research question focused on participants' experiences infusing mathematical creativity into their teaching practice. This question was addressed through analysis of participant-created lessons, researcher reflections after professional development sessions, transcripts of postlesson interviews, and artifacts collected from classrooms and professional development sessions. These data sources helped me gain insight into the challenges and successes participants faced as they developed and implemented lessons that promoted creative thinking in mathematics.

Preprogram Survey

The researcher-created, preprogram survey was administered to participants and provided basic information regarding number of years teaching, what certifications they hold, and initial ideas about creativity (Appendix C). This data also provided a brief overview of participants' experiences with general and mathematics-specific professional development, opportunities for collaboration with colleagues and consideration of creativity. Because of my interest in developing meaningful professional development and having participants from across the district, I sought to understand participants' past experiences with professional development and what they valued for their own teaching.

Semi-Structured Interviews

Semi-structured interviews are a mixture of open-ended and structured questions, focusing on participants' lived experiences and stories (Merriam & Tisdell, 2015). Preprogram (Appendix D) and postlesson (Appendix E) semi-structured interviews were conducted to understand participants' views about creativity in mathematics teaching and learning, their experiences participating in the professional development sessions, and creating and implementing lessons focused on creativity. Each interview was recorded and transcribed.

The preprogram interviews were conducted in January, prior to the start of the professional development program, to better understand teachers' conceptions of mathematics teaching and learning prior to their participation in the professional development program. Eleven postlesson interviews were conducted in June, after teachers implemented their mathematically creative lessons, to understand how teachers' views about mathematical creativity evolved, their experiences participating in the professional development program, and how they described creating and implementing a lesson focused on mathematical creativity. Pamela was interviewed in July because she was overwhelmed in June with teaching online, grading, and finalizing the school year. Using a semi-structured postlesson interview helped me better understand how participants viewed their experiences and provided opportunities for them to reflect on their participation and engagement in the program.

Lesson Plan and Student Work

Each teacher or pair of teachers submitted two lesson plans after the culmination of the professional development program. I initially planned to observe teachers' implementation of one lesson but due to the 2019 pandemic and switch to online learning, this was not possible. Participants were not meeting synchronously with students at that time. Instead, they were posting assignments on Google Classroom and students were required to check in twice per day, once at 9:30am and again at 1:30pm. Teachers posted assignments but had limited synchronous interaction with students other than typing in an online chat. Because lessons were developed after the switch to online learning, the lesson plans were activities or a series of tasks intended to infuse ideas of creativity into mathematics. Participant-created lessons provided insight into their attempt to be creative and promote students' mathematical creativity, and instances of participant implementation of themes of creativity discussed during the professional development program.

Participants were also asked to provide student work on the mathematically creative lessons. Viewing student work helped me to better understand the response to lessons and any teacher feedback that was provided. Particularly, I asked participants during the postlesson interview to point to a student they felt was particularly creative. Having access to student work allowed me to determine if participants' ideas about student work aligned with traits and dispositions of mathematical creativity.

Professional Development Session Recordings

All professional development sessions were recorded and transcribed. I analyzed data throughout the professional development program and adjusted the content of each session in accordance with what I found from the session recordings. After each professional development session, I reviewed my notes and conducted initial analysis by noting important instances from the audio. I collected and analyzed participant-created artifacts from the professional development sessions (e.g. mathematics problems, written reflections) to understand their process and participation throughout the professional development program. The artifacts from the professional development sessions, including participant reflections from three sessions, helped me understand teachers' use of the ideas presented in the professional development and their musings about the topics discussed.

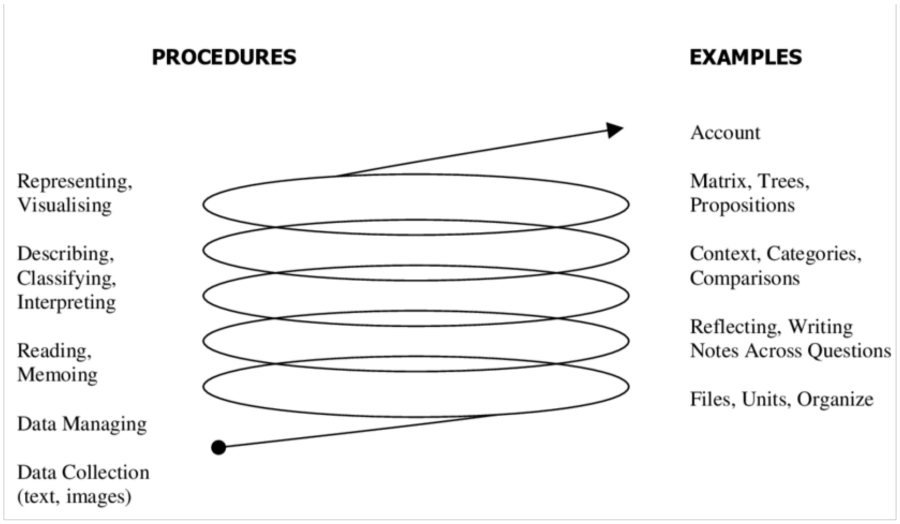
Data Analysis

Aligned with my theoretical framework, understanding creativity on a spectrum of little-c to Big-C creativity (Beghetto & Kaufman, 2007a; Csikszentmihalyi, 1996, 1998) and promoting ideas of creativity among students (Sternberg, 2006; Robert J Sternberg & W.M. Williams, 1998), I analyzed the data using an adapted version of Schacter, Thum, and Zifkin's (2006) lesson observation protocol (Appendix F) described in detail in the following section. Throughout data analysis, I used constant comparative data analysis (Glaser & Strauss, 1967) and categorical aggregation (Creswell, 2007; Yin, 2018), establishing patterns and finding correspondence among categories through the comparison of various data sources. In doing so, I first searched for patterns or concepts that emerged and applied a ground up approach employing inductive reasoning. Although first developed for grounded theory research, the constant comparative method of data analysis is appropriate for case-study research because it requires the researcher to compare segments of data with one another to identify similarities and differences.

Data analysis was inductive as I sought to understand teachers' experiences throughout the professional development program, not prove a preconceived theory. In analyzing data and making meaning of how participating teachers employed creativity in planning lessons and implemented themes of creativity in their classrooms, I visualized the process as an ongoing data analysis spiral (see Figure 3)

Figure 3

Image of Data Analysis Spiral (Creswell, 2007, p. 151)



Creative Teaching Framework (CTF)

The CTF (Schacter et al., 2006) is a 19-item lesson observation protocol designed to evaluate the frequency and quality with which classroom teachers provide opportunities for their students to be creative. In developing the CTF, researchers identified five areas that increase the likelihood of creative output. According to the instrument developers, teaching behaviors that elicit creativity among students include: (a) explicitly teaching creative thinking strategies, (b) providing opportunities for choice and discovery, (c) encouraging intrinsic motivation, (d) establishing a learning environment conducive to creativity, and (e) providing opportunities for imagination and fantasy.

I used this protocol to frame my initial analysis of the postlesson interviews for any mention of the participant-created lesson, finding categories among the data and further understanding these distinctions. However, because this is a lesson observation protocol, I was also looking for other themes and distinctions among the data as I applied it to participant created lesson plans. Although the CTF is an observation protocol, it was appropriate for analyzing postlesson interviews and participant created lessons because I was assessing how the teachers think about and incorporate elements of creativity in practice or in reflecting about their practice. The five areas, *thinking strategies, choice and discovery, motivation, environment,* and *imagination and fantasy*, became part of my code book to describe any mention of the lesson during the postlesson interviews and participant created lessons. The adapted protocol was used for analysis of participant created lesson plans, preprogram and postlesson interviews, and professional development sessions.

To adapt the lesson observation protocol for use in analyzing lesson plans, I highlighted themes from the CTF that might be observable in the teacher developed tasks, student work, and postlesson interview. I noted differences in the lesson plan analysis versus an in-person lesson observation, highlighting portions of the protocol that might only be visible through observation, and used the revised protocol (Appendix G) to review one postlesson interview transcript, including any instances where the participant created lesson was mentioned, and the accompanying lesson plan, to further adapt the protocol. As I coded the accompanying lesson plan, I determined that the traits of creativity needed to be included in the *thinking strategies* category. The *thinking strategies* theme was expanded to incorporate the specific focus on traits of mathematical creativity introduced during the professional development sessions (see Figure 4). The red boxes indicate traits of creativity from the literature that were added to the CTF protocol category, *thinking strategies*.

Figure 4

Initial adaptation of Schacter et al. (2006) protocol for post-program interview analysis

Level 3 Code	Description	Level 4 Code	Description
Thinking Strategies	Teaching Creative Thinking Strategies	Non-algorithmic	Non-algorithmic decision making, divergent thinking
		Novel solution	Generate novel solutions to problems
		Mathematical elegance	Strive for mathematical elegance
		Explain reasoning	Explain reasoning
		Mathematical Connections	Be curious about mathematical connections
		Continue to explore	Continue to explore after a problem has been solved
		Insightful solutions	Develop insightful solutions to a problem
		New questions	Formulate new questions for other students to consider
		Multiple solutions	See multiple ideas/solutions to a problem
		Persevere	Perseverance on mathematics tasks
		Metacognition	The teacher explicitly teaches metacognitive strategies.
		Other	

The process of data analysis was continuous and recursive. First level codes indicate the data source that was analyzed (e.g. lesson plan, professional development, preprogram interview). Second level codes indicate the topic being discussed (e.g. during the preprogram interview [level 1], the teacher is discussing students, state standards, or teaching practice [level 2]; during the professional development sessions [level 1], teacher mentions online teaching [level 2]). Third level codes were adapted from the CTF protocol and additional codes were generated from the data and added to the code book. Codes were developed through an iterative process, reviewing and revising codes from the data through initial and retrospective analysis, and coding with initial, affective and axial perspectives (Creswell, 2007; Saldana, 2016). After coding the first postlesson interview, each subsequent read through of this and other data sources was completed with the first transcript in mind and codes were added and revised as needed. Table 5 provides a detailed list of the research question that is addressed by each data source and how the data source was analyzed to answer the questions.

Table 5

Overview of the Data Source and Data Analysis used to Answer each Research Question

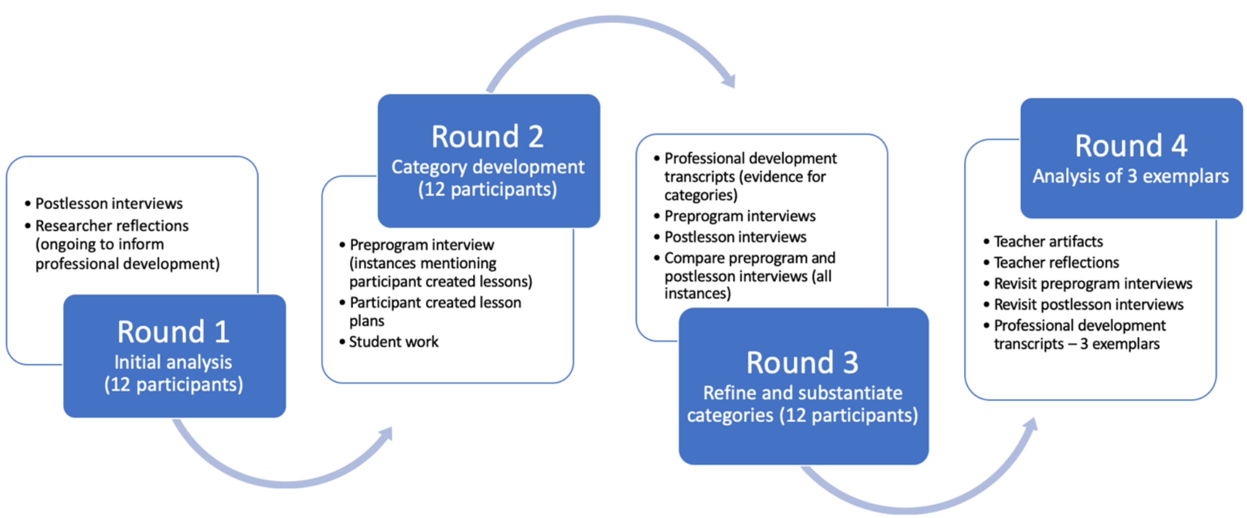
Research Question	Data source	Data Analysis
RQ 1: How do in-service mathematics teachers' conceptions of mathematics and mathematical creativity evolve as they participate in a professional development program that focuses on mathematical creativity?	Preprogram and postlesson interviews Preprogram survey Audio of professional development sessions Artifacts from professional development sessions	Initial and axial coding (Saldana, 2016) of audio recording transcripts Attribute coding (Saldana, 2016) Open coding of audio recording transcripts Open coding to search for themes of creativity
RQ 2: What are in-service mathematics teachers' experiences infusing creativity into their mathematics lesson(s) as they participate in a professional development program?	Postlesson interview Lesson plan Classroom artifacts Audio of professional development sessions Artifacts from professional development sessions	Affective (Saldana, 2016) and open coding of audio recording transcript Observation protocol adapted from (Schacter et al., 2006) Open coding of lesson Open coding of audio recording transcripts Open coding to search for themes of creativity
RQ 2a: How do the ideas of fostering mathematical creativity explored in a professional development program transfer to in-service teachers' practice?	Lesson plan Classroom artifacts	Observation protocol adapted from (Schacter et al., 2006) Open coding of lesson
RQ2b: How do teachers describe the development and implementation of lessons that require creativity and encourage creativity among students?	Researcher's reflection on professional development sessions Postlesson interview	Open coding of journal Initial and axial coding (Saldana, 2016) of audio recording transcript

Because of the variety of data sources, a mixture of data analysis procedures including initial and focused coding was employed. I organized the data with all teacher responses for a particular data source into one document (e.g. all teacher lesson plans were in different tabs in one document). I conducted initial analysis applying the adapted protocol and looking for additional themes. Having all responses for each data source organized in this way allowed me to easily revise codes and see similarities within each data source, and to examine and explore similarities and differences among the data (Strauss & Corbin, 1998). Focused coding followed initial coding to identify major categories and themes that developed (Saldana, 2016).

Participant created artifacts from the professional development program and classrooms were analyzed beginning with "jottings and analytic memos" (Creswell, 2007, p. 61) that documented my impressions from the material to gain insight into what is unobservable on the day of lesson observation. After subsequent read throughs of participant artifacts, the researcher developed code book was applied and new codes were added as needed. See Figure 5 for more details regarding the stages of data analysis.

Figure 5

Data Analysis Process



Category Development

To investigate the data, I first looked at the case of teachers participating in the professional development program as a whole. I started by transcribing all eight professional development sessions, and each participant's preprogram and postlesson interview. Next, I analyzed participant created lesson plans and preprogram interviews for instances where teachers mentioned the development or implementation of the lesson plan, to gain understanding of participants' views about mathematics and mathematical creativity, and their experiences planning and implementing a lesson on mathematical creativity. Using the CTF protocol, I applied codes to each data source and adapted the protocol based on the data as described above.

Based on my initial analysis of participant created lesson plans and postlesson interviews, I grouped participants by how they described creativity and mathematics. Through this analysis, three groupings based on participants' views of mathematical creativity emerged. Originally groups were defined as the shift participants experienced in their thinking about mathematical creativity throughout the professional development program. However, as I analyzed the data more closely to get a better understanding of each participant's individual experiences, reactions, and teaching practices, the themes that emerged were in reference to how teachers talked about mathematics, creativity, and teaching and learning. These groupings were *adherence to traditional teaching practices (traditional), appreciation of teaching for creativity (creative but hesitant), and teaching for creativity (creative)*.

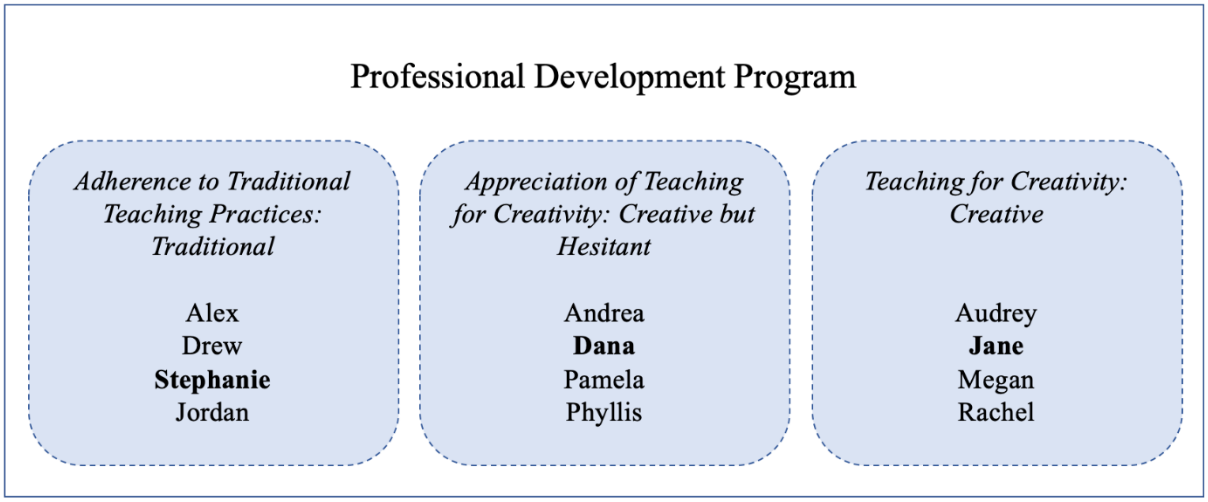
To categorize teachers, I then reviewed professional development session transcripts looking for evidence of these categories, and compared participants' responses to preprogram and postlesson interviews where the same questions were asked. An exemplar was chosen to represent each of the three categories. To choose these exemplars, I reviewed postlesson interviews and professional development session transcripts looking for participants who were representative of the category as well as offered regular participation in the sessions. I further analyzed participants artifacts, reflections, professional development transcripts, and preprogram and postlesson interviews for each exemplar. More detail about what exemplified these categories will be discussed in the results section.

Choice of Exemplars

After categorizing the 12 teachers into one of the three groups mentioned above, I identified one teacher from each group as an exemplar (Figure 6). To select these exemplars, I reviewed researcher reflections from the professional development sessions and preprogram and postlesson interviews, teacher created lessons, student work, and professional development session transcripts. I was looking for teachers who were representative of their respective groups, were vocal throughout the professional development program, and offered interesting insights regarding mathematics and creativity. Stephanie, Dana and Jane were selected from the *traditional, creative but hesitant,* and *creative* categories respectively. Although I could have selected another participant to represent each category, the participants' chosen were the most vocal in the professional development sessions and provided more detail regarding their ideas about mathematics, creativity, and teaching and learning than others. I followed these exemplars through the professional development sessions to dive deeply into their understanding and thoughts regarding mathematical creativity in middle school classrooms. I analyzed their responses on three reflections and other relevant artifacts from the professional development sessions, and revisited their preprogram and postlesson interview transcripts, lesson plans, and student work.

Figure 6

Participants Embedded in the Single-Case of the Professional Development Program



Note. Exemplar names are listed in bold

Positionality

Because research is shaped by researcher and participants (England, 1994), the identities of each person involved in the research impacts the research process (Bourke, 2014). As the researcher and professional development facilitator, teachers may have felt pressure to agree with the ideas presented during the professional development sessions. To address this, I created a welcoming and honest culture for our sessions and phrased questions throughout the study so as to not lead teachers to agree with the ideas put forth in the sessions and instead ensure that they were comfortable to be honest about their experiences, regardless of whether they differed from those that were presented. Because this is a relatively small district with one mathematics supervisor, teachers may have felt apprehensive about sharing struggles they face within the district or with the professional development material. To alleviate this, I reminded teachers that their participation in this study was voluntary and their ideas would not be shared with their supervisor or others, as any reporting of results will be anonymous. As a former middle school mathematics teacher focused on finding alternative ways to present ideas and as a researcher interested in the development of mathematical creativity, at times I was discouraged by struggles teachers had with implementing these ideas into practice. To mitigate this potential bias, I participated in a critical friend group, which was comprised of other mathematics education doctoral students, to review my data and coding schemes to ensure honest discussions about my views of the data and expectations of the teachers during data analysis.

Credibility and Triangulation

Stake (1995) argued that a, "Qualitative case study is highly personal research" (p. 135) with the researcher as the main instrument of data collection and analysis (Merriam & Tisdell, 2015). Because of this, it is important to establish the study's validity and researcher's credibility. To establish validity, minimizing the misrepresentation of the case, I used multiple data sources, as described above. To verify the reliability of these codes, I had an additional researcher verify my codes by reviewing 20% of the preprogram and postlesson interview transcripts. To do this, I first documented how many lines were included in each of the 12 preprogram and 12 postlesson transcripts. I then used an online random number generator to choose a starting line number for each transcript. I calculated 20% of each transcript and provided the end line number for each section. A second coder, a recent graduate of the mathematics education doctoral program, was asked to code 20% of each transcript based on the random starting point. Miles & Huberman (1994) suggest that interrater reliability between 80% and 90% is acceptable. After three transcripts were coded by the second coder, the researchers met to discuss discrepancy and further identify the meanings of codes. The second coder then coded the remaining postlesson transcripts. The researchers met again to discuss and note differences that might be evident in the preprogram interviews. After the second coder analyzed all preprogram and postlesson transcripts, we found there was interrater reliability of 82%, therefore, the reliability of these codes is justifiable.

To establish researcher credibility, I kept a researcher journal to record and reflect on all interactions with participants during the preprogram and postlesson interviews and professional development sessions. To triangulate and further substantiate my findings, I conducted respondent validation (Merriam & Tisdell, 2015) to establish reliable participant perspectives. Because of the nature of ongoing professional development, I had the opportunity to revisit ideas brought up in previous sessions to ensure participants saw themselves reflected in my descriptions. I discussed my understandings of each participant's ideas during the postlesson interviews to allow participants to substantiate or dispute my claims and further validate my findings.

CHAPTER 4: RESULTS

In this section I first report the findings for the main unit of analysis, the 12 teacher participants in the professional development program, organized by research question. Because of the special circumstances under which the second half of the professional development was held, I dedicate one section to participants' experiences teaching in an online environment and how creativity played a role. Next, I explore how these results led to the identification of three participant categories within the main unit of analysis: *adherence to traditional teaching practices (traditional), appreciation of teaching for creativity (creative but hesitant),* and *teaching for creativity (creative)*, and describe how each category was defined. Lastly, I present one exemplar participant from each category and analyze the similarities and differences among them.

To investigate my research questions, I first focused on participant created lessons and student work, adapting Schacter et al.'s (2006) lesson observation protocol as explained in the methods section. I then analyzed the preprogram and postlesson interviews to further develop the code book and identify themes among the data. The themes that emerged from the data were participants' views of mathematics, creativity, and teaching and learning. After reviewing all preprogram and postlesson interviews, participants' lesson plans, and student work, I re-analyzed the lesson plans with the updated code book and identified instances where these themes were evident as participants developed and implemented mathematically creative lessons.

Results from the Main Unit of Analysis

The results presented in the following section address the research questions:

1. How do in-service mathematics teachers' conceptions of mathematics and mathematical creativity evolve as they participate in a professional development program that focuses on mathematical creativity?
2. What are in-service mathematics teachers' experiences infusing creativity into their mathematics lessons as they participate in a professional development program?
   a) How do the ideas of fostering mathematical creativity explored in a professional development program transfer to in-service teachers' practice?
   b) How do teachers describe the development and implementation of lessons that require creativity and encourage creativity among students?

Teachers' Conceptions of Mathematics, Creativity, and Teaching and Learning

To answer my first research question regarding how teachers' conceptions of mathematics and mathematical creativity evolved throughout their participation in the professional development program, I coded the data using the Schacter et al. (2006) protocol. The themes that emerged were participants' differing views of mathematics, creativity, and teaching and learning.

Theme 1: View of Mathematics

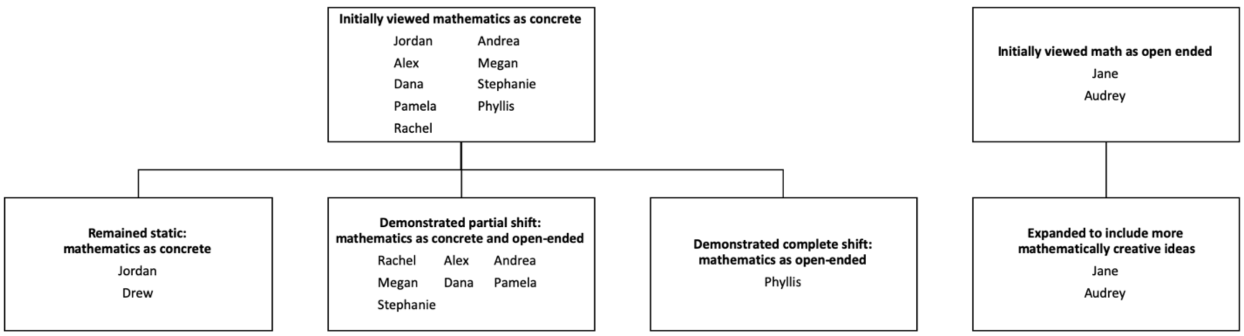
In order to understand how teachers defined mathematics, I analyzed the code, *teaching practices* (level 2), *mathematics* (level 3). The analysis of this code suggested that there were differences in how participants viewed, discussed, and defined mathematics. In particular, participants discussed mathematics on a spectrum of concrete to open-ended. Aligned with research on mathematical creativity, viewing mathematics as open-ended is one way to exhibit mathematically creative ideas (Csikszentmihalyi, 1996; Schacter et al., 2006; Sternberg, 1985; Sternberg & Lubart, 1996). In this section, I describe teachers' differing views of mathematics and highlight the similarities, differences, and shifts in these views as they participated in the professional development program.

Mathematics on a Spectrum of Concrete to Open-Ended. My initial analysis of *teaching practice* (level 2), *mathematics* (level 3), revealed that participants described mathematics on a spectrum of concrete to open-ended. Prior to the professional development program, nine participants described mathematics as concrete and mentioned the importance of knowing the steps to solve a problem, two participants identified the application of mathematics in real life situations with no mention of mathematics as concrete, and one teacher did not respond directly to the question but provided evidence of seeing mathematics as concrete. After participating in the professional development program two participants continued to view mathematics as concrete. However, the majority of teachers demonstrated a shift in their thinking from the traditional view of mathematics as concrete to a more creative one that included open-endedness.

Among the nine participants who originally described mathematics as concrete, two continued to think about mathematics as solely concrete, six demonstrated a partial shift in their thinking as they continued to see mathematics as concrete but also expanded this idea to include levels of open-endedness in mathematics, and one demonstrated a complete shift. After her participation in the professional development program, Phyllis, who originally viewed mathematics as concrete, no longer held this view. Instead, she highlighted the open-ended nature of mathematics in that its purpose was to make sense and be critical of information. The two teachers who initially thought of the open-ended nature of mathematics built upon their original ideas. Figure 7 further describes the evolution of participants who initially viewed mathematics as concrete prior to the professional development program.

Figure 7

Evolution of participants' views of mathematics as concrete



After participating in the professional development program, the nine participants who viewed mathematics as both concrete and open-ended thought about and described these ideas in varying levels of detail. While some participants noted that mathematics was less about the steps than they originally believed, others detailed the importance of having students recognize the application and usefulness of mathematics outside of the classroom. For example, in the preprogram interview, Dana defined mathematics as concrete noting, "I love the more I guess concrete and black and white examples as opposed to reading the chapter books and writing the essays...when I try to compare the subject matter, I just enjoy working with the numbers" (line 14). However, after participating in the professional development program, her view expanded to also include ideas of mathematics as open ended. Although she held onto the idea of mathematics as concrete, she also recognized the importance of the open-ended nature of the subject. Dana commented that she, "like[d] the formulas behind [math. I personally always liked the black and white behind it" (Postlesson Interview, line 85). However, she also questioned:

> Where else can I get them to think and get them to not just focus on a number, that it's more than just the number because we're going to use this in other areas of our life. And I

> also now opened up a section going forward for math in the real world per topic, and they'll be doing research on what careers they can utilize these standards with. So, it's not just a math class anymore. (Postlesson Interview, line 98)

In this instance, Dana demonstrated beliefs of mathematics both as concrete and open ended in that she described the importance of students' connecting mathematics to areas outside of the classroom, namely future careers.

This was different from Jane and Audrey who initially saw mathematics as open-ended but expanded upon these ideas in the postlesson interviews to include more detailed connections. After participating in the professional development program, these two participants described the open-ended nature of mathematics as making connections within and outside of mathematics. For example, Audrey noted that there are not always, "perfect solutions" (Postlesson Interview, line 137). Instead, there might be "thousands of solutions" and mathematics is connected to real-life applications and skills. Similarly, after participating in the professional development program, Jane thought that, "everything is connected in some way. There's always a way to figure out a math problem. There's never one direct way there's like, especially with this [professional development program], there's different approaches in order to figure things out" (line 102). Audrey's realization that there might be many solutions to one mathematics problem and Jane's identification of everything in mathematics as being connected are examples of viewing mathematics as open-ended with complicated, sometimes unclear solutions and approaches to problem solving. This connection between mathematics and real-life applications is an example of seeing mathematics as more open-ended and focused on problem solving versus a concrete problem with a definite answer.

Overall, the analysis of the code *teaching practice*, *mathematics*, identified three ways participants thought about mathematics on a spectrum of concrete to open-ended after participating in the professional development program. These categories included those that continued to think about mathematics as solely concrete, those who viewed mathematics as concrete but also recognized it as open-ended, and those that saw mathematics as open-ended without restriction.

Theme 2: View of Creativity

Similar to participants' views of mathematics, there were varying degrees in participants' views of creativity on a spectrum from traditional to creative. To further understand how teachers described their ideas about creativity and their recognition of mathematical creativity in varying ways, I drew from my analysis of the category *thinking strategies* (see Table 6), adapted from the Schacter et al. (2006) protocol, to address instances when teachers elicited creative ideas such as *identifying multiple solutions to a problem, engaging in non-algorithmic decision making,* and *continuing to explore after a problem has been solved*. The category *thinking strategies* also included traits of mathematical creativity from literature: *explain reasoning, develop novel solutions to problems,* and *strive for mathematical elegance* (Chamberlin & Moon, 2005; DeHaan, 2009; Sheffield, 2008). Participants tended to discuss the category *thinking strategies* more than any other third level code.

Table 6

Thinking Strategies

Level 3 Code	Level 4 Code	Level 4 Description
Thinking Strategies	Non-algorithmic	Engage in non-algorithmic decision making
	Novel solution	Generate novel solutions to problems
	Mathematical elegance	Strive for mathematical elegance
	Explain reasoning	Explain reasoning
	Mathematical Connections	Be curious about mathematical connections
	Continue to explore	Continue to explore after a problem has been solved
	Insightful solutions	Develop insightful solutions to a problem
	New questions	Formulate new questions for other students to consider
	Multiple solutions	Identify multiple ideas, solutions, or approaches to a problem
	Persevere	Persevere in problem solving

A deep dive into this level 3 category allowed me to explore the depth to which participants discussed these ideas in the postlesson interviews, implemented these ideas into their lesson plans, and described this category in relation to their teaching practice and ideas about students. I determined the frequency of the level 4 codes embedded within *thinking strategies*. Doing this allowed me to further understand in detail the ways in which participants varied in their views of creativity.

Some participants mentioned level 4 codes less frequently than others. If discussed, these participants tended to do so in relation to only four of the ten codes: *explain reasoning*, *identify multiple solutions, formulate new questions,* and *be curious about mathematical connections*. When mentioned, these participants discussed the ideas on a surface level, were unable to describe how it might be realized in practice, and did not effectively incorporate these ideas in their lesson plans. In contrast, other participants described a greater number of level 4 categories, including: *non-algorithmic, mathematical elegance, explain reasoning, mathematical connections, continue to explore, new questions, multiple solutions,* and *persevere,* suggesting a more robust understanding of mathematically creative ideas. Among these teachers, there was again a difference in the depth to which they discussed these ideas. Participants who discussed a greater number of level 4 codes, differed in their implementation of these ideas in their lesson plans and discussion of student work. While some participants did not include these ideas in their lesson plans or teaching practice, others embedded these ideas into their lesson but continued to describe a traditional view of creativity. Still others discussed how getting students to attend to these traits of mathematical creativity (e.g. *be curious about mathematical connections*) might work in practice and included ways to elicit these ideas from students in their lesson plans.

One way in which participants conveyed their view of creativity was by providing opportunities for students to reflect on learning and make connections among concepts. Rachel exemplified this when she developed a table to encourage students to, "compare the areas when the length and the base, and the width and the height were the same, and they could see that it was a half" (Postlesson Interview, line 15). This addition to her original lesson provided an opportunity for students to reflect on learning and further highlight connections among formulas.

One interesting finding was that level 4 codes, *explain reasoning* (20% of instances) and *be curious about mathematical connections* (18% of instances) were identified more than any other category. Participants viewed these traits of mathematical creativity as important for middle school mathematics students. However, participants differed in the degree in which they implemented these ideas. While some participants provided opportunities for students to explain their reasoning and think about mathematical connections, and described instances where students engaged in this type of thinking as it was presented in the professional development program, others discussed these ideas on a surface level, not describing what it might look like in practice or misinterpreted the meaning of the trait of mathematical creativity.

In thinking about these differences, I provide examples of two contrasting views of teachers' description of the code *be curious about mathematical connections*. For example, to describe a lesson about finding area of irregular figures, Audrey noted:

> I actually gave them graph paper…and I asked them to cut triangles out of it and try to create a new shape. So, they had to do rectangle[s] or something. So it makes them think and…for me, I think it was more engag[ing] because they were able to understand that if you take a parallelogram, and you rearrange [it] into rectangle, then you can take [the] parallelogram formula to find area and rearrange to find [area of a] rectangle formula. So, they can see [the] connection that sometimes shapes could combine and create the composite shapes…so now I don't think it's a waste of time. I think it's a creative way of thinking. (Postlesson Interview, line 48)

In this instance, Audrey described how she encouraged students to be curious about connections among formulas by designing an activity to help them recognize these connections.

In contrast, Jordan described an activity where students designed a room using shapes. For Jordan, this activity "allowed [students] to say, 'oh, well, yeah, there's shapes everywhere.' And I think that definitely sparked…the curiosity about mathematical connections [in] the real world" (Postlesson Interview, line 34). Although Jordan viewed this activity as encouraging students to *be curious about mathematical connections*, this only required students to use various shapes to design a room and therefore this category was instantiated on a surface level rather than identifying connections among different mathematics topics within the problem. Jordan's view of students being curious about mathematical connections by recognizing shapes in a superficial way was different from Audrey's view that helped students be curious about mathematics in a more comprehensive, meaningful way. Three categories emerged regarding different levels of specificity, buy-in, and implementation of mathematically creative ideas among the 12 participants.

Creativity as Artistic and Fun, or Embedded in the Task Itself. After participating in the professional development program, participants discussed creativity differently. While some continued to see creativity as an artistic product, game, or fun activity, disjoint from traits of mathematical creativity, others described creativity as embedded in a task or lesson. After participating in the professional development program, participants continued to view creativity as an artistic product. However, six participants focused more on traits of mathematical creativity as embedded in tasks and students' responses. For example, after participating in the professional development program, Andrea described creativity as an artistic product when she noted:

> When I see that they're having trouble, if we have a little quiz and they didn't understand it, I'll have them create a little poster. How would you teach this to a fifth-grade student that's coming up next year? So, it gives them a chance to think about it and be creative and come up with a little informational poster that will help them as well as someone else. (Postlesson Interview, line 116)

In this example, the design of a poster, rather than the mathematics, was a way for students to be creative. Instead of focusing on how this activity might get students to make connections among mathematical topics, explore different approaches to a problem, or develop new questions for other students to consider, Andrea focused on the artistic nature of designing a poster. Similarly, Alex also described the artistic nature of students' products. She noted that, "You have some kids who write right in the Google Doc, and I'm like, ugh! As me reading it, I'm just like, okay, I scroll it. But the ones who make presentations and do the effects. I'm like, aw" (Postlesson Interview, line 98). This focus on presentations with effects was more appealing to Alex and again demonstrated the artistic nature of creativity.

Creativity as a Fun Game or as a Challenging Task. Similarly, when asked about connections between mathematics and creativity, some participants noted that creativity might be exemplified in a game or developing ways for students to think of math as fun. Thinking about creativity as fun, Stephanie noted creativity is when:

> A student doesn't see [math] as work. Or, and they see it...they thought [Stephanie's participant-designed lesson] was fun. It was fun. I think that's the creativity. I think if they don't view it as a chore, then it was successful in being creative, because I think the worksheets, the textbook, all that boring stuff, that's a chore for them." (Postlesson Interview, line 179).

Similar to Stephanie, several participants also continued to focus on infusing creativity into mathematics as fun. For example, Alex described a creative project as fun, absent of challenging mathematics. She noted, "I gear towards projects, because I think they're fun, and different, and letting them express themselves however they want. But I guess I'm not really thinking more of the mathematical aspect, it's more of the social component" (Postlesson Interview, line 178). Although some teachers focused on creativity on a surface level in the form of games, activities, or group work, others described mathematics and creativity as intertwined and at times inseparable.

While some participants continued to focus on an activity as fun absent of thought-provoking mathematics, others viewed it as fun and challenging. For example, in the postlesson interview, Pamela noted that, "making the connection of the basics [and] being more independent and being more challenged and creative to get them at a higher level of thinking" (line 191) when she described the challenging nature of mathematical creativity. These differences among participants' ideas about mathematics further highlighted a distinction of three categories in how participants viewed creativity.

Adoption or Misinterpretation of Traits of Creativity. As previously described, participants thought about, interpreted, and implemented the traits of creativity introduced in the professional development program in different ways. While some participants spoke about the traits in detail and adopted these ideas in practice, others misinterpreted traits and continued to think about creativity in a traditional way or as disjoint from mathematics.

When participants misinterpreted the traits of mathematical creativity introduced during the professional development program, they either labeled something as mathematically creative that was not or spoke about an idea on a surface level without evidence of how it might be implemented effectively in practice. For example, Alex noted that a lesson plan on probability required students to *be curious about mathematical connections* by having them repeat the experiment with a different sized basket. The trait of creativity, *be curious about mathematical connection,* is meant to encourage students to make meaningful, content-related connections among topics, disciplines, or personal interests (Sheffield, 2008). Reflecting on this lesson during the postlesson interview, Alex noted that the trait of mathematical creativity, *be curious about mathematical connections*, was evident in the lesson because this lesson required students to shoot a ball into a basket and record how many times they made the shot. Alex noted:

> Because I had them do it twice, once with whatever size basket they had, and then I had them try to do it again with a smaller wastebasket...I asked, "How could a smaller basket change the probability?" So, without saying it, I was hoping that they would kind of

> come up with an answer on their own and then do the math for it. But I realize right now, I didn't specifically say that before you fill out the chart and do it, what do you think can happen? And then compare it to your actual outcome. (Postlesson Interview, line 33)

She also noted that, "[Students] can apply this to just throwing a ball and I was hoping some of the kids would maybe even connect it to...sports, basketball, soccer, making a goal" (Postlesson Interview, line 36). In these instances, Alex described *being curious about mathematical connections* as having students repeat an experiment. However, the lesson plan did not encourage students to recognize or explore this idea, nor did it identify ways for students to make meaningful connections between the two experiments. Alex did not encourage students to *be curious about mathematical connections* between experiments or in connection to other topics. In the postlesson interview, Alex noted that she wanted students to connect probability with sports. Although it might be interesting, engaging, and creative to have students think about connections between probability and sports, this was not explicit in the way questions were structured in the lesson plan and instead seemed to be misaligned with the depth intended by the category, *be curious about mathematical connections*.

Conversely, Megan interpreted the traits of creativity as intended and developed a lesson on exponential functions which included the traits: *strive for mathematical elegance, explain reasoning, be curious about mathematical connections, formulate new questions for other students to consider,* and *engage in non-algorithmic decision making,* as central to her lesson. The lesson challenged students to think about the following questions:

- What information do you need to be able to attempt to solve this problem?
- How would you begin to approach this task?

- What equation would you use to solve this task? Or what method (if not using an equation)?
- Is your equation linear, quadratic, exponential, or something else?
- Is there more than one way to solve this problem?
- Can you create and solve your own paper folding problem?

Megan fostered students' creativity by designing a lesson that encouraged students to be independent, explore ideas about exponential functions through discovery, and explore multiple ways to solve the problem. The task was situated in a real-world context in which students had to think about ways to approach the problem without an algorithm.

While some teachers misinterpreted elements of mathematical creativity in their lesson planning, others implemented them as intended and the mathematically creative ideas were a central part of the planning of the lesson. This demonstrated the different degrees to which teachers addressed traits of mathematical creativity presented in the professional development program.

Making Meaningful and Lasting Mathematical Connections. The majority of participants identified mathematical creativity as a way to help students make meaningful and lasting mathematical connections. There was an overall shift in that by the end of the professional development program participants described the role creativity could have in helping students make mathematical connections and therefore make a lasting impact on student learning. For example, Pamela noted the importance of having students understand a concept beyond memorizing steps. She saw mathematical creativity as a way to help students achieve understanding versus memorization when she noted:

> Well, if [using creativity in mathematics teaching] does happen, it will stick with the student more. And then we, maybe we won't have these problems at all, they didn't learn that or they don't understand fractions or they don't know how to add decimals, or oh, I went to the store and the computer broke and the person couldn't give me change. Well, maybe by having them do these types of lessons...from year to year, it'll stick. (Postlesson Interview, line 209)

Pamela came to recognize the importance of infusing creativity in mathematics in order to help students make meaningful and lasting connections with the content.

Similarly, Megan thought that infusing creativity into mathematics was, "beneficial because then they're learning different ways to approach problem[s] which, in the long run I think can be helpful and beneficial to them" (Postlesson Interview, line 130). These ideas substantiate participants' overall view that allowing, encouraging, and supporting students to think creatively in mathematics, instead of memorizing information for a test, might have a lasting impact on their understanding and overall learning of mathematics. Although the majority of teachers saw a benefit of infusing creativity in mathematics, their views of teaching and learning and the extent to which they implemented mathematically creative ideas in practice continued to vary.

Theme 3: View of Teaching and Learning

Literature on mathematical creativity identifies a need for teachers to encourage students to make mistakes, take risks, and support one another in their explorations (Ball, 1992; Levenson, 2013; Nadjafikhah et al., 2012; Neumann, 2007). In order to identify the ways in which participants thought about teaching and learning mathematics, I analyzed professional development session transcripts, preprogram and postlesson interview transcripts, and lesson plans for instances of level 3 codes, *environment* and *choice and discovery*. The results of this analysis highlighted differences in participants' focus on students' product or problem-solving process, ownership of learning, and teaching creatively versus teaching for creativity. These findings presented in the following section highlight the differences in participants' views of teaching and learning.

In a creative mathematics learning environment, teachers implement tasks that allow students to choose from various activities, discover mathematics, and create artifacts to explore the content. Participants' comments about the learning environment were categorized as *environment* and *choice and discovery* (see Table 7). Among these instances, various levels of detail existed from focusing on a direct answer to designing activities where students had to discover and create with minimal instruction.

Table 7

Environment and Choice and Discovery Code Descriptions

Level 3 Code	Level 3 Description	Level 4 Code	Level 5 Description
Environment	Environment Conducive to Creativity	**Support**	The teacher is supportive and encouraging of students' non-conformist and unusual ideas.
		Incorrect	The teacher is tolerant of ideas that do not lead to the correct answer.
		Atmosphere	The teacher develops an atmosphere focused on inquiry, curiosity, exploration, and self-directed learning.
		Risks	The teacher supports and encourages risk-taking and makes students aware that they will not be penalized for failing.
		Independence	The teacher emphasizes a sense of independence and responsibility for learning.
		Reflect	Opportunity for student to reflect on learning
Choice and Discovery	Opportunities for Choice and Discovery	**Activities**	The teacher creates learning scenarios where students can choose from one of several activities.
		Discovery	The teacher creates activities where students have to discover the answer by examining various models and ideas.
		Create	The teacher creates activities where students have to create an original artifact and present this artifact as a potential new solution to a problem.

Importance of the Process Versus the Product. There were varying degrees of participants' views about the importance of an answer (product) or the approach (process) in mathematics. Some participants described a more traditional practice of focusing on students' final answers over their problem-solving process. Others described mathematics as having multiple approaches to a problem and highlighted the importance of having students explore these different approaches but also described challenges that hindered this in practice. Still others highlighted the nature of mathematics as an elegant or multifaceted process, sometimes describing students' approaches as similar to that of mathematicians. These differences highlighted a distinction in the degree to which participants valued the product or problem-solving process.

Some participants described students' approach to a problem as important, yet continued to focus on the product in their description of students' work or in their discussion of mathematics and mathematical tasks. For example, Alex described a shift in thinking about mathematics from product oriented to process oriented noting that at the outset of the program she thought, "more about [the] final product, projects, and now I'm thinking about the process of it" (Postlesson Interview, line 139). However, when discussing students' work, Alex remained focused on students' products. For example, in discussing the importance of students' problem-solving processes, Alex noted that the lesson might be altered in future iterations to focus more on, "the self-discovery, the connections, because of course that is important, but I always just want to see the final outcome, which is important as well" (Postlesson Interview, line 100). Although the students' problem-solving process was noted as important, Alex continued to focus on students' final products and a singular, correct answer. For example, Alex noted:

I: You were saying…no matter how a student approaches the problem, as long as it's the right answer, you're fine [with their approach]. But, what do you do when a student approaches a problem in a way that you maybe weren't expecting?

Alex: But they're correct?

I: Yeah.

Alex: I'm probably impressed, and I probably talk about it…with the class. But then sometimes I'm like, should I share this? Or should I just let it be a moment for the child, because sometimes other kids get confused. So, I kind of have to like feel it out first. (Postlesson Interview, line 129)

This displayed Alex's continued focus on the correct, final answer, and described a reluctance to share students' different approaches with the class because it might be confusing for others.

There were also instances where teachers who initially viewed mathematics as only having a final answer shifted these ideas after participating in the professional development program to include the importance of the context in which a problem was approached or solved. For example, after participating in the professional development program, Rachel who initially discussed mathematics as a series of steps and procedures, came to recognize the importance of the process and understanding students' approaches to a problem. She commented, "that's something again that I will work on in September, the idea of stop worrying about getting it right or wrong. Let's focus on what we do know and go with that and build on that (Postlesson Interview, line 177). Rachel collaborated on a lesson that focused on students' problem-solving processes and their ability to make connections among mathematical concepts. She noted the importance of going beyond the step by step procedures to instead ask students to identify, "How did you arrive at your answer? Why did this happen? Compare this. It was more specific where before it was, justify your answer, [after] it was more specific in these lessons with the questions that I was asking" (Postlesson Interview, line 64). She also noted the importance of having students explain their thinking beyond describing the steps they took to answer the problem and asked more specific questions about students' process instead of procedural steps.

Ownership of Student Learning. Research suggests that one way to foster an environment conducive to creativity is for the teacher to take on the role of a guide rather than an authority (Elam & Duckenfield, 2000; Neumann, 2007). While some participants were more traditional in their role as an authority, others tended to view themselves as a guide and commented on the importance of fostering student independence. There was a distinct difference in participants' ownership of student learning. In some instances, participants described traditional, authoritarian environments where the teacher had authority and ownership of student learning, whereas others demonstrated more democratic environments where students had ownership of learning and were encouraged to be independent. Participants' lesson plans reflected their views of themselves as an authority or as a guide. For example, those who viewed their role as an authority tended to develop lessons where the teacher provided step-by-step instructions to solve a set of problems and students were required to take notes from a premade presentation. In contrast, other participants attempted to encourage students to discover mathematics through exploration, embraced multiple solutions to a problem, and allowed student independence in their development of solutions.

Mistakes and Risk Taking. Another way to demonstrate an environment conducive to creativity is for teachers to support students in their approach to solving a problem, even if that process leads to an incorrect answer, and view mistakes as an opportunity to learn. Participants described an environment that was conducive to mathematically creative ideas by viewing mistakes as learning opportunities and supporting students to be inquisitive, curious, and self-directed in addressing these mistakes (Table 7). The majority of participants highlighted the importance of mistakes and learning from mistakes in mathematics. However, some participants tended to take ownership of students' mistakes as a personal failure for the teacher to address and fix. These participants were concerned with students' mistakes as a representation of how they as teachers failed, exemplifying ownership over student learning.

In contrast, others focused on students' existing knowledge and worked with students to help them make sense of the content. For example, Rachel noted, "maybe they didn't get all the problems, correct…but they knew…the concept" (Postlesson Interview, line 97). She also noted that she wanted students to move past viewing an answer as wrong and instead commented, "I want them to get away from the idea of getting a problem wrong and let's focus on what you did right in the problem and let's go from there" (Postlesson Interview, line 96). Although some participants viewed mistakes as important, they did not provide evidence that they were reflecting on mistakes as a way to encourage students to take risks, reflect on their learning, or be independent. However, others saw mistakes as an opportunity to collaborate and help students gain conceptual understanding.

For some participants, there were more frequent instances of supporting students to take risks, be independent, and supportive of helping them identify and correct mistakes. Participants described classroom environments that mirrored these ideas in various ways. For example, many participants described the importance of mistakes and celebrating students' various approaches to problems. Audrey described an activity where both she and students had to take risks. She reflected:

> For volume and for surface area…kids created the shapes, they were able to measure things and obviously everyone cut differently so measurements were different. So they came up with [an] idea how to find surface area without me teaching them. They came up with it [on their] own. Without the [professional development] sessions, I would be afraid to trust them to do it on their own. I would probably give them instructions first and

> explain how to do it. But with you, I took a chance…let's see what they can see on their own. So, it gave me encouragement to do risky things in math. (Postlesson Interview, line 20)

The professional development program encouraged Audrey to take risks and foster risk taking among students. Audrey came to view the importance of allowing students ownership of learning and guiding them in making connections without direct teacher instruction.

At times, participants described an environment that was not conducive to creativity and instead focused on correcting mistakes for the state test. While some participants focused on the importance of correcting mistakes before the state test, others focused less on the state test and more on the importance of students learning mathematics for their future careers or lives. Some participants were tied to the curriculum pacing, restricted by time, and concerned with covering material before the state test. These barriers within the classroom environment hindered the level of participants' buy-in to foster students' mathematical creativity.

There were also differences in how teachers viewed students' mathematical capabilities as well as who they saw as having creative potential. While some participants held a more traditional view of gifted students' as having the ability to be creative, others saw infusing creativity in mathematics as a way for all leaners, particularly those who struggle, to gain access to the subject.

Teaching Creatively Versus Teaching for Creativity. Prior to participating in the professional development program, participants tended to focus on their ability to be creative in teaching mathematics. Aligned with existing literature, my analysis suggested that at the outset of the professional development program, participants thought about creativity from the teacher perspective with little or no thought about how students might be creative in mathematics or how teachers might foster students' creativity. Even when specifically prompted to think about student creativity during the preprogram interview, many participants continued to reference how they might teach creatively rather than how a student might be creative in mathematics.

One example of teaching creatively is when a teacher is focused on ways to alter their teaching practice by designing tasks, implementing a lesson, or revisiting a concept to attend to students' needs. For example, prior to her participation in the professional development program Jordan noted:

> If you're not [creative], sometimes students don't understand the concepts the first time you teach it. So being creative in the way that you are explaining the concepts or allowing them to think outside the box or break it down into pieces…sometimes we teach a lesson, you could see the stone faces on there and they're just like, "Oh no, this is not gonna work." We have to go back to drawing boards and I have to go home and think creatively or think, try to find a way of understanding my students and finding a way to teach to them a way in which they're gonna understand this concept. (Preprogram Interview, line 38)

This focus on how to alter her instruction to teach students who are still unclear about a topic, is focused on the ways in which Jordan was creative in presenting material differently. Even when asked how a student might exemplify creativity she responded about the ways she might change her practice to keep students engaged and involved in the lesson.

Throughout their participation in the professional development program, some teachers began to identify ways students might be creative and distinguished between teaching creatively and teaching for creativity. After participating in the professional development program, the majority of participants demonstrated recognition of teaching for creativity to include ways students might be creative and the role creativity had in helping students make meaningful mathematical connections. There was an overall recognition of how students might be creative in that after participating in the professional development program, participants discussed potential benefits of having students think creatively and what they as teachers can do to foster this creativity. For example, Rachel stated that:

> I used to think student creativity was group work and artsy assignments…where now I think creativity is more getting students to explain their thinking and their mathematical process…I think it's very important for them to explain it, but then me to really reflect on it and read it and take it and do something with it. (Postlesson Interview, line 156)

For Rachel, initially creativity was artistic projects whereas after her participation in the professional development program she identified how she might foster students' creativity by having them explain their thinking and describe the problem-solving process. She was creative in that she developed ways to encourage students to do this and she reflected on and used their ideas to inform her teaching practice.

The majority of participants came to recognize the possibility for students' creativity. Although all teachers described an evolution of beliefs about creativity in that they began to think about how a student might be creative in their approach or response to a mathematical task, and how the teacher might foster students' creativity, this was discussed in varying degrees and at times not implemented in practice. While some teachers designed lessons that fostered student creativity, others continued to focus on the product, or implemented the lesson in a traditional manner. In the following section I describe results for research question two focused on participants' experiences designing and implementing mathematically creative lessons.

Introducing Creativity into Mathematics Lessons

To answer my second research question, I reviewed participant-created lessons, student work, and postlesson interview transcripts for any mention of participants' lessons. The second research question was:

2. What are in-service mathematics teachers' experiences infusing creativity into their mathematics lessons as they participate in a professional development program?
    a) How do the ideas of fostering mathematical creativity explored in a professional development program transfer to in-service teachers' practice?
    b) How do teachers describe the development and implementation of lessons that require creativity and encourage creativity among students?

During the last two sessions of the professional development program, participants were tasked with designing a lesson inspired by mathematical creativity. They were asked to infuse elements of mathematical creativity that were discussed throughout the professional development program into their lesson planning.

There were varying degrees to which participants implemented ideas of mathematical creativity into their lessons. While some teachers designed the lessons around ideas of mathematical creativity, others adapted an original lesson plan to incorporate traits of creativity. Still others presented an existing, unchanged lesson. Some participants viewed an existing lesson as creative, making no change to the original lesson plan, whereas others designed a new lesson with the ideas of mathematical creativity as central to their development.

There were also differences in how teachers described the development and implementation of the lesson. While some teachers continued to implement traditional instruction with step-by-step guidelines, others took on the role of a guide and allowed students to work with fewer restrictions in their implementation of the lesson. In the following sections I describe the development and implementation of participant-created lessons separately, then highlight the major transition from in-person to online teaching and learning.

Developing Mathematically Creativity Tasks

Throughout the professional development program, teachers were encouraged to bring mathematically creative ideas into their classrooms (e.g. journaling, questioning strategies, opportunities for student discovery), reflect on the experience, and improve the ideas for future implementation. Ultimately, teachers were tasked with designing lessons that infused ideas of creativity in mathematics. Having students journal, discover, and create gave students ownership of learning and helped teachers to reflect and revise these ideas as they developed their own lessons infusing creativity in mathematics. The tasks introduced during the professional development program helped teachers recognize the importance of understanding students' reasoning, even when a student has the correct answer, and consider these ideas as they developed or transformed existing lessons to include mathematical creativity.

As participants developed lessons focused on principles of mathematical creativity, they differed in how they designed their lesson. Although the majority of participants created new lessons, others took a previous lesson and revised it based on the traits of creativity. Still others presented an unchanged lesson from previous years. There was a difference among those participants who chose to create new lessons. Some participants who developed new lessons to implement with students started with the mathematical content to design their lesson, then identified where traits of mathematical creativity might be apparent. Others used the traits of creativity as a way to design the lesson and decided on the content afterward. For example, Jane and Rachel began with the topic of area and volume then thought about how they would include mathematically creative ideas into the lesson. In contrast, Andrea first relied of the traits of creativity and designed her lessons with these traits in mind. She noted, "looking at the list of traits…how can I apply those different aspects…to make it interesting for the kids" (Postlesson Interview, line 310). All participants designed their lessons using one of these two methods.

Although teachers were asked to send me the lesson plans for feedback prior to implementation, only five of the 12 teachers complied. The lesson plans that were submitted for feedback (Audrey, Jane, Megan, Phyllis, Rachel) tended to be more aligned with ideas of mathematical creativity before receiving feedback, and participants' responses to researcher comments pushed their thinking further to develop more creative lessons. With the exception of Andrea, participants who did not receive feedback (Alex, Andrea, Dana, Drew, Jordan, Stephanie, Pamela) tended to design lessons that were less inclusive of mathematically creative ideas. This might be because the participants who sent me lessons for feedback were more involved and had greater buy-in to the ideas presented in the professional development.

Influence of Professional Development Program on Lesson Design. Participants' experience during the fourth professional development session helped them to take on the teacher role and think about how they might design tasks for their own classrooms. The fraction task (**Error! Reference source not found.**) encouraged participants to see direct connections between this task and their own classrooms. Phyllis found that that fraction task, "helped [students] to visualize [fractions]…to get them to understand how it looks, because sometimes it's very abstract for them... I think using that would work really well" (Professional Development Session 4, line 136). Similarly, during the postlesson interview, she commented that when completing the fraction task herself, she tried to put the tiles together in different ways.

She reflected that she drew on ideas from the fraction task in designing her own lesson. Phyllis noted that the fraction task:

> Was very hands on and I was trying to think of something that they would do without any [manipulatives]…I was trying to think of what do they have that at home? Well, they all have a computer or a phone and they can time it on their computer and then do the exercises themselves. (Postlesson Interview, line 24)

Because of online learning Phyllis was unable to provide students with manipulatives. Instead, she designed a task that required students to use take measurements at different intervals to write and graph ordered pairs using numerical patterns. Her lesson drew on ideas of creativity including providing opportunities for choice and discovery. She accomplished this by presenting students with tasks that allowed them to choose different activities, intervals, and tools for collecting and reporting their results. She also encouraged students to be curious about mathematical connections when she asked students what would happen if the interval were changed as well as visualizing the information graphically. She reflected on students' excitement completing the lesson and noted that she, "would love to keep it up, and especially since Audrey…she's the sixth-grade math teacher, so, if I do something [creative] maybe she could carry it on" (Postlesson Interview, line 111). Phyllis' experience taking ideas from Session 4 to design her lesson was similar to other participants' lesson designing experience. While many participants recalled the activity from Session 4, others cited the importance of journaling.

Lesson Design and Journaling. The professional development program was centered around encouraging participants to make connections between the ideas presented throughout the sessions and their own classrooms. Between Sessions 5 and 6, participants were encouraged to have students journal and reflect on the experience. Several teachers highlighted the importance of having students journal in mathematics after completing the task. Megan described students journaling and creating their own problems as a way to reflect on proportional reasoning. Having students create new problems for other students to consider helped Megan:

> See common misconceptions that I was missing maybe. Because they kept, even in their journals, they were saying things that maybe weren't proportional. Or they were making problems that weren't actually a proportional relationship, they were something else, so it helped me kind of see their misunderstandings and their misconceptions. (Postlesson Interview, line 16)

Journaling was something Megan had not done prior to participating in the professional development program. However, she planned to use journaling in the future, beginning at the start of the next school year, to help foster student reflection and encourage them to develop new questions for others to consider.

Many participants also chose to include journaling or reflective questioning similar to the ones posed in the journaling activity into their lesson designs. Megan's exponential function task required students to watch a video and predict how many folds it would take for a stack of paper to reach the moon. She included reflective questions, similar to those that were included in the previous journal prompt, in her lesson design. For example, Megan asked students to think about what other information they might need to solve the problem and how they first thought about approaching the task. These reflective questions were creative in that they required students to think about mathematical connections and reflect on their problem-solving process. Other participants also implemented reflective questioning in their lesson design as a way to encourage student creativity.

Implementing Mathematically Creative Lessons

After shifting to online teaching and learning, teachers in the Tarnot district were not permitted to meet with students in synchronous sessions. Instead, they used email or a protected online chat to engage with students. The nature of asynchronous meetings impacted how teachers implemented mathematically creative lessons. The degree to which participants implemented mathematically creative lessons varied in that some teachers continued to demonstrate traditional teaching strategies whereas others used the online learning environment as a way to foster students' independence. Participants who exemplified traditional lesson implementation recorded Screencastify videos of themselves completing a task and provided step by step instructions or relayed a series of procedures for students to mimic. Those who tended to demonstrate more creative lesson implementations used the online environment to encourage students to respond to and discuss one another's solutions, work independently, and discover mathematics with less teacher direction.

For example, Andrea created an assignment where students were encouraged to respond to one another and discuss their approaches to a problem. She used an online chat where students posted answers to a prompt and pushed one another's thinking by responding and debating answers. The activity had multiple answers which encouraged students to *be curious about mathematics* and *continue to explore after a problem had been solved*. Andrea noted that she structured the chat to allow students to see one another's responses which allowed students to view and comment on other students' ideas. Andrea described students experiences as engaged and excited. She noted, "different students saw different [answers]…they're like, "Oh yeah, I didn't notice that!" And they were…seeing what they saw, but then when they're hearing what their peers were [saying]" (Postlesson Interview, line 27). This allowed students to see different perspectives for solving a problem and build off of their peers' initial ideas.

Teachers described benefits to implementing lessons focused on mathematical creativity. For example, several participants noted that students were more engaged in the tasks and felt that implementing tasks focused on mathematical creativity allowed the content to be more personal for students. Participants also noted that they enjoyed implementing these lessons because it was fun and different than traditional lessons and it allowed them to gain different insight into students' thinking. However, some participates were disappointed students' responses and attributed this to online learning as well as students' inexperience with the type of thinking required of mathematically creative tasks.

Participants described challenges implementing mathematically creative tasks. For example, a few participants described time as an issue because they were only checking in with students twice a day and were unable to recreate the in-person teaching and learning experience. In contrast, others came to recognize that implementing mathematically creative lessons did not require extra time. Rather, it was easily built into the content by shifting questioning strategies which allowed them to elicit students' creativity. Some participants struggled to release control to students whereas others implemented lessons taking on the role of as a guide. In particular, Pamela noted that she was uncomfortable allowing students to have more independence because she feared that they would fail and be discouraged if left on their own. However, others felt that the independence was beneficial and important for student learning. Megan commented, "it's more beneficial for me, even just letting them explore things more and try to come to their own conclusions first before going over it together as a whole group" (Postlesson Interview, line 66). All participants noted the challenge online learning presented for many reasons (e.g. limited interaction with students, student technology issues). However, the majority of participants looked forward to implementing their lessons focused on creativity in person next year.

Transition to Online Teaching and Learning: Mid Program Shift

The COVID-19 pandemic required a shift from in person to virtual meetings for the remaining four professional development sessions. This change in modality for implementing the professional development program was not planned. However, the facilitator explored the idea of creativity within this unusual and immediate shift to online teaching and learning. In response to a survey, participants described how they were required to be creative in the new environment of online teaching and learning. They noted that they had to find alternate ways to present content while not being able to interact with students in a synchronous platform. They did this by encouraging student independence, drawing on new resources for the online environment, and demonstrating flexibility in their approach to teaching and learning.

This online learning environment shifted the way teachers thought about engaging students in mathematics. For example, the majority of teachers noted the importance of student independence and many realized that their previous teaching style impeded student independence. In the online learning environment, students had difficulty taking responsibility for their learning as they lacked the needed independence for completing and engaging with mathematical tasks. Many participants highlighted an immediate and apparent need to foster students' independence and recognized the importance of understanding students' reasoning, steering away from traditional instruction.

The online learning environment impacted teachers in different ways. In some instances, online learning hindered ideas of mathematical creativity whereas in others it encouraged teachers to be creative and foster creativity among students in designing lessons that fostered student independence without the opportunity for synchronous teaching interactions. Some teachers attempted to recreate traditional teaching by delivering content through step-by-step instructions through Screencastify videos. However, others developed ways to have their students explain their reasoning and collaborate asynchronously through discussion forums, building upon other students' responses, and arguing about different approaches and solutions to problems.

Although the transition to remote professional development sessions was not designed by the researcher, it allowed for both affordances and drawbacks to the professional development program itself. Unforeseen benefits included allowing participants to collaborate more effectively in small breakout rooms with less distraction from other groups and more input from the facilitator. Drawbacks included issues facilitating tasks in an online setting because participants had internet connection issues and difficulties finding and manipulating files. Had the remote meetings been planned at the outset, the sessions might have been structured differently and tasks that would have been more amenable to an online setting would have been chosen.

Description of Categories Within the Main Unit of Analysis

The analysis of data from the 12 participants in the professional development program led me to identify the degree to which teachers personalized, identified with, and instantiated ideas of mathematical creativity into their practice. This resulted in the development of three categories, *adherence to traditional teaching practices (traditional), appreciation of teaching for creativity (creative but hesitant),* and *teaching for creativity (creative)* (). In the following section, I describe these categories in more detail.

Table 8

Participants Organized by Category

Adherence to Traditional Teaching Practices (Traditional)	Appreciation of Teaching for Creativity (Creative but Hesitant)	Teaching for Creativity (Creative)
Alex	Andrea	Audrey
Drew	Dana	Jane
Jordan	Pamela	Megan
Stephanie	Phyllis	Rachel

Adherence to Traditional Teaching Practices (Traditional)

Teachers in this category continued to teach within traditional paradigms (e.g. viewing only advanced students as creative, taking ownership of student learning, providing step-by-step procedures) and in some cases interpreted mathematical creativity differently than how it was introduced in the professional development program. Although no substantial change was evident in my analysis, some participants in this category relabeled their teaching as creative.

One example of relabeling her teaching as creative was when Stephanie noted that after participating in the professional development program, she recognized the things she was already doing in her classroom as creative. However, her lesson plan was focused on students' artistic design over the mathematics process and did not attend to the traits of mathematical creativity. In particular, she noted that, "The first part of designing the truck had really nothing mathematical...I said [to the students], this is just about following directions, being fun and creative. And you're getting just as many points as you [are] for the math" (Postlesson Interview, line 24). This was similar to other participants in this category who continued to view creativity as only artistic and developed lessons mainly void of mathematical creativity.

Participants in this category viewed their role as being mainly responsible for student learning and although they attempted to teach creatively by implementing fun activities, manipulatives, or artistic projects, there was little evidence that participants were able to tie these activities to mathematical creativity, require students to be curious about and discover mathematics, or identify multiple approaches to a problem by thinking deeply about the content.

Appreciation of Teaching for Creativity (Creative but Hesitant)

Participants in this category saw mathematical creativity as important for student learning. However, a barrier existed that prevented teachers from weaving mathematical creativity into their everyday practice. These participants mentioned a need to promote student creativity by having students *explain reasoning* and *be curious about mathematical connections*. However, they also described these ideas as impractical or designed lessons that did not elicit these ideas from students. The implementation of mathematically creative ideas was inhibited by barriers such as constraints of state testing, curriculum requirements, time needed for students to complete and teachers to grade mathematically creative tasks, and student achievement levels.

Participants in this category described mathematically creative ideas but tended not to present information in a creative way. Instead, they continued to implement tasks focused on the steps to solve a problem, procedural memorization strategies over conceptual understanding, and continued to have ownership over student learning. Although participants acknowledged the usefulness of infusing creativity into mathematics for students' deeper understanding and learning the material, they were hesitant to fully implement these ideas into their practice. A contradiction existed in participants' views and implementation of mathematically creative ideas. While participants in this category were able to identify the importance of mathematical creativity and in some instances attended to mathematically creative ideas, mathematical creativity was not embedded throughout their teaching practices.

Teaching for Creativity (Creative)

Participants in the *teaching for creativity (creative)* category provided evidence of teaching practices, views of student learning, and lesson development that were aligned with research on teaching for creativity and fostering students' mathematical creativity. Traits and principles of mathematical creativity were evident in how mathematics and creativity were defined and their views on teacher and student roles in the classroom. Participants' lesson plans included activities and questioning strategies that encouraged students to think differently about mathematics, allowed for student discovery, and fostered student independence.

The language in the lesson plans and postlesson interviews provided evidence that the teachers' role was seen as a guide instead of as the authority of knowledge. For example, one teacher noted a shift from giving formulas for students to enter numbers into and calculate an answer, to having students explore connections between formulas and discover similarities and differences, deepening students' conceptual understanding of the topic. Participants in the *creative* category engaged students in challenging problems, used journaling to better understand how students approached a problem, made connections to things outside of the math classroom, and identified students as responsible to solve and discover mathematics. Participants highlighted the importance of having students demonstrate their understanding of a topic and were interested in students' problem-solving process rather than a final answer.

Conclusion

The analysis of the whole case, 12 participants in the professional development program, led to the identification of three distinct categories in how teachers viewed and approached mathematics, creativity, and teaching and learning. To further differentiate the three categories that emerged from the data, I chose to further describe one exemplar from each group based on their overall representation of the group and their vocal participation in the professional development sessions. In the following section I report on the results of a deep analysis of each exemplar and further describe the aforementioned themes that arose among the analysis of the group of participants in the professional development program and to distinguish among the three categories, *traditional, creative but hesitant,* and *creative*.

Results from the Three Exemplars

The results presented in the following section more thoroughly describe the categories *traditional, creative but hesitant,* and *creative*. Through the analysis of the exemplars I highlight the main similarities and differences among the categories, *traditional*, *creative but hesitant*, and *creative*.

Stephanie, Adherence to Traditional Teaching Practices (Traditional)

> Sometimes I feel like I missed the target with what she's talking about, with being creative, because I don't consider myself creative at all, but then when I have it worked out, I guess it is just that simple. (Professional Development Session 8[2], line 94)

Stephanie is a 7th grade teacher at Center school with 18 years of teaching experience. She holds K-12 and middle school mathematics certifications. Her opportunities to collaborate with colleagues include monthly district meetings and weekly grade level meetings but she has not had the opportunity to meet with cross-grade level mathematics teachers. Although Stephanie viewed her teaching practice as creative, her lesson implementation remained focused on an artistic product as opposed to mathematically creativity ideas. At times throughout the professional development program, Stephanie mentioned strategies she defined as creative. However, these ideas did not yet align with research on mathematical creativity and there is not sufficient evidence in her lesson plan or discussions regarding teaching and learning that she is convinced of the importance of implementing these ideas in practice. A further exploration of Stephanie's view on mathematics, creativity, and teaching and learning is provided in the following section.

View of Mathematics

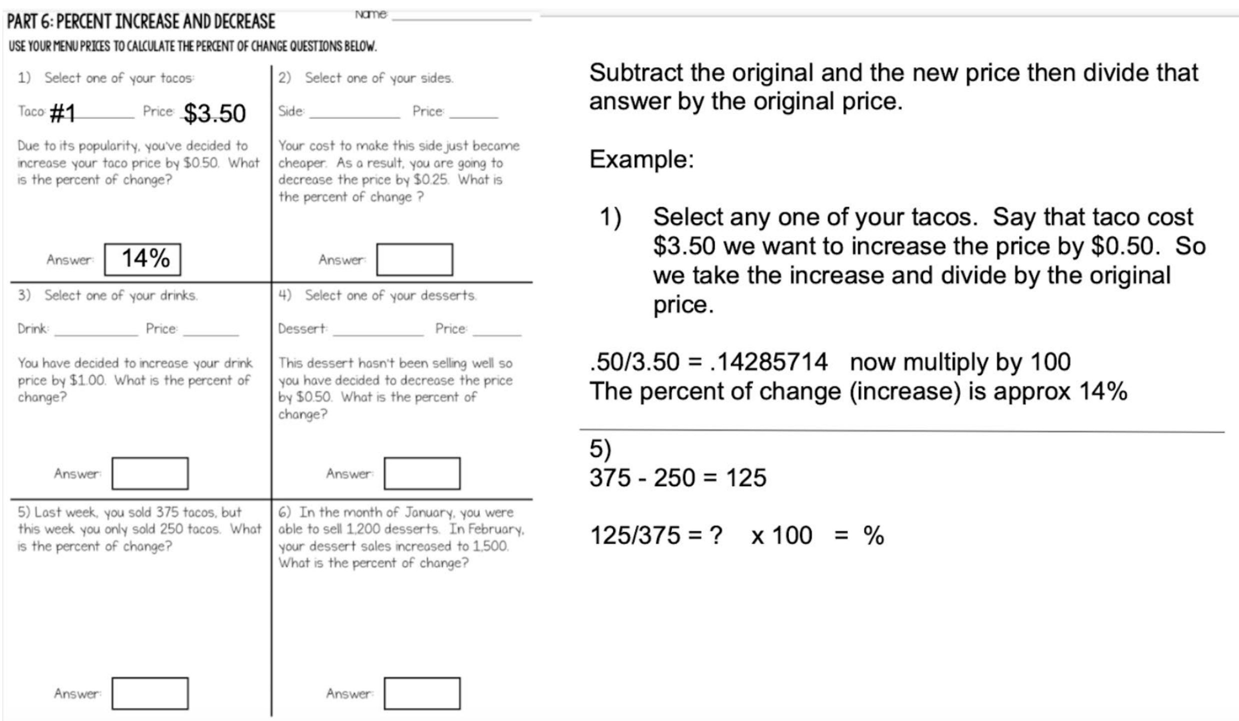
Both prior to and after the professional development program, Stephanie viewed mathematics as concrete and enjoyed that math had definite answers, particularly at the seventh-grade level. At the outset of the professional development program, she described liking mathematics because, "this is the answer" (Preprogram Interview, line 17). Even after participating in the professional development program, she explained that, "the math that they require someone at this time [seventh grade] is all definitive, they have answers. They know they have to get to [the] answer" (Postlesson Interview, line 154). She also noted that open-ended questions, where mathematical creativity might be evident, were too time consuming and instead felt that straightforward answers allowed students to demonstrate their understanding of a concept. This is aligned with a traditional view of mathematics as concrete, definite, and final.

Stephanie's view of mathematics as concrete was also exemplified in a discussion of nonstandard measurement during the final professional development session. Teachers were asked to revise a traditional task to be more aligned with mathematical creativity. Specifically, teachers were asked to create a mini-lesson addressing the same topic from the problem in an intentionally creative way by purposefully including one trait of creativity. The traditional task was, "A circular mirror has a diameter of 12 inches. What is the area, in square inches, of the mirror? What is the circumference of the mirror?" Stephanie's group developed a task on nonstandard measurement using household objects. Dana, another participant, suggested that they present material without using numbers. Building upon this idea, Stephanie mentioned nonstandard measurement, but she did not like the idea herself. She noted, “I always thought it [nonstandard measurement] was stupid. Because I like exact; but I get it how they could gauge things so maybe that’s something that you could do is even give them little Legos” (Professional Development Session 8[2], line 74). She noted that nonstandard measurement would be helpful for visual learners but commented that she, “would hate this, I would hate measuring with things, I want a ruler. Give me a ruler” (Professional Development Session 8[2], line 87). Although Stephanie agreed to include nonstandard measurement as part of their task, she did not provide evidence that these ideas would be useful or observable in her teaching practice. Instead, she gave insights into her discomfort with the exploratory nature of mathematics and described the importance of continued focus on straightforward answers.

Her views of mathematics as concrete and definite were also apparent in her lesson plans. Stephanie’s first lesson had seven parts which required students to design a virtual taco truck, take someone’s food order, and calculate percentages for various scenarios. The first two parts of the lesson focused on the design of the taco truck and required students choose items for a menu. The third part of the lesson required students to take orders from six customers. The last four parts of the lesson presented students with different scenarios to calculate sales tax and discounts based on the order and prices on their menu. For example, one question read: “Customer three found a hair in their taco! To make up for it, you gave 40% off their order, but they still had to pay a 4.5% sales tax. What is the new cost of the order?” (Lesson Plan). The five other questions on this page were similar in nature and required students to calculate given discounts or increases. Other tasks required students to calculate the percent of change from one price to another after a price was increased by a certain amount. For example, students were directed to select one taco, provide the price of the taco, and calculate the percent of change when it increased by $0.50. In the margin of the page, Stephanie also provided examples using arbitrary numbers to show the procedure for setting up and calculating each set of questions. See Figure 8 for an excerpt from Stephanie's lesson plan.

Figure 8

Excerpt from Stephanie's Taco Tuck Lesson Plan



PART 6: PERCENT INCREASE AND DECREASE

Name: ______

USE YOUR MENU PRICES TO CALCULATE THE PERCENT OF CHANGE QUESTIONS BELOW.

1) Select one of your tacos:

Taco: #1 Price: $3.50

Due to its popularity, you've decided to increase your taco price by $0.50. What is the percent of change?

Answer: 14%

2) Select one of your sides.

Side: ______ Price: ______

Your cost to make this side just became cheaper. As a result, you are going to decrease the price by $0.25. What is the percent of change ?

Answer:

3) Select one of your drinks.

Drink: ______ Price: ______

You have decided to increase your drink price by $1.00. What is the percent of change?

Answer:

4) Select one of your desserts.

Dessert: ______ Price: ______

This dessert hasn't been selling well so you have decided to decrease the price by $0.50. What is the percent of change?

Answer:

5) Last week, you sold 375 tacos, but this week you only sold 250 tacos. What is the percent of change?

Answer:

6) In the month of January, you were able to sell 1,200 desserts. In February, your dessert sales increased to 1,500. What is the percent of change?

Answer:

Subtract the original and the new price then divide that answer by the original price.

Example:

1) Select any one of your tacos. Say that taco cost $3.50 we want to increase the price by $0.50. So we take the increase and divide by the original price.

.50/3.50 = .14285714 now multiply by 100
The percent of change (increase) is approx 14%

5)
375 - 250 = 125

125/375 = ? x 100 = %

For each of the four sections that included mathematics, students were required to complete a straightforward calculation for an end result. Stephanie self-recorded a Screencastify video that included detailed steps for designing a taco truck in Google slides and specific directions for completing the activity. When reflecting on the lesson, Stephanie noted that the math involved following directions and connected this skill to completing word problems. Students received equal points for the taco truck design and the mathematics. This view was consistent with Stephanie's definition of mathematics as concrete and a series of definite answers in that students were required to follow directions and give direct, calculated answers, absent of conceptual understanding of the content. She described the creative portion of this activity as students' design of the taco truck and viewed the mathematics as a separate part of the task.

The importance of following directions was further highlighted in her discussion of students' products. She commented, "I still had kids leave out phone numbers… So, it's all about the following directions…it may not be a math skill, but it's a skill required" (Postlesson Interview, line 24). Even though Stephanie provided students with a Screencastify of step-by-step instructions, and these instructions were outlined on the assignment itself, she noted that students still did not follow directions.

Stephanie's lesson plan was implemented in previous years. Although teachers were encouraged to do so, Stephanie did not submit her lesson to me for feedback prior to implementation and noted that no changes were made to the original lesson based on mathematical creativity before she implemented this with her students. She also noted that only basic mathematics calculations were required of students. This decision to present me with unchanged, existing activities might be because she misinterpreted the definition of mathematical creativity as it was presented during the professional development program, viewed her existing activity of designing a taco truck as creative, and was rigid in her teaching practice, unable to consider suggestions. Her traditional view of mathematics as concrete and definite, and continued focus on the importance of following directions rather than developing conceptual understanding was consistent with those in the *traditional category.*

View of Creativity

Stephanie also continued to describe traditional views of creativity even after participating in the professional development program. An environment conducive to creativity is one where the teacher is supportive of risk taking and students' ideas, even those that do not lead to the correct answer. Teachers in a creative environment emphasize student independence and provide opportunities for reflection. In a creative learning environment, the teacher develops an atmosphere focused on inquiry, curiosity and self-directed learning.

Stephanie identified her teaching as creative by presenting material to students in different ways. However, these different ways remained aligned with traditional views of creativity as artistic, fun, and disjoint from mathematics (Bolden et al., 2010; Patston et al., 2018). Prior to participating in the professional development program, she noted, “I just feel like if you’re not evolving, changing, capturing them and I guess me creatively thinking how to do it differently, they’re not going to listen, they’re not going to be involved, they’re not going to care" (Preprogram Interview, line 65). This further demonstrated Stephanie’s view of teaching creativity in that she was creative in how presented material to keep students engaged. Although Stephanie was confident (4 on a scale of 5) to implement these ideas in her classroom, her view of a mathematically creative task was different from that presented throughout the professional development program. After participating in the professional development program, she continued to describe creativity as artistic, fun, and void of conceptual connections in mathematics. She also focused solely on how the teacher might be creative which was representative of Stephanie’s overall view of creativity.

Creativity as Artistic. At the outset of the professional development program, Stephanie viewed art, poetry, and music as creative. She also noted entrepreneurs and inventors as creative. She explained:

> There’s something about people who go and take their ideas, like your Bill Gates and your Steve Jobs and you’re like, Oh my God. Like who saw this coming? They did like in

> a garage. That to me is creative. Of course, people were always going to write songs and poetry and that's always going to be creative" (Preprogram Interview, line 73).

Although Stephanie recognized innovative ideas and divergent thinking exemplified in technological advances, she continued to view art as creative. Initially, she saw music, poetry, and inventions as creative. This idea did not evolve throughout her participation in the professional development program. Instead, she distinguished between her definition of creativity and the facilitator's definition when she noted:

> I don't think it's your definition of creative in the sense like I think of creative. I always think of artsy when I think of creative, but I do think that there's a level of creativity in math, basic, but I think a lot of kids lack it, they just accept it as God. They're like this is it, this is the answer. (Postlesson Interview, line 142)

Stephanie agreed that there was a basic level of creativity in mathematics but also argued that seventh grade students lack creativity and instead focus on the final answer. This comment on students' view of mathematics as definite and disjoint from creativity is aligned with Stephanie's view and presentation of mathematics as concrete and definite. Because Stephanie presented material in one way and focused on students' final product, students also came to view mathematics as focused solely on an answer.

Stephanie highlighted that student creativity was exemplified in the artistic product of the taco truck lesson rather than through traits of mathematical creativity or students' problem-solving process. She noted that when implementing the lesson, she "started with the design portion. So, I started with the creativity...I think you're starting out that creative part [of] drawing, the coloring, the creating a menu…[Students] actually really loved it..." (Postlesson Interview, line 21). Stephanie viewed creativity as artistic and described it as a way to engage students. Engaging students might be a positive outcome of implementing mathematical creativity into middle school teaching and learning. However, the goal of creativity is to encourage students through meaningful mathematical explorations rather than surface level artistic presentations.

Infusing creativity with mathematics should support students in making meaningful and lasting connections with the material. However, when asked about the mathematical objectives for her creative lesson, Stephanie described using easier numbers and focused on creativity in mathematics as less challenging. She shared:

> The second part of creating the menu was basic math, you have to have something and it can't be more than $5, and I had kids make tacos [for] $15 and I'm like the direction[s] said you can't have more than five. So again, following directions and that was basic math, adding up their total. So that was the first part of the project, and I don't really think the beginning part had a ton to do with math, other than following directions. (Postlesson Interview, line 24)

For Stephanie, the professional development program allowed her to identify things she was already doing as creative rather than promoting creativity among students. Placing a focus on following directions, although important at times, is not considered creative in literature on mathematical creativity. For Stephanie, mathematical creativity made the content easier rather than more challenging. This is different than how ideas of mathematical creativity were presented throughout the professional development program.

This task had the potential to be mathematically creative in that students might have explored mathematical connections between percents in varying scenarios, explained their reasoning as they solve percent problems, created and built off of other students' ideas, or depicted their reasoning visually. However, no attempt was made to have students think deeply about mathematics in a creative way. Instead the focus of the lesson was on the artistic nature of the taco truck design, following directions, and completing straight forward calculations.

Creativity as Fun. Another way Stephanie defined creativity in mathematics was presenting math as a game and making the content fun rather than boring. For example, she noted:

> I think if a student doesn't see [math] as work, or like the taco truck, they didn't see that as work, they thought it was fun. It was fun. I think that's the creativity. I think if they don't view it as a chore, then it was successful in being creative, because I think the worksheets, the textbook, all that boring stuff, that's a chore for them. And I think they're very turned off to it. (Postlesson Interview, line 178)

This idea of the taco truck lesson as different from boring textbook problems was further described in how this task was implemented in past years. She commented that when the taco truck lesson was implemented in the past, Stephanie would bring in nachos and make it a celebration. This example further highlights Stephanie's view of creativity as fun for students. Stephanie also distinguished between boring worksheets and implementing activities that had students focus less on mathematics and more on a task or game. Although Stephanie identified the taco truck lesson as something that was fun and students enjoyed, there was no evidence that this lesson required or encouraged students to think creatively or develop conceptual understanding of the material.

Similarly, when talking about her second lesson on integers, Stephanie again referenced creativity as a game. She commented, "I think that's being creative, taking a boring lesson on integers and making it a game" (Postlesson Interview, line 176). The lesson Stephanie presented to me was a virtual escape room that required students to answer questions involving integers in order to unlock the next question and complete the challenge of escaping the room. This activity on integers included placing integers on a number line and calculating basic operations with integers. Stephanie noted that students liked this lesson because, “they were able to do it on a calculator and they didn’t have to think much. They were able to plug in, positive, negative, they didn’t have to think” (Postlesson Interview, line 78). Stephanie’s view of creativity as a game also eliminated any challenge and instead focused on engaging students in a simple task.

Viewing mathematics as a game has the potential for mathematical creativity in that a game might provide ways for students to discuss and explain their reasoning, continue to explore ideas, or develop new questions in mathematics. However, Stephanie did not expand upon how the idea of math as a game might be creative, nor did she make connections to challenge required of mathematical creativity. While the mode of delivering content as a game might have been different from a traditional worksheet, the mathematical demand was unchanged and she did not purposefully attend to creativity.

Similar to thinking about how portraying mathematics as fun might engage students, Stephanie also viewed implementing activities where students get up from their seats, as an example of creativity. She commented:

> It’s like just something as simple as chart paper, putting questions up. I’ll have the kids go up with different color markers and they have to contribute an answer, it’s silly, but I guess that’s creative in the sense that they don’t realize what they’re doing because they will just want to get up. (Preprogram Interview, line 65)

On its own, having students leave their seats or contribute an answer to communal chart paper is not inherently creative. Instead, creativity might be exemplified in having students expand initial ideas on the chart paper, develop novel approaches to a problem, or be curious about mathematical connections. Stephanie's view of creativity as artistic or as a game, might be further explained by Stephanie's misinterpretation of mathematical creativity.

Misinterpreting Mathematical Creativity. Although it was not evident in her lesson plan, Stephanie identified her lesson and teaching as creative. When asked what traits of mathematical creativity were evident in her lesson design, Stephanie identified that students provided multiple solutions to a problem, engaged in non-algorithmic decision making, and continued to explore after a problem had been solved. Although she viewed these as evident in her lesson plan, she misinterpreted them to mean something different than how these ideas were presented in the professional development program. When asked what traits of mathematical creativity were included in her lesson, she commented:

S: General [creativity] definitely because they just had to, well, they had...multiple solutions to the problem. The problem was, they had to find people to take the order...So they had to take someone's order. I definitely think...non-algorithm[ic] decision making. The continue to explore was big because they...took the order and then it wasn't just done, then they had to take the order and do stuff with it...there was a hair in this, take 40% off and you still had to do tax and you were giving [a] discount or things like that. So there was more, they had to explore after a problem had been solved.

I: Mhmm. And then in the non-algorithmic decision making, was that...a little bit of their freedom to create the menu and figure out the pricing...there [were] no...instructions?

S: Nope, they had to come up with everything and they just had to follow [a] guide, like the tacos couldn't be more than $5. (Postlesson Interview, line 29)

Stephanie misinterpreted the traits of mathematical creativity when she noted that her lesson plan had students investigate multiple solutions to a problem, engage in non-algorithmic decision making, and continue to explore after a problem had been solved. When thinking about seeing multiple solutions to a problem, she noted that students were required to ask different people for their order. This is different from how multiple solutions are defined in the literature where students might come up with several different ways to solve a problem or see that there is more than one answer to a problem. Simply making up prices for items and taking someone's order would not be considered exploring multiple solutions to a problem. Similarly, Stephanie misinterpreted non-algorithmic decision making. According to literature, non-algorithmic decision making refers to students working through a problem without a formula or algorithm (Ervynck, 1991). However, Stephanie defined it as students being given freedom to develop items for a menu. Stephanie noted that students engaged in non-algorithmic decision making by choosing the price of tacos, yet this was not mathematical. Instead, the mathematics required was calculating percentages in a concrete manner and using a formula with no exploration of mathematical ideas.

Stephanie also misinterpreted the idea of having students develop questions for others to consider. She noted that students were required to develop new problems when she stated:

> I didn't even tell them, "be easy, make your taco a dollar, make it two." [I] have people making it $1.47 which then, when you're starting to do your percentages, they realize, "Oh my God, I'm getting these long answers." And then they're asking me, "well, how do I round money?" I'm like, Oh my God, how many decimal places does money have?

> So, they did have that...flexibility in a sense, they really created it themselves. They created their own problem. The problem was there but they created their own numbers. And so they made the problem hard or easy based on their numbers and some kids figured it out. Some kids, you could see, because I could see all their edits, so they started doing the math, they went back and changed the prices to make it easier. (Postlesson Interview, line 33)

Students were asked to solve six problems that required them to calculate a specific percentage based on their original order. Although students decided on the cost of the taco order, they were inputting these numbers into the teacher created problems. Stephanie described this as problem posing. However, this is different from mathematical creativity that provides opportunities for students to create a new problem for others to consider.

She also misinterpreted ideas of mathematical creativity when asked to reflect on changes she could make to her lesson to include mathematically creative ideas. She commented that students could create their own questions but misinterpreted the meaning of this idea. Although developing new questions is considered mathematically creative, Stephanie's discussion of this tended to be more traditional with set parameters and as separate from rather than central to the activity. She reflected:

> I definitely think you can make a question…it's really just more about [the] thought process and less about the answer. Like for example, I guess I could ask a question, how would you do sales tax? And I'll have some kids do the two-step and I'll have some kids do the one-step. So it's really, there [are] multiple answers to a problem. So I do think I could do that to foster discussion, but I think overall, the curriculum itself that they learn

> they have answers. I think that's right. I think that sounds good. (Postlesson Interview, line 155)

Although Stephanie noted that she could ask a question about sales tax and allow students to complete the problem in one or two steps, there is no extension of how this problem required students to recognize multiple approaches to the problem, or highlighted the importance of students' differing approaches. Again, Stephanie misinterpreted the traits of mathematical creativity and focused on doing a problem in one- or two-steps.

Her discussion of steps seemed to be related to the idea of simplifying a two-step process into one-step. For example, she described an instance where she explained how to find the total of something after sales tax by calculating 107% of the original price rather than calculating 7% of the price then adding that to the original price in order to calculate the total. This is different from mathematical creativity that is focused on students' process and conceptual understandings over reiterating procedural steps.

Although Stephanie did not see creativity as difficult to implement, her view of herself as creativity was also misaligned with research on mathematical creativity. During the postlesson interview, Stephanie noted that she did not realize what she was already doing, in terms of presenting material differently and working with each new group of students' needs, as creative. She described the term "get creative" to mean having students complete a project, never thinking that, "the different styles of presenting material would be thought of as creative" (Postlesson Interview, line 174). This example highlighted that Stephanie remained static in her teaching practices but relabeled things she was already doing as creative. She believed that she was creative:

> Just by doing things a little differently. I don't think it takes much to be creative, as much as I thought it did. I thought it would be a ton of work. I thought it was gonna be so hard but really when you think about it, the little stuff I do anyway so, I never thought of it. Just something as simple as, I love the "I have, who has" cards. I actually just placed an order for more of them because it's interactive, the kids like it. I could do it as a whole class, I could do it as a small group…and they really get into it." (Postlesson Interview, line 176)

Stephanie continued to teach similarly to how she had prior to her participation in the professional development program yet re-labeled her actions as creative. Although she did not add ideas of mathematical creativity into her teaching practice in the ways that were offered during the professional development program, she now saw her teaching and existing, unchanged activities as creative.

Stephanie viewed projects as creative and commented that they were appropriate only after all concepts were covered and state testing was complete. She described introducing projects, "normally [in the] fourth marking period after testing, I do a lot of work and group projects" (Professional Development Session 5, line 2040). This was reiterated in the postlesson interview when she commented, "In June, really after state testing, I go right into project-based learning… I don't find [project-based learning] practical for the whole year, every lesson" (line 89). Viewing creativity as separate from mathematics content is a common traditional view where projects and student collaboration are introduced after skills are covered and state testing is complete. It is also not evident if the projects implemented after the state test attended to mathematical creativity. Although she explained that projects were introduced later in the year, she did not view the idea of shifting her questioning or having students explore topics throughout the year as practical and instead saw it as more difficult to be creative in middle school.

View of Teaching and Learning

Stephanie's view of teaching and learning extended from the idea of introducing projects and creativity after state testing. She also focused on students' product rather than their process of solving a problem, tended to take ownership of student learning, and characterized students based on their mathematics achievement levels. These ideas, aligned with a traditional view of teaching and learning, are further explored in the following section.

Focus on Student Product Over Process. After participating in the professional development program, Stephanie noted the importance of focusing on students' problem-solving process over their final answer. However, she felt that in attending to the process, she was less rigid than she was prior to her participation in the professional development program. She noted:

> My expectations [of students] changed. Because I felt like I softened a little bit. I was always a little rigid...if I didn't see certain [test] scores, then they didn't learn and it was always my fault. Just thinking well, so I'm going to blame myself, then I need to change how this stuff is presented, how and what I'm grading. (Postlesson Interview, line 222)

When she noted that she "softened a little bit" she did so in terms of students' grades. She also demonstrated ownership over student learning when she took responsibility for students' low test scores. She focused on the product of the final test score rather than what students learned as they engaged in the problem-solving process. Although she recognized the importance of students' problem-solving process, she often discussed student products or getting the correct answers, highlighting a disconnect between her recognition of students' problem-solving process as important and her continued focus on the final product in the classroom. This focus on a final answer as the ultimate goal in completing a problem was further substantiated in her lesson design and implementation. Part of the lesson implementation included a teacher-created Screencastify video where Stephanie provided step by step instructions on how to design the taco truck in Google Slides, demonstrated different ways to add clip art to the project, and provided completed examples of mathematics for students to replicate.

The focus on students' products shifted slightly throughout her participation in the professional development program where she began to identify the importance of students' problem-solving processes. During the preprogram interview, Stephanie noted that her expectations of students were to "just get it right" (Preprogram Interview, line 94) and that attempting to get students to understand why something works is, "pointless" because it would confuse students. She noted that conceptual understanding, "has to happen naturally and I think everyone is trying to force it the other way because everyone has an opinion and those people usually aren't in the classroom" (Preprogram Interview, line 97). Her expectations of students changed during the course of the professional development program as she began to see the point of understanding students' process. However, the focus on students' processes remained on procedural steps rather than conceptual understanding. She discussed the importance of scrap work when she questioned: "Am I really grading just for 100% accuracy?...Am I really just looking for that final answer? Like I never used to collect scrap work, your answer is wrong, your answer is wrong. I started collecting scrap work this year" (Postlesson Interview, line 222). In this instance, Stephanie recognized the importance of students' scrap work. However, there is no indication that collecting scrap work was an attempt to focus on students' problem-solving processes but rather for Stephanie to identify where students made an error in completing the steps to get to a final answer.

Prior to participating in the professional development program, Stephanie did not mention students' problem-solving process. There was a slight shift after participating in the professional development program in that Stephanie recognized students' problem-solving process as potentially important. However, her lesson plan, implementation, and discussion of student work continued to focus on students' product. She came to recognize students' incorrect answers as a learning opportunity but continued to focus on the final answer, and did not provide feedback on students' process. She commented, "So, I'll tell them your answer is wrong. But you need to figure out why it's wrong and don't come back to me with the same answer" (Postlesson Interview, line 224). Stephanie, when collecting scrap work or identifying a students' incorrect answer, continued to focus on the product and procedural process rather than conceptual understanding. This idea was also represented in her view student learning.

Ownership of Student Learning. During the postlesson interview, Stephanie was asked to think more deeply about the traits of creativity and how she might include them in her lesson. When prompted by the researcher, she agreed that the last two, teacher-created questions could be removed for students to develop their own. However, she seemed reluctant to give students control to create a new problem and instead described the guidance she would provide for students to ensure they were following the specifics of her intended topic. When asked how she might incorporate mathematical creativity by having students explain reasoning or be curious about mathematical connections, Stephanie responded:

> You're picking my brain in an area I'm not good at. I'm very concrete. For them to explain the reasoning…I could have given them a few questions like, you know, the 40% off tax and stuff like that and then I...could have said develop your own question, your own scenario for one of the menu options. I can do that next year, that's a good idea...

> there were all those silly sets of questions, but maybe I could have left the bottom two blank and said, for person...five and person six, come up with your own scenario for your own percentages and solve your problem. That's probably how I could have done [it] as opposed to just giving them the question, but giving them at least some to see different format[s], different percentages being used, different tax percentages being used, different discounts, and that might have some kids might have actually marked it up, as opposed to discount so I think leaving those two open, I could...actually do that next year to develop and let them pick their own question and have them solve it themselves as well. (Postlesson Interview, line 42)

Although formulating new problems for other students to consider is an example of mathematical creativity as described above, providing the opportunity in this way would not get students to explain their reasoning or be curious about mathematical connections as Stephanie believed it would. Stephanie noted that she would, "give [students] the guide...this page has to have discounts, or sales tax, or both, this page has to have markup this one, the percentage of change" (Postlesson Interview, line 114). Although Stephanie saw a benefit in allowing students to create their own problems, she was reluctant to release control to students and instead described a clear structure for them to create their own problems within set parameters. Although research on creativity acknowledges a need for constraints and recognizes that there are times when creativity is not appropriate, Stephanie's set guidelines exemplify a traditional teaching paradigm and teacher ownership over students' products which makes implementing creative ideas challenging.

There was potential for mathematical creativity in Stephanie's taco truck lesson, but it was not realized and instead there were missed opportunities in implementing mathematically creative ideas. One way to foster creativity among students in this lesson might be to provide guidelines that student-created problems must involve their previous work and have to do with percentages. However, the guidelines might be less structured than Stephanie described to direct them to having markup or percent of change in a specific place or noting the topic for the question (calculating sales tax). Allowing students to have the freedom to create problems regarding the task might help them make connections among different types of percent problems or generate other questions regarding percentages.

Another way to foster student creativity is to provide an opportunity for students to share and solve each other’s problems. This would allow for students to critique, rework, and improve their problems based on peer and teacher feedback. The idea of students creating their own problem is creative. However, Stephanie added specifications and described removing some but “not a ton” of teacher created problems. This exemplified her continued ownership of student learning. She viewed the teacher created problems in the taco truck lesson as arbitrary. Therefore, taking out one or two problems and allowing students to create their own would not impact the lesson itself. Stephanie seemed restricted to the view of creativity as having students develop their own problems, but she did not attend to this fully or to the many other ways teachers might be creative and foster students’ creativity that were presented throughout the professional development program.

Stephanie noted her responsibility for student learning in the preprogram and postlesson interviews. At the outset of the study, Stephanie held herself responsible for student learning and although she noted the importance of mistakes, she viewed them as an opportunity for her, rather than students, to fix those mistakes. After participating in the professional development program, she continued to hold herself responsible for student learning in that she described how she helped them fix errors. Although she viewed herself as more open to giving students a chance to make corrections, the nature of the corrections still demonstrated ownership of student learning. For example, she noted:

> I never was a second chance person, you never got to do a correction. Now when they hand it in, if I see those mistakes, I look at them more. I look at their scrap paper and I'll pull them up and I'll say, "Listen, I'm not gonna tell you the answer, but I'm gonna show you the area that's wrong and needs to be checked. Go back and check problem one, this part, and see if you can come up with a different answer. (Postlesson Interview, line 224)

Stephanie began to view students' scrap work but she continued to have ownership over student learning in that she identified an incorrect answer for students and provided an opportunity for them redo it and bring it back to her for review.

For Stephanie failure was important because mistakes highlighted something for the teacher to fix. She noted that she tried to tell students that mistakes are good. However, she also commented that it is difficult to change students' mindsets (Teacher Artifact). The focus of student mistakes continued to be about Stephanie's teaching rather than students' learning as there was no evidence that she encouraged students to learn from their mistakes or rewarded them for taking risks. Instead, mistakes were seen as an opportunity for Stephanie to fix the error or redirect students back to the correct set of steps.

Although Stephanie noted the importance of mistakes, she took ownership of fixing these mistakes. For example, she compared her view of the final score over how students got to the answer but noted that she tells students if their answer is wrong so they have an opportunity to fix it. She explained, "So for me it...used to be about the score, but now...it's not even about the number that they end up with, it's more about, can they fix it after I tell them it's wrong, if that makes sense" (Post Interview, line 225). In this instance, Stephanie took ownership of student learning by telling students when an answer was wrong. She described students' understanding as their ability to fix a mistake once she identified the problem.

Stephanie recognized mistakes as creative because you have to be creative and confident in order to self-assess. She noted:

> I do [think there is creativity in recognizing mistakes]. I think being self-assessing has, you have to be creative enough self-assess, you also have to be confident enough to self-assess, because I think with someone like me it's a blow to your ego if you're wrong. I hate when I put something on the board and it's wrong because my brain was already thinking of the next thing. And the kids are like, that's wrong and I'm like darn it! So, I do think that that creative is being able to self-assess, fix, because it's very easy just to take what's in the calculator and just write it down and just accept it. I think you have to think to yourself and be creative enough to go wait a minute, I don't think that that looks right. I should try it again. (Postlesson Interview, line 142)

Stephanie described how she did not like to make mistakes or be wrong. Particularly, she seemed to take it personally when students identified her mistakes. This again demonstrated Stephanie's focus on the product rather than the process.

In thinking about students' products, she mentioned the state test and her fear of having poor test scores. Stephanie noted, "I also know what has to be done and learned in a certain timeframe, the constraints of that state testing gets to me" (Postlesson Interview, line 128). She also had concerns about students' performance on the state test. In terms of pacing and teaching seventh-grade material, she commented:

> We got to move on because I was always about the testing because I was always so petrified, petrified of that score coming in and not being good. But then what I realized after really thinking about it, because I also never thought about it, if the process is wrong now, the processes is gonna be wrong on the [state] testing. So, I'll get better scores if I just pay attention more to the process. (Postlesson Interview, line 224)

She continued to identify students' process as steps for solving a problem instead of as understanding and discovering the mathematical ideas within a concept.

Although Stephanie came to recognize the importance of students correcting mistakes, she continued to take ownership over identifying students' mistakes and allowing them a chance to fix it. She described the importance to focus on the process, yet she is continued to focus on the final outcome. The process seemed to be following the correct steps to come to an answer rather than students demonstrating independence and ownership over the development of ideas and the learning process.

Characterization of Students. A large amount of research on gifted students' mathematical creativity exists (e.g. Aiken, 1973; Sriraman, 2005; Zenasni et al., 2016). However, viewing creativity as only accessible for high achieving students is detrimental for all learners, particularly those who struggle. Prior to participating in the professional development program, Stephanie noted that struggling students are more creative than higher achieving students because of their willingness to try different strategies and take risks. She noted that this was because struggling students:

> Are so used to almost having to, I don't want to say fake it, but try to just get through, survive and they're like, "How would I do this?" And they're more willing to take a risk because they really have nothing to lose. (Preprogram Interview, line 52)

Literature on mathematical creativity identifies encouraging risk taking as a characteristic of a creative person (Sternberg, 1985; Robert J Sternberg & W.M. Williams, 1998) and an element of a creativity teaching and learning environment (Ball, 1992; Nadjafikhah et al., 2012; Schacter et al., 2006).

Stephanie's initial view of struggling students as creative was contradicted throughout the professional development program and postlesson interview. During the first professional development program, teachers were presented with order of magnitude problems (Appendix B) which are defined as estimation problems of measurement or numerosity (e.g., how big is chalk dust?) for which the goal is to be within an order of magnitude of the actual value. Participants were encouraged to discuss various approaches, scenarios, and parameters that might impact their final estimate. When discussing order of magnitude problems, Stephanie commented, "Yeah, well you're gonna have your higher thinkers start asking all those questions and your lower thinkers are just gonna sit there and go [looks up to the ceiling]" (Professional Development 1a, line 78). Her view of students' creative ability was described in more detail during the second professional development session. Stephanie focused on a need to challenge high achieving students, rather than struggling students. She argued that the focus of intervention efforts is often placed on struggling students to raise their test scores, whereas high achieving students are not challenged. She shared:

> I think more of the focus has always been pushed to what are we going to do with these [low achieving] kids, all of the help seems to be centered around them, but nothing seems to be done about peaking the kids who are up there [high achieving]. (Professional Development Session 2, line 122)

Stephanie's belief about the type of student who might be creative in mathematics may have shifted because of the level of difficulty in the tasks presented during the professional development program. Similarly, when presented with a task during the third professional development session (see Figure 9), teachers were asked to think about alternative ways to approach this problem and discuss this problem with students. In thinking about the problem, Stephanie referenced a need for students to have a high level of knowledge to solve this problem. This contradicted her initial view of struggling students as more resourceful and creative than high achieving students.

Figure 9

Multiple Approach Task, Professional Development Session 3

A girl scout troupe baked a batch of cookies to sell at the annual bake sale. They made between 100 and 150 cookies. One fourth of the cookies were lemon crunch, and one fifth of the cookies were chocolate macadamia nut. What is the largest number of cookies the troop could have baked?

Student response:
x represents the number of cookies.
$\frac{x}{4}$ represents the number of lemon crunch cookies.
$\frac{x}{5}$ represents the number of chocolate macadamia nut cookies.
$x - \frac{x}{4} - \frac{x}{5}$ represents the remainder of the cookies baked. Then,

$$100 < \frac{x}{4} + \frac{x}{5} + \left[x - \frac{x}{4} - \frac{x}{5}\right] < 150$$
$$2{,}000 < 5x + 4x + 20x - 5x - 4x < 3{,}000$$
$$2{,}000 < 20x < 3{,}000$$
$$100 < x < 150$$

…but this results in no new information beyond $100 < x < 150$.

Stephanie anticipated how her students might respond to the question when she noted:

I think I would have some higher thinkers going along the lines of…"Wait a minute, okay, so I know it's going to be between a 100 and a 150." And I think their wheels would start turning. I think the problem is when you run into your lower level and your

> mid-level learners, where they're afraid to risk because they don't want to get it wrong. They kind of leave the paper blank and they start with the "I don't know." I feel like this is where I jump in and help too much. And I start with the prompts and, "Well, what could you do? What do you notice?" I highlight information. "Well, I see fractions." "Okay, what do you know about fractions?" (Professional Development Session 3, line 286)

This is contradictory to her initial belief that lower achieving students were more creative than high achieving students because they were willing to try different strategies as opposed to being concerned about getting an answer correct.

Her characterization of students during the professional development program remained static. In the postlesson interview, she continued to view advanced students as creative because they are able to make connections whereas struggling students do not have enough knowledge to approach tasks creatively. However, this idea of creativity as challenging was contradictory to her lesson development which required students to choose taco prices and did not include the challenging aspect of mathematical creativity that was presented throughout the professional development program. Although she deemed this lesson as creative, and choosing taco prices would be accessible to everyone, her view regarding advanced students' abilities to make connections contradicted her initial idea. Stephanie noted:

> I feel like I can't [have students explore] now because we have such low functioning and then the high functioning in the same class. I almost feel like it's hard to be creative, because it's two different levels of creative and I don't know how to balance that...what can I do to balance that in groups that are just so high and low? (Postlesson Interview, line 132).

Stephanie mentioned “two different levels of creative” which seemed to reference students’ ability levels. Earlier in the postlesson interview she noted that advanced students might be creative in how they make connections, whereas struggling students would not be able to do so. Because of the differences in student ability levels, Stephanie would have to develop two lessons, one mathematically creative lesson for advanced students and another, simpler lesson, for struggling students.

Similarly, throughout the professional development program, Stephanie believed students should be grouped based on their ability level. She found having various student achievement levels in one classroom difficult and instead felt that students should be leveled based on achievement. She described:

> [One year] I had a really high functioning group, because we split them based on ability for a couple of years. That was nice. It was rough because when you get the slow class, you hit a brick wall, but when you had that advanced class, I did that [had students figure out the reason for a procedure]. And I said...I want to erase the steps, can anyone just give me one step to do this? It took a little bit, within 40 minutes they had it and it was unbelievable they did it themselves. (Postlesson Interview, line 131)

This further highlighted Stephanie’s view of advanced students as capable of tasks that required mathematical creativity. Stephanie feared that if she allowed students to have more time to explore, student behavior would become an issue. She noted, “If you let them too long and they don’t know what direction to think in, now you have behavior [issues]…if you’re not monitoring [manipulatives] and constantly engaged, they’re playing with them, drawing on them” (Professional Development Session 3a, line 143). Stephanie identified an issue with having students take ownership of an activity. She viewed most students’ critical thinking skills as, “not at the level they should be. I find most of my students want to be spoon fed information and if I ask them to exert any extra effort they become agitated and frustrated" (Professional Development Session 6 reflection).

Instead of viewing creativity as a way for all learners to access mathematics, Stephanie viewed it as a skill requiring extensive knowledge and one that would not be engaging enough to keep students focused.

> I think we as professionals feel like we know what's best for what's in front of us at the time. But then, you have all these outside sources telling you, "This is how it needs to be focused now. This is student-led classrooms." Teachers are taking a step back and I'm thinking, "Then why do I have all these degrees?" You know? "What do you need me for if it's so..." And I get having it student-centered, but I don't always agree in student-led. And that's a big problem also. And taking the risk for me, is letting go of the reins. The risk is pulling out those manipulatives and making equations with fraction tiles, flipping them upside down, writing on them, or using algebraic tiles and the kids sitting there, going, "What do I do with these?" That's a big risk because it's out of my comfort zone. Big risk letting them think too long, behavior problems. I just think there's a ton of risk when you try to be creative. (Professional Development Session 3a, line 286)

Stephanie noted that it was uncomfortable to give students more ownership over their learning by having them explore with manipulatives or think about a problem. She identified student behavior and her own discomfort with a student-led classroom as barriers to implementing ideas of mathematical creativity into her teaching practice.

Conclusion

Stephanie was chosen to exemplify the *traditional* category because as seen from above, she tended to view creativity as artistic, noted that creativity would be represented in the form of a fun game, absent of challenge, and described creativity as not possible until content from the curriculum had been presented to students. Stephanie also viewed mathematical creativity as accessible only to high achieving students. These ideas were representative of participants in the, *traditional* category, of the main unit of analysis.

When prompted by the researcher, Stephanie was able to identify ways she might restructure her lesson to include mathematical creativity. However, when discussing mathematical creativity, she misinterpreted the ideas and instead thought of creativity as a way to simplify mathematics instead of encourage students to develop deep, meaningful, and lasting connections. Evidence is lacking to show how Stephanie might fully implement these ideas into her teaching practice and she continued to exemplify teacher ownership over student learning. Stephanie saw creativity as accessible only to high level students, which is aligned with a traditional view of the classroom and of creativity. Stephanie was conflicted in her responses about student abilities, focused on product over process, and was not convinced of the importance of mathematical creativity at the middle school level.

Dana, Appreciation of Teaching for Creativity (Creative but Hesitant)

> I just try to take what I'm supposed to be doing, and make it more interactive for them…I have a lot more hands on activities this year, they were walking a number line instead of just seeing one on a paper… I think getting as many what they thought were games, tasks to get them to enjoy math was definitely important for me this year from this professional development [program] as well. (Postlesson Interview, line 173)

Dana is a sixth-grade teacher in Center School and at the time of the professional development program had been teaching for 16 years. She had extensive experiences with professional development outside of the Tarnot school district, attending workshops to support struggling students, implement growth mindset in mathematics classrooms, and work with the Institute of Student Achievement (ISA) to create a vision for Center School. She had collaborated with colleagues through monthly mathematics and science meetings, and met with in-school grade level partners weekly. Her interest in participating in this professional development program was to collaborate on engaging activities that connected multiple state learning standards.

Dana was chosen as the exemplar for the *appreciation for teaching creatively (creative but hesitant)* category because although a contradiction existed in her views of mathematics, creativity, and teaching and learning. She established a classroom environment where mathematical creativity might flourish and described some tasks that had the potential to be mathematically creative. However, she continued to describe a traditional view of mathematics as concrete and creativity as artistic. She also emphasized student- and teacher-made products over problem-solving process, and stressed the importance of developing skills for the state test. These elements presented a barrier which limited Dana's implementation of the ideas presented in the professional development program into her teaching practice.

View of Mathematics

Dana highlighted the significance of seeing multiple approaches to a problem in mathematics and described the importance of students' problem-solving processes. However, she also identified mathematics as concrete and emphasized ways to help students memorize rather than conceptually understand concepts. She was placed in the category *creative but hesitant* because she demonstrated a contradiction in her beliefs about mathematics.

To help students identify multiple approaches to a problem, she implemented "What Does Not Belong?" cards as a lesson opener for proportions. Dana described that these cards provided various ways to identify why something might not belong in a set. Dana described an example of the activity where the ratio 3:4 is shown four different ways (e.g. ratio, proportion, rate, percent) and used different font colors. Students were required to describe differences among the four to identify that there are multiple answers they might draw upon. She commented that this activity:

> Gives an opportunity for a lower level child to say "three to four" because it's red, and some say that. But then others will say, well, this is written as a rate...it just gives them an opportunity where they're all correct, but to have dialogue. And that's when they realize, wow, there's multiple different ways to get a correct response. (Preprogram Interview, line 33)

She shared this activity again in her reflection after the second professional development session. She noted that these cards helped students identify multiple approaches to solving problems and recognize that some problems have multiple answers.

Similarly, Dana described activities where she gave students the answer to a problem, required them to explain the reasoning behind the answer, and determine the problem-solving process to arrive at that answer. Dana provided an answer to a state released test question and encouraged students to:

> Turn it around and say, how did I get to that response? Why did I choose certain words out of the word problem and what operations were used? But not only why is *a* correct if

> I circled *a*, why is *b* wrong, and *c* wrong, and *d* wrong? So that they understand what's not correct and why. (Preprogram Interview, line 44)

She referenced the importance of student understanding again in the postlesson interview when she noted that students worked on state released test questions. Students were required to, "write the answer and circle the important information and jot down the why. How did I get to that result? And why is that the correct answer? So, it was a way of kind of working backwards" (line 176). In these instances, Dana saw mathematics as a process and identified the importance of having students explain their reasoning. However, she focused more on the importance of strategies to do well on the state test and less about exploring the problem-solving process for mathematics learning. Focusing on the problem-solving process and identifying why an answer is correct is mathematically creative in that it highlights the importance of students' conceptual understanding over the product. It also requires students to participate in the mathematically creative behaviors, explaining reasoning and making mathematical connections. However, the questions chosen for this activity seemed to be focused on test taking strategies rather than conceptual understanding to prepare students for the state test in April. It seemed that Dana was working within the constraints of the curriculum and focused on the state test, but tried to so do in a creative way.

During the professional development program, participants were asked to take a traditional task and recreate it by attending to traits of mathematical creativity. Discussions throughout the professional development program encouraged teachers to think about understanding students' problem-solving process by presenting tasks that had multiple solutions, required them to analyze and describe alternate ways to approach problems, and think about when it might be beneficial to focus on the process over the product.

Although Dana saw mathematics as a process with multiple approaches and answers to problems, she also believed it was concrete and consisted of a series of rules to memorize. Prior to her participation in the professional development program, Dana described mathematics as concrete and compared mathematics to reading and writing. She noted, “I love the...concrete and black and white examples as opposed to reading the chapter books and writing the essays...when I try to compare the subject matter, I just enjoy working with the numbers” (Preprogram program Interview, line 14). She also highlighted the importance of vocabulary and focused some of her instruction on strategies to memorize rules. For example, she recalled:

> We say the big bully dividend, we call the dividend the big bully. Why? Because he goes in the jail. So, you’ll see my children will never screw up a division problem, where so many do. You have two numbers, they see it as two numbers. I don’t remember who goes inside that house, but my [students], no. The big bully always comes first. He thinks he’s tough, throw him in the jail. Now the security guard is outside of the jail. So, we get to decimals. That security guard cannot be a decimal. He needs to be a whole number, a big strong security guard. (Preprogram Interview, line 35)

In this instance, Dana provided students with this arbitrary story, absent of any mathematical basis, about where to place the divisor, dividend, and quotient in a long division. This focus on vocabulary and strategies to help students memorize rules remained static throughout her participation in the professional development program and was particularly evident in professional development sessions four and five.

During the fourth professional development session, another participant noted difficulties she was experiencing with one student’s struggle to add and subtract fractions. Dana described the four-square method which focused on a procedure of adding and subtracting fractions, circumvented the need for students to conceptually understand fractions, and instead focused on a procedure for memorization. Dana also indicated, “I use whole brain learning. It’s like a cheer, so they actually stand up and physically use their body to memorize the rules. And you will see them doing that during a test” (Professional Development Session 4, line 43). When thinking about the shift to virtual learning, Dana noted, “Keeping it definitely more simplified for them, sticking to the skills and getting in as much vocabulary and skills as we can for the remainder of what we have to teach, is working for me” (Professional Development Session 5, line 57). In these instances, Dana again provided strategies for memorization without mathematical basis.

These examples demonstrated a contradiction in Dana’s beliefs about mathematics. Although she noted the importance of having students explain their reasoning and described mathematics as a process, in practice, there was contradicting evidence that Dana viewed mathematics as concrete and presented ways to help students to remember procedures with no conceptual connections, particularly for the state test. A contradiction existed between her discussion of the problem-solving process and her implementation of strategies to help students memorize rules.

View of Creativity

Dana also demonstrated a contradiction in her view of creativity. For Dana, her view of creativity as exemplified through games that were fun for students remained static throughout the professional development program. She also described creativity as helping students develop conceptual understanding in mathematics. However, there was a contradiction between her description of creativity and her continued focused on students’ artistic products. These contradictions about creativity were evident among all participants in the *creative but hesitant* category.

Before participating in the professional development program, Dana described her teaching as creative and was focused only on ways she taught creatively. She noted that she was creative in the design of lessons and in her presentation of content and defined creativity as:

> A teacher who doesn't sit and open up one workbook page a day and go to the next. Who comes up with the stations and there is a technology station and there's a hands-on station...So doing a lot of those extras, I would say makes the classroom more creative and more engaging. (Preprogram Interview, line 61)

However, after participating in the professional development program she included how students might also be creative. She noted, "I think I'm more open now to the student side of creativity, not just the teacher...going back to letting them show what they know, making up their own problems" (Postlesson Interview, line 120). This idea of giving students an opportunity to create their own problems is an example of mathematical creativity.

Although Dana saw creativity more broadly after participating in the professional development program, at times she was unsure of how to implement mathematically creative tasks from the professional development program into her teaching practice. She noted:

> I loved some of those questions, those out of the box questions about, remember we had like the Dunkin Donuts ones? But some of those I think wouldn't necessarily, maybe I just didn't know how to fit them into what I needed to get out of [students] this year, and before testing and with the quarantine. (Postlesson Interview, line 45)

There was a possibility of infusing mathematically creative tasks into her classroom practice. However, Dana was unsure of how to apply tasks presented in the professional development program directly into her classroom or draw on ideas from the professional development program to create her own mathematically creative lessons. Although the goal of the professional development program was to encourage teachers to explore mathematical creativity through challenging tasks rather than provide tasks that might be directly applicable to their classrooms, Dana highlighted that there was not a direct take-away of implementing these problems with sixth grade students.

Creativity as Art and Design. Prior to participating in the professional development program, Dana provided an example of what she defined as a creative decimal activity and connected it to generating holiday wish lists and developing an artistic product. She described students' artistic products as creative when she commented:

> Instead of just doing pages and pages of decimals, I gave them a budget and a log sheet to fill out where they had magazines and they had to go shopping and pick out items up to that budget and cut them out and glue them on a stocking. (Preprogram Interview, line 63)

For Dana, the creative components of this activity included students cutting out items from magazines, gluing them on a stocking, and choosing their own prices for items. Dana also described a graphing activity where students decorated and designed a graph. She noted, "They're getting the skill, but for them it's through something for the month, whether it be *Happy New Year* or *Let it Snow*, and they get to decorate it and design it" (Preprogram Interview, line 20). Although these might be examples of engaging activities, they do not necessarily foster students' mathematical creativity.

After participating in the professional development program, Dana described seeing beyond an artistic interpretation of creativity and focusing instead on content. However, it was not clear that this view aligned with her practice. Instead, Dana continued to focus on the artistic nature of creativity and described visual representations and the design of student products as creative. Although Dana noted that creativity was not necessarily “bells and whistles” (Postlesson Interview, line 104) referring to teachers’ presentation of mathematics, she described students’ artistic products at length. She viewed creativity as more than the design of a product when she commented:

> When I look at an assignment, the creativity I’m looking for is more the creativity in the content...where others might look for colors and the shapes and the images and what kind of background they use, if it’s virtual and I’m looking for the creativity in the content. (Postlesson Interview, line 110).

However, this idea was contradicted during the professional development sessions and in her lesson plan and discussion of student work. Throughout the professional development program, Dana submitted student samples from various activities. She described one creative assignment that required students, “to create a Google slideshow of their favorite topic and make posters of it, and include rules and creativity and examples. And again, they loved it. And for me it’s a review for each of them on the topic” (Professional Development Session 5, line 135). In this instance, creativity was defined as design of the slideshow and posters to demonstrate knowledge of rules, further highlighting the artistic nature of the product. This focus on design and rules is absent of mathematical creativity and instead is aligned with a traditional view of creativity as artistic.

In several instances during the postlesson interview and the second half of the professional development program, Dana described virtual classrooms she created using Google Slides at length. She noted that students, “almost feel like now that it’s like a video game and they can click around and get to all the tasks that they need to do” (Postlesson Interview, line

141). She viewed these virtual rooms as more engaging and fun for students. During the sixth professional development session Dana commented:

> One thing…I've been working on morning, noon and night is I created a brand new Google classroom that will be up and running for September…I made my own group per standard with below, on level, and above level. So that's all there for them. And then it has a section for virtual posters, virtual tools, GoMath, Screencastify videos [that] I made myself, and IXL videos I made myself, and then Khan Academy videos and…Each module ends with an escape room that I'm working on with another teacher, and video. So it's a jam packed module and it completely would include everything that they would get for me if I was in the classroom. (line 425)

She also described her and others teachers' virtual room designs when she noted, "we're all trying to go with the creativity aspect in this room. You'll see every possible subject area down to speech designed. Phys ed [physical education], oh my goodness, the phys ed rooms are gorgeous" (Postlesson Interview, line 247). She identified how these virtual rooms were beneficial for students because the teacher-created notes would exist after the conclusion of a lesson and she described the importance of having a virtual classroom for every subject. Although these rooms might be beneficial for students to reference, there was not evidence that they were aligned with mathematical creativity. Instead the development of these rooms remained focused on creativity as artistic and design oriented.

Similarly, her view of creativity as artistic was also evident in her design of a geometry lesson. Dana's lesson plan was not submitted to me prior to implementation. In this lesson on finding the area and volume of different shapes and figures, traditional tasks were implemented including, using formulas to find the area of irregular polygons and the volume of rectangular prisms, and using nets to find surface area. Dana included three challenge questions for this lesson:

1. How does the area relate to the perimeter?
2. Why is ½ part of the formula for the area of a triangle or trapezoid?
3. When calculating surface area, is it easier with a three-dimensional figure or net? Why?

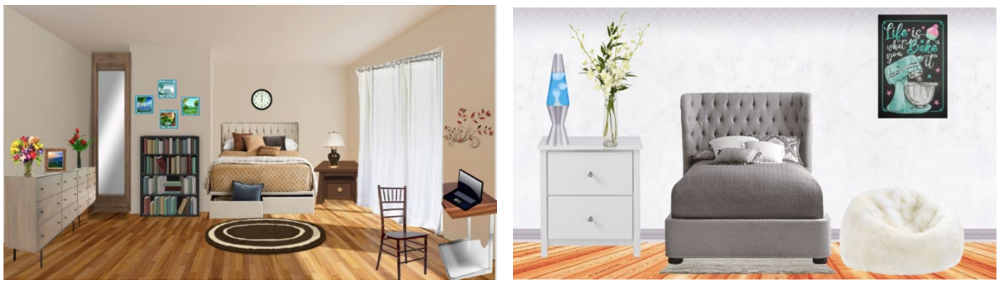
These challenge questions were separate from the lesson itself. While these questions might help students make connections in a mathematically creative way by recognizing connections between formulas, the main lesson focused more on students' application of formulas in straightforward contexts. The first slide of the assignment described seven elements needed to receive a grade of 100 which included:

- Colorful Fonts/Fun Backgrounds
- Creativity
- Formulas AND Completed Problems with steps for ALL SHAPES
- Virtual Tool Examples
- Pictures
- Answers to all questions in COMPLETE sentences
- ALL 13 Slides are COMPLETE

The focus of this lesson was more about the visual representation of the product over the mathematics and no attention was paid to students' mathematics. Further evidence of Dana's focus on creativity as artistic was the portion included for extra credit. Students were tasked with designing a virtual representation of their dream bedroom (see Figure 10). This visualization was artistic in nature and the students' product did not highlight mathematics in any way. However, the student samples Dana shared were of students' virtual room designs rather than mathematics.

Figure 10

Samples of Dana's students' virtual room design



Her lesson plan and discussion of student work was aligned with her view of creativity as artistic. In an email communication with me, Dana further detailed this virtual room design when she noted that:

> Students are currently working on a geometry project. Part of the assignment was to create architect plans for a dream bedroom. I gave them an additional task for extra credit. This task had them design a three-dimensional dream bedroom after learning about how I designed their three-dimensional classroom...I am blown away by their creativity and showing their true personalities. (05/24/2019)

Although the design portion of the assignment was included as extra credit in the lesson plan, this was the main focus of the postlesson interview discussion about the lesson, and Dana only shared student samples of their room designs, void of mathematics content.

When Dana shared student examples with me, I replied asking her to think about connections to mathematical creativity. I questioned:

> What mathematics content, still Geometry? Are they covering? I'm interested to know if this is helping them to dive deeper into the mathematics you're working on or if they are

> better able to pose and answer questions. What do you think about how this project is impacting their math learning?

Dana replied that virtual rooms were fun and noted that she planned to incorporate more mathematics in future implementations of the activity. She noted, “Honestly, for this year it was more of a creative ‘fun’ activity to bring their architect plans to life. Next year we will use these rooms to solve for surface area and volume” (05/27/2019). This recognition of creativity as art is aligned with traditional views of creativity. Although Dana noted her intention to include mathematics in the future, she did not elaborate on what that might look like in practice or how it would incorporate mathematically creative ideas.

After participating in the professional development program, Dana recognized the potential for creativity among struggling students. However, this view of student creativity continued to be aligned with an artistic view of creativity. She noted that students who struggled in mathematics had more creative designs for the geometry lesson than higher achieving students. When asked to point to specific student samples she viewed as creative, she noted:

> Even in the building of the virtual rooms, again, the students who struggled mathematically, their rooms were gorgeous, like one of them brought tears to my eyes. This particular 92 [numerical grade] student, the window was literally over the bed and there were items in the room with a white box around them that she wasn’t getting transparent or utilizing the tools I gave her to take the background out. It was interesting. (Postlesson Interview, line 72)

In this instance, although this lesson allowed Dana to appreciate the strengths of struggling students, she continued to focus on students’ design products. She also explained that high achieving students wanted direct questions that required specific answers. She described one students' experience creating the virtual room when she commented:

> You know who struggled on this? Higher level, 90s and above. I had one particular girl who couldn't figure it out, that she has to use her own problems. She wanted to open that slide, see a problem, and ace this assignment. And multiple times I had to say, you have to pull your own tools. Use the virtual tools, use the internet, pull your own shapes, make up your data. (Postlesson Interview, line 70)

Dana highlighted that higher-level students were unsure of how to complete the virtual bedroom design task because they were looking for a problem to solve, possibly questioning whether mathematics was involved. However, there is no evidence that the product involved questions that required students to engage in the problem-solving process or deeply consider mathematical creativity.

Creativity as a Game. Dana initially identified creativity to be in the form of a game and this belief remained static even after her participation in the professional development program. During the preprogram interview, she described creativity as fun games and hands-on activities. For example, she highlighted:

> Any activities dealing with numbers that you can make fun for them would be your creativity. Going back to the card games instead of a worksheet...when we get to operations with integers, actually walking a number line instead of just seeing it, but getting your body involved, using the whole brain learning, I would say is a way to connect even just your rules of fraction operations with their body and they're able to retain it. So, there are so many mathematic[ally] creative activities, but they'd have to be implemented and designed. (Preprogram Interview, line 63)

Similarly, when asked what she enjoyed about mathematics, she noted, “coming up with games, different card games...bringing in the pizza boxes to have them measure for area and perimeter, just the hands-on experiences that I can bring with each standard or each topic is what I enjoy” (Preprogram Interview, line 14). Although the professional development program did not present games as mathematically creative, Dana described one of her takeaways from the professional development program as incorporating games into her lessons. The content of the professional development program did not include a focus on games and although designing and implementing games has the potential to be mathematically creative, a game in itself is not necessarily creative.

After her participation in the professional development program, Dana continued to view creativity as fun games. When asked how she developed or chose tasks, she noted:

> I just try to take what I’m supposed to be doing and make it more interactive for them, whether it be with each other or with me. I have a lot more hands-on activities this year even, they were walking a number line instead of just seeing one on a paper, but they would roll dice and walk out a number line and follow order of operations, left to right. They were rolling out dice and not being at the hands of making their own order of operation problem when we did that, but depending on the color you rolled and the number, that’s what you have to write on the board and the color represented the operation. So, it was a game for them. Playing cards and made fractions and did fraction war and then changed it to ratio war. (Postlesson Interview, line 173)

Dana’s description of hands-on, interactive activities included using dice and playing cards. Although there is potential for a game to be mathematically creative in that it might foster students to explain their reasoning or express different approaches to a problem, simply implementing a game without added elements that encourage students to see multiple solutions to a problem, explore non-algorithmic decision making, or explain their reasoning, is not inherently creative.

It was unclear what Dana viewed as mathematically creative about fun games. For example, she described "getting as many, what they thought were games, tasks to get them to enjoy math was definitely important for me this year from this [professional development program] as well" (Postlesson Interview, line 174). She also referenced games, "as a way to help students understand the basics of standards, developed card games for students to learn exponents and review content, and implemented whole group scavenger hunts to introduce and review lessons" (Professional Development Session 2 Reflection). Although she described games as helpful for students to understand mathematics, Dana did not elaborate on what was mathematically creative about implementing games other than allowing students to have fun and not realize they were learning mathematics.

Misinterpretation of Mathematical Creativity. Dana described ways in which she created a classroom where mathematical creativity could be implemented. However, she also misinterpreted ideas presented during the professional development program when she described that creativity involved choosing more difficult numbers or misinterpreting the spectrum of creativity. During the second professional development session, teachers were introduced to literature on the spectrum of creativity (Beghetto & Kaufman, 2007a; Csikszentmihalyi, 1996, 1998; Maslow, 1967). Although this idea resonated with Dana, she misinterpreted this idea to mean sharing artistically designed products with other teachers.

Halfway through the professional development program, Dana identified herself as creative and was confident (5 on a scale of 1 to 5) in her ability to implement mathematically creative lessons into her classroom. Dana described creativity as evident in students' use of more complicated numbers such as decimals or integers. She described higher level students' creativity as developing, "a math problem with volume and instead of length, width, and height being 2, 4, and 1, but you know 2.768*3.124, and I would, in a math aspect, say more creative data is something with fractions and decimals instead of your base 10 numbers" (Postlesson Interview, line 105). She substantiated this view later when she noted that students' use of integers was creative because, "they were comfortable enough to add in the negatives" (Postlesson Interview, line 126). Dana believed she was teaching creatively in that she created developed stations for students, implemented whole brain learning activities, and allowed students to choose their own learning path. However, she misunderstood ideas of students' mathematical creativity as focused on using fractions, decimals, and integers, rather than whole numbers.

She believed that sharing the virtual classroom with other teachers in her school and district was representative of Pro-C creativity (Beghetto & Kaufman, 2007a). She described creativity as sharing ideas in a community of teachers. Dana described creativity as:

> I can do something and think I was creative, but then someone else does it and I pull other ideas from theirs that I want to incorporate into my item and think, wow, I thought I was okay but I wasn't because there's somebody out there always more creative. (Postlesson Interview, line 103)

In this instance, Dana described how she drew on and shared ideas with other teachers in the community. This is related to the spectrum of creativity literature that outlined the varying ways one might be creative in sharing an original idea or artifact within a community. Although this could be creative, Dana focused on the virtual classroom design to engage students, and although she saw creativity as adding to and sharing ideas with others, this was in reference to the product of the virtual classroom rather than something mathematically creative or that might foster students' creativity.

To develop these virtual classrooms, Dana collaborated with teachers in Center school, within Tarnot school district, and outside of the district through a Facebook group dedicated to designing virtual classrooms. She referenced this collaboration as creative twice during the postlesson interview. She first noted:

> Another huge take back was the day we worked on different types of problems [in the professional development session] but then how can we go beyond the classroom. We talked about getting the problem out into the school and then beyond the school and getting the problem out into the district. That's what I have to show you. (Postlesson Interview, line 47)

This comment was specifically about the virtual classroom design. Later, Dana described the creation and future use of these virtual rooms at length. Although sharing a product with the community would be considered creative on the spectrum of creativity, the product itself that Dana was eager to share, was not mathematically creative. At the end of the interview, she shared her virtual classroom with me. This virtual classroom included classroom norms, schedules, and posters, mostly absent of mathematical content.

Dana commented on the spectrum of creativity again when she recalled her understanding that creativity is when a product is brought, "out of the classroom and into the school and then out of the school and into the district" (Postlesson Interview, line 146). Although sharing ideas with the community has the potential to be creative, Dana's product itself was disjoint from mathematical creativity and therefore sharing it with her colleagues at the school and district levels would not be considered mathematically creative. Instead she shared the design of a product which she considered creative.

When asked what traits of creativity were exemplified in the geometry lesson, Dana was unsure but noted, “Well, would you call the non-algorithmic decision making when they built those rooms? The virtual rooms, like they made their architect plans 3D, because there was no data involved there” (Postlesson Interview, line 21). She also identified creativity in that students were required to, “explain reasoning, yes. They had to answer all of the challenge questions for each slide” (Postlesson Interview, line 21). These challenge questions were separate from the task of designing the virtual room. This exemplified a complex situation where Dana may have misinterpreted what it means to be creative. Although she noted a focus on mathematical creativity, she discussed the online platform and design of virtual rooms where students can access resources to build the room, rather than explore mathematical creatively by having students investigate connections or develop insightful solutions to a problem.

Although Dana may think she is teaching creatively by allowing students to choose their learning path, implementing stations, and presenting content through fun activities or games, she does not seem to be teaching for creativity. Instead she viewed using decimals rather than integers as creative and continued to focus on the ways she might be creative in her teaching, particularly through her development of the virtual classrooms.

View of Teaching and Learning

Before participating in the professional development program, Dana celebrated student mistakes as learning opportunities and encouraged students to make connections between mathematics and real-world situations in a way that is consistent with mathematical creativity. Existing literature identified this type of classroom environment as conducive to fostering mathematical creativity (Neumann, 2007; Sternberg, 2006; Sternberg & Lubart, 1996; Robert J Sternberg & W.M. Williams, 1998). Although Dana celebrated students' mistakes and encouraged connections in mathematics, she connected correcting mistakes to performing well on the state test and provided direct instruction and teacher created notes and videos. This exemplified a contradiction in her beliefs about teaching and learning that was representative of the *creative but hesitant* category.

Real World Connections. One way to foster creativity among students is to encourage them to see mathematics outside of the classroom and make connections between mathematics and real situations (Ervynck, 1991). At the outset of the professional development program, Dana noted the importance of applying state standards to real-world contexts. She noted that she "constantly tr[ied] to take whatever standards we're working on, as many as I can, and apply it to the real world" (Preprogram Interview, line 47) and viewed geometry as the most creative discipline because of the hands-on, real world nature of the content.

During the professional development program, teachers were tasked with having students journal about the mathematics they see around them and write mathematical questions based on these observations. Dana described journaling as a way to help students explain their reasoning and think about the purpose of mathematics outside of the classroom. Dana's existing ideas about encouraging students to make real world connections in mathematics allowed her to implement some of the ideas from the professional development related to encouraging students to make connections between mathematics outside of the classroom into practice.

For example, after participating in the professional development program, Dana highlighted ways students could make connections to their own lives instead of, "just handing them that worksheet sheet of math problems" (Postlesson Interview, line 133). To do this, she implemented tasks that required students to use fractions to create and adjust recipes and explore unit rates by researching and comparing products in ShopRite fliers. The professional development program focused on the importance of having students make connections between mathematics and areas outside of the classroom which prompted Dana to think about further connections between mathematics and real-world contexts. She commented:

> Where else can I get them to think and get them to not just focus on a number, that it's more than just the number because we're going to use this in other areas of our life? And I also now opened up a section going forward for math in the real world per topic, and they'll be doing research on what careers they can utilize these standards with. So, it's not just a math class anymore. (Postlesson Interview, line 98)

Dana highlighted the importance of making connections within mathematics content instead of seeing mathematics as topics that were disjoint from one another. She noted, "I think just getting them to think outside the box, getting them to question themselves and question their thinking and then applying it, applying it to other areas of math is huge" (Postlesson Interview, line 130). These instances provided evidence of mathematically creative ideas where Dana encouraged students to make connections among mathematical topics and between mathematics and real-world contexts (Sheffield, 2008).

Ownership of Student Learning. Although Dana identified the importance of having students make real-world connections, there were instances where Dana's ideas about teaching and learning were more aligned with traditional teaching, namely teacher ownership of learning. Dana saw her teaching as flexible and adapted her instruction based on students' needs and described her teaching practice as, "a work in progress, it's an art, it's not a science, it's not black and white...just changing all the time with the flow of what they need" (Preprogram Interview, line 86). Although Dana saw teaching and learning as a work in progress, she had ownership over student learning and placed a strong emphasis on the state test.

During the preprogram interview, Dana described a situation where she directed students to approach a word problem correctly and not be "tricked" by the language of the problem. This idea of students' being aware of tricks on the state test was also evident in the sixth and seventh professional development sessions. Dana described a scenario on the state test where tables were incorporated into word problems to trick students. This direct instruction of students' approaching a problem so as to not be tricked contradicted her attempt to have students lead the class and discover material. Instead of having students work through the problem themselves, Dana described a situation where she primed students when she commented:

> We were working on percent word problems and they'll give you a ratio of cherry ice pops but then ask you for the percent of grape. So, before we even get to the question, I say to them, okay, two thirds are cherry...what is the ratio of grape [ice pops]? And they'll say, well, one third, and then we read the question and they say, 'Oh, they're trying to trick us.' They gave us information...my overall example is, did they give us information about dogs and ask about dogs, or did they give us information on dogs and ask about cats? And they said, 'Well, this one they're asking about the cats, it's the opposite.' Okay, so do you want to use the ratio that's in the problem, or do you want to use the one that you came up with? And they said, 'No, no, we're going to use the one we came up with'...So we discuss a lot of that because I explain to them that they don't want to get tricked on the state test. (Preprogram Interview, line 46)

Although she developed activities for students to be responsible for their own learning, in this instance she directed students to identify what the answer required so they were not tricked by the wording of the problem. This view of teacher ownership of learning was again evident in the postlesson interview when Dana described the delivery of mathematics content and an emphasis of learning for the state test. After shifting to virtual learning, Dana created Screencastify videos and notes for students' step by step procedures. She noted that students:

> Are watching my Screencastify videos and I have my notes already done for them, where I used to instruct in front of them and write the notes on the board and they would copy, and that took time. So now I have it all done for them, it's all separated by chapter. I have virtual tools and virtual posters and everything they need for [a] resource base library. (Postlesson Interview, line 150)

Developing pre-made notes and videos is aligned with the traditional view of a teacher providing information for students. At times there was evidence of teaching creatively and fostering student creativity. However, this was not always the case as Dana also described instances where she directed student learning. This contradiction was evident for participants in the *creative but hesitant* category.

Although Dana provided evidence that she directed student learning, she also noted that she encouraged students to be independent with little teacher input as they worked through an activity. She described:

> Making sure that they get the standard and the skill, but handing the reins, we're told to hand the reins to them and letting the children run the classroom. And today, my students ran the classroom. I didn't say much. I walked around, I watched, I did my 10-minute introduction, showed them a new hands-on activity so they knew the rules, and then, they went, and had the day. And that's a risk for me, it's not what I'm used to, but it's the new way. (Professional Development Session 3a, line 132)

Although she fostered student independence in this case allowing students to “run the classroom” and build expressions with physical manipulatives, she also described situations where she had ownership over student learning. Dana was uncomfortable with allowing students to run the classroom and viewed it as a risk to have students discover mathematics with little teacher input. This is potentially because she was not convinced that students would learn effectively with less teacher instruction and this would negatively impact results on the state test.

Mistakes as an Opportunity to Learn for State Testing. Literature suggests that an environment conducive to creativity is one where students are not penalized for making mistakes and the teacher is tolerant of ideas that lead to incorrect answers (Schacter et al., 2006). These characteristics were evident in Dana’s view of mistakes as an opportunity for students to revise answers and learn collaboratively. However, there was also evidence of Dana’s continued focus on fixing mistakes for the state test rather than student learning.

Prior to participating in the professional development program, Dana saw students’ mistakes as learning opportunities. In the preprogram interview, she noted that, “The first several days when I group lessons together, I tell them, you’re not being graded numerically because it’s okay to make mistakes. I want you to learn from your mistakes” (line 76). She described the importance of student collaboration in order to learn from others’ mistakes. One way she allowed students to learn from one another was through:

> A ‘DUMP and RETRIEVE method’ where students dump what they know on assessments and retrieve that information the following day as they collaborate with a partner to discuss solutions and ‘whys’. This method allows students to not worry about mistakes on day one and answer the questions at hand without the stress of a number grade hurting their average. (Professional Development Session 2 Reflection, line 12)

Dana continued to see mistakes as important and encouraged student collaboration. However, she emphasized the importance of correcting mistakes for the state test. Dana required, "corrections to be made in pen so they can see it and learn from each other's mistakes as opposed to their own...we try to correct that before assessments and before the state test, so that they're comfortable and confident" (Preprogram Interview, line 76). She continued to focus on the importance of fixing mistakes for the state test rather than students' conceptual understanding. This exemplified how the implementation of mathematically creative ideas might be hindered by Dana's traditional views of teaching and learning.

Dana also offered that although she was originally an early childhood teacher, because of her students' high state test scores, she realized, "now I'm this middle school teacher and seeing the results of the state test every year is when I realize like, this is where I'm meant to be" (Preprogram Interview, line 11). Her view of the state test was also apparent in the seventh professional development session when she described virtual classrooms that she and other colleagues were creating. She noted, "Every room has a spot for the state released questions" (Postlesson Interview, Line 223) and mentioned several times throughout the professional development program and again in the postlesson interview about the importance of making sure students are not tricked on the state test.

Characterization of Students. A contradiction was also evident in Dana's characterization of students. Prior to her participation in the professional development program, Dana described the learning process as one that involved students in making choices and conferencing with peers. During the preprogram interview, Dana explained how she co-created rubrics with students for a graphing assignment, gave them a choice in what activity they felt they are ready for next, and had students conference with peers to discuss incorrect answers after assessments. For example, “on day four of every lesson, [students] get to choose...their path. Do you need a review...or are you ready for a challenge? ...So it’s, it’s a way for them to take control of what they need” (Preprogram Interview, line 58). Although Dana developed a system where students chose their activity, there is no evidence that the activities themselves were mathematically creative. One way to structure a creative environment is to allow for student choice. However, Dana implemented tasks that were potentially absent of mathematical creativity and recognized that with the switch to virtual learning students were not independent. She described an immediate need to foster students’ independence and commented that with the switch to virtual learning, teachers:

> Definitely learned one thing, we did not prepare them, I don’t think anyone prepared them to read and read directions like they needed to do now. They’re used to it being said to them. The Google Classroom, they didn’t enjoy reading their directions in the morning and seeing their assignments. (Postlesson Interview, line 143)

She described a need to make students more independent. The switch to virtual learning highlighted this need because Dana was not present to guide students, deliver content, or describe instructions as had been done during in-person teaching.

Throughout the professional development program and switch to online learning, Dana realized the importance of, “giving them more creativity and... giving them the ability to solve their own problems. So, I guess it would just go back there that they can put in their own data and show us what they know” (Postlesson Interview, line 153). However, her lesson plan and discussion of student work did not demonstrate this idea in practice, highlighting a contradiction between mathematically creative and traditional ideas.

Conclusion

Participants placed in the *creative but hesitant* category provided evidence that at times they exemplified mathematically creative ideas, whereas other times they described traditional views of mathematics, creativity, and teaching and learning. Dana is an exemplar of the *creative but hesitant* group because although there are elements of mathematical creativity evident in her descriptions of her teaching (e.g. beliefs about students and classroom culture), there were also traditional elements of mathematics and creativity (e.g. focus on rules and memorization, define creativity as an artistic product, emphasis on stat test).

Dana demonstrated characteristics of both traditional and creative categories but was inconsistent in her view of one or the other. Even after her participation in the program, Dana continued to focus on creativity as artistic and often referenced students' creative product over the mathematical processes involved in an activity. She viewed creativity as important for student learning but noted it was challenging for teachers because, "It's more work...but I'm willing to do it, but it's a lot more work to open up 68 different assignments, rather than one assignment with the same answer key" (Postlesson Interview, line 190). These examples highlighted that although Dana created a classroom environment that might allow for the implementation of mathematical creativity, her definition of creativity remained focused on design and her desire for classroom structure created a barrier for the full implementation of these ideas.

Jane, Teaching for Creativity (Creative)

> Elegance for me is like madness in the classroom. You're solving it one way, he's getting it another way, and just bouncing off of each other. But I think that's mathematical

elegance, like I don't know, I just think mathematicians are kind of crazy, so that chaos is beautiful. (Professional Development Session 2, line 124)

Jane is an eighth-grade teacher at Park Elementary School with nine years of teaching experience. She holds certifications in K-12 mathematics, 6-8 mathematics, 6-8 science, principal and supervisor. She attends monthly grade level and cross district mathematics meetings and has had extensive experience with professional development outside of Tarnot district including education leadership classes, training for the Go Math curriculum, use of calculators in algebra, and IXL program. Go Math is a Houghton Mifflin Harcourt K-8 mathematics curriculum used by the Tarnot school district and IXL is a personalized online learning program provided for all K-8 mathematics students in the Tarnot school district. She collaborates with colleagues through monthly grade level professional learning communities and monthly mathematics faculty meetings. Jane hoped that this professional development program would help her engage students who lack confidence in mathematics.

The description of mathematical elegance as beautiful chaos encapsulated Jane's overall view of mathematics, creativity, and the connections between the two. Jane's ideas about her ability to be creative, implementation of creativity in her teaching practice, and view of student creativity evolved throughout her participation in the professional development program and was evident in her mathematically creative lessons and discussions about teaching and learning.

Prior to participating in the professional development program, Jane provided evidence of a flexible teaching style, described the importance of helping students make connections within mathematics, fostered student independence in preparation for high school, and demonstrated flexibility in her teaching practice. These existing beliefs about teaching and learning allowed her to expand upon and embrace characteristics of teaching creatively and teaching for creativity.

Throughout her participation in the professional development program, Jane developed mathematically creative lessons and reflected on the experience to improve the lessons. Her ideas about teaching and learning expanded to further align with mathematical creativity. Because of these shifts, an intricate interconnectedness of Jane's ideas about mathematics and creativity existed and her views of creativity, mathematics, and teaching and learning. In this section, I present the results of the evolution of Jane's views on creativity, mathematics, and teaching and learning, as they grew into mathematically creative concepts.

View of Mathematics

Prior to participating in the professional development program, Jane viewed mathematics as part of everyday life and described the importance of helping students see real-world connections in mathematics. For example, Jane commented:

> My favorite thing about math is that you can incorporate it into every aspect of your life. So, I actually brought this up in class today, we're doing systems of equations next week, and I was like, that is something you guys will really use in your life. And they're looking at me like I'm crazy. But it is, you don't realize that you're using math in certain decisions that you're making, but you are. So, it's something that you really do need to know in order to better yourself and to make sure that you're on the right track and make good decisions financially and otherwise, in the future. (Preprogram Interview, line 8)

This recognition of connections between mathematics and students' lives is one of many ways to exemplify creativity in mathematics. The ideas presented during the professional development program encouraged Jane to expand her initial definition of mathematics to include other mathematically creative ideas such as the importance of having students recognize different approaches to solve problems and incorporating ways to challenge and motivate students to figure out a problem.

Jane's initial belief about the importance of making connections in mathematics expanded as she participated in the professional development program. She continued to describe mathematics as applicable to everyday life and also recognized specific ways that she could help students personalize these connections. She noted, "everything is connected in some way, there's always a way to figure out a math problem. There's never one direct way, there's, especially with this [professional development program] I guess there's different approaches in order to figure things out" (Postlesson Interview, line 102). After participating in the professional development program, she viewed math as "creative in itself" (Postlesson Interview, line 124), described mathematical creativity as making personal connections to students' future careers, and highlighted a need for math in everyday life. This was similar to other teachers in the *creative* category.

View of Creativity

At the outset of the professional development program, Jane did not identify as creative. Instead, she noted, "I think I'm the least creative person ever" (Professional Development Session 1b, line 191) and, "if you give me an idea, I can work with it...but if you expect me to think of these ideas on my own…" (Professional Development Session 3a, line 154). This discontinuation of her sentence suggested that Jane was not confident in her ability to develop creative tasks and although she felt she could implement someone else's task, she did not believe she could generate creative tasks herself.

Jane's idea of herself as creative evolved throughout her participation in the professional development program. In her reflection from the fourth professional development session, Jane began to believe that she could be creative. She noted, “I see that I can be creative mathematically and thus feel more comfortable with engaging students in this way” (Professional Development Session 4 Reflection). Although she saw that she could be creative midway through the professional development program, she was still hesitant about implementing these ideas in practice. She commented that, “I am getting comfortable, just not there” and cited eighth grade student behavior as a challenge for implementing mathematically creative tasks (Professional Development Session 4 Reflection). However, by the conclusion of the professional development program, Jane fully embraced the idea of being creative in her classroom when she commented:

> There’s so much material to get through, there’s so much that we don’t have time to be creative. But we do have time to be creative, it’s just like putting it out there a different way. Instead of...that straightforward like three times three, what is it? Having them solve things. (Postlesson Interview, line 43)

Jane came to view the possibility of infusing creativity in mathematics by restructuring how to ask questions and present material and therefore no longer cited student behavior as a barrier to implementing mathematically creative ideas. This view of creativity made it possible for Jane to implement these ideas into her teaching as she came to see creativity and mathematics as interconnected.

In the *view of mathematics* section just prior, Jane described mathematics as creative. This was also true about creativity as she also came to view creativity as something that was inherently part of mathematics. During the preprogram interview, Jane had difficulty describing connections between mathematics and creativity. Although she noted that students solving problems in different ways could be considered creative and described mathematics as a puzzle, she was unsure of explicit connections between mathematics and creativity. This view shifted after participating in the professional development program as Jane came to view mathematics as inherently creative. She commented, “I just feel like math is creative in itself...People can always use math to figure out basically anything they want to” (Postlesson Interview, line 122). When asked if she thought creativity was something tangential to the curriculum or could be incorporated throughout the school year, she noted that these ideas could be weaved into each eighth-grade topic and might help students remember and apply mathematical ideas, highlighting Jane’s view of creativity as important for teaching mathematics.

Creativity as More than Art and Design. Jane demonstrated an evolution of her beliefs from seeing creativity traditionally as art and design to recognizing inherent connections and benefits of mathematical creativity. At the outset of the professional development program, Jane focused on creativity as art and design. When asked about challenges teachers might face when trying to implement creativity in mathematics, Jane was concerned about the physical materials needed to be creative and the financial burden that infusing creativity in mathematics might place on teachers. When thinking about how to incorporate creativity in mathematics, one concern she noted was:

> To find ways to have a creative lesson or have them do something creative that isn’t going to bog me down financially. Because I think when you think of ‘Oh, I have to be creative,’ now I have to go out and buy stuff. And it’s hard. Where do you draw the line? (Professional Development Session 1b, line 163)

Jane initially focused on creativity as an artistic product rather than the questioning and thinking elements of mathematical creativity. This view of creativity shifted after her participation in the professional development program as she came to view the importance of challenging and engaging students through mathematical creativity, evident in her description of embedding creativity throughout the school year and with each topic.

Creativity as Challenging Mathematics Content. During the professional development program teachers explored difficult but engaging problems. This helped Jane recognize mathematical creativity as challenging rather than focused on design. For example, during the second professional development session, participants were presented with the problem, “How can you cut eight pieces of cake from a round cake with three straight slices of a knife, without moving any pieces?” This problem encouraged participants to be curious about mathematical connections and develop insightful solutions to a problem. I provided teachers with PlayDoh and a plastic knife to model their thinking. However, only two teachers chose to use it as they solved the problem. This is one example from the professional development program where there was an element of fun or play but the thought-provoking mathematics task was central to the exploration and the materials were not necessary for them to engage in thinking about the problem.

Another example of an interesting, challenging task presented during the professional development program was order of magnitude problems (see Appendix B). These estimation problems challenged participants to think about how to define the context of the problem (e.g. Is the second question referring to one Dunkin Donuts, the entire company, or something else?) and encouraged them to demonstrate non-algorithmic decision making, explain reasoning, discuss varying approaches to the problem, be curious about mathematical connections, and continue to explore after the problem has been solved. Jane’s participation in the professional development program helped her realize that, “you can still use those challenging problems and it’s fun in a way” (Postlesson Interview, line 106). Engaging in these and other problems during the professional development program and being asked to think about how they might mirror these ideas to encourage students with interesting, engaging, and challenging mathematics tasks in their own classrooms, helped Jane realize the importance of challenging students through creativity in mathematics rather than simplifying the content through a game.

After participating in the professional development program, Jane distinguished between the idea of creativity as a game or activity and instead saw creativity as presenting mathematics differently for students by implementing questioning strategies that challenged them to think. She described existing activities as fun but noted that they lacked the challenge of mathematical creativity. For example, she described, "we have the escape rooms, we do things...that the kids find fun and they find it creative, but it's fun and it's not as challenging...I think creativity brings challenge" (Postlesson Interview, line 139). In this instance, Jane highlighted the importance of using creativity in mathematics to challenge students and foster deep understanding of the content. Jane highlighted the importance of challenging students through mathematically creative tasks and distinguished between superficial creativity of an artistic product or a game and instead identified that mathematically creative tasks required students to think deeply about the content.

The professional development program helped Jane come to view creativity as challenging students by developing engaging questions and activities to foster student discovery and curiosity. She also distinguished between fun activities normally implemented by teachers and those similar to the ones presented during the professional development program. This idea demonstrated that by including thought provoking elements, fun and engaging mathematics activities could also be creative, and that fun does not necessarily mean mathematical. This is consistent with literature that mathematical creativity challenges students to be fluent in their use of basic knowledge, flexible in their approach to problems, original in thinking in a unique way and generalizing ideas (Lev-Zamir & Leikin, 2011). Jane's revised view of creativity in mathematics helped her to adopt and implement many of the ideas discussed during the professional development which I will explore in the following section.

Adoption of Mathematical Creativity. Throughout her participation in the professional development program, Jane came to understand and appreciate traits of mathematical creativity. She collaborated with Rachel, another participant, to design, implement, and reflect on mathematically creative lessons. They used the traits of mathematical creativity as a starting point to develop their lessons. In this section, I present the results on how Jane came to appreciate and adopt mathematical creativity into her teaching practice by describing the lessons' connections to creativity, and ideas for future implementation of this lesson.

Jane and Rachel developed two lessons on volume of three-dimensional figures. The first lesson on volume of cones and cylinders was an extension of a previous lesson that was introduced four months earlier. Prior to implementing this lesson, Jane expected that students would remember the formulas previously introduced and engage with the material as a review and extension of the original lesson. However, after implementing the lesson, Jane realized that the first lesson did not help students make lasting connections with the material. She noted, "We covered [the formulas] earlier in the year. So, I think they probably truly forgot, which then shows like just learning the formula isn't as helpful" (Postlesson Interview, line 28). Jane came to recognize that the initial, traditional practice of having students memorize formulas without connection to the material was not successful for student learning. Later in the interview, she referenced this idea again when she reflected:

> That's why I think I chose volume too because I did give it to them previously… I gave them a packet and on volume, but who really remembers plugging into the formula? And

> who really can apply it now to real life problems that we're kind of initiating with? And it was drastically different. I'm sure on those packets they were getting 90s and 80s because it's just, here's the formula, look at the formula, plug it in, use a calculator. But being creative in the approach of giving it to them and applying it to something not as straightforward as we normally would, definitely shows, can they do this? And that's important because if they don't feel like they have to persevere in math at any stage, they're just going to give up even more so than they would give up previously. (Postlesson Interview, line 133)

Jane's participation in the professional development program helped her to realize the importance of having students explore and develop mathematical ideas, independent of the teacher, in order to make lasting connections with the material. During the professional development program participants were asked to think about how having students think deeply about the content in mathematically creative ways might help them make lasting connections with the material. Jane reflected on why students might lack understanding in mathematics noting, "It makes you understand why some of the grades are so low, their diagnostic levels are so low, because they like immediate answers, instead of really thinking out a problem" (Postlesson Interview, line 56). This idea of creativity in mathematics as thinking deeply about a problem, discovering relationships, and making connections within mathematics is important for students to be able to make lasting connections and recall these ideas in the future. These ideas are aligned with research on mathematical creativity that requires students to explain their reasoning, make connections to other disciplines, and take risks in problem solving.

Jane and Rachel used ideas associated with creativity that were discussed throughout the professional development program (e.g. explain reasoning, develop insightful solutions, be curious about mathematical connections) to design their lessons. Jane noted that as they developed the lessons, they focused more on how students might approach the problems being presented than she had in developing lessons previously.

The first lesson encouraged students to recognize and identify relationships between formulas for cones and cylinders. Students were first asked to think about connections between formulas for area of triangles and rectangles and to make connections to volume formulas for triangular and rectangular prisms. Students reviewed area and volume formulas for triangular and rectangular prisms and then were asked journal about the similarities and differences of the formulas for the area and volume of both figures. Then, they were asked to discover volume formulas for cylinders and cones and think critically about the application of these ideas in other situations. For example, students were presented with virtual objects with circular bases and asked to determine the formulas for cones and cylinders. They also extended the lesson for future in-person implementation to include geomodel folding shapes to provide another way for students to explore these connections. Instead of memorizing formulas, as was expected in the previous presentations of this content, students were expected to investigate different figures, generate connections between area and volume for both solids, and determine the formulas for volume of cylinders and cones.

The traits of creativity that Rachel and Jane identified in their lesson were encouraging students to be curious about mathematical connections, persevere in problem solving, and making mathematical connections by identifying relationships among formulas. To get students to be curious about mathematical connections, they began with having students reflect on connections with triangular and rectangular prisms, and built up to an exploration of cones and cylinders. They structured questions that guided students to identify and discover relationships between formulas. Jane noted that in making mathematical connections:

> We were trying to see with volume, could we connect it to anything else? With the connectedness one and we knew obviously connected to area, so when we originally did the triangles and rectangles, I feel like that's a little bit easier for students because you can relate the formulas for area and then you're like, hey, you went from 2D now you're going 3D, you're just adding one more dimension to get to volume to make a connection. (Postlesson Interview, line 67)

Schacter et al. (2006) identified one way to demonstrate mathematical creativity is by encouraging students to make mathematical connections. The questions Jane and Rachel posed in their lesson helped students to do this. Jane identified that students were required to think more, "than what they typically would because they couldn't just plug it into a formula and get the answer. They got parts of the answers, but then they couldn't answer the questions fully unless they really thought about it" (Postlesson Interview, line 84).

Jane saw the lesson as challenging, thought provoking, and interesting. Because of the challenging and interesting nature of the lesson, Jane expected students to persevere in completing the tasks. Jane noted:

> It is a little bit of a challenge when you throw something that they're not comfortable with….so, any problem that I think that we present to them that's not straightforward, has to have that trait of you're gonna have to persevere in order to get through this. (Postlesson Interview, line 68)

Although Jane identified perseverance as a trait embedded in the lesson, there was little evidence in the lesson plan itself that included ways to encourage student perseverance. Although the lesson itself did not encourage students to persevere, during the implementation of the lesson Jane commented in the online chat as students worked to encourage them to continue working on the activity. Had the lesson been implemented in person, there may have been more opportunities for Jane to elicit this students' perseverance. To encourage students to persevere, Jane and Rachel might have included an opportunity for students to share and compare answers, comment on one another's approaches to the problem, or journal about a challenge they faced and how they overcame it as they completed the task. This highlighted that although teachers in the *creative* category embraced and implemented many ideas from the professional development, continued support is needed because these ideas were novel for all participants.

Journaling. During the sixth professional development session, participants were asked to have students journal about mathematics, provide student samples, and reflect on the experience over a two-week period. The purpose of having students journal was to help them personalize mathematics, notice mathematics outside of the classroom, be curious about mathematical connections, and in some cases explain their reasoning or approach to a problem. Jane discussed the importance of journaling for her students and found that it was a way to understand their approaches to problems and perspectives about a topic. Part of her first lesson was to have students journal about the connections between volume formulas. In reflecting on students' journaling, Jane described how students made connections between mathematics and their daily lives as well as understood the importance of the topic outside of the classroom.

The lesson required students to compare dimensions of a cone and a cylinder. Jane posed the question, "If a cone and a cylinder had the same dimensions, which would have a greater volume and why?" Then, she asked them to identify different 3-dimensional figures around their house and in the world and also discuss why volume might be an important for them learn

(Professional Development Session 6 Reflection). Jane felt that these journal prompts helped students continue to think about mathematics content outside of class, explain their reasoning, and make connections to mathematics in their personal lives. Personalizing the content and making connections to things outside of the classroom also allowed for students to potentially make lasting connections with the material. Jane saw journals as a way to foster student independence and help them make connections between their own lives and mathematics in general. She used journaling as a way to encourage students to reflect on mathematics in their daily lives as well as to gain an understanding of students' interests. She noted:

> I thought [journaling] was a nice change of pace. Because...I wasn't just giving them outright problems to solve, it was more thinking behind how they use math in everyday life, and just conversations that we don't typically have, came about from it. But they got very personal with some of their examples of their questions, talking about their past and how they lived in one country, and this is what they did, and it just led to different types of conversations, which I think really made it more so like a journal, which I liked for some of my students. (Professional Development Session 7, line 30)

These aspects of mathematical creativity were evident in how Jane discussed the development of future lessons and how journaling might be even more effective in the future.

Collaboration. Participants were encouraged to collaborate when developing lessons that infused creativity in mathematics. Out of all professional development participants, Jane and Rachel were the only two teachers who chose to collaborate on their lesson. Jane viewed the opportunity to collaborate with Rachel as beneficial because it allowed her to think more about how to draw out mathematically creative ideas, see the lesson from other perspectives, and develop lessons focused on student discovery and independence. In previous experiences with other professional development programs, Jane did not have opportunities to collaborate with colleagues. She noted:

> We also haven't been presented the opportunity to create the lesson together and reflect on it together and bounce it off of each other and really talk about it. I see Rachel once every three months, four months, and we do talk on the phone here and there, but...we talked more during this professional development than we've talked in the previous years. (Postlesson Interview, line 40)

In the post lesson interview, Jane referenced how developing the lesson with Rachel helped them to push one another's thinking and view the lesson from the student perspective. She noted that she and Rachel thought about:

> The misconceptions that [students] might have about what's being asked of them, we could really bounce it off of each other. If I didn't bounce it off of somebody…I would have just planned it like 1, 2, 3...and just went with it the first time. But having to write it out and really go through it with somebody else, I had to think about a longer, which needed to happen. (Postlesson Interview, line 60)

In this instance collaboration pushed Jane to anticipate students' responses to a problem in a way that she had not done previously. This is an example of teaching for creativity in that teachers thought about the lesson from the students' perspective and pushed one another to think about how to incorporate traits of mathematical creativity into the lesson and identify what misconceptions students might have prior to implementing the lesson. For Jane, this was possible because she of her collaboration with Rachel.

Jane implemented this lesson with her students first. After this implementation, Jane and Rachel reflected on and adjusted the lesson to include a table that further encouraged ways for students to think about the similarities and differences among formulas. They reflected on the overall lesson again after Rachel's implementation to further improve the lesson. Jane recalled, "we even talked about it after her class did it and we're like, okay, next time we'd definitely do something like this again, but we need to like do A, B, and C, a little bit differently as well" (Postlesson Interview, line 60). Jane's participation in the professional development program and the opportunity to collaborate with another participant helped her to successfully imbed mathematically creative ideas into this geometry lesson.

For Jane, the experience of being encouraged to collaborate in a professional development program was different from other experiences. She described:

> The fact that we really did have to talk about it with each other and we had the chance to talk about it with each other…We had to think outside of our own comfort zones which like I said, we don't really do often, we just present kids with material and we'll help them in person see things a little bit clearer and easier to explain in person but I've never really got that in depth with the lesson with somebody else…so that was definitely beneficial. (Postlesson Interview, line 76)

The professional development program encouraged participants to collaborate, share ideas, and build upon one another's responses during the sessions as they worked through challenging tasks. This was intended to model the type of environment participants might create in their classrooms to allow for and encourage students' mathematical creativity.

View of Teaching and Learning

Jane came to appreciate and implement ideas of mathematical creativity into her teaching practice. In the following section I provide evidence of the evolution of Jane's views on teaching and learning and how she thought about the implementation of mathematical creativity into her teaching practice. I describe how Jane's views of creativity helped her to recognize the importance of students' problem-solving process, her characterization of students, and her transformation from teaching creatively to teaching for creativity.

Prior to beginning the professional development program, Jane demonstrated a flexible teaching style. For example, she described a mini-lesson on converting improper fractions into mixed numbers and noted, "you have discussions with them, and it might take you on a tangent, but if it's a good one, it's okay. So you appreciate that and you go with it" (Preprogram Interview, line 117). In this instance, Jane described ways she was flexible in her teaching philosophy. She reiterated this idea during the second professional development session when she noted that lessons can divert from the original plan if needed.

In designing and implementing their mathematically creative lessons, Jane and Rachel made a deliberate attempt to shift their teaching practice from asking straightforward questions to developing questions that encouraged student discovery and independence. Initially when the topic was first introduced four months prior, Jane reflected, "it was more like a here's with the formulas plug them in and go with it. It wasn't the discovery part of learning. It was more like just here it is. Go for it" (Postlesson Interview, line 63). Later, Jane identified:

> We're trying to present problems in different ways than we normally would. Because we're both kind of straightforward, here's the problem, solve it. Here's the formula, solve it. And this made us say like, "Oh, try to relate to real life, try to do something," a little bit different in the presentation of the material to have them stumble upon an answer themselves rather than saying like do ABC to get to the answer. (Postlesson Interview, line 18)

This demonstrated a shift from Jane's original ideas about teaching and learning at the outset of the professional development program. Initially, she presented material in a straightforward way where students were expected to input numbers into formulas and calculate a result. However, after participating, she realized the importance of having students make connections and discover answers to understand formulas.

Jane planned to adapt her instruction to model mathematically creative ideas by including non-algorithmic questions, going beyond simply solving straightforward equations or problems, and having students discuss different approaches to solving a problem. After participating in the professional development program, Jane identified the role of creativity in teaching mathematics as more important than she originally did at the outset of the professional development program. She commented:

> I think it's much bigger than I originally thought it was [laughs]. I think that it's important to infuse it in the traits that we were talking about...to challenge the kids a little bit more and have them write out their thoughts. At the end of the day is probably more important than them getting that straight forward problem, and just like done, 100, you get it. (Postlesson Interview, line 130)

During the professional development sessions, Jane described an attempt to present problems differently than she had prior to her participation to elicit students' mathematical reasoning and curiosity, and encouraged students to explain their reasoning and think deeply about the mathematics content. She came to believe that infusing creativity into mathematics, "gets people to think," and helps both teacher and students make connections among topics.

In the sixth professional development session, the facilitator presented participants with the idea that textbooks often present ratio and proportion problems in a traditional manner using recipe and paint mixing examples. Participants were tasked with working in small groups to draw on mathematical creativity to develop proportion problems and think about how they might present proportions differently. During the postlesson interview, Jane distinguished between textbook problems labeled “real-world” and mathematically creativity tasks that required students to make personalized connections to mathematics. She commented:

> I think we’ve been so [focused on] 21st century skills and as teachers, they make you just zone in on relating everything to real life and connecting it to the real world for the kids. And sometimes that’s just tiring…what if I don’t want to be this farmer on a land with acres, just present it in a different way that’s challenging but fun and just gets you thinking a little bit more than the farmer on land problem. (Postlesson Interview, line 106)

This comment exemplified Jane’s realization that textbook problems often do not help students make the meaningful connections intended because they are out of context, and instead offered that challenging, creative, mathematical content and encouraging students to personalize connections with mathematics might be more significant for student learning.

After participating in the professional development program, Jane compared her previous teaching style as straightforward and recognized a shift to instead focus on student understanding through mathematically creative tasks. In the postlesson interview, she reflected on how her teaching practice evolved as she participated in the professional development program:

> I was more a streamline, just do it this way, get it done, and [I] didn’t really think about it from other perspective, but I think being creative as a teacher now, I kind of have to put a little bit more thought into some of the lessons...or challenge problems that I present to

> the students to kind of get them going and to make sure they're actually understanding the content at hand. (line 122)

This is representative of a shift in Jane's thinking from straightforward to including mathematically creative ideas.

Focus on Student Process Over Product. Jane highlighted differences between the volume lesson she and Rachel developed and previous lessons she implemented on the same topic. Lessons on volume prior to this professional development program included activities where students filled containers with sand or water was hands-on in that students were able to work with the material. Jane felt their new lesson on volume was more challenging and thought provoking for students. In reflecting on this lesson, she noted, that the mathematically creative lesson, "wasn't obviously hands-on, it was just more thought provoking for them, but it was definitely a challenge for them" (Postlesson Interview, line 65). This demonstrated a shift in Jane's thinking from the beginning of the professional development program when she described that creative mathematical tasks required supplies and added time to be implemented.

Because of the nature of virtual learning, the volume exploration was not hands-on. However, Jane felt their new lesson was more challenging for students because of the depth of thinking and reasoning it required. Jane realized that only presenting students with a series of problems to enter numbers into a formula was not enough for students' understanding or their ability to apply it to different situations later in the year. She noted:

> Some of them get [understand] the formulas and I feel like most math people are like just plug it in and go with it and be done with it, but some [students] need to see it a little bit more, and we're trying to get them to see it a little bit differently (Postlesson Interview line, 65).

Through this lesson, Jane noted that she and Rachel were trying to get students to think more deeply, not just input numbers into a formula to calculate an answer and focus instead on the problem-solving process rather than the final product.

Ownership of Student Learning. To prepare her eighth-grade students for high school, Jane described the importance of fostering student independence by using a hands-off approach and encouraging students to persevere when solving problems. Initially, to foster student independence, a typical day in her classroom included students opening a daily agenda on their Chromebooks and following instructions. However, the agenda was often a set of step by step instructions (e.g. turn in your homework, take out your textbook, work on certain problems), rather than tasks that required students to think differently about mathematics. She described student independence in terms of students' ability to turn in assignments on time rather than implementing mathematically creative activities that fostered student independence for their learning. As she participated in the professional development program, her ideas about student independence evolved and Jane began to view student independence as less teacher directed instruction to allow for student discovery.

Although Jane believed in the importance of student independence, in the preprogram interview she estimated that the majority of students adapted their approach to solve a problem to the way she presents material. After her participation in the professional development program, she came to believe that it was not only important to foster student independence in the sense that they were expected to be responsible for turning in assignments and staying on task, but also in their ability to think about mathematics and devise their own approaches to solving problems.

She noted the importance of building students' confidence in mathematics and did so by creating a welcoming environment that encouraged students' questions and embraced different approaches to problems. In helping to build students' confidence, she highlighted a classroom environment where mistakes were encouraged noting, "Mistakes happen, mistakes are encouraged, learning from a mistake is the best way to learn" (Professional Development Session 2 Reflection). She also believed that it was important for the teacher to help students recognize mathematics as less intimidating because many students come to eighth grade lacking confidence in the subject. After the professional development program, Jane spoke more about the importance of student discovery and having students take ownership of their own learning. Although she noted that this process was uncomfortable for herself and her students, it was necessary to allow students to struggle and engage with the material for learning.

Characterization of Students. Overall, Jane was disappointed in her students' responses to the lessons. She attributed students' struggles to implementing these lessons toward the end of the school year and in an online setting. She also reflected that this was the first time she asked students to think in a mathematically creative way, which presented a challenge for students. She felt that beginning with this type of questioning from the beginning of the year would allow for student growth and comfort with mathematically creative ideas that placed more responsibility on students.

Jane commented that, "by eighth grade we've lost a lot of kids. So, I think [mathematical creativity] is not only obviously important in middle school…but they definitely have to be brought into our curriculum a little bit more" (Postlesson Interview, line 133). Introducing these ideas earlier in students' education, as Jane suggested, might help them to be more comfortable with the type of thinking required of mathematically creative tasks. She also noted the importance of having these mathematically creative ideas introduced in earlier grades so students were more comfortable with these ideas by eighth grade. When thinking about her introduction of mathematically creative ideas, she recalled, “I think this time it was like a culture shock for them” (Postlesson Interview, line 137). Jane believed that students struggled with the lesson because of the depth of knowledge it required of them. This was different from how they had been taught previously both in previous grades and earlier that school year. Jane reflected that in the future she would need to, “ease them into this new… approach to learning. Because…not saying anybody’s a bad teacher, [but] I don’t think any of us have really done something like that before” (Postlesson Interview, line 137). It was evident that for Jane, students needed support and practice with the implementation of mathematical creativity because of the level of challenge that these tasks required.

Jane realized that struggling students might benefit from introducing creativity in mathematics. She described some students as “lost” and referenced weaving mathematically creative lessons throughout each marking period to help build student confidence in mathematics, particularly for students who are struggling. She thought it would be beneficial to choose, “two or three lessons for marking period one to kind of infuse the creative aspect of giving them a new approach to being presented with a problem or a content that we’re learning or standard that we’re learning (Postlesson Interview, line 156). Jane saw creativity as interwoven with mathematics content.

This idea of infusing creativity in mathematics to allow for all students to have access to the subject is not often highlighted in existing research which recognizes gifted students as mathematically creative. However, it is also important to think about and explore how infusing creativity in mathematics might offer opportunities for struggling students to increase participation and better understand the material. Jane identified the importance of infusing creativity into mathematics to allow struggling students access to the content and providing ways for struggling students to participate in mathematical activities by exploring and celebrating alternate solutions to problems.

Teaching for Creativity. There was a shift in how Jane defined teacher and student creativity. At the outset of the professional development program she focused on teaching creatively and did not view creativity in mathematics as “the most important” but noted that teachers:

> Have to be somewhat creative in order to get [students’] attention and to engage them a little bit. Especially with this generation, they’re so tech savvy, they’re so all over the place...So you have to think of ways to get them moving at all times and just every couple minutes, shift their focus. (Preprogram Interview, line 76)

When asked how students might be creative, she noted that students might think creatively to understand a problem but focused on the teacher’s presentation of material instead of ways students might be creative themselves. She noted, “Some kids will just understand like cut and dry, seven plus seven is 14, but some kids have to see it many ways and they have to see it in different...types of ways in order to fully understand” (Preprogram Interview, line 92). This is representative of a view of teaching creatively by presenting material to students in a variety of ways to reach all learners.

This definition teaching creatively by presenting material differently to engage students shifted after her participation in the professional development program. After the professional development she commented on both teaching creatively and teaching for creativity when she described, “I guess the thought process on how students go about solving a problem and how teachers, now how teachers implement the lesson and their approach teaching content is definitely something that...people can be more creative with” (Postlesson Interview, line 122).

Prior to the professional development program, challenging mathematics, if presented, was considered separate from instruction and included as a bonus problem on a test. After participating in the professional development program, Jane viewed challenging students through creativity as imbedded in, rather than separate from, mathematics.

Future Implementation of Mathematical Creativity. Throughout the professional development program Jane was reflective about her teaching practice and how she might incorporate ideas of mathematical creativity into her teaching. For example, she elaborated on another participants' comment about students' lack of curiosity in mathematics when she said:

> I agree with you about, what'd you say about the creativity? They're not curious on many things. But I think it's kind of like, we're not really curious either. We have a game plan, we got to stick to the game plan. But when you throw questions, you throw up the past one that we just did, the one from last week [the cake problem], it provides for an opportunity to be a little bit curious, to provoke kids to have that creative thinking outside of what we're doing...You're thinking mathematically without thinking mathematically. That, I think the kids would be a little bit more open to speaking. (Professional Development Session 2, line 118)

Her ability to reflect honestly on her own shortcomings may have allowed her to think about and implement new ideas immediately and effectively into her teaching practice.

Jane noted that because of the depth of thinking that mathematical creativity required of both students and teachers, it was necessary to plan these lessons in advance. In particular, Jane identified ways that she might be successful in implementing mathematical creativity into future lessons. She noted:

> I would have to plan, maybe I'd spend the summer planning out some of those activities so that I could grab them on the go…the morning of if needed. But I would definitely have to plan that out previously I couldn't just throw a problem, even the cake one at them without thinking about it previously myself and thinking about what they think about it. Just taking into consideration, like how it would play out. Whereas I don't, not that I don't take into consideration how it maps out, but some of the things are so straightforward that you don't have to take that into consideration. (Postlesson Interview, line 207)

In this instance, Jane thought about how she might continue to implement the ideas from the professional development sessions in the future. For example, Jane felt that it was difficult to expect students to think differently about mathematics and engage with material in a creative way midway through the year. Therefore, at the start of next school year she intended to introduce journaling to get students into the habit of thinking differently and exploring ideas in mathematics rather than answering straightforward questions. Jane recognized that introducing these ideas at the beginning of the school year might help students become more comfortable with this type of thinking.

Conclusion

Throughout her participation in the professional development program, Jane's ideas about mathematics, creativity, and teaching and learning evolved and expanded to include mathematically creative traits, dispositions, and practices. Participants in the *creative* category tended to have mathematically creative views of the classroom including the importance of student independence, discovery, and making connections among topics within mathematics and between mathematics and students' personal lives. As Jane exemplified, participants in this category demonstrated the ability to implement traits of mathematical creativity into their lesson plans and viewed mathematical creativity as intertwined with middle school mathematics.

Findings of Cross-Case Analysis

I looked across the findings from the three exemplars' views of mathematics, creativity, and teaching and learning. I took these findings and conducted a cross-case analysis, analyzing similarities, differences, and relationships specifically around the research questions:

1. How do in-service mathematics participants' conceptions of mathematics and mathematical creativity evolve as they participate in a professional development program that focuses on mathematical creativity?
2. What are in-service mathematics participants' experiences infusing creativity into their mathematics lessons as they participate in a professional development program?
    a. How do the ideas of fostering mathematical creativity explored in a professional development program transfer to in-service participants' practice?
    b. How do participants describe the development and implementation of lessons that require creativity and encourage creativity among students?

This analysis allowed me to highlight and compare the important aspects of the three exemplars' stories. Through this analysis, I further elucidate the differences among the three categories identified in the main unit of analysis: *adherence to traditional teaching practices (traditional), appreciation of teaching for creativity (creative but hesitant),* and *teaching for creativity (creative)*. As noted in the results for the main unit of analysis, participants discussed mathematics, creativity, and teaching and learning in varying levels of detail, and designed and implemented lessons intended to incorporate creativity into the teaching and learning of mathematics differently. In the following section I describe the similarities and differences among the exemplars related to their views of mathematics, creativity, and teaching and learning, discuss these ideas in terms of their lesson design and implementation, and outline how the professional development program may have impacted participants.

Conceptions of Mathematics, Creativity, and Teaching and Learning

The three participants discussed mathematics, creativity, and teaching and learning on a spectrum of views that aligned with traditional practices to creativity. **Error! Reference source not found.** provides an overview of these similarities and differences among the three exemplars. Stephanie's *(traditional)* ideas about mathematics, creativity, and teaching and learning tended to be traditional in that she defined mathematics as concrete, creativity as artistic, and characterized only advanced students as having the potential to be creative. Similarly, Dana *(creative but hesitant)* thought about mathematics as concrete and creativity as artistic. However, she differed in that she also discussed mathematics as focused on students' process rather than product and creativity as potentially beneficial for struggling students. She was conflicted in her views about mathematics, creativity, and teaching and learning throughout the professional development program which was also reflected in her lesson plan. At times Dana demonstrated traditional views of mathematics and creativity whereas other times she described elements of mathematical creativity and an environment where mathematical creativity might be possible. Jane *(creative)*, demonstrated growth in her ideas about mathematics and creativity. While she initially viewed mathematics as connected with daily life and highlighted the importance of student independence and responsibility for learning, she did not view herself as creative and thought of creativity as an artistic product. However, unlike Dana or Stephanie, through the professional development program, Jane came to view mathematics and creativity as interconnected and at times inseparable. Because of this, Jane's views on creativity, mathematics, and teaching and learning naturally grew into mathematically creative concepts and she described connections between mathematics and creativity.

Table 9

Overview of Exemplar's Views on Mathematics, Creativity, and Teaching and Learning

Categories in which participants differed	**Stephanie's Views** ***(Traditional)***	**Dana's Views** ***(Creative but Hesitant)***	**Jane's Views** ***(Creative)***
View of Mathematics	Concrete and definite, separate from creativity	*Important to generate conceptual understanding* but continued focus on memorization	Connected to students' lives, open-ended, and *interconnected with creativity*
View of Creativity	Creativity as artistic	Creativity as artistic	Creativity as artistic and *interconnected with mathematics*
	Creativity as straightforward, fun, and less challenging	Creativity as straightforward and fun	Creativity as *challenging* but fun
	Mathematical creativity interpreted differently than introduced in professional development	Mathematical creativity interpreted differently than introduced in professional development	Mathematical creativity as *imbedded throughout and intertwined with mathematics*
View of Teaching and Learning	Focus on state test	Importance of real-world connections but focused on state test	Importance of real-world connections, no focus placed on state test
	Focus on student product	*Discussed student process* but focused on product	Focus on student process
	Participant ownership of student learning	Participant *and student ownership* of student learning	*Student ownership of learning*
	Mistakes as an opportunity for participant to fix	Mistakes as learning opportunities to be fixed before state test	Mistakes as a positive learning opportunity for conceptual understanding

	Initially described struggling students as creative, shifted to see *only gifted students as creative*	Gifted and *struggling students as creative*	Gifted and *struggling students as creative*
	Viewed herself as teaching creatively but misinterpreted the meaning of doing so	Teaching creatively but focused on students' and participants' artistic products	Teaching creatively and *teaching for creativity*
Purpose of Professional Development Program	Reaffirmed her teaching practice as creative	Reaffirmed her teaching practice as creative and developed new activities, although continued focus on art	Collaborated with colleagues and developed new insights to implement mathematically creative tasks and foster these ides among students
Alignment between views and teaching practice	Traditional views aligned with traditional teaching practice as Stephanie continued to focus on procedures and definite answers	Contradiction between creative views and teaching practice, described the importance of conceptual understanding, but continued to focus on procedural tasks	Mathematically creative views aligned with teaching practice

[1] Italics indicate a shift in participants' views prior to and at the conclusion of the professional development program

In conducting the cross-case analysis, I first if investigated shifts in each exemplar's thinking was evident in their discussions of mathematics. I then explored their ideas about creativity. Lastly, I describe shifts in their views on teaching and learning. In the following section, I present the results regarding how Stephanie, Dana, and Jane viewed mathematics, how these views changed throughout their participation in the professional development program, and describe the main similarities and differences among them.

View of Mathematics

One theme that came through among the three exemplars was the extent to which participants saw mathematics on a spectrum of concrete to open-ended. The participants differed in how they initially viewed mathematics on this spectrum. At the outset of the professional development program, Stephanie and Dana both viewed mathematics as concrete whereas Jane initially thought of mathematics as connected to students' lives outside of the classroom. Stephanie's view of mathematics remained static after her participation in the professional development program whereas Dana and Jane's views evolved. Although there was evolution in Dana and Jane's views, differences among all three participants' views of mathematics continued to exist.

Stephanie's initial view of mathematics as concrete did not change throughout the professional development program. Instead, she continued to view mathematics as having a definite answer, particularly at the middle school level, and described herself as concrete in her thinking about the subject. Stephanie was uncomfortable with the exploratory nature of mathematics presented throughout the professional development program and continued to view herself as concrete in her thinking. Therefore, she had difficulty seeing mathematics as open-ended and instead remained static in her description of mathematics as concrete and definite.

In contrast, Dana's view of mathematics shifted after participating in the professional development program in that she described the importance of having students develop a conceptual understanding of mathematics. However, in practice she continued to offer procedural strategies to help students memorize a set of steps that were absent of conceptual understanding. A contradiction in Dana's beliefs about mathematics existed where she described both the importance of students' conceptual understanding, yet continued to focus on procedures in her presentation of mathematics. Dana initially described mathematics as focusing on students' problem-solving process. However, a contradiction was evident in her view of mathematics where she remained focused on procedural tricks for student memorization.

Prior to participating in the professional development program, Jane outlined a flexible teaching style and described the importance of students' making connections to their own lives. Jane's initial view of mathematics expanded from seeing the importance of mathematics in students' everyday lives to include mathematically creative ideas. After participating in the professional development program, Jane continued to see mathematics as connected to daily life and expanded this to include mathematics as elegant, chaotic, and interconnected with creativity. Because Jane initially described mathematics in a creative way as connected to students' lives, she did not have far to shift to get to her final view of mathematics as creative. Jane's views of mathematics naturally grew into seeing mathematics as elegant and "creative in itself" because she initially described ideas that aligned with mathematical creativity.

View of Creativity

The participants' ideas also varied regarding their views of creativity on a spectrum from artistic to mathematical. At the outset of the professional development program, all three participants described the artistic nature of creativity and focused on how a teacher might be creative in their presentation of mathematics. After participating in the professional development program, Stephanie's ideas remained static but Dana and Jane's ideas evolved. Stephanie continued to describe creativity as artistic and separate from mathematics. In contrast, Dana described the importance of mathematical content when thinking about creativity. However, she also described her own and students' design products in detail, which were void of mathematical thinking. In contrast, rather than focusing on a product, Jane came to recognize creativity as more than art and design, and instead described mathematical creativity as challenging and involving inherent connections between them.

Creativity as Artistic

Prior to participating in the professional development program, all exemplars described the artistic nature of creativity. While Stephanie viewed art, poetry, and music as creative, Dana and Jane described activities that required materials (e.g. glue, crayons) and focused on the artistic design of a product. While this view remained static for Stephanie, Dana and Jane's views evolved after participating in the professional development program. Stephanie continued to focus on creativity as solely artistic, Dana described connections between creativity and mathematics but continued to focus on students' artistic product, and Jane viewed creativity as mathematical rather than artistic.

Initially, Stephanie saw art, poetry, and music as creative and described creative activities as ones that required students to focus on design rather than mathematics. After the professional development program, she continued to describe the artistic nature of creativity. For Stephanie, creativity was a way to engage students by drawing their attention to art and design, separate from mathematics. This was evident in her lesson plan where students designed a virtual taco truck, chose prices for items, and calculated the costs of these items including sales tax and discounts. For this assignment, points were allocated separately for creativity and mathematics and the two were equally weighted. When Stephanie described the taco truck lesson, she focused on following directions in designing the taco truck and completing straightforward problems with teacher-made examples, rather than on mathematically creative ideas. This task might have been mathematically creative if Stephanie provided opportunities for students to explore connections between percents, explain their reasoning as they solved percent problems, build upon other students' ideas, or depict their reasoning visually. However, the focus of the lesson was on the taco truck design, following directions, and completing straight forward calculations.

Similarly, Dana initially described projects that included cutting and pasting items onto a holiday stocking and coloring graphs to unveil secret messages, making the artistic design the central focus of the activity. After participating in the professional development program, Dana described the importance of seeing beyond an artistic design and instead focused on the mathematical content. Although there were instances where Dana described creativity as getting students to explain their reasoning and focus on the content of students' work, she also continued to discuss artistic products, particularly in relation to virtual classroom designs she, her colleagues, and students developed. For example, her lesson plan encouraged students to design their dream bedroom and Dana was impressed by student designs and abilities to create these rooms, but did not comment on mathematics related to the task.

Prior to participating in the professional development program, Jane defined creativity as art and design. She worried about the cost of materials when thinking about infusing creativity into mathematics and discussed artistic products as creative. However, Jane's ideas evolved throughout her participation in the professional development program in that she came to see mathematics and creativity as intertwined and no longer described creativity as artistic when thinking about mathematics. Instead, she discussed how she came to view the two as closely connected and described that infusing creativity in mathematics challenged students to think deeply about the content.

Creativity as Fun Versus Challenging

At the outset of the professional development program, all three participants viewed creativity in mathematics as being fun or represented as a game. After participating in the professional development program, all three participants continued to view creativity in mathematics as something fun for students. However, they differed in whether they saw mathematical creativity as engaging students in fun and easy or fun and challenging tasks. For example, Stephanie focused on students' enjoyment of a task and described creativity in mathematics as fun rather than boring. For example, she described students getting out of their seats to add ideas to chart paper as creative. She noted that creativity was:

> Something as simple as chart paper, putting questions up…I'll have the kids go up with different color markers and they have to contribute an answer like that's, it's silly, but I guess that's creative in the sense that they don't realize what they're doing because they will just want to get up in the old just wanna do it. (Preprogram Interview, line 65)

Although having students contribute to chart paper has the potential to be creative, the task itself was not necessarily challenging. Rather, a focus was placed on students being engaged because it was different than textbook problems instead of being a challenging task. Similarly, Dana described how creativity could be implemented as a game, puzzle, or hands-on experience. Dana noted that she tried to take the content from the curriculum and, "make it more interactive for them… they were walking a number line instead of just seeing one on a paper, but they would roll dice and walk out a number line and follow order of operations, left to right" (Postlesson Interview, line 173). In contrast, Jane noted that adding mathematical creativity to a task makes a fun task challenging and attended to the depth of thinking required of mathematically creative tasks. The difference among the exemplars was the degree to which they saw a fun mathematical task as challenging.

While Stephanie and Dana continued to view fun activities as creative and a break from instruction, Jane recognized the importance of thought-provoking tasks within the fun activity.

Stephanie viewed mathematical creativity as making the content easier for students by removing challenging numbers (e.g. decimals or fractions). For example, Stephanie noted that students were required to choose their own cost of tacos in her lesson. She recalled, “I didn’t even tell them, be easy, make your taco a dollar, make it two, have people making it $1.47 which then…when you’re starting to do your percentages, they realize, oh my God, I’m getting like these long answers” (Postlesson Interview, line 33). She also highlighted that the lesson included basic math and following directions rather than challenging students to think creatively. Stephanie noted:

> Creating the menu was basic math, you have to have something and it can’t be more than $5, and I had kids make tacos [for] $15 and I’m like the direction[s] said you can’t have more than five, so again following directions, and that was basic math, adding up their total. (Postlesson Interview, line 24)

Stephanie demonstrated a view of creativity as simplifying mathematics and placed in the background to the artistic focus of the activity. Dana reported similar ideas about creativity as using easier numbers in problems. She described a struggling student choosing an easy number, 10, in order to complete a task. This demonstrated a focus on mathematical creativity as less challenging, different from how it was introduced in the professional development program. For Dana, struggling students could choose easier numbers for the problems they created. However, she also noted creativity in mathematics added a challenging component. She recalled an activity where students reviewed geometry:

> I was able to see their creativity and know that they truly understood the depth behind each geometry topic. Where instead of just handing them a worksheet of area, okay they can multiply two numbers, let’s move on. But now they created it and they worked

through the challenge question and designed a…new room with the same area. (Postlesson Interview, line 61)

Although Dana viewed creativity as potentially challenging by engaging in activities that differed from a worksheet, she continued to keep challenge questions disjoint from the lesson itself, and viewed creativity as mainly separate from mathematics.

In contrast, Jane's views of creativity evolved from seeing creativity as fun to recognizing the thought and consideration needed for mathematically creative tasks. Similar to Dana, initially, Jane viewed challenging questions as separate from the lesson and introduced them as bonus problems on a test instead of incorporated throughout the lesson and in-class activities. For example, in the postlesson interview, Jane reflected that prior to participating in the professional development program, "I probably wouldn't have presented [challenging questions] to them until a test, like one problem on a test, and that would be a bonus problem, it wouldn't be a regular problem for them to think through" (line 83). This view of challenging questions as bonus problems on a test evolved as she participated in the professional development program to include mathematical creativity as a way to help students engage in thought provoking activities and foster deep conceptual understanding through challenging problems. Her views shifted to include mathematically creative ideas within the existing curriculum and adapted problems that encouraged students' creativity throughout modules rather than separately on a test. For example, Jane commented, "we do have time to be creative, it's just like putting it out there a different way instead of like I said that straightforward like three times three, what is it? Having them solve things." She also though about using creative examples to, "relate them to our own content to create something to weave into…every topic that we do" (Postlesson Interview, line 92). After participating in the professional development program Jane no longer saw mathematics and creativity as disjoint and instead included creativity throughout lessons to challenge student thinking.

Jane highlighted a need for students' deep understanding of concepts and described infusing creativity in mathematics as a way to encourage students' conceptual understanding. She noted the importance of using mathematical creativity to challenge students and get them to think differently than she had previously required of them. These ideas were also evident in their lesson design which focused on volume and required students to question, discover, and make connections in mathematics. In this lesson, Jane described that she was trying to encourage students to make connections between area and volume. The lesson required students to journal about the similarities and differences of area (triangle, rectangle) and volume (triangular prism, rectangular prism) formulas. This was extended to have students think about objects with circular bases. Students were asked to determine the volume formulas for cones and cylinders and describe similarities and differences among them. The activity challenged students to make connections among formulas and be independent in their exploration.

Misinterpretation or Integration of Mathematical Creativity

All exemplars discussed various traits of mathematical creativity throughout the professional development program and postlesson interviews. Stephanie highlighted that her lesson required students to explore multiple solutions to a problem, engage in non-algorithmic decision making, and develop new questions for others to consider. Dana described the spectrum of creativity as a major takeaway from the professional development program and something that she recalled in her development of virtual classrooms. Jane attempted to help students make lasting connections with the material by encouraging students to explore and develop connections among concepts independent from the teacher. She moved away from implementing tasks with straightforward problems that required students to calculate an answer and toward getting students to discover mathematical relationships, explain their reasoning, and take risks when solving problems.

Although each exemplar described traits of mathematical creativity and discussed creativity incorporated into their lesson, Stephanie and Dana misinterpreted these ideas whereas Jane integrated them as intended by the professional development facilitator. Stephanie misinterpreted what it meant to have students attend to these traits and instead implemented tasks focused on basic mathematics understanding rather than mathematically creative ones. The spectrum of creativity, introduced in Session 2, resonated with Dana and she discussed this idea several times during the postlesson interview. However, she misinterpreted the spectrum of creativity to mean sharing an artistic design of virtual classrooms with other participants and colleagues rather than focusing on creative mathematical content. In contrast, Jane adopted many ideas introduced during the professional development program, namely having students explain reasoning, discover relationships among topics, and be independent. She also encouraged students to make connections within and outside of the mathematics classroom, aligned with how they were presented in the professional development program.

View of Teaching and Learning

Similarities and differences also existed in the exemplars' discussion of teaching and learning. While Stephanie and Dana described the state test as important throughout the professional development program, Jane did not seem to be impacted by the pressures of the state test. The three exemplars initially thought about mathematics and creativity as disjoint in that there was not enough time to include creativity. This view remained static for Stephanie and Dana but shifted for Jane in that she recognized that creativity could be implemented by altering existing tasks. Jane initially presented straightforward problems for students to input numbers into formulas which was evident in her discussion of the first-time volume was introduced to students prior to participating in the professional development program. However, she came to focus on students' process and the importance of helping them make connections to discover answers to a problem.

Focus on State Test

Stephanie and Dana were often focused on ideas of teaching and learning that would positively impact students' scores on the yearly state test given each April. At the outset of the professional development program, Stephanie and Dana mentioned the importance of learning for the state test and focused on students' test scores. Due to the COVID-19 pandemic, the state test for the 2019-2020 school year was cancelled on March 24, 2019. This announcement happened between our last in-person session (March 11, 2019) and first remote meeting (April 8, 2019). Even after participating in the professional development program and the state test being cancelled, Stephanie and Dana continued to emphasize the state test in that they viewed fixing students' mistakes prior to the state test and highlighted that their students' test scores were generally high. For example, Stephanie noted that in the past, although she had requested to teach fifth-grade mathematics, the mathematics director chose not to move her to because her students' test scores were high. Stephanie was hindered by her concern about students' performance on the state test and was focused on covering all the material before the test in April. Similarly, Dana noted that although she was previously an elementary teacher, when she saw how well her sixth-grade students achieved on the state test, she knew that she was meant to teach at the middle school level. In contrast, Jane did not mention the state test at any point prior to, during, or after the professional development program.

Stephanie, Dana, and Jane all discussed mistakes as an opportunity for learning. However, they differed in how they approached correcting these mistakes and why the mistakes needed to be corrected. Stephanie took responsibility for students' mistakes and explained that student failure was her failure and through re-teaching a topic or directing students on making corrections to a single problem, she was able to ensure students understood the concept. Dana also described the importance of correcting mistakes throughout the school year, before the state test rather than a focus on students' conceptual understanding. Dana referenced the state test several times throughout the professional development program. For example, in the sixth and seventh professional development sessions, participants were asked to create a task on ratios and proportions that was aligned with ideas of mathematical creativity. In her group, Dana suggested that they reword a state-released question, "because I know we always want to hit those" (Professional Development Session 7[1], line 11). During the seventh professional development session, she also described how questions on the state test trick students and she developed ways for students to memorize procedures so as to not be tricked. She viewed mistakes as an opportunity to learn for the state test. This continued focus on the state test demonstrated its importance for Dana as the central purpose for learning and highlighted a barrier that at times prevented her from implementing mathematically creative ideas into practice.

In contrast, Jane's teaching practice was not restricted by the state standardized test at any point in the professional development program, preprogram or postlesson interviews. She did not reference the state test at any point. Instead, Jane focused on mathematical creativity as a way to help students improve their conceptual understandings and to remember, apply, and make connections among topics when re-introduced later in the school year or in future high school courses. The differences among the exemplars regarding the importance of the state test impacted how they thought about teaching and learning in general. The exemplars also differed in their focus on students' products or problem-solving process, ownership of student learning, and teaching creatively versus teaching for creativity.

Focus on Product Versus Process

Participants' focus on product versus process tended to be evident in their lessons designed to include mathematical creativity and they discussed students' products and problem-solving processes differently in the interviews. After participating in the professional development program, Stephanie, Dana, and Jane emphasized the importance of drawing attention to students' process and not only focusing on the product or final answer. However, in practice and further discussion of students, it was evident that Stephanie and Dana continued to highlight the importance of product over process, whereas Jane discussed both as important and developed ways for students to reflect on their problem-solving process.

Initially, Stephanie did not mention students' processes, rather she focused only on their final answer. Stephanie did not ask students to explain their reasoning and focused only on correct and incorrect answers. However, after participating in the professional development program, she described the importance of reviewing students' scrap work. This suggested that she came to see the process as important in students' mathematics learning. However, in reviewing students scrap work, Stephanie remained focused on errors for her to fix and a way for her to get students to the correct, final answer, viewing students' process as the procedural steps they took to arrive at their answer. In contrast, Dana highlighted the importance of process both prior to and after the professional development program. Initially, she described the importance of having students explain their reasoning and for her to understanding their process. However, even after her participation in the professional development program, she continued to focus on the final, artistic product and tricks for memorizing steps, and discussed students' process as a series of steps rather than conceptual understanding.

Jane did not discuss students' product or process directly throughout the professional development program or postlesson interview. However, her lesson plan included a component for student reflection through journaling where students were prompted to identify and explore their problem-solving processes. In having students journal about their approach to problems, Jane was able to assess both their final answer and process to arrive at their answer. Jane was interested in students' conceptual understanding of the problem-solving process, not just the procedural steps they completed to arrive at their answer.

Ownership of Learning

The three exemplars noted the importance of student independence but differed in the level of ownership of student learning they encouraged and provided instruction in different ways. Stephanie viewed student independence as important, particularly after the shift to virtual learning. Although Stephanie noted student independence as important, this was not evident in her discussion of teaching and learning, nor were opportunities for student independence apparent in her lesson design. Instead, Stephanie was reluctant to allow students freedom to be independent and discover mathematics because she was initially cautious about behavior management issues in person and was not confident that students would be successful on their own. She also noted that many seventh-grade students did not have enough mathematics knowledge to be independent and instead would allow other students in the group to do the work for them as they sat quietly without participating. She viewed high achieving students as having the ability to be creative whereas lower achieving students would not persevere or attempt mathematically creative tasks. She was concerned that if she introduced mathematically creative tasks, "the kids who are slower just quit and wait for the faster kids to do it" (Postlesson Interview, line 133). She also described that, "it's hard to be creative, because it's two different levels of creative and I don't know how to balance that…what can I do to balance that in groups that are just so high and low?" (Postlesson Interview, line 134).

Stephanie demonstrated a contradiction in her view of student creativity. Although she described creativity as simple tasks that included basic mathematics, she also commented that mathematical creativity was too complicated for struggling students and that only higher achieving students would be able to think at the level required of mathematical creativity. This view of students shows Stephanie's ownership of student learning where they needed structure, teacher guidance, and rules to engage in challenging tasks.

In contrast, Dana and Jane discussed creating an environment where mathematical creativity might flourish in that they described the importance of student independence, described a supportive environment where students were encouraged to take risks, and teachers were tolerant of ideas that did not lead to the correct answer. However, they differed in how these ideas were implemented in practice. Although Dana viewed creativity as useful for struggling students to share their knowledge in different ways, this was mainly focused on an artistic product rather than mathematics. In practice, Dana at times provided opportunities for students to choose their learning path and conference with peers to make test corrections. However, she continued to direct students in approaching a problem and provided premade notes and videos for students to review. Jane also noted that struggling students might benefit from mathematical creativity and saw student independence and building students' confidence as important for preparing her students for high school. After participating in the professional development program, Jane began to allow for student discovery and additional opportunities for student ownership of learning. In contrast with Dana who continued to provide direct instruction and have ownership of student learning, Jane's focus on student independence and student discovery was evident in her lesson plan that required students to think and discover ideas on their own.

Teaching Creatively Versus Teaching for Creativity

Initially, all three participants described teaching creatively in how they presented mathematics in different ways for diverse learners (e.g. auditory, kinesthetic, visual). In particular, Dana viewed creativity as the participant's presentation of material and described the many ways in which she demonstrated creativity in her classroom. For example, she described various tasks that allowed access points for all levels of students to dialogue about mathematics. Similarly, Jane thought about creativity in mathematics as presenting material in a way that addressed different types of learning styles and revisited ideas or included things external to the curriculum to ensure all students were prepared with the background knowledge needed for eighth-grade mathematics.

When selecting tasks, Stephanie searched for existing lessons or activities through *Teachers Pay Teachers* or *Pinterest*, websites where people share lessons, ideas, and resources for teaching and learning. She reflected that she was not creative because she could not develop ideas on her own. Similarly, Jane initially described herself as, "the least creative person" and was unsure that she would be a good candidate for this professional development program because of the focus on creativity. Conversely, Dana viewed herself as a creative teacher prior to participating in the professional development program because she presented material to students in a variety of ways. Although there was no evidence that a shift in Stephanie's teaching practice existed after her participation in the professional development program, she came to view her teaching as creative and relabeled things she already did in the classroom as creative. Similarly, although Dana misinterpreted some of the ideas presented in the professional development program and continued to focus on students' artistic products, she viewed her teaching as creative and noted that she was confident to implement the ideas presented in the professional development into her teaching practice.

After participating in the professional development program, Stephanie continued to focus only on the ways in which a teacher might be creative. Dana also continued to consider herself as creative describing how she designed lessons. However, she also added that students might be creative by showing her what they know through a less teacher-directed assignment and allowing students to develop new questions for others to consider. Comparatively, Jane focused on mathematics itself as creative and described the ways she might be creative. However, she also identified how she might foster students' creativity by having students reflect on their problem-solving process and discover connections in mathematics.

The similarities and differences among the exemplars were also evident in their design and implementation of mathematically creative lessons. All three exemplars submitted lessons and student work as requested by the professional development facilitator. However, they differed in how they designed the lesson, what they adjusted for implementing the lesson online, and the depth to which they included creativity.

Development and Implementation of Lessons

At the end of the professional development program, participants were tasked with submitting two lessons that exemplified the ideas of mathematical creativity presented in the professional development program. Participants were encouraged to use ideas of mathematical creativity to design a new lesson or to revise an existing one. The three exemplars developed and implemented their lessons using various mathematically creative ideas from the professional development program (Table 10). The exemplars differed in how they designed and implemented these tasks. Although participants were asked to submit their lessons to me for feedback prior to implementation, Stephanie and Dana implemented their lessons prior to submitting them. Therefore, I was unable to provide feedback on these lessons prior to their implementation. In contrast, Jane submitted her lessons to me and addressed my feedback, and also reflected on the lesson with Rachel after she implemented it with her students to adjust for future implementation.

Table 10

Details of Exemplars' Design and Implementation of Mathematically Creative Lessons

	Stephanie	Dana	Jane
Feedback	Not submitted for feedback	Not submitted for feedback	Submitted for feedback
Design	Submitted an existing, unchanged lesson	Developed new lessons for virtual learning but did not address mathematical creativity	Used the traits of creativity as the basis of her lesson design.
Implementation	Provided students with a Screencastify video, step by step examples with the mathematics worked out for each set of questions.	Provided students with a Screencastify video including instructions to design virtual rooms using Google Slides	Provided the task to students. Used the online chat to encourage them and provide feedback.

Stephanie relabeled her existing teaching practices and activities as creative. This was evident in the lessons she submitted in that although they were not altered to attend to mathematical creativity, she described them as creative. She revised the lesson so that it was suitable for virtual learning. However, the changes remained aligned with traditional teaching practices. These changes included a Screencastify video and students completing Google Slides presentations rather than doing the activity on paper. This allowed Stephanie to implement the lesson in a virtual, asynchronous setting, but did not address mathematical creativity. Dana developed new tasks for the virtual learning environment, but similar to Stephanie, she did not consider ideas of mathematical creativity until the postlesson interview when the facilitator asked what mathematically creative ideas were represented in the lesson. In contrast, Jane used the traits of creativity to design the lessons and collaborated with Rachel to think deeply about connections to creativity. The exemplars differed in the mathematical focus of the lesson, their approach to developing the lessons, and the degree to which they fostered students' creativity.

Jane collaborated with Rachel on her lesson development and reflected on the implementation twice, once after Jane implemented it with her students and again after Rachel's implementation. When reflecting on Jane's implementation, they discussed changes to the lesson plan to support students' success with the lesson. This collaboration was important in that it pushed Jane and Rachel's thinking about how to challenge students and be creative as they planned the lesson. Jane also noted that because of her collaboration with Rachel she thought more about how students might approach the task than she had when planning lessons individually. Jane reflected that collaborating with Rachel helped her to think more deeply about the lesson. Thinking about how students might approach the lesson was not evident in Stephanie or Dana's lessons.

Stephanie and Dana provided Screencastify videos and detailed instructions on how to complete the tasks embedded in their lessons. Stephanie included a separate step by step example of how to calculate each set of questions as well as a detailed recording of instructions for designing the taco truck. Similarly, Dana also included a Screencastify video for guidelines on how to design the virtual rooms in Google Slides. In contrast, Jane provided the lesson activity to students and encouraged discussion through an online chat with no further instruction of the steps they should take to complete the task, leaving it open for students to explore.

Conclusion

The results presented in the cross-case analysis further highlight the differences among the categories: *traditional, creative but hesitant,* and *creative*. The three exemplars' participation in the professional development program, combined with their initial views about mathematics, creativity, and teaching and learning, allowed them to engage with ideas presented during the professional development program to varying degrees from traditional to creative and each provided evidence of how they viewed connections between creativity and mathematics. While Stephanie and Dana described connections between mathematics and creativity, these were not evident in their lesson development or implementation in the same way that they were in Jane's lesson. While Stephanie and Dana continued to see mathematics as creativity as disjoint, Jane came to see the two as interconnected.

All three participants personalized the ideas presented in the professional development program in different ways. Stephanie did not seem to make changes to her teaching practice but instead relabeled what she was already doing as creative. Dana's initial ideas of herself as creative remained static even after her participation in the professional development program. Although she seemed to teach creatively by developing different activities for virtual learning, she did not teach for creativity in that tasks did not require student creativity. In contrast, Jane seemed to reorganize how she thought about presenting material to students in order to incorporate mathematical creativity into her teaching practice.

Although there is evidence that mathematically creative ideas might benefit students' mathematics learning (Sriraman & Lee, 2011; Yaftian, 2015), there is limited research on the development and implementation of mathematical creativity among middle school teachers. Findings from this study add to this limited body of research on creativity in mathematics education. The analysis of the exemplars allowed me to zoom into Schacter et al. (2006) protocol and identify the ways in which participants viewed mathematics, creativity, and teaching and learning. I analyzed participants' experiences participating in the professional development program and identified the evolution of their ideas focusing on their experiences developing and implementing mathematically creative lessons. In the following chapter I report on what these results mean for the diverse ways in which participants instantiated ideas of mathematical creativity into their teaching practice and fostered students' creativity in mathematics.

CHAPTER 5: DISCUSSION

In this chapter, I first consider the results in the context of the research on creativity specifically highlighting connections to the research on mathematical creativity. Rhodes (1961) categorized creativity using four strands (4Ps): person, process, product, and place. A creative person is able to connect ideas, see similarities, have flexibility, be inquisitive, and question norms. The creative process in classrooms might be how teachers and students engage in formulating or solving a problem, or developing new information about a concept. For a teacher, a creative product could be a lesson plan or project, whereas for a student the product may be represented in multiple responses to a question or the discovery of a new idea. The development of products and engaging in a creative process happen within an environment. Place refers to the environment in which creative activities are possible.

The findings of this study provide insight into each of the 4Ps and the interconnection among them. The results highlight the combination of participants' personal characteristics, lesson plans, problem-solving processes, and the environment that allowed teachers to foster creativity among middle school mathematics students. It also highlights the challenges participants faced when designing and implementing mathematically creative lessons and how professional development programs can attend to these challenges to further support teachers as they implement elements of mathematical creativity into their teaching practice.

The results presented in the previous chapter provided evidence of how participants conceived of mathematics and mathematical creativity, how their ideas evolved as they participated in a professional development program, and how they described designing and implementing lessons that include mathematical creativity. The first set of results identified the 12 participants' ideas regarding mathematics, creativity, and teaching and learning. Within these three groupings, participants' views varied on a spectrum of traditional to open-ended which led to the development of three participant groupings within the main unit of analysis, *adherence to traditional teaching (traditional), appreciation for teaching for creativity (creative but hesitant),* and *teaching for creativity (creative)*. From these groupings I chose one exemplar who was representative of the group (Table 8). The results from each exemplar and the cross-case analysis further highlighted the differences among these groups.

Table 8

Participants in each of the three groupings

Adherence to traditional teaching (traditional)	**Appreciation for teaching for creativity (creative but hesitant)**	**Teaching for creativity (creative)**
Alex	Andrea	Audrey
Drew	Dana	Jane
Jordan	Pamela	Megan
Stephanie	Phyllis	Rachel

In the following section I discuss the findings from my research, implications of the study, and next steps for future research. First, I will consider the findings within the context of the 4Ps. Then, I will describe my other contributions to the field including an overlap and interconnection among the 4Ps, the spectrum of creativity, and the triangular theory of creativity, the modification of Schacter et al.'s (2006) Creative Teaching Framework (CTF), and a revised professional development model for using creativity in the teaching and learning of K-12 mathematics. Last, I will discuss limitations of the study and recommendations for professional development programs and future research.

Contributions to Research

As noted above, the 4Ps of creativity (Rhodes, 1961) categorized definitions into four categories. In the following section I discuss how the results of my research fall within each of these P's and how these results highlighted an overlap among investment theory, triangular theory, and the spectrum of creativity within the context of the 4Ps.

Person

There are various characteristics associated with a creative person. Sternberg and Lubart's (1991) investment theory of creativity identifies six attributes: (a) intelligence, (b) knowledge, (c) thinking styles, (d) personality, (e) motivation, and (f) environment (See **Error! Reference source not found.** for a detailed description of attributes). Of these six attributes, I organize the first five under *person*, and discuss the last attribute, environment, separately as it is applicable to *place*. These attributes were evident among participants in this study to varying degrees. While some participants demonstrated these attributes, others discussed them but did not provide evidence of these ideas in practice. Still others did not demonstrate these attributes in any context and struggled to implement mathematically creative ideas into their teaching practice.

Table 11

Investment Theory: Attributes of Creativity (Sternberg & Lubart, 1996)

Attribute	Description of Attribute
Intelligence	Redefining a problem: generating ideas
	Evaluating the quality of ideas
	Promoting and refining ideas
Knowledge	Domain specific
	Informal
Thinking Style	Legislative
	Flexible
Personality	Perseverance
	Risk taking
Motivation	Intrinsic
	Extrinsic
Environment	Encourages, supports, and rewards creative thinking

In investment theory, Sternberg and Lubart (1996) identified that a creative person demonstrates a confluence of the aforementioned attributes of creativity in order to defy the crowd. The results of this study indicate that those who did not demonstrate these attributes of creativity had difficulty implementing elements of mathematical creativity into their teaching practice.

Participants in the *creative* group tended to exemplify attributes described in investment theory (e.g. legislative thinking style, willingness to grow, willingness to take risks, promoting and refining ideas). For example, Jane demonstrated flexibility, risk taking, persistence, innovation, and growth as she participated in the professional development program and developed lessons that included mathematical creativity. She engaged in refining ideas by reflecting on students' experiences with journaling and extending journaling to have students formulate new questions. She also reflected on improvements to her mathematically creative lesson for future implementation. Similarly, Audrey was willing to take risks and overcome obstacles. She implemented order of magnitude problems and redesigned a lesson on volume and surface area for her fifth-grade students. Audrey discussed how it was a risk for her to develop and implement these activities because she provided an opportunity for student independence, which she had not done in the past. The professional development program encouraged Audrey to allow for student independence. She noted, "without the sessions, I would be afraid to trust [students] to do it on their own…But with you, I took a chance…let's see what they can see on their own. So, it gave me encouragement to do risky things in math" (Postlesson Interview, line 21). The professional development program supported Audrey's willingness to try new things and take risks, demonstrating growth in her teaching practice toward mathematical creative ideas. These examples were similar to attributes demonstrated by other participants in the *creative* group and allowed them to incorporate elements of mathematical creativity into their teaching practice.

In contrast, participants in the *traditional* group exhibited attributes that were misaligned with investment theory. For example, Stephanie was inflexible and intolerant of ambiguity. She demonstrated an unwillingness to take risks and instead demonstrated an executive thinking style. This executive thinking style was evident in that she enjoyed rules and felt comfortable implementing tasks designed by others rather than developing them herself. It was evident that she had difficulty designing mathematically creative lessons and implementing ideas from the professional development. Instead, she relabeled her teaching as creative without making substantial changes to her practice. Similarly, Jordan described herself as concrete and demonstrated an intolerance of ambiguity. She also struggled to implement mathematical creativity into her teaching practice and remained focused on students' artistic product for her lesson activity on finding area of irregular polygons. Teachers in the *traditional* group tended to portray attributes that were misaligned with investment theory and therefore had difficulty implementing mathematically creative ideas in practice.

The results indicate that it would be beneficial to collect supplementary data on participants' existing *personality* and *thinking style* attributes at the outset of a professional development program to encourage those who tend to be concrete, intolerant of ambiguity, and unwilling to take risks in a more effective way and encourage them to buy into mathematically creative ideas.

Triangular Theory of Creativity

In developing triangular theory, Sternberg (2018) argued that in order to be creative one must defy oneself, the crowd, and the Zeitgeist. Defiance is most closely related to the attribute, willingness to take sensible risks, from investment theory. Defying oneself, the crowd, and the Zeitgeist imposes a need for one to be willing to take risks because convincing oneself or others of a novel idea can be discouraging. In order to engage in defiance, you must also have a confluence of attributes as outlined in the investment theory. Participants' varying attributes from investment theory influenced the degree to which they were able to defy themselves, the crowd, and the Zeitgeist. The results of this study indicate that when these attributes were evident in a person, such as those in the *creative* group, it allowed them to engage in one or more levels of defiance and engage in mathematical creativity. However, misalignment of attributes among participants in the *creative but hesitant* and *traditional* groups hindered them from demonstrating defiance at any level. In the following section I draw on triangular theory and provide examples of how participants in each group engaged in defiance and demonstrated or struggled to implement teaching creatively and teaching for creativity.

Participants in this study demonstrated defiance aligned with triangular theory at varying levels. Those in the *creative* group tended to defy themselves and their teaching practice to implement ideas of mathematical creativity into their lesson planning and activities. Participants in the *creative but hesitant* group at times discussed ideas that defied oneself or the crowd, yet in their lesson development and discussion of students did not demonstrate defiance. Participants categorized as *traditional* tended not to demonstrate defiance and instead remained static in their thinking about creativity, mathematics, and teaching and learning. In the following section I address each type of defiance separately and connect how participants demonstrated this defiance or were unable to in part because of missing attributes of investment theory.

Defying Oneself and One's Teaching Practice. As noted above, in order to defy oneself one must be willing to take risks and persevere. Participants in the *creative* group tended to demonstrate defiance of themselves and their teaching practice in that they were more flexible and willing to take risks whereas those categorized as *creative but hesitant* described taking risks and time where they defied themselves, but did not do so in practice. In contrast, those categorized as *traditional* tended to not discuss or implement ideas that required defiance of oneself.

Jane was able to engage in defying herself in part due to her legislative thinking style. She demonstrated a flexibility in that she often planned lessons the day before class, rather than the prior week, and was open to, "discussions with [students], and it might take you on a tangent, but if it's a good one, it's okay. So, you appreciate that and you go with it" (Preprogram Interview, line 117). This flexibility may be one reason that Jane came to see the importance of mathematical creativity for students and demonstrated defiance of herself and her existing teaching practice to include mathematical creativity. Rachel also demonstrated defiance of oneself. At the outset of the professional development program, Rachel described her teaching as traditional and noted that she provided step by step instructions for students. However, she was looking for ways to adjust her teaching to be less traditional and provided evidence of her willingness to grow. Throughout the professional development program Rachel defied her initial teaching practice by trying to implement mathematical creativity with students through journaling, questioning, and designing a lesson that required students to explore and demonstrate independence.

Teachers in the *creative but hesitant* group often described ways they defied their original teaching practice to include mathematical creativity. However, there was little evidence that participants altered their teaching practice to focus on elements of mathematical creativity. Instead, they continued to teach as they had prior to participating in the professional development program. For example, Pamela highlighted the importance of having students discover ideas on their own. However, she designed activities that did not foster student discovery and continued to think of creativity as projects, separate from the curriculum. Although projects might be considered creative, Pamela continued to implement projects in June, after all fifth-grade content was covered. She described the importance of mathematical creativity for student learning, but did not demonstrate these ideas in practice. This might be because their ideas of creativity did not align with their teaching and they remained rigid in ways participants in the *creative* group did not.

In contrast with participants in the other two groups, Stephanie demonstrated an executive thinking style in that she enjoyed implementing systems, rules, and others' tasks. She described herself as concrete and did not like when students identified her mistakes. Specifically, Stephanie noted, "With someone like me it's a blow to your ego if you're wrong. I hate when I put something on the board and it's wrong…and the kids are like, that's wrong and I'm like darn it!" (Postlesson Interview, line 142). This is exemplary of an inability to engage in defiance of oneself and therefore Stephanie may have been hindered in engaging in levels of defiance or adjusting her teaching practice to include mathematical creativity.

Stephanie and other participants in the *traditional* group did not demonstrate defiance. Instead, there seemed to be a lack of buy in or misinterpretation of the ideas presented in the professional development program. Alex initially held traditional views about mathematic, creativity, and teaching and learning. After participating in the professional development program, she continued to see mathematics as concrete, creativity as artistic, and teaching and learning as teacher centered. She remained focused on mathematics as having one correct answer and noted that if a student approached a problem differently that she presented it, she would be hesitant to share it with the class. She questioned, “Should I share this? Or should I just let it be a moment for the child, because sometimes other kids get confused. So, I kind of have to like feel it out first” (Postlesson Interview, line 129). This is representative of a lack of defiance of her teaching practice which may have prevented her from demonstrating creativity.

Defying the Crowd. Defying the crowd is described as presenting something to a group that differs from previously held beliefs of that group. In this study, the crowd might be defined as the participants in the professional development program. Some participants in the *creative* group demonstrated instances of defying the crowd when they noted that the pacing guide was meant to be loosely followed and it was more important for students to develop understanding of the material rather than cover the content for an assessment. They also did not highlight the state test as a constraint for creativity and instead developed ways for themselves and students to think differently about mathematics. In contrast, other teachers continued to focus on rules and algorithms, in some cases absent of conceptual understanding, and described the importance of students’ state test scores. Other examples of defying the crowd include the impact of the state test and views of who is capable of creativity.

Defying the Curriculum and State Test. Jane engaged in defying the crowd when she noted how she and other teachers had not been engaging in practices that fostered students’ lasting connections in math or developing their curiosity. For example, during the second professional development session, Jane described that she, and other teachers, did not demonstrate curiosity or require it of students. She noted, “[Students] are not curious on many things. But I think it’s kind of like, we’re not really curious either. We have a game plan, we got to stick to the game plan” (Professional Development Session 2, line 118). In this case, Jane exemplified defying the crowd as she critiqued her and other teachers’ lack of curiosity in teaching mathematics. She attributed teachers' lack of curiosity as a potential reason for students' lack of curiosity.

Similarly, some teachers in the aforementioned groupings also demonstrated a lack of defiance of the crowd in that they were note yet able to defy the curriculum or district pacing guide. There seemed to be a tension between teaching practice and curriculum. Participants noted that they skipped explorations or modified the order of the curriculum because it was not aligned with what needed to be covered for quarterly district assessments or the yearly state test. Traditionally teachers cite time as a constraint due to state testing and curricular pacing deadlines. The state test became crutch for them to continue teaching the way they were because of the state test even when state test was cancelled demonstrating that Dana and Stephanie did not engage in defying the crowd.

Participants in the *creative but hesitant* and *traditional* groups demonstrated a continued focus on the state test. Although the state test was cancelled on March 24, 2020 due to the COVID-19 pandemic, some participants continued to see the state test as a constraint, demonstrating a lack of defiance of the crowd. For example, Stephanie and Dana described themselves as good teachers because their students performed well on the state test. Even after the state test was cancelled, Dana designed virtual classrooms where each room had state released questions. She also continued to discuss how she prepared her students for the state test by practicing test taking strategies to minimize the likelihood of students getting tricked by the wording of questions. Similarly, Stephanie struggled to allow students to be independent in their exploration of mathematics because of the constraints of the state test. She noted, "I have such a control issue because I also know what has to be done and learned in a certain timeframe, the constraints of that state testing gets to me" (Postlesson Interview, line 128). Although she also described wanting to move away from having the state test shape her room, she did not provide evidence that this would be possible.

One commonly held view of creativity is that gifted students are able to be creative whereas struggling students are not. Dana defied the crowd in that she saw struggling students as having creative potential. However, this creative potential included a continued focus on artistic products rather than challenging mathematics problem-solving or products as creative.

Defying the View that Only Gifted Students can be Creative. Typically, research is focused on gifted students' mathematical creativity (Sheffield, 2008). However, this professional development program recalled Mann's (2009) appeal to include creativity in mathematics providing all students access to mathematics. While participants in the *traditional* group tended to view creativity as only achievable by gifted students, teachers in the *creative but hesitant* and *creative* groups said they saw struggling students as able to shine because of mathematically creative ideas.

Many participants categorized students based on ability (high, middle, and low), and described levels of support students needed: none, some guidance, and extra support, respectively. While participants in the *traditional* group tended to view creativity as only achievable by gifted students, other teachers said they saw struggling students as able to shine because they used creativity. Teachers described that all students, particularly those who struggled, seemed to benefit from journaling as it encouraged them to see mathematical connections, explain their reasoning, and generate new problems for others to consider. Teachers described that journaling provided an opportunity to understand students' misconceptions differently than in other contexts. This provides evidence for Mann's (2020) claim to focus not only on mathematical creativity for gifted students but for all students. The results of this study highlighted the need to implement creativity in mathematics for the betterment of all students and a defiance of the crowd's belief that creativity is only accessible to gifted students.

Defying Traditional Views of Creativity and Mathematics. As presented in the results chapter, participants differed in their view of creativity on a spectrum of traditional, seeing creativity only as artistic, to creative, identifying connections between creativity and mathematics. While some participants saw creativity as connected to art and disjoint from mathematics, others came to understand creativity as interconnected with mathematics. Seeing the two as connected demonstrates defiance of the crowd because it goes against common views of mathematics and creativity. Similarly, some teachers came to see mathematics as less definite and more open-ended.

Among participants in the *traditional* group, there seemed to be some misconceptions about creativity as artistic, fun, and easy. Some teachers continued to associate creativity with art and described it as involving fun games that were removed from mathematics content and instead focused on the artistic, design nature of the activity. Teachers in the *creative* group also connected creativity to fun but incorporated a challenging component, seeing the two as equally important rather than disjoint.

Even after participating in the professional development program, all teachers continued to describe creativity as fun. This may be because some of the tasks presented in the professional development program were engaging for participants (e.g. order of magnitude problems, the cake problem, the fraction task). Participants in the *traditional* and *creative but hesitant* groups tended to describe creativity as implementing less challenging tasks and often described projects with design elements as creative. In contrast, those in the *creative* group described creativity as fun and challenging and often described the possibility of embedding creativity throughout the curriculum. Affording teachers an opportunity to explore mathematics in a fun but challenging way may be beneficial for their own exploration of content as well as their teaching practice and ability to foster creativity among students.

This study suggests that teachers varied in their conceptions of mathematics on a spectrum of concrete to open ended. While some teachers remained static in their view of mathematics as concrete, others who initially viewed math as concrete came to understand it as somewhat open-ended. Still others at the outset of the professional development program viewed math as connected to students' daily lives and expanded this idea to include the importance of helping students make meaningful connections to the content. This is aligned with research focused on preservice teachers' awareness of creativity in mathematics (Shriki, 2010). In that study, preservice teachers engaged in tasks that encouraged them to rethink their conceptions of mathematics teaching and learning. Findings suggested that preservice teachers developed ideas aligned with mathematical creativity in part due to the environment in which they engaged with the tasks. This study extends these ideas to the complexity of in-service teachers' conceptions of mathematics and highlights the ways demonstrating a fear of taking risks might prevent teachers from defying traditional ideas about mathematics.

Defying the Zeitgeist. The focus of this study was about teachers recognizing the importance of mathematical creativity in their classrooms and fostering students' creativity, focusing on little-, mini-, and Pro-c creativity rather than Big-C contributions to the field of mathematics or defiance of the Zeitgeist. However, the potential for participants in the *creative* group to defy the Zeitgeist might have been possible. Although the majority of participants did not defy the Zeitgeist in a substantial way, some participants came to see the importance of altering their teaching practice (defying themselves and the crowd). Initially, teachers saw the two as disjoint. However, after participating in the professional development program, some teachers came to view mathematics and creativity as interconnected. This opposition to the commonly held cultural belief that creativity may only be found in art and is often not recognized in mathematics is an example of a start to defying the Zeitgeist.

Throughout the professional development program, teachers were encouraged to think about defying the Zeitgeist. However, teachers individually did not seem to defy it on their own as this is a practice that takes much more experience, effort, buy in, and time. Similar to Shriki's (2010) findings, comparatively to the length of teachers' careers, their participation in the fourth month professional development program was very short. Possible long-term effects might not have been evident at this time. Those who bought into the ideas presented throughout the professional development program, as well as those who were still working toward implementing these ideas, may alter their teaching practice in the future and come to implement mathematical creativity, altering the Zeitgeist of traditional mathematics teaching paradigms, and in turn may influence others to take on similar challenges, defying the teaching culture within and beyond the district.

Process

Teachers' approach to incorporating creativity varied from discomfort to enthusiastic about completely revamping their approach (e.g. teaching creatively versus teaching for creativity) in developing lessons on mathematical creativity. This process of coming to see the importance of fostering students' creativity and developing mathematically creative lessons was uncomfortable and challenging for participants.

The majority of participants came to understand creativity in mathematics as more than their own presentation of material (e.g. introducing similar problems to explain a concept to struggling students, providing multiple ways to help different types of learners). Many participants described ways they might be creative but also saw the benefit of fostering students' creativity. However, not all teachers who described the benefit of fostering students' creativity did so in practice. This is reminiscent of findings that have suggested that teachers' beliefs and practices at times are disjoint (Polly et al., 2013). Creativity in mathematics is often a new concept for teachers. Therefore, a need exists to support teachers as they engage in the process of understanding benefits of mathematical creativity for students and what this looks like in practice.

In their work with preservice elementary teachers, Bolden et al. (2010) found that participants held narrow beliefs about mathematics with little room for imagination and did not describe mathematics as creative. However, after participating in the course, preservice teachers seemed to shift to see mathematics as having the potential for creativity but remained unsure what this might look like in practice. The results of my study extend Bolden et al.'s (2010) findings that participants engaged in a process as they reconceptualized mathematics, creativity, and connections between the two. However, some participants were unsure of what creativity in practice might look like, suggesting that a situated professional development program focused on mathematical creativity can help teachers identify ways to implement mathematical creativity in practice.

In thinking about teachers' process to see mathematics as creative, there was also an important focus on students' problem-solving processes throughout the professional development program. Traditionally mathematics is seen as "a digestive process rather than a creative one" (Dreyfus & Eisenberg, 1996, p. 258) in that teachers demonstrate a method for solving a problem and students practice with similar problems. However, through this professional development program teachers were encouraged to highlight students' problem-solving process. This is aligned with research on creativity which highlights the importance of having students develop multiple approaches to solve a problem and convince one another of their ideas (Sternberg, 2006; 2018). This study filled a need to encourage teachers to teach creatively and foster creativity among students invoking characteristics of creativity including seeking new methods to solve problems, making decisions about appropriate approaches, and convincing others of their ideas (Sternberg, 2006, 2018).

Product

A creative product is often associated with something novel, innovative, and useful (Sternberg & Lubart, 2000). For this research, the product was participant-created lessons and views on teaching and learning that relied on principles of mathematical creativity. As noted in Chapter 2, of the 4Ps, *product* is viewed as the most objective because it is often in tangible form (Rhodes, 1961). However, there were nuances to how teachers designed lessons and established classroom environments to facilitate or hinder opportunities for creativity.

Teachers differed in how they generated lessons that incorporated creativity and mathematics. While some applied traits of mathematical creativity to existing lesson plans, others used the traits to decide on the content of the lesson. Still others did not attend to traits of mathematical creativity until prompted in the postlesson interview. The participants' lessons were relabeled by the teacher as creative but remained unchanged from original lessons. These findings align with research regarding teachers' implementation of creativity into lesson plans and note that programs geared toward teachers should aide in the development of mathematical creativity and implementation of classroom knowledge into practice (Bolden et al., 2010; Shriki, 2010). They argued that although teachers viewed creativity as original or novel, they were unable to include mathematical creativity in their lesson plans.

This professional development program aided some teachers' development of mathematically creative lessons and implementation of this knowledge in practice. Participants in the *creative* group adapted ideas of mathematical creativity from the professional development program. However, the results also highlight a need for further support participants as not all teachers were able to implement these ideas into their lesson plans.

Place

In this section I recall the last attribute of investment theory, environment, described as encouraging, supporting, and rewarding creative thinking, and connect this idea with other research on creative environments (Amabile, 1990; Ball, 1992; Borko, 2004; Schacter et al., 2006). For the purposes of this study, place is described as the context of the professional development program A creative place for teachers and students is one where members are encouraged to collaborate with colleagues, reflect on ideas, and develop new ideas. In classroom environments, teachers should encourage students to take risks, make mistakes, and be independent in the development of solutions to problems.

As noted above, this professional development provided opportunities for teachers to collaborate with colleagues, reflect on their experiences implementing ideas introduced during the sessions, and develop new ideas that incorporated mathematical creativity. The design of this professional development program attended to the call for professional development programs to allow for participant collaboration (Borko, 2004; DuFour, 2004). Also, a portion of each session was reserved for teachers to collaborate on various mathematical tasks and revise traditional tasks to be more mathematically creative. This supportive environment engaged teachers in solving challenging problems and encouraged them to think about the type environment that might allow them to foster creativity among students.

Promoting Collaboration

Teachers described several benefits of collaborating with colleagues during the professional development program. For Jordan, collaborating with other middle school teachers across the Tarnot district allowed her to see vertical alignment among curricula at different grade levels. She described that the professional development gave her the opportunity to, "see other teachers…the 6th, 7th, and 8th grades, to see the topics that they were covering and the way that they would implement it in their classroom, inspired me to kind of do the same in 5th grade" (Postlesson Interview, line 120). The environment created by the professional development program allowed for Jordan and others to collaborate with teachers with whom they otherwise had little interaction and identify similarities and differences among the grade-level content.

While some participants described benefits of collaboration during the professional development sessions, others used the sessions as a starting point for collaboration and continued this collaboration outside of the sessions. Rachel and Jane used the opportunity to share ideas and become more creative as they collaborated on their lesson design. Both Rachel and Jane were categorized in the *creative* group and their lesson plan and implementation attended to mathematically creative ideas. This highlights the importance of including partner or small group work in professional development programs. Collaborating with colleagues provided an opportunity to help one another engage in defiance and develop more creative lessons than they might have individually.

Allowing for Reflection

Throughout the professional development program, participants had opportunities to reflect on how the ideas presented might translate into their classroom practice. They also had opportunities to implement ideas into their classrooms (e.g. journaling) and reflect on the experience to improve these ideas. For example, teachers built on journaling by having students use their journals to create problems based on the mathematics they noticed outside of the classroom. The environment created throughout the professional development program encouraged teachers to reflect, revise, and reimplement mathematically creative ideas. In providing opportunities for reflection, teachers were able to think about successes and challenges they faced when implementing these ideas and refine these ideas for more successful implementation in the future.

Situated Within an Authentic Context

This professional development program was situated within teachers' authentic classroom contexts allowing for the implementation of these mathematically creative ideas immediately and with their students rather than being told about situations where ideas were implemented with other students in vastly different environments. Several teachers highlighted the benefit of having the professional development program intertwined with their real classrooms. For example, Stephanie commented,

> Well, I think that when you go to a PD…they're giving you what their philosophy is, I always leave there with something, but I also always leave there very just, 'Ugh, it's not going to work.' Because I immediately look at the environment that they taught and they taught in, Perfect Ville Pleasantville with the perfect students in front of them…we have tough kids and these kids carry baggage…when I would go to these PDs that I would get

> a little bit from it, but I never walked away going, 'God, I want to use everything.' I feel like here, this gave us, because it was all Tarnot people, all people who understand what we're dealing with, that we are in an urban environment, that we are dealing with kids who have issues bigger than we can ever imagine because we never lived it. (Postlesson Interview, line 230)

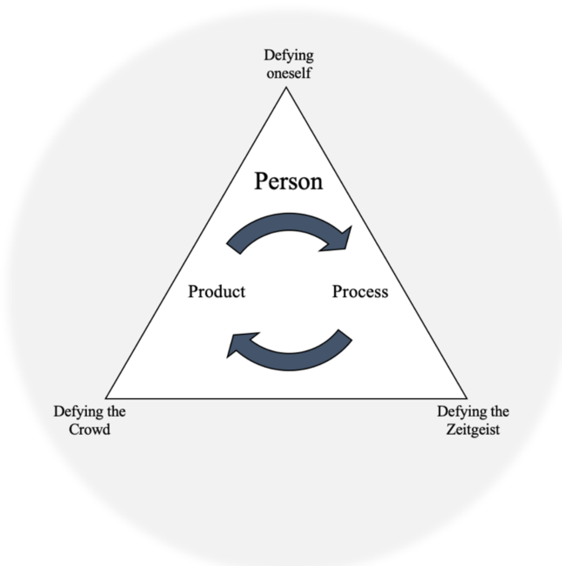
This further substantiates the need for teachers to participate in professional development programs that are set within real classroom contexts. Although not all teachers were able to develop their own mathematically creative lessons or drastically alter their teaching practice, they did find it beneficial to bring ideas from the professional development program into their classrooms and reflect on the experience.

Overlap of Research on Creativity

As noted above, Rhodes (1961) analyzed various definitions of creativity to develop the 4Ps. Therefore, the categories tend to be viewed as discrete. Although they are often viewed as discrete, these categories do not stand alone and instead are interconnected. As described above, a person engages in a process to develop a product. The person, process, and product are situated within a place. Although some researchers (Sternberg, 2018) have identified overlaps among the categories, little research describes the deep interconnection among the 4Ps or how other theories of creativity interact with the 4Ps in the context of mathematics. Through the results I considered all three theories together and identified an overlap of the 4Ps and triangular theory (Figure 11). I also found that there is a spectrum or levels of defiance that participants demonstrate when thinking about, implemetning, and fostering mathematical creativity in classrooms. This spectrum was in relation to other participants in the professional program but also individual for each person in the degree to which they defied their own ideas about mathematics, creativity, and teaching and learning.

Figure 11

Revised 4P Model (Rhodes, 1961)



A person engages in a process of defying oneself, the crowd, or the Zeitgeist, to develop a new product. This process happens within the context of a place. Figure 11 demonstrates the overlap of the defiance ones engages in and the context of 4Ps of creativity and illustrates what this might look like in the context of teaching mathematics.

Extension of the Creative Teaching Framework (CTF)

As described in chapter three and outlined in chapter four, I adapted the CTF (Schacter et al., 2006) lesson observation protocol to account for instances of creativity in participant-created lesson plans and semi-structured interviews. Some considerations in adapting the protocol for lesson plans versus observations were:

1. What about the CTF protocol is specific to classroom observations?

2. What changes across platforms? Why?
3. What stays the same across platforms? Why?
4. How does this research inform adaptations to the CTF protocol?

It was also important to consider how the lesson plans in some instances became standalone activities versus complete lessons because teachers were required to meet with students in asynchronous, online meetings.

Fundamental changes were made to the *thinking strategies* category of the protocol. The *thinking strategies* category is noted when the teacher: (a) leads students in activities that require them to generate and record multiple ideas, (b) teaches students creative thinking strategies such as divergent thinking, brainstorming, using analogies, redefining a problem, or synectics, or (c) explicitly teaches metacognitive strategies. These three components of the original protocol overlapped with research on creativity in mathematics and were reworded to be more aligned with literature. The components of the *thinking strategy* category became: engage in non-algorithmic decision making, generate novel solutions to problems, strive for mathematical elegance, explain reasoning, be curious about mathematical connections, continue to explore after a problem has been solved, develop insightful solutions, formulate new questions for others to consider, see multiple solutions to a problem, and persevere. These elements were added to gain a broader understanding of instances where teachers demonstrated ideas presented in the professional development program. The wording of descriptions for the remaining categories (*opportunities for choice and discovery, intrinsic motivation, environment conducive to creativity,* and *imagination and fantasy*) was changed as needed to focus more on instances appropriate for lesson design rather than lesson observation. For example, "The teacher creates learning scenarios where students can choose from one of several activities" was changed to "the teacher provides students the opportunity to choose from one of several activities." This updated description was evident in lesson plans whereas identifying instances where a teacher created learning scenarios was not visible by analyzing lesson plans.

Opportunities to account for counter examples were integrated into the CTF protocol to understand when the lesson plan did not attend to the category or were misinterpreted creative ideas. These counter examples were denoted with "(-)". Identifying instances where teachers misinterpreted mathematically creative ideas was important for my purposes because understanding these misconceptions might help professional development facilitators design programs to specifically mitigate existing misconceptions. Schacter et al. (2006) designed the protocol to understand teachers' demonstration of creativity in practice. My study extended the use of this protocol to not only understand when teachers are being creative or fostering creativity among students, but also when they are misinterpreting these ideas. Results from the use of the extended protocol have the potential to inform how professional development programs can support teachers in appropriate implementation of creative thinking and learning.

Professional Development Model for K-12 Teaching and Learning Through Creativity

This study contributes a model for a specific professional development program focused on mathematical creativity that meets the standards for effective professional development. The results from this study amplify what we already know about successful professional development, provide more specific examples of how a professional development program for in-service teachers functions in the context of developing teachers' and students' creativity in mathematics, and offer contributions to in-person and online professional development models, particularly when using creativity as a lens for teaching and learning mathematics.

Loucks-Horsley et al. (2009) recommended that effective professional development: (i) is focused on employing teaching strategies that are accessible to all students, not just those deemed mathematically gifted, (ii) develops teachers pedagogical content knowledge, (iii) provides engaging, active learning, (iv) provides opportunities for collaboration, and (v) supports changes in student learning such as assessment, curriculum, and classroom culture (p. 11). This professional development program took these principles into consideration and addressed ways to provide all students access to mathematics through creativity in ongoing sessions held over a four-month period. The professional development program engaged teachers in challenging mathematics and focused on developing a classroom culture to support students' mathematical creativity. As a result, some teachers' minds were changed about creativity and they came to see it is a way to be inclusive and accommodating of all learners whereas others struggled with the implementation of these ideas in practice.

Researchers have described the importance of fostering a sense of community among teachers, an important element of this professional development program (Borko, 2004; DuFour, 2004). Fostering a sense of community was especially pertinent for teachers in this professional development program due to the COVID-19 pandemic and transition to online teaching. This professional development program, outside of the school setting and absent of administrators, contributed to a peer-focused community. Findings suggest that teachers benefited from the opportunity to communicate with one another during a time of crisis and there was continuity in that the teachers relied on one another through the professional development program by sharing resources, successes, challenges and fears. They reflected on the importance of having the support of other teachers to confide in and learn alongside who were facing similar challenges. Through the professional development program, teachers became resources for one another and found having this community to rely on to be beneficial. Findings suggest that teachers needed a place for dialogue and that they appreciated having this community during the COVID-19 pandemic and drastic shift to online teaching. This contributes to our understanding of what supports teachers need, in particular among a community of teachers facing similar challenges.

Another feature of effective professional development programs is a focus on teachers' content and pedagogical content knowledge (Borko, 2004; DuFour, 2004). This professional development program included challenging tasks for teachers to engage in mathematical creativity and encouraged teachers to implement these and other ideas into their classrooms. To address the call for professional development programs, the features of this professional development program included having teachers work on mathematics, implementing ideas from the professional development program into their classrooms, reflecting successes and challenges of this implementation, feel uncertain the way students might when presented with mathematics in this way, collaboration.

Limitations

This study provided an opportunity for teachers to collaborate as they thought about connections between mathematics and creativity. The results of this study suggest that some teachers demonstrated a shift in their teaching practices to include mathematical creativity, others recognized the importance of these ideas but remained static in their teaching practices, and still others remained static in their views and teaching practices. This highlights a need for continued support through a revised model of this professional development program.

One limitation of the professional development program was its timing. After working with students for five months prior to attending professional development program, participants stated that changing their teaching practice and students' approach to mathematics midyear was challenging. Although results demonstrated that teachers in the *creative* group were able to alter their teaching practices midyear to foster students' creativity, many participants felt once the norms of the classroom were established it was difficult to change their teaching practice.

Extenuating Circumstances of the COVID-19 Pandemic

The COVID-19 pandemic and transition from in-person to online professional development sessions presented several limitations of this study. Halfway through the professional development program, schools were closed due to the COVID-19 the pandemic, and the professional development program and K-12 teaching shifted online. Because of this shift, there was a one-month gap between Sessions four and five. This created an interruption in the purposefully scheduled meetings. Because the fifth session was pushed back by two weeks, the remaining sessions and teachers' lesson implementation continued into June. In planning the professional development, I scheduled for lessons to be implemented mid-May, prior to end of year stressors common in K-12 teaching. In June, students were fatigued from online learning and in some instances, teachers reported that they were turning in assignments at a lower return rate than they had been at the outset of the online shift in March. Concluding the professional development program closer to the end of the school year presented a limitation because it was a more stressful time of year for teachers and students were not as engaged as they had been previously.

Another limitation of this study, impacted by the COVID-19 pandemic, was the cancellation of the state test on March 24th, 2020. This cancellation may have taken pressure off of teachers which might have allowed them to implement mathematical creativity more freely. The cancellation of the state test might have alleviated pressure to finish the text book or cover specific material that would have been on the state test. Teachers in *creative* group may have had a stronger focus on state test had it not been cancelled, potentially changing the results of my study.

The shift to online sessions changed the professional development program completely. This shift was a limitation because the initial intent was for in person sessions, and I did not envision an online context when developing the program. The focus of the professional development program strayed slightly from my original intent. In planning the last three sessions, I drew on the influx of online resources more heavily than I did prior to the switch to online in an attempt to support teachers as they navigated the new and complicated online teaching environment. Session five became more of a focus group interview, different than the other seven sessions. In this session, teachers were invited to discuss how creativity involved risks and how the shift to online teaching required them to be creative. However, teachers also shared resources for online teaching and challenges they faced as they shifted to this new modality. This session was less focused on the objectives of the professional development program, which were to have teachers engaged with challenging mathematics tasks and develop ideas to implement with students, and more focused on teachers supporting one another with this drastic change.

Another limitation related to online teaching was that participants were required to redesign lessons for the asynchronous, online setting. Participants were required to be creative in ways they may not have considered prior to the shift to online teaching as this platform required developing new ways to interact with students. For example, some teachers used discussion boards to foster student collaboration and others considered the context of online teaching as a way to structure activities that fostered student independence. However, for some participants, this transition also highlighted traditional teaching in the form of pre-recording videos of them teaching a lesson in an attempt to recreate the in-person setting. Online teaching was an added challenge for teachers and in some cases, teachers were overwhelmed with this shift, unable to also attend to ideas presented in the professional development program.

Data Collection

This professional development program and data collection were conducted during an atypical academic year which changed the nature of the professional development, data collection, and results. Therefore, I cannot be sure how the data would have been different if the professional development program and teaching and learning had remained in person, with the state testing in place and teachers meeting synchronously with students.

As noted above, the online transition had an impact on the second half of the professional development program. Because of the shift to remote teaching and learning and because synchronous class meetings were prohibited, I was not in a position to observe the actual implementation of mathematically creative lessons. It was necessary to take teachers' descriptions and physical lesson plans at face value. Although I was unable to observe lessons, I relied on the use of secondary sources including: teacher-created lesson plans, professional development program reflections, and post-program interviews to establish reliable results. Conducting a deep analysis of the lesson plans allowed me to gain insight into teachers' understanding of the ideas presented in the professional development program and their ability to design tasks that included these ideas.

Although lesson enactment is important to observe how teachers implement these ideas in practice, had I been able to observe a class, it would have been contrived because teachers would have invited me to view a class that was going to be deliberately geared toward creative thinking in mathematics. Although contrived, observing a class would have provided more information about the classroom culture that allowed for creativity, beyond just the task design and would have lessened any discrepancies between belief and practice. For example, teachers could have discussed the importance of students taking risks, but in practice not create an environment that fostered risk taking. Although I was unable to account for this potential discrepancy, I was able to identify how teachers thought about and designed lessons regarding mathematical creativity and it would be impossible for teachers to enact a mathematically creative lesson unless the planning was aligned with mathematical creativity. Therefore, lesson plan analysis allowed me to gain an understanding of how they designed an activity and described elements of creativity in mathematics.

Positionality

Another limitation was my positionality as professional development facilitator and researcher. Bourke (2014) argued that in qualitative research, the researcher impacts the research process. Teachers may have felt pressure to agree with the ideas presented during the professional development sessions and provide responses to interview questions that they though were aligned with the goals of the professional development. To mitigate this, during each professional development session I provided opportunities for teachers to share challenges they faced when thinking about mathematical creativity in practice and elements that would make these ideas difficult to implement in practice.

Implications and Future Research

Throughout this project, I refined the professional development program, taking teachers' comments, critiques, and experiences into account. I adapted the CTF (Schacter et al., 2006) for analysis of teacher descriptions of mathematical creativity and teacher-created lessons. I also contributed to the literature on teaching and learning in middle school mathematics, creativity in mathematics, and professional development to support teachers in their development and implementation of these ideas. This research helped me better understand what mathematics lessons that incorporate creativity look like for middle school teaching and learning and what supports are needed to encourage teachers in their implementation of this model of teaching.

Recommendations for Professional Development Programs that Encourage Creativity

Results from this study are aligned with principles outlined by Loucks-Horsley et al. (2009), particularly in terms of providing ongoing support for teachers and encouraging collaboration and a sense of community among participants. I extend these ideas to also consider the time of year to implement professional development programs, extend time allotted for each session, provide further support for teachers implementing mathematically creative ideas in practice, and foster collaboration and community among participants.

Timing and Location of Professional Development Sessions

Results indicate that professional development programs might be most beneficial if implemented at the beginning of the school year. Some teachers commented they would have preferred to meet weekly and have more tasks in between sessions. Meeting weekly and requiring more challenges for teachers to complete in their classrooms and reflect on between sessions might help teachers better understand and design their own lessons.

At times, particularly in the online environment, small group collaboration was cut short due to the one-hour scheduled time. Longer sessions would allow more time for teachers to engage in each planned component of the program (engaging with mathematics, reflecting on things they tried in their practice, collaborating to develop questions, activities, and lessons focused on mathematical creativity). Extended time for sessions would also allow the facilitator to provide detailed, ongoing feedback for each group of teachers. More time collaborating on tasks and ongoing facilitator feedback might have strengthened participants' understanding of and willingness to implement these ideas in practice.

Rachel noted that she enjoyed meeting in the sessions online because it was a way for her to continue to collaborate with other teachers in the district. This professional development program provided a way for teachers to continue to connect when they otherwise were isolated, facing the unfamiliar challenge of online teaching. The has implications for online professional development programs to develop opportunities for teachers to collaborate across the district. The nature of online professional development allows for teachers to collaborate more easily and mitigates some challenges of finding a central meeting place that requires teachers to travel.

Further Support for Implementation of Mathematical Creativity in Practice

Although participants in the *creative* group demonstrated instances of teaching creatively and teaching for creativity, they could still benefit from continued support. For example, when discussing students' work from a geometry lesson, Jane noted that the lesson required students to persevere. However, there was no evidence that her lesson plan or the implementation of the activity actually required this of the students. Although challenging tasks might inherently require student perseverance, they do not ensure that students will persevere. Instead, the teacher might include ways to encourage student perseverance such as structuring time for independent and collaborative work so students can share and build on each other's initial ideas. Because these ideas were novel for all participants, further support is still needed to have teachers, even those who were in the *creative* group, to encourage elements of mathematical creativity in practice.

As with any new idea, allowing for an incubation period where teachers try things on their own might be beneficial in understanding teachers' conceptions of mathematics, creativity, and the implementation of these ideas in practice over time. It is possible that teachers in the *creative but hesitant* and *traditional* groups required more time to process the ideas presented in the professional development program. It also might be beneficial to understand this processing time to encourage buy-in of these ideas. Developing a more in-depth understanding of those in the *traditional* group might expose a different set of processes, such as incremental steps, needed for them to buy-in that could be embedded in professional development programs focused on creativity.

Those who demonstrated fewer investment theory attributes (e.g. flexibility, willingness to take risks) and instead tended to be concrete in their thinking might need more time and extensive support to align their teaching practice with elements of mathematical creativity. It might take longer for those who demonstrated an unwillingness to take risks to buy-into these new ideas. Providing incremental steps involving less risk might better support teachers as they alter their teaching practice. In thinking about incremental steps, it might be that the professional development programs are implemented in sections throughout the school year, with facilitator support in between each section. The professional development facilitator might continue to provide teacher support in their classrooms as they try an idea or collection of ideas presented in each section. This might help teachers be more comfortable to try some of the ideas introduced in the professional development program.

Community and Collaboration

As noted in the contribution section, teachers benefited from participating in the PD during the crisis of the COVID-19 pandemic. Because of this, school districts might plan to generate ways for teachers to have a supportive environment with someone from outside of the district who holds no authority over them, but can provide guidance for teachers. Based on the finding that teachers benefited from this setting outside of school during the shift to online, I recommend that districts design these communities for teachers to rely on in the event of a crisis or need. Often things happen within schools that present challenges for teachers but department meetings are not the place to discuss these challenges. This community allowed teachers to talk freely in a safe place without a supervisor present which is not typical in school districts. When a professional development program is held, often a supervisor or district leader is present hindering teachers' willingness to be open and share challenges. This may need to be reconsidered.

The results also indicated that creativity was encouraged more when teachers chose to collaborate on their lesson development. Teacher collaboration was beneficial for furthering Rachel and Jane's ideas as they designed mathematically creative lessons. Future professional development programs might purposefully group teachers and more strongly encourage collaboration on lesson design and implementation. Providing more opportunities for teachers to reflect, report on, revise, and reapply these lessons might also help them as they develop their own lessons.

Initially, I planned to have teachers share their lessons within our community and collaborate during the professional development program to revise the lessons. However, the shift to online learning and the nature of the tasks made this less possible. Providing an opportunity for teachers to critique and expand upon their initial ideas might allow teachers to recognize other's creativity, exemplifying mini-c creativity. Having the opportunity to critique and expand other's ideas, altering their teaching practice long term to include mathematical creativity would exemplify Pro-c creativity.

In order to further solidify the importance of creativity in mathematics for those who did not yet alter their teaching practice, it might be beneficial to have teachers read and critique research on ideas aligned to similar populations of students involving mathematical creativity. Examples of middle school students' success in part due to mathematical creativity might help them recognize the importance and usefulness of infusing creativity in their classrooms and provide them with other ideas to alter for their own practice.

Recommendations for Future Research

This research addressed a need to explore how in-service teachers' conceptions of mathematics and mathematical creativity evolved as they participated in a professional development program focused on creativity and how these ideas transferred into teachers' practices. Because this professional development model is unique, more research is needed to analyze connections between creativity and curricula, teachers' implementation of ideas in person, and students' experiences with mathematical creativity. More sustained engagement with the revised program could help the field gain a more detailed understanding of these ideas in practice and its impact on students.

As stated above, at times participants described skipping explorations or modules in the curriculum so they could spend more time on other concepts. These explorations seemed to be aligned with mathematically creative ideas of student discovery and independence. Because of this, it would be beneficial to study the intended versus enacted curriculum, when the intention of the curriculum is to promote creativity, and highlight how teachers engage with and implement the curriculum. If curricula have creative elements, it is possible that teachers need support to enact it as intended with a purposeful focus on mathematical creativity. Curriculum is a major player in teachers' decision making and it would be interesting to better understand connections between the GoMath curriculum and mathematical creativity, and what support teachers need to enactment the curriculum as intended while focusing on creativity. Professional development programs can support teachers as they consider how to supplement or adapt the given curricula but more research is needed about the connections to creativity.

This research focused on middle school teachers' implementation of mathematical creativity in practice. Future research should also consider students' experience with these ideas to understand the support students need in explaining reasoning, being curious about mathematics, discovering solutions, and demonstrating independence. Some teachers noted that getting students to think differently about mathematics is challenging because it had not been done in earlier grades. Given this challenge, research should also focus on implementing adjusted models of this professional development program for different elementary and high school levels. In fostering these ideas throughout K-12 education, teachers and students might become more comfortable with and further refine these ideas. This would also allow for vertical alignment and more continuity throughout K-12 mathematics teaching and learning, strengthening students' comfort and ability to engage in challenging mathematically creative tasks.

The 12 participants' engagement in the professional development program focused on creativity in mathematics teaching and learning and offered opportunities for teachers to challenge their ideas about mathematics, creativity, and teaching and learning. The participants embraced the ideas presented throughout the professional development program to varying degrees. Similarly, participants demonstrated the implementation of these ideas into their classroom practices at varying levels. Further research is needed to understand the benefits of creativity in mathematics and how professional development programs can support teachers in the implementation of these ideas.

Conclusion

The results presented in this study indicate differences among the 12 participants and the placement of them into three categories: *traditional, creative but hesitant,* and *creative*. Their participation in the professional development program and initial views about mathematics, creativity, and teaching and learning, allowed them to implement the ideas presented during the professional development program to varying degrees from traditional to creative. Those in the *traditional* category tended to align with more traditional paradigms (e.g. seeing only gifted students as creative, having ownership of student learning, viewing mathematics and creativity as separate). Participants in the *creative but hesitant* category described connections between mathematics and creativity but these ideas were not evident in their lesson development or implementation. In the *creative* category, participants came to see mathematics and creativity as interconnected, described ways creativity could be incorporated with middle school mathematics content, and designed and implemented mathematically creative lessons.

Findings from this study add to the limited body of research on creativity in mathematics education. In particular, results indicated an overlap of the 4P's of creativity (Rhodes, 1961), the spectrum of creativity (Beghetto & Kaufman, 2007), and triangular theory (Sternberg, 2018). This study also highlighted adaptations to the Schacter et al. (2006) protocol for lesson plans, posed recommendations for professional development programs, and suggested supports for teachers when thinking about and implementing ideas of mathematical creativity in practice.

References

Aiken, L. R. (1973). Ability and creativity in mathematics. *Review of Educational Research, 43*(4), 405-432.

Akgül, S., & Kahveci, N. G. (2016). A study on the development of a mathematics creativity scale. *Eurasian Journal of Educational Research, 16*(62). https://doi.org/10.14689/ejer.2016.62.5

Amabile, T. M. (1983). The social psychology of creativity: A componential conceptualization. *Journal of Personality and Social Psychology, 45*(2), 357-376.

Amabile, T. M. (1990). Within you, without you: The social psychology of creativity, and beyond. In M. A. Runco & S. Pritzker (Eds.), *Theories of Creativity* (pp. 61-69). Sage.

Baer, J., & Kaufman, J. C. (2005). Bridging generality and specificity: The amusement park theoretical (APT) model of creativity. *Roeper Review, 27*, 158–163.

Bahar, A. K., & Maker, C. J. (2011). Exploring the relationship between mathematical creativity and mathematical achievement. *Asia-Pacific Journal of Gifted and Talented Education, 3*(1), 33-48.

Balka, D. S. (1974). The development of an instrument to measure creative ability in mathematics. *Dissertation Abstracts International, 36*(1), 98.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator: The Professional Journal of the American Federation of Teachers, 16*(2).

Ball, D. L., & Cohen, D. K. (1999). Developing practices, developing practitioners: Toward a practice-based theory of professional development. In L. Darling-Hammond & G. Syke (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 30-32). Jossey-Bass.

Beghetto, R. A. (2007). Does creativity have a place in classroom discussions? Prospective teachers' response preferences. *Thinking skills and creativity, 2*(1), 1-9.

Beghetto, R. A., & Kaufman, J. C. (2007a). Toward a broader conception of creativity: A case for "mini-c" creativity. *Psychology of Aesthetics, Creativity, and the Arts, 1*(2), 73.

Beghetto, R. A., & Kaufman, J. C. (2007b). Toward a broader conception of creativity: A case for "mini-c" creativity. *Psychology of Aesthetics, Creativity, and the Arts, 1*(2), 73.

Benderly, B. L. (June, 2014). *Luck, brilliance, and persistance*. American Association for the Advancement of Science. https://www.sciencemag.org/careers/2014/06/luck-brilliance-and-persistence

Blank, R. K., & de las Alas, N. (2009). *The effects of teacher professional development on gains in student achievement: How meta analysis provides scientific evidence useful to education leaders*. Council of Chief State School Officers.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Bolden, D. S., Harries, T. V., & Newton, D. P. (2010). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics, 73*(2), 143-157. https://doi.org/10.1007/s10649-009-9207-z

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Bourke, B. (2014). Positionality: Reflecting on the research process. *The Qualitative Report, 19*(33), 1-9.

Briscoe, C., & Peters, J. (1997). Teacher collaboration across and within schools: Supporting individual change in elementary science teaching. *Science Education, 81*(1), 51-65.

Brown, C. A., Carpenter, T. P., Kouba, V. L., Lindquist, M. M., Silver, E. A., & Swafford, J. O. (1988). Secondary school results for the fourth NAEP mathematics assessment: Algebra, geometry, mathematical methods, and attitudes. *The Mathematics Teacher, 81*(5), 337-397.

Carlson, M., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent problem-solving framework. *Educational Studies in Mathematics, 58*.

Centers for Disease Control and Prevention. (2018, October 9). *Prion diseases*. Centers for Disease Control and Prevention (CDC). https://www.cdc.gov/prions/index.html

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Journal of Secondary Gifted Education, 17*(1), 37-47.

Chval, K., Abell, S., Pareja, E., Musikul, K., & Ritzka, G. (2008). Science and mathematics teachers' experiences, needs, and expectations regarding professional development. *Eurasia Journal of Mathematics, Science & Technology Education, 4*(1).

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education*. Routledge.

Collinson, V., & Ono, Y. (2001). The professional development of teachers in the United States and Japan. *European Journal of Teacher Education, 24*(2), 223-248.

Creswell, J. W. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (2nd ed.). Sage Publications.

Cropley, A. J. (2000). Defining and measuring creativity: Are creativity tests worth using? *Roeper Review, 23*(2), 72-79.

Csikszentmihalyi, M. (1996). *Creativity: Flow and the psychology of discovery and invention*. Harper Collins.

Csikszentmihalyi, M. (1998). Reflections on the field. *Roeper Review, 21*, 80-81.

Darling-Hammond, L., & McLaughlin, M. W. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan, 76*(8), 597-604.

DeHaan, R. L. (2009, Fall). Teaching creativity and inventive problem solving in science. *CBE Life Science Education, 8*(3), 172-181. https://doi.org/10.1187/cbe.08-12-0081

DÍaz-Obando, E., Plasencia-Cruz, I., & Solano-Alvarado, A. (2003). The impact of beliefs in student's learning: An investigation with students of two different contexts. *International Journal of Mathematical Education in Science and Technology, 34*(2), 161-173.

Doerr, H., Goldsmith, L., & Lewis, C. (2010). *Professional development research brief: Mathematics professional development*. National Council of Teachers of Mathematics.

Donovan, S., & Bransford, J. (2005). *How students learn: History, mathematics, and science in the classroom*. National Academies Press.

Dreyfus, T., & Eisenberg, T. (1996). Mathematical thinking. *The nature of mathematical thinking*, 253-284.

DuFour, R. (2004). What is a "professional learning community"? *Educational Leadership, 61*(8), 6-11.

Elam, K. G., & Duckenfield, M. (2000). *Creating a community of learners: Using the teacher as facilitator model*. National Dropout Prevention Center.

England, K. V. L. (1994). Getting personal: Reflexivity, positionality, and feminist research. *The Professional Geographer, 46*(1), 80-89.

Ervynck, G. (1991). Mathematical Creativity. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 42-53). Springer Netherlands. https://doi.org/10.1007/0-306-47203-1_3

Frank, M. L. (1988). Problem solving and mathematical beliefs. *Arithmetic Teacher, 35*(5), 32-34.

Gajda, R., & Koliba, C. J. (2008). Evaluating and improving the quality of teacher collaboration: A field-tested framework for secondary school leaders. *NASSP Bulletin, 92*(2), 133-153.

Garofalo, J. (1989). Beliefs and their influence on mathematical performance. *The Mathematics Teacher, 82*(7), 502-505.

Geist, E., & Hohn, J. (2009). Encouraging creativity in the face of administrative convenience: How our schools discourage divergent thinking. *Education, 130*(1).

Givvin, K. B., & Santagata, R. (2011). Toward a common language for discussing the features of effective professional development: The case of a US mathematics program. *Professional Development in Education, 37*(3), 439-451.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory*. Aldine.

Goldberg, D. E. (2008). Last word: Bury the cold war curriculum. *ASEE Prism, 17*(8).

Guilford, J. P. (1950). *Fundamental statistics in psychology and education* (2nd ed.). McGraw-Hill.

Guilford, J. P. (1968). Creativity, intelligence, and their educational implications. *San Diego, CA: EDITS/Knapp*.

Guskey, T. R. (1991). Enhancing the effectiveness of professional development programs. *Journal of Educational and Psychological Consultation, 2*(3), 239-247.

Guskey, T. R. (2000). *Evaluating professional development*. Corwin Press.

Hadamard, J. (1945). *An essay on the psychology of invention in the mathematical fields*. Dover Publications.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice*. Jossey-Bass.

Haylock, D. W. (1987). A framework for assessing mathematical creativity in school chilren. *Educational Studies in Mathematics, 18*(1), 59-74.

Hershkovitz, S., Peled, I., & Littler, G. (2009). Mathematical creativity and giftedness in elementary school: Task and teacher promoting creativity for all. *Creativity in Mathematics the Education of Gifted Students*, 255-269.

Hong, E., & Aqui, Y. (2004). Cognitive and motivational characteristics of adolescents gifted in mathematics: Comparisons among students with different types of giftedness. *Gifted Child Quarterly, 48*(3), 191-201.

Hong, M., & Kang, N. (2010). South Korean and the US secondary school science teachers' conceptions of creativity and teaching for creativity. *International Journal of Science & Mathematics Education, 8*(5), 821-843.

Karwowski, M., Jankowska, D. M., Brzeski, A., Czerwonka, M., Gajda, A., Lebuda, I., & Beghetto, R. A. (2020). Delving into creativity and learning. *Creativity Research Journal, 32*(1), 4-16.

Kaufman, J. C., & Beghetto, R. A. (2013). In praise of Clark Kent: Creative metacognition and the importance of teaching kids when (not) to be creative. *Roeper Review, 35*, 155–165.

Kaufman, J. C., & Sternberg, R. J. (2007). Resource review: Creativity. *Change, 39*(4), 55-58.

Kaufman, J. C., & Sternberg, R. J. (2010). *Cambridge handbook of creativity*. Cambridge University Press.

Kennedy, M. M. (2016). How does professional development improve teaching? *Review of Educational Research, 86*(4), 945-980.

Kim, H., Cho, S., & Ahn, D. (2004). Development of mathematical creative problem solving ability test for identification of the gifted in math. *Gifted Education International, 18*(2), 164-174.

Kloosterman, P., & Cougan, M. C. (1994). Students' beliefs about learning school mathematics. *The Elementary School Journal, 94*(4), 375-388.

Koestler, A. (1967). The act of creation. In D. B. Lindsley & A. A. Lumsdaine (Eds.), *Brain function, vol. IV: Brain function and learning* (pp. 327-346). University of California Press.

Krajcik, J., Blumenfeld, P., Marx, R. W., & Soloway, E. (1994). A collaborative model for helping science teachers learn project-based instruction. *Elementary School Journal, 94*(5), 483-498.

Krajcik, J. S., & Blumenfeld, P. C. (2006). Project-based learning. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences*. Cambridge.

LaRocque, M. (2008). Assessing perceptions of the environment in elementary classrooms: The link with achievement. *Educational Psychology in Practice, 24*(4), 289-305. https://doi.org/10.1080/02667360802488732

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. *Creativity in Mathematics and the Education of Gifted Students, 9*, 129-145.

Leikin, R., Subotnik, R., Pitta-Pantazi, D., Singer, F. M., & Pelczer, I. (2013). Teachers' views on creativity in mathematics education: An international survey. *ZDM Mathematics Education, 45*(2), 309-324.

Lev-Zamir, H., & Leikin, R. (2011). Creative mathematics teaching in the eye of the beholder: Focusing on teachers' conceptions. *Research in Mathematics Education, 13*(1), 17-32. https://doi.org/10.1080/14794802.2011.550715

Levenson, E. (2013a). Exploring Ava's developing sense for tasks that may occasion mathematical creativity. *Journal of Mathematics Teacher Education, 18*(1), 1-25. https://doi.org/10.1007/s10857-013-9262-3

Levenson, E. (2013b). Tasks that may occasion mathematical creativity: teachers' choices. *Journal of Mathematics Teacher Education, 16*(4), 269-291.

Levenson, E., Swisa, R., & Tabach, M. (2018). Evaluating the potential of tasks to occasion mathematical creativity: Definitions and measurements. *Research in Mathematics Education, 20*, 273-294. https://doi.org/10.1080/14794802.2018.1450777

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2009). *Designing professional development for teachers of science and mathematics*. Corwin Press.

Lubart, T. I., & Sternberg, R. J. (1998). Creativity across time and place: Life span and cross-cultural perspectives. *High Ability Studies, 9*(1), 59-74.

Ma, X., & Kishor, N. (1997). Assessing the Relationship between Attitude toward Mathematics and Achievement in Mathematics: A Meta-Analysis. *Journal for Research in Mathematics Education, 28*(1). https://doi.org/10.2307/749662

Mace, M.-A., & Ward, T. (2002). Modeling the creative process: A grounded theory analysis of creativity in the domain of art making. *Creativity Research Journal, 14*(2), 179-192.

Mann, E. L. (2005). *Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students* (Publication No. 3205573) [Doctoral dissertation, University of Connecticut] ProQuest Dissertations and Theses Global.

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30*(2), 236-260.

Mann, E. L. (2009). The search for mathematical creativity: Identifying creative potential in middle school students. *Creativity Research Journal, 21*(4), 338-348.

Mann, E. L. (2020). Mathematics. In M. Runco & S. Pritzker (Eds.), *Encyclopedia of creativity* (3rd ed., Vol. 2, pp. 80-85). Elsevier, Academic Press.

Marshall, J. C., Horton, R., Igo, B. L., & Switzer, D. (2009). K-12 science and mathematics teachers' beliefs about and use of inquiry in the classroom. *International Journal of Science and Mathematics Education, 7*(3), 575-596.

Maslow, A. (1967). The creative attitude. In R. L. Mooney & T. A. Rasik (Eds.), *Explorations in creativity*. Harper and Row.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. Josey-Bass Publishers.

Merriam, S. B., & Tisdell, E. J. (2015). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Merriam-Webster. (2021). *Defy*. Merriam-Webster.com/dictionary. https://www.merriam-webster.com/dictionary/defy?utm_campaign=sd&utm_medium=serp&utm_source=jsonld

Muijs, D., & Reynolds, D. (2015). Teachers' beliefs and behaviors: What really matters? *The Journal of Classroom Interaction, 37*(2), 3-15.

Muis, K. R. (2004). Personal epistemology and mathematics: A critical review and synthesis of research. *Review of Educational Research, 74*(3), 317-377.

Mumford, M. D., Baughman, W. A., Maher, M. A., Costanza, D. P., & Supinski, E. P. (1997). Process-based measures of creative problem-solving skills: IV. Category combination. *Creativity Research Journal, 10*(1), 59-71.

Munakata, M., & Vaidya, A. (2012). Encouraging creativity in mathematics and science through photography. *Teaching Mathematics and Its Applications: International Journal of the IMA, 31*(3), 121-132.

Munakata, M., & Vaidya, A. (2015). Using project-and theme-based learning to encourage creativity in science. *Journal of College Science Teaching, 45*(2), 48-53.

Munakata, M., & Vaidya, A. J. P. (2013). Fostering creativity through personalized education. *Primus, 23*(9), 764-775.

NACCCE. (1999). *All our futures: Creativity, culture and education*. DfEE.

Nadjafikhah, M., Yaftian, N., & Bakhshalizadeh, S. (2012). Mathematical creativity: Some definitions and characteristics. *Procedia - Social and Behavioral Sciences, 31*, 285-291. https://doi.org/10.1016/j.sbspro.2011.12.056

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Author.

National Governors Association Center for Best Practices. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.

National Research Council. (1996). *National science education standards*. The National Academies Press.

NCES. (2017). *Data point: Beginning college students who change their majors within 3 years of enrollment*. https://nces.ed.gov/pubs2018/2018434.pdf

Neumann, C. J. (2007). Fostering creativity: A model for developing a culture of collective creativity in science. *EMBO Reports, 8*(3), 202-206.

New Jersey Department of Education. (2019). *National Center for Education Statistics*. Retrieved October 20 from https://nces.ed.gov/ccd/districtsearch/district_detail.asp?Search=2&details=1&ID2=3401260&DistrictID=3401260

Panaoura, A., & Panaoura, G. (2014). Teachers' awareness of creativity in mathematical teaching and their practice. *Issues in Undergraduate Mathematics Preparation of School Teachers, 4*, 1-11.

Papert, S. (1972, 1972/07/01). Teaching children to be mathematicians versus teaching about mathematics. *International Journal of Mathematical Education in Science and Technology, 3*(3), 249-262. https://doi.org/10.1080/0020739700030306

Patston, T. J., Cropley, D. H., Marrone, R. L., & Kaufman, J. C. (2018). Teacher implicit beliefs of creativity: Is there an arts bias? *Teaching and Teacher Education, 75*, 366-374.

Plucker, J. A., & Beghetto, R. A. (2004). Why creativity is domain general, why it looks domain specific, and why the distinction doesn't matter. In R. J. Sternberg, E. L. Grigorenko, & J. L. Singer (Eds.), *Creativity: From potential to realization* (pp. 153–168). American Psychological Association.

Plucker, J. A., Beghetto, R. A., & Dow, G. T. (2004). Why isn't creativity more important to educational psychologists? Potentials, pitfalls, and future directions in creativity research *Educational Psychologist, 39*(2), 83-96.

Poincaré, H. (1948). *Science and method*. Dover.

Polly, D., McGee, J. R., Wang, C., Lambert, R. G., Pugalee, D. K., & Johnson, S. (2013). The association between teachers' beliefs, enacted practices, and student learning in mathematics. *Mathematics Educator, 22*(2), 11-30.

Pólya, G. (1962). *Mathematical discovery: On understanding, learning, and teaching problem solving* (Vol. 1). John Wiley and Sons.

Prusiner, S. B. (1997). *Stanley B. Prusiner - Biographical: The Nobel Prize in Physiology or Medicine 1997*. https://www.nobelprize.org/prizes/medicine/1997/prusiner/facts/

Purnomo, Y. W. (2017). The complex relationship between teachers' mathematics-related beliefs and their practices in mathematics class. *The New Educational Review, 47*(1), 200-210.

Rhodes, M. (1961). An analysis of creativity. *The Phi Delta Kappan, 42*(7), 305-310.

Richards, R. (2007). Everyday creativity: Our hidden potential. In R. Richards (Ed.), *Everyday creativity and new views of human nature* (pp. 25-53). American Psychological Association.

Runco, M. A. (2004). Everyone has creative potential. In R. J. Sternberg, E. L. Grigorenko, & J. L. Singer (Eds.), *Creativity: From potential to realization* (pp. 21-30). American Psychological Association.

Runco, M. A. (2007). *Creativity theory and themes: Research, development, and practice*. Elsevier.

Runco, M. A. (2014). *Creativity: Theories and themes: Research, development, and practice*. Elsevier.

Said-Metwaly, S., Van den Noortgate, W., & Kyndt, E. (2017). Approaches to measuring creativity: A systematic literature review. *Creativity. Theories–Research-Applications, 4*(2), 238-275.

Saldana, J. (2016). *The coding manual for qualitative researchers* (3rd ed.). Sage.

Schacter, J., Thum, Y. M., & Zifkin, D. (2006). How much does creative teaching enhance elementary school students' achievement? *The Journal of Creative Behavior, 40*(1), 47-72.

Scher, L., & O'Reilly, F. (2009). Professional development for K–12 math and science teachers: What do we really know? *Journal of Research on Educational Effectiveness, 2*(3), 209-249.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370).

Schrauth, M. A. (2014). *Fostering mathematical creativity in the middle grades: Pedagogical and mathematical practices* (Publication No. 3665425) [Doctoral dissertation, Texas State University]. ProQuest Dissertations and Theses Global.

Sheffield, L. J. (2008). Questioning mathematical creativity – questions may be the answers. 5th International Conference on Creativity in Mathematics and the Education of Gifted Students, Haifa, Israel.

Shriki, A. (2010). Working like real mathematicians: Developing prospective teachers' awareness of mathematical creativity through generating new concepts. *Educational Studies in Mathematics, 73*(2), 159-179. https://doi.org/10.1007/s10649-009-9212-2

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics, 14*(1), 19-28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *Zentralblatt fuer Didaktik der Mathematik, 29*(3), 75-80.

Simonton, D. K. (1984). Artistic creativity and interpersonal relationships across and within generations. *Journal of Social Psychology, 46*(6), 1273.

Spangler, D. A. (1992). Assessing students' beliefs about mathematics. *The Mathematics Educator, 3*(1).

Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? *Journal of Secondary Gifted Education, 17*(1), 20-36. https://doi.org/10.4219/jsge-2005-389

Sriraman, B. (2009). The characteristics of mathematical creativity. *The Mathematics Educator, 14*(1), 19-34.

Sriraman, B., & Lee, K. H. (2011). *The elements of creativity and giftedness in mathematics*. Sense Publishers.

Stake, R. E. (1995). *The art of case study research*. Sage Publications.

Sternberg, R. J. (1985). *Beyond IQ: A triarchic theory of human intelligence*. Cambridge University Press.

Sternberg, R. J. (1988). A three-facet model of creativity. *The nature of creativity*, 125-147.

Sternberg, R. J. (1999). A propulsion model of types of creative contributions. *Review of General Psychology, 3*(2), 83-100.

Sternberg, R. J. (2006). The nature of creativity. *Creativity Research Journal, 18*(1), 87-98. https://doi.org/10.1207/s15326934crj1801_10

Sternberg, R. J. (2018). A triangular theory of creativity. *Psychology of Aesthetics, Creativity, and the Arts, 12*(1), 50.

Sternberg, R. J., & Lubart, T. I. (1991). An investment theory of creativity and its development. *Human Development, 34*(1), 1-31.

Sternberg, R. J., & Lubart, T. I. (1995). *Defying the crowd: Cultivating creativity in a culture of conformity*. Free Press.

Sternberg, R. J., & Lubart, T. I. (1996). Investing in creativity. *American Psychologist, 51*(7), 677-688. https://doi.org/10.1037/0003-066X.51.7.677

Sternberg, R. J., & Lubart, T. I. (2000). The concept of creativity: Prospects and paradigms. In R. J. Sternberg (Ed.), *Handbook of creativity* (pp. 93-115). Cambridge University Press.

Sternberg, R. J., & Williams, W. M. (1998). Teaching for creativity: Two dozen tips. *Center for Development and Learning*. http://www.uf.bg.ac.rs/wp-content/uploads/2015/10/Master7.2015.pdf.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Sage.

Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration*. Ministry of Education.

Underhill, R. (1988). Focus on research into practice in diagnostic and prescriptive mathematics: Mathematics learners' beliefs: A review. *Focus on learning problems in mathematics, 10*(1), 55069.

Wallas, G. (1926). *The art of thought* (15th ed.). Harcourt, Brace and Company.

Wayne, A. J., Yoon, K. S., Zhu, P., Cronen, S., & Garet, M. S. (2008). Experimenting with teacher professional development: Motives and methods. *Educational Researcher, 37*(8), 469-479.

Wilkens, J. L. M., & Ma, X. (2003). Modeling change in student attitude toward and beliefs about mathematics. *The Journal of Educational Research, 97*(1).

Yaftian, N. (2015, 6/2/2 June 2015). The outlook of the mathematicians' creative processes. *Procedia - Social and Behavioral Sciences, 191*, 2519-2525. https://doi.org/10.1016/j.sbspro.2015.04.617

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage.

Zenasni, F., Mourgues, C., Nelson, J., Muter, C., & Myszkowski, N. (2016). How does creative giftedness differ from academic giftedness? A multidimensional conception. *Learning and Individual Differences, 52*, 216-223.

Appendix A

Overview of Professional Development Sessions

Session	Creativity Topic	Session Activities	Challenge for teachers
1	Introduce goals of the professional development	Engage in mathematical questions determining numbers that fit criteria (e.g. have a sum of 9 and a difference of 2).	Look for textbook examples to confirm or refute Sir Ken Robinson's theory on schools and creativity
	Introduce CMS project and idea behind modules	Collaborate to solve fermi problems (e.g. How many cups of coffee does Dunkin Donut sell per day? How many ping pong balls fit in a Boeing 747?)	
		Watch Sir Ken Robinson – schools and creativity video	
2	Introduce spectrum of creativity (Beghetto & Kaufman, 2007; Csikszentmihalyi, 1996, 1998; Maslow, 1967)	Work in groups of 2 to solve the cake problem: How can you cut eight pieces of cake from a round cake with three straight slices of a knife, without moving any pieces?	Between sessions: keep track of ideas, things you tried or what you'd like to try related to mathematical creativity and any resources you might need to implement these ideas.
	Compare teacher definitions of creativity (from pre-interviews) to researcher defined traits.	Discuss the promotion of creativity among students: What does your classroom culture look like? Can creativity flourish in your classroom? Are there times when it is impossible? Why or why not?	
	Multiple solutions or approaches to a problem as creative	Engage in multiple solution task: Jam problem Mali produces strawberry jam for several food shops. She uses big jars to deliver the jam to the shops. One time she distributed 80 liters of jam equally among the jars. She decided to save 4 jars and to distribute jam from these jars equally among the other jars. She realized that she had added exactly ¼ of the previous amount to each of the jars. How many	

		jars did she prepare at the start? Find as many solutions as possible.	
		Generate list of things teachers want to try in their classrooms and reflect on the place of creativity in their classrooms.	
3	Big-C, little-c creativity Connect to Agnes Denes exhibit at the SHED Revisit spectrum of creativity (Beghetto & Kaufman, 2007; Csikszentmihalyi, 1996, 1998)	Show list of things teachers want to try to motivate their work during the upcoming 2 weeks before the PD session meets again.	Reflect on spectrum of creativity and how it relates to their lesson development ideas. How do you envision the spectrum of creativity in the lesson you are developing? How would you encourage or would *students* exemplify these types of creativity? In what ways might *you* exemplify these types of creativity in the planning or implementation of the lesson?
		Engage in activity designed to highlight differences along the spectrum of creativity.	
		A girl scout troupe baked a batch of cookies to sell at the annual bake sale. They made between 100 and 150 cookies. One fourth of the cookies were lemon crunch, and one fifth of the cookies were chocolate macadamia nut. What is the largest number of cookies the troop could have baked? Student response (algebraic approach) results in no new information. How would you respond to this student?	
4		Finding 1 activity – teachers work in groups to complete activity. Discuss what this might look like in middle school classrooms? How does thinking about fractions in this way differ from the traditional ideas? How might this help students better understand and remember fraction properties?	
		Teachers create problems based on the manipulative they were given	
		Discussion regarding what creativity in middle school mathematics classrooms looks like.	

5	Review spectrum of creativity, traits of mathematical creativity, and environment that allows for the promotion of creativity among students.	Opportunity to reflect, discuss, the switch to remote learning, struggles and benefits, and how creativity fits in to this format of teaching and learning (more like a focus group interview format)– professional development community support	
6	Teaching creatively versus teaching for creativity	Proportional reasoning – given a bar diagram, think about: - What are all of the representations associated with this visual? - Write at least five problems using this ratio (at least two of which are problems that are not proportional). Be sure to write problems in different forms (e.g. table, word-problem, graph, percent, fraction). Come up with problems that elicit different approaches and/or solutions. - In the notes of your slide, provide the solution to the problem, write a brief summary of how you think a student will approach the problem and what misconception they might have (or how this problem addresses a common misconception).	Have students journal daily about proportional reasoning (or relevant content) and complete a task similar to the one we did together.
7	Traits of mathematical creativity Problem posing and productive struggle	Continue proportion activity from previous session	Have students begin (or continue) to pose problems about content you're covering. This might be generated from their journal entries.

8	Demonstrate connections between each activity we did throughout the sessions and their connection(s) to mathematical creativity	Each group will be given the same problem: A circular mirror has a diameter of 12 inches. What is the area, in square inches, of the mirror? What is the circumference of the mirror? 1. Create a mini-lesson addressing the same topic addressed in the problem but in an intentionally creative way. 2. Purposefully tie in at least 1 trait of creativity (listed on the next slide). 3. Write a few sentences about each trait of creativity you address and how it is evident in the mini-lesson you designed. 4. Reflect with your group about your process of developing this mini-lesson.	Design at least 2 intentionally creative lessons focusing on at least one trait of mathematical creativity and purposefully promote students' creativity. Reflect on your own creativity as you develop and implement the lesson. Collect and share student work.

Appendix B

Order of Magnitude Problems

Order of Magnitude, or Fermi problems, are estimation problems of measurement or numerosity for which the goal is to be within an order of magnitude of the actual value. Choose at least two from the list below to think about, discuss, and determine a reasonable estimate. Where did you start? How did you come up with your answer? How does this represent creativity?

- How big is chalk dust?
- How many cups of coffee does Dunkin Donuts sell per day?
- How many ping pong balls fit in a Boeing 747?
- How many STEM teachers are there in [state]?
- How much does an average cumulus cloud weigh?

Appendix C

Preprogram Survey

Thank you so much for your interest in my study. This survey will help me select participants and begin scheduling the professional development sessions that works best for the group. I ask that if you agree to participate in the study you are committed to attending all of the professional development sessions as I will do my best to choose a day that works for everyone. This survey should only take you about 15 minutes. I would really appreciate if you could get this completed by Friday, December 23rd, but if you're bogged down with holiday preparations, I can accept responses until January 2nd. As a reminder, this study will involve two interviews, eight professional development sessions, and one lesson observation. If you participate in all of these, you will receive a certificate for 8 hours of professional development and a stipend of $500.

1. How many years have you been teaching?
2. How many years have you been teaching middle school (grades 6-8) math?
3. What certification(s) do you currently hold? (for example, K-12 Math, K-8 math)
4. Are you the main teacher in your classroom?
5. What type of general professional development have you attended in the last 2 years?
6. What type of mathematics professional development have you attended in the last 2 years?
7. What do you think is important in a professional development program for middle school mathematics teachers?
8. How often do you develop new lessons, on your own or with colleagues? Please give an example of the last time you did this.
9. What opportunities do you have to collaborate with your colleagues? (within your school, other mathematics teachers, within grade level, across district)?
10. Please give examples of what you think of as being creative?
11. Have you ever considered mathematics as creative? If yes, how? Please be as specific as possible.

Appendix D

Teacher Preprogram Interview Questions, adapted from Schrauth (2014)

The purpose of this interview is to better understand your ideas about the teaching and learning of mathematics. Before we begin, do you have any questions for me? The first set of questions has to do with your beliefs, are you ready to begin?

Beliefs about mathematics

1. What do you like about mathematics?
    a. Why did you become a middle school mathematics teacher?
2. What is mathematical reasoning?
3. What do you do when a student approaches a problem in a way that you weren't expecting?
4. Describe a situation when a student struggled with a particular concept and what you did to engage with them mathematically?
5. What do you do if you don't initially understand a student's approach?

Beliefs about creativity in mathematics

6. How would you define creativity?
7. Describe connections you see between mathematics and creativity.
8. What is creativity's place in teaching mathematics? Learning?
9. How do you promote student creativity?
10. What is the most creative discipline you can think of? And why?

Instruction

11. What does one of your typical classes look like?
    - What is your general approach to teaching math?
    - How are students expected to behave as learners?
    - How do you expect students to interact with their classmates? You?
    - What are the expectations for doing math?
12. How do you decide what mathematics to teach on a day to day basis?
    a. How do you select or develop mathematical tasks?
    b. What characteristics of tasks are important to you? (e.g. open-endedness, option for multiple responses/approaches to solving a math problem)
    c. How do you decide which materials to use?
    d. What input do you have in selecting instructional materials and resources? [who is involved in materials selection (e.g., teacher, school, district), accessibility to resources]

Is there anything else you'd like to add that about your ideas on teaching and learning mathematics, or other things you feel are important for me to know about your classroom as I develop content for our upcoming professional development sessions?

Appendix E

Teacher Postlesson Interview Questions, adapted from Schrauth (2014)

The purpose of this interview is to better understand your experience participating in the professional development program focused on infusing creativity into the teaching and learning of mathematics and to hear about your experience designing lessons with creativity in mind. My original plan was to observe your lesson in person then meet with you to reflect about the role of creativity in your teaching, but since we weren't able to do this, I tailored these interview questions to make the most out of this change. The first set of questions is for me to better understand your experience planning and implementing a lesson focused on creativity.

Lesson Implementation

1. What part(s) of the PD influenced the development of your lesson on [insert topic of teacher lesson] and why?
2. What part(s) of the PD helped you directly in the planning and implementation of the lesson? Why?
3. What part(s) of the PD helped you indirectly in the planning and implementation of the lesson? Why?
4. What part(s) of the PD wasn't transferable to your practice? Why?
5. What was your experience planning and implementing the lessons that infused creativity into the teaching and learning of mathematics?
    a. Explain your rationale for how you decided on and created this lesson.
    b. What traits of creativity did you draw on in your lesson design? How does this lesson attend to these traits?
    c. How were you creative in the design of the lesson?
    d. How were you creative in the implementation of the lesson?
    e. How were your students expected to be creative?
        i. Can you point to a student in particular that you felt was creative?
        ii. What part of this student's work do you think was the most creative?
    f. What were the benefits of implementing the lesson with the lens of creativity?
    g. What were the struggles you experienced?
6. How do you think the lesson went? (student understanding, lesson implementation)
    a. What would you change to improve this lesson?
7. How do you feel students reacted to the lesson?

The next set of questions is about the professional development program itself and how it influenced your beliefs about mathematics and creativity.

8. In the pre-interview, you said that you like math because [insert teacher-specific response from pre-interview]. How would your answer change after participating in the PD program?
    a. Do you see mathematics less as a set of right and wrong answers and more about the thinking and creativity behind the subject? In what ways?
9. What is mathematical reasoning? In what ways has your definition changed since your participation in the professional development program?
10. What do you do when a student approaches a problem in a way that you weren't expecting? How is this different since your participation in the professional development sessions?

Creativity

11. In the pre-interview you defined creativity as [insert teacher-specific response from pre-interview]. How would you change your answer after participating in the PD program?
12. Describe connections you see between mathematics and creativity.
13. What is creativity's place in teaching mathematics?
14. What is creativity's place in learning mathematics?
15. How do you promote student creativity?
    a. How has this changed since your participation in the professional development sessions?
16. What is the most creative discipline you can think of? And why?

Instruction

17. A lot has changed with the switch to remote learning. Thinking about this new format and your experience in the PD program in mind:
    - What does one of your typical classes look like? How has this changed since your participation in the professional development program?
    - What is your general approach to teaching math? How did the PD influence this? How did the switch to online learning influence this?
    - How are students expected to behave as learners? How did the PD influence this? How did the switch to online learning influence this?
    - How do you expect students to interact with their classmates? You? How did the PD influence this? How did the switch to online learning influence this?
    - What are the expectations for doing math? How did the PD influence this? How did the switch to online learning influence this?
18. How do you decide what mathematics to teach on a day to day basis? Has this changed since your participation in the professional development program? How?
    a. How do you select or develop mathematical tasks?
    b. What characteristics of tasks are important to you? (e.g. open-endedness, option for multiple responses/approaches to solving a math problem)
    c. How do you decide which materials to use?

Professional development

19. How does this professional development experience compare to other experiences you've had with professional development?
    a. How has this professional development helped you with the transition to online teaching and learning?
20. What did you take away from your participation in the professional development program?
21. What would you change about these sessions?

What else would you like to add that pertains to your ideas about teaching and learning mathematics or other things about this PD we didn't already discuss?

Appendix F

Original Lesson Observation Protocol (Schacter et al., 2006)

Teaching Creative Thinking Strategies	**Tally of instances/Researcher notes**
The teacher leads students in activities that require them to generate and record multiple ideas.	
The teacher teaches students creative thinking strategies such as: divergent thinking, brainstorming, using analogies, redefining a problem, or synectics.	
The teacher explicitly teaches metacognitive strategies.	
Opportunities for Choice and Discovery	
The teacher creates learning scenarios where students can choose from one of several activities.	
The teacher creates activities where students have to discover the answer by examining various models and ideas.	
The teacher creates activities where students have to create an original artifact and present this artifact as a potential new solution to a problem.	
Intrinsic Motivation	
The teacher focuses students' attention by reinforcing the importance of the task, and not extrinsic factors such as grades and rewards.	
The teacher recognizes, values, and reinforces creative thinking and creative processes.	
The teacher links rewards with the creative process, not solely the result.	
When using rewards, the teacher emphasizes the importance of the task rather than completing the task for the reward.	
Environment Conducive to Creativity	
The teacher is supportive and encouraging of students' non- conformist and unusual ideas.	
The teacher is tolerant of ideas that do not lead to the correct answer.	

The teacher encourages creative thinking and creativity in tasks that do not necessarily require creativity.	
The teacher develops an atmosphere focused on inquiry, curiosity, exploration, and self-directed learning.	
The teacher supports and encourages risk-taking and makes students aware that they will not be penalized for failing.	
The teacher emphasizes a sense of independence and responsibility for learning.	
Imagination and Fantasy	
The teacher explains how imagination and fantasy can lead to changing existing ideas into original creations.	
The teacher creates activities that engage students in using imagination and fantasy.	
The teacher creates learning experiences where students apply imagination and fantasy to real world situations and problems.	

Note: Creative teaching frequency scaled from 1 to 5 with 5 indicating high frequency. Creative teaching quality scaled from 1 to 3 with 3 indicating high quality.

Appendix G

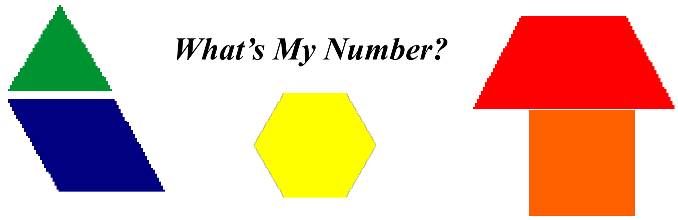
Adapted Protocol for Data Analysis (Schacter et al., 2006)

Thinking Strategies: Teaching Creative Thinking Strategies	**Non-algorithmic**	Non-algorithmic decision making
	Novel solution	Generate novel solutions to problems
	Mathematical elegance	Strive for mathematical elegance
	Explain reasoning	Explain reasoning used to solve a problem
	Mathematical Connections	Be curious about mathematical connections
	Continue to explore	Continue to explore after a problem has been solved
	Insightful solutions	Develop insightful solutions to a problem
	New questions	Formulate new questions for other students to consider
	Multiple solutions	See multiple ideas/solutions to a problem
	Persevere	Perseverance
	Metacognition	The teacher explicitly teaches metacognitive strategies.
Choice and Discovery: Opportunities for Choice and Discovery	**Activities**	The teacher creates learning scenarios where students can choose from one of several activities.
	Discovery	The teacher creates activities where students have to discover the answer by examining various models and ideas.
	Create	The teacher creates activities where students have to create an original artifact and present this artifact as a potential new solution to a problem.
Motivation: Teacher values the students' process over the end result	**Task**	The teacher focuses students' attention by reinforcing the importance of the task, and not extrinsic factors such as grades and rewards
	Process	The teacher recognizes, values, and reinforces creative thinking and creative processes (not just giving the answer)
	Assessing creative process	The teacher links rewards with the creative process, not solely the result.
	Assessing against task	The teacher emphasizes the importance of the task rather than completing the task for the reward.
Environment: Environment	**Support**	The teacher is supportive and encouraging of students' non- conformist and unusual ideas.

Conducive to Creativity	**Incorrect**	The teacher is tolerant of ideas that do not lead to the correct answer.
	Atmosphere	The teacher develops an atmosphere focused on inquiry, curiosity, exploration, and self-directed learning.
	Risks	The teacher supports and encourages risk-taking and makes students aware that they will not be penalized for failing.
	Independence	The teacher emphasizes a sense of independence and responsibility for learning.
	Reflect	Opportunity for student to reflect on learning
Imagination and Fantasy	**Original**	The teacher explains how imagination and fantasy can lead to changing existing ideas into original creations.
	Engage	The teacher creates activities that engage students in using imagination and fantasy.
	Real world	The teacher creates learning experiences where students apply imagination and fantasy to real world situations and problems.

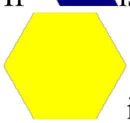
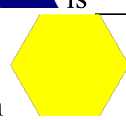
Appendix H

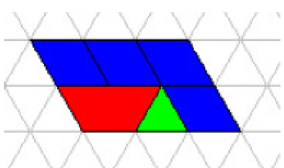
Professional Development Session 4 Fraction Activity

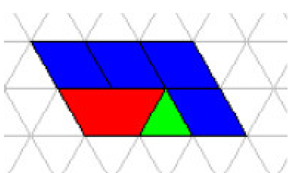
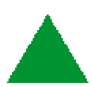


What's My Number?

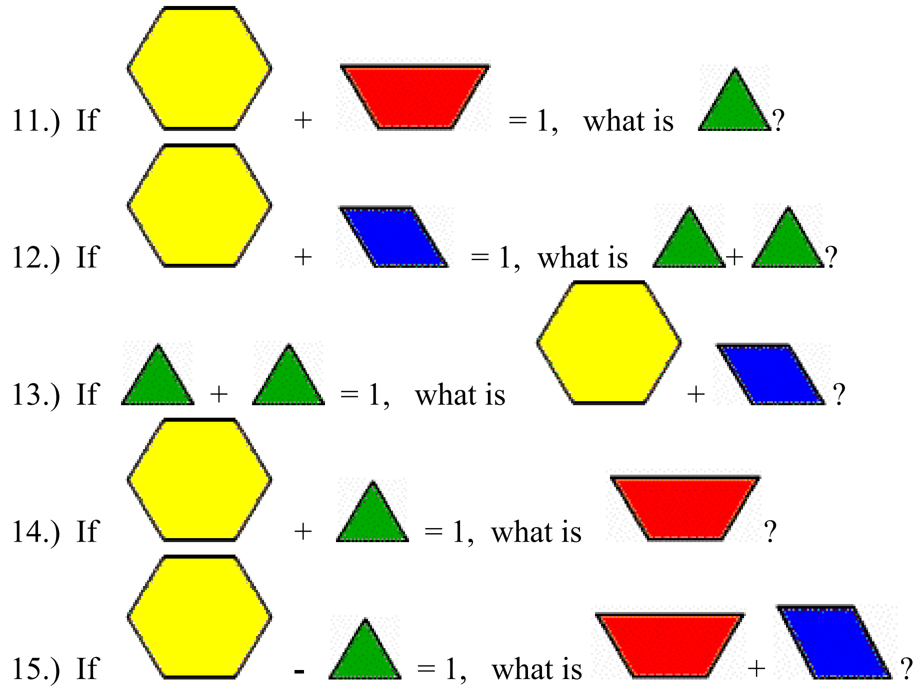
Objective: Students will name the size of an object given the size of 1 unit.

1. If  is 1/2 of a unit then draw 1 unit.
2. If  is 1/2 of a unit then draw 2 1/2 units.
3. If  is 1/2 of a unit then draw 1 unit.
4. If  is 1/3 of a unit then draw 1 unit.
5.  is 3 units then  is _____________.
6. If  is 1 unit then  is _____________.
7. If  is 1 unit then  is _____________.
8. If  is 1 unit then  is _____________.
9. **EXTRA!**

If  is 1 then  is ____________.

If  is 1 then  is _____________.

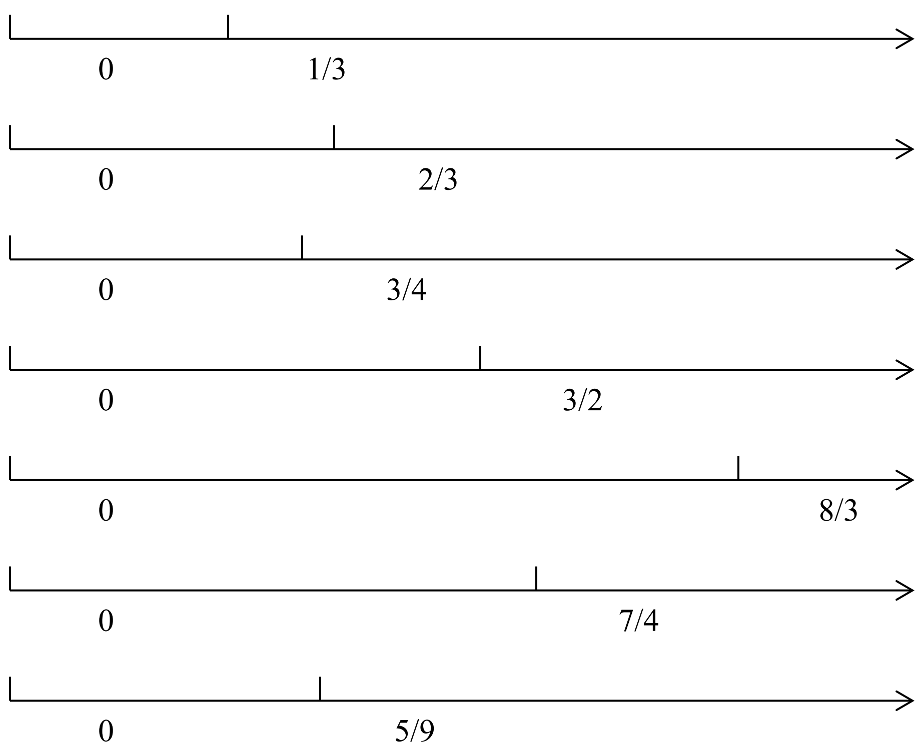
10.) Make up some of your own problems. Have your partner answer them!



16.) Make up some more addition/subtraction problems!

<u>Finding 1</u>

On each line, mark the approximate location of 1 (it will be different for each line), then use your manipulative to model each example.



Did you develop a procedure that works for every problem? Describe in detail how you would find the location of 1 for any fraction *a*/*b*. (e.g., would you break the line into parts? How many? Then what would you do? Why are you doing each step?)