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Transitioning Secondary Mathematics Pedagogy Towards Reform-Oriented Practice Through Coteaching

Jessica Tybursky Nuzzi
Montclair State University

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Jessica Tybursky Nuzzi

Montclair State University

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Dissertation Chair: Dr. Eileen Fernández

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We hereby approve the Dissertation

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of

Jessica Tybursky Nuzzi

Candidate for the Degree:

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Graduate Program:
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Dissertation Committee:

[redacted]

Dr. Eileen Fernández
Dissertation Chair

Certified by:

[redacted]

Dr. M. Scott Herness
Vice Provost for Research and
Dean of the Graduate School

Jan. 22, 2021

Date

[redacted]

Dr. Steven Greenstein

[redacted]

Dr. Eileen Murray

Abstract

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Reform standards in mathematics education have called for classrooms that are student-centered and that incorporate problem solving and reasoning for meaningful learning. After decades of reform efforts involving multiple stakeholders, research indicates that most classrooms remain teacher-centered and procedurally focused, due to the complexity of concerns and competing intentions that teachers face in their work. Coteaching, a commitment between two teachers to coplan, coenact, and coreflect on lessons, can serve as an ongoing, sustained, focused, integrated, reflective professional development structure that supports teachers towards growth. The theoretical constructs used to describe possible growth towards reform orientations in teaching secondary mathematics include craft knowledge gained in practice, shifts in actions of the teaching habitus, and zones of proximal development among coteachers. This study questions how coteachers can serve as resources for each other's professional growth towards reform-oriented pedagogy. Through a qualitative, naturalistic inquiry, a case study methodology was used to focus on one pair of coteachers in a year-long study of their high school Algebra 1 coteaching. The analytic framework includes the stages of development of forming, norming, storming, and performing, as well as the process, content, and levels of reflection. Findings regarding teachers' actions as resources towards reform-oriented teaching include modeling and observing teaching, sharing ideas for teaching, communicating with and challenging each other, and creating alternatives for teaching. These actions, along with reflection, demonstrate

possibilities for transitioning teaching practices towards interactive, intellectually engaging pedagogy through purposeful collaboration and deep reflection on, and in, practice.

Keywords: secondary mathematics teaching, professional growth, reform-oriented teaching, coteaching

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Dedication

This dissertation is dedicated to my family. To Frank, my complement, who lives by the motto, “You do you,” and has supported me in every possible way to accomplish what I set out to do. To Liliana and Emilia, my loves, may you grow to love learning and recognize the power of education. To Mom and Dad, my role models and counselors, who instilled in me the will to strive for greatness through hard work and commitment.

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Chapter 1: Introduction

Reform Standards: Necessary, Not Sufficient

In the teaching and learning of mathematics, an intense debate has long existed regarding what content should be taught and how it should be taught. After the curricular reforms of the 1960's (New Math) and 1970's (Back to Basics), the National Council of Teachers of Mathematics (NCTM) rebuilt their initiatives to introduce problem solving and meaningful mathematics learning into classrooms (NCTM, 1980) and launched their *Curriculum and Evaluation Standards for School Mathematics* (1989). This document was intended to "make mathematics accessible to all students" based on "what could work more effectively and equitably" (Hekimoglu & Sloan, 2005, p. 37). Subsequent standards documents (e.g., National Research Council, NRC, 2001; National Governors Association, NGA, 2010) also called for student-centered classrooms in which learners reason and justify in solving problems while building conceptual understanding and making meaning of their world using mathematics (Brodie, 2010). Given that all these standards were written or endorsed by education's central stakeholders (teachers, professors, researchers, and policy makers), we would expect that these documents' ideas would have taken hold in America's classrooms.

However, in a review of studies on mathematics reform, Cohen and Mehta (2017) found that most classrooms remain teacher-centered and most mathematics instruction remains procedural, "both common features of the teaching the reforms tried to change" (p. 663). The construction of these documents has been a necessary step, but they have not been sufficient to create the changes intended to build the mathematical literacy reformers sought. Although there are many factors that drive reform efforts, research has shown that the greatest impact on student achievement is, in fact, the classroom teacher (Darling-Hammond, 2000). This study will take a

close look at possibilities for transitioning teaching practices that would sufficiently integrate the standards' ideals.

A Focus on Teachers of Mathematics

Mathematics teachers are largely trained according to forward-thinking standards, yet the gap between this training and classroom instantiation of the standards persists. The following sets of standards have been widely used in pre-service training and referenced among in-service teachers, giving them credibility as a lens through which to view how mathematics teachers are being directed to structure their classroom practices. An understanding of the standards documents informs this study's perspective on mathematics teaching and learning. Teaching in a way aligned to the standards outlined below will be referred to throughout the study as *reform-oriented practices*.

Standards for Teaching and Learning Mathematics

In their first set of standards, the National Council of Teachers of Mathematics (NCTM, 1989) called for new goals to help students "gain *mathematical power*" (p. 5). For students, the goals include learning to value mathematics, becoming confident in their abilities, becoming problem solvers, and communicating and reasoning mathematically. As students "read, write, and discuss" (p. 5) mathematics, they can explore, conjecture, and logically reason in order to build understanding. These basic tenets promoted by NCTM can serve as a foundation for what can be considered *student-centered teaching,* a central component in this reform, where students are actively engaged with each other and their teacher, and not passively receiving instruction.

NCTM (2000) further built on these ideas in their *Process Standards* of problem solving, reasoning and proof, communication, representation, and connections. Armed with problem-solving strategies, students can leave high school "with the disposition, knowledge, and

strategies to deal with the new challenges they will encounter" (NCTM, 2000, p. 334). For students to acquire a repertoire of mathematical strategies, they must engage with "mathematics as a sense-making discipline rather than one in which rules for working exercises are given by the teacher to be memorized" (p. 334). Therefore, teachers are responsible for fostering knowledge and attitudes in their students that will develop productive dispositions and heuristic strategies. In addition, habits of mind for reasoning through mathematical situations, exchanging and reflecting on ideas, and representing and linking mathematical ideas are also critical for providing students with opportunities to develop deep understandings of mathematics.

In *Adding it Up*, the authors from the National Research Council and the Mathematics Learning Study Committee (NRC, 2001) describe five "interwoven and interdependent" (p. 116) strands of mathematical proficiency. Two strands frequently discussed are *conceptual understanding* and *procedural fluency*. Conceptual understanding elicits the deep, rich understandings sought in learning mathematics, "an integrated and functional grasp of mathematical ideas" (NRC, 2001, p. 118). Procedural fluency includes the knowledge of procedures and goes beyond the application of routine algorithms to include "when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently" (NRC, 2001, p. 121). The strands not often referenced are *strategic competence* (the ability to formulate, represent, and solve problems), *adaptive reasoning* (the ability to relate and adjust to concepts and situations), and *productive disposition* (the ability to perceive mathematics as worthwhile and to view oneself as an effective doer of mathematics). The five strands of mathematical proficiency convey an image of what it means to know mathematics by integrating various aspects of thinking and reasoning regarding mathematical learning.

In general, teachers tend to focus on procedural aspects of mathematical proficiency because procedural knowledge is a clear and measurable output of student learning. Procedures have also been tested through high-stakes assessments, through which teachers are often evaluated based on their students' scores. However, educational research has shown that procedural knowledge alone does not support retention and transfer of knowledge in the long run (Rittle-Johnson & Alibali, 1999). As described by the interrelatedness of the five strands for mathematical proficiency, other aspects have to be intertwined with procedural knowledge for students to be productive, successful, and life-long learners and users of mathematics. Conceptual understanding is vital to support the underlying knowledge of why procedures work and why they are applicable, to lead to procedural fluency. Unless teachers of mathematics simultaneously develop each of the five strands with students and connect them to each other, students will continue to struggle to achieve their mathematical potential.

More recently, the eight *Standards for Mathematical Practice* (Mathematical Practices, NGA, 2010) were published as part of the Common Core State Standards as practices that "rest on important 'processes and proficiencies'" (p. 6). The *Mathematical Practices* reference the NCTM (2000) *Process Standards* and the NRC (2001) strands of mathematical proficiency. The Mathematical Practices include making sense of problems and persevering in solving, reasoning abstractly and quantitatively, constructing viable arguments and critiquing others, modeling, using tools strategically, attending to precision, making use of structure, and looking for or expressing repeated reasoning. This latest document again enlists teachers to provide students with opportunities to engage in mathematical thinking.

The area of focus of the NCTM standards is student-centered teaching where students gain mathematical power by making sense of situations using problem-solving heuristics and

building knowledge. The authors of *Adding it Up* (NRC, 2001) also focus on the importance of learners' adaptiveness and productive dispositions in their mathematical work, alongside procedural and conceptual understanding. The *Mathematical Practices* extend these areas of focus to specify perseverance in problem solving (e.g., Schoenfeld, 2014), reasoning and argumentation (e.g., Brodie, 2009), and modeling (e.g., Blomhøj, 2009). As these goals have developed over time and informed mathematics teaching, they have become more specific and directed for practice in the field. Since "teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions" (NCTM, 2000, p. 18), it is of vital importance for teachers of mathematics to understand the implications of the standards' areas of focus and ways they can be enacted in their classrooms.

Reducing the Theory-Practice Gap to Achieve Reform

With three decades worth of efforts to reform mathematics education and no widespread, significant changes, we have to ask: is it even possible to accomplish reform ideals? Mary Kennedy (2005) proposed five hypotheses as reasons that teachers struggle to enact reform efforts in everyday instruction: teachers' knowledge; teachers' beliefs; teachers' dispositions or attitudes; teaching circumstances and systemic constraints; and, the hypothesis not commonly considered, "the (reform) ideals themselves may be unattainable or may actually impede practice" (p. 12). Understanding teaching practices, how teachers account for these practices, and teachers' ideas that motivate practices were the foci for Kennedy's (2005) study of 499 teaching episodes across 45 teachers, from which she thematically summarized teachers' lines of thinking, teachers' intentions, and teachers' areas of concern in their practice. Kennedy found that teachers often act with multiple intentions, related to goals, fears, aspirations, obligations, or personal

needs, and that these may compete internally with each other, creating conflicts or "knots" in teachers' thinking (Wagner, 1987).

Whereas reform ideals may be somewhat present in practice, they are "barely visible in the complex landscape of competing intentions and the multiple areas of concerns that are important to teachers" (Kennedy, 2005, p. 61). In order to cope with top-down initiatives, teachers may change their "descriptive and explanatory language to account for teaching without requiring any change in the praxis of teaching" (Roth, 2002, p. 57). Furthermore, when the discourse about changes in action are open to interpretation and limited in application to the particular contexts in which teachers work, it can be justified that teachers are enacting the reform in their own way, even though the intention of the reform does not reach students.

For teachers to truly engage in the intended reform ideals, collective responsibility and active participation must be involved in building an understanding of the reform and design for practice, otherwise "changes made to the classroom context would always be experienced as an imposition" (Roth, 2002, p. 152). In their Interconnected Model, Clarke and Peter (1993) propose that professional change occurs through the "mediating processes of reflection and enactment" (p. 167). To decrease the gap between the reform ideals and teachers' practices and accomplish professional growth, reflection and enactment must be integrated into reform efforts moving forward.

Addressing the Inability to Meet the Calls of Reform

Educational research, at large, is focused on creating environments that are student-centered for optimal learning. As Danielson and McGreal (2000) describe, there has been a shift from behaviorism towards constructivism in education research. Unfortunately, we currently have a workforce of teachers who have grown up with, and only experienced, teaching and

learning through a largely behaviorist perspective. The policies that have been engrained into our system support teacher-centered, structured classrooms as a result of the "effects research" (Danielson & McGreal, p. 13). Based in systems such as the Hunter model (Hunter, 1982), teachers are evaluated on a regular basis, which perpetuates the valuation of certain teacher behaviors. Because public education does not place a priority on developing "reform teaching," supports that would encourage such a shift also are limited. This suggests teachers will continue teaching exactly as they were taught and continually *struggle to shift* their pedagogy towards a student-centered philosophy.

Thus, while we recognize "good teaching" to include "critical thinking, problem solving, lifelong learning, collaborative learning, and deeper understanding" (Danielson & McGreal, 2000, p. 14), this is largely not happening in U.S. classrooms. One lens for examining the divide that exists between what is known to be good teaching and the reality of most mathematics classrooms is the field's approach to *professional teacher development*: rather than top-down initiatives and imperatives for how classrooms *should* look (Kennedy, 2005; Wagner, 1987), the current study examines the proposition that teachers require opportunities to experience, learn, and experiment *in practice*. As professionals, active participation and time to reflect on actions (Schön, 1983) can provide experiences from which to draw in future practice. In order to determine how this can best be accomplished, a deep understanding of teach*ers* and teach*ing* is required. This study aims to gain further understanding of mathematics teachers in action, working towards achieving reform-oriented goals by reducing the gap between theory and practice, with the ultimate goal of providing classroom environments in which students can more actively engage in mathematical reasoning.

Personal Significance

I have witnessed the inability to achieve reforms in my own teaching practice. I am a high school mathematics teacher who:

- experienced largely teacher-led, rather than student-centered, lessons as a mathematics student in a public education system in the Northeastern part of the United States in the 1980's and 1990's,
- was educated in undergraduate and master level mathematics education courses according to the NCTM (2000) *Process Standards*,
- has pursued doctoral studies and participated in many professional development opportunities available to her, and,
- has strived to incorporate collaborative, problem-based learning in her classroom towards students making mathematical meaning throughout eight years of full-time teaching prior to this study.

Yet, I felt my instantiation of what we know to be good teaching was just not hitting the mark. Although observers of my lessons provided limited feedback and praised my teaching abilities, an inner tension arose for me, particularly after doctoral coursework and my reading of literature, regarding how I could enact what *I knew* to be best practices within the realities of a classroom over a 180 day school year. I have always questioned how mathematics teachers can accomplish "thinking classrooms" (Liljedahl, 2016) where students are engaged in quality tasks, arguing and justifying with each other in order to establish mathematical truths for themselves, particularly at the high school level.

I entered the teaching profession with dreams of a classroom with high student engagement and students making connections with the world around them using mathematics.

However, it seemed I was losing my dream to the "system," possibly due to demands for curricular rigor, the pressure of state testing, a traditionally focused textbook, and a lack of model examples of and collegiality around student meaning making. I felt this loss of my dream as an inner tension and found myself at a crossroads in my career. As a practitioner, I believe in these reforms, but struggled to instantiate them.

As a result, this study is generally framed as an investigation into the inner workings of the experience of classroom teachers of mathematics at the high school level. Not only am I the author of this dissertation, but also am a participant in the research, attempting to instantiate the reforms by working in a full-time, authentic teaching role. This study investigates how secondary mathematics teachers can work together to transition their teaching towards reform-oriented pedagogy.

Chapter 2: Review of the Literature and Theoretical Framework

The following chapter will discuss literature relevant to knowledge and beliefs and connect this work to the *practice* of teaching. By considering ways in which teachers enact their knowledge and beliefs in practice through the constructs of *craft knowledge* and *habitus*, I will then propose a professional development structure and theoretical framework as a way to develop teachers' practice towards achieving reform-oriented teaching.

Knowledge and Beliefs in Teaching Mathematics

Teaching is a unique profession in that everyone experiences teaching from the other side of the desk as students for a number of years, observing and making decisions about what teachers do. However, this is done without understanding the analytical decisions made by teachers in their pedagogy. Lortie (1975) described the recognized stages of socialization into teaching: formal schooling (as education students), mediated entry (practice or student teaching), and learning-while-doing (on the job). Through his chapter on *The Limits of Socialization*, Lortie demonstrated that "socialization into teaching is largely *self-socialization*; one's personal predispositions are not only relevant but, in fact, stand at the core of becoming a teacher" (p. 79). That is, teachers learn on the job and from other teachers, and are not largely influenced by education courses or supervisors. Therefore, "idiosyncratic experience and personal synthesis" (Lortie, p. 79) overshadow "commonly held, empirically derived, and rigorously grounded practices and principles of pedagogy" (p. 79).

Lortie (1975) shows us that teachers learn through their experiences, both as a student and as a teacher. Teachers largely learn from how they are taught and subsequently, through their own practice and from their students, which leads to the question: How do teachers gain

knowledge while they are in their practice? Before this can be answered, the question of *What is knowledge for teaching*? must be addressed.

Forms of Knowledge for Teaching

Unlike many other professions, such as medicine or law, there is no formal knowledge base for teaching (Hiebert et al., 2002). Prior to Shulman (1986), educational research did not have a structured way of describing the types of knowledge that teachers bring to and develop through teaching that is specific to pedagogy. In addition to general pedagogical knowledge, Shulman (1986) suggested three distinguishing *categories* for content knowledge: *subject matter content knowledge* as understanding the structures of the subject matter; *pedagogical content knowledge* (PCK) as the "ways of representing and formulating the subject that make it comprehensible to others" (p. 9); and *curricular content knowledge* as the "full range of programs designed for teaching [and] the variety of instructional materials in relation to those programs" (p. 10).

Ball et al. (2008) and Hill et al. (2008) further conceptualized the domain of mathematical knowledge for teaching using two of Shulman's (1986) categories in their resulting structure. The Mathematical Knowledge for Teaching (MKT, Ball et al., 2008) framework is composed of two separate and specified arenas, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge, which can be utilized to categorize and quantify mathematical knowledge needed for teachers. With different research goals in mind from Shulman or others, the MKT framework does not necessarily address the use of these types of knowledge *in action*; that is, the subcategories of SMK and PCK may only provide a static view of what the teacher can currently summon (Nuzzi et al., 2020).

Shulman (1986) also put forth three *forms* of teacher knowledge through which the three categories can be expressed: *propositional knowledge* (principles, practical maxims, ideological norms); *case knowledge* (prototypes, precedents, and parables); and *strategic knowledge* (the reasoning behind decisions and actions chosen from many options, which may be inherently contradictory or incompatible). Strategic knowledge "extend[s] understanding beyond principle to the wisdom of practice" (Shulman, p. 13) by building on both propositional and case knowledge and allowing for a "metacognitive awareness" (p. 13). Shulman (1986) related that this "reflective awareness of how and why one performs complicates rather than simplifies action and renders it less predictable and regular" (p. 13). With these forms of knowledge, Shulman recognized that knowledge required for teaching is more complicated than simply knowing the content: professionals must hold knowledge of varied categories and forms and be able to strategically call upon useful knowledge for given situations.

Wilson et al. (1987) wrote: "We believe that the transformation of subject matter knowledge is at the heart of teaching in secondary schools" (p. 117). The authors described that PCK is not just a set of representations of subject matter but "is characterized by a way of thinking that facilitates the generation of these transformations" (p. 115). The teacher must adapt instruction and reflect on classroom experiences to plan, teach, and acquire new knowledge (Wilson et al., 1987).

Cochran et al. (1993) considered Shulman's (1986) transformation of knowledge as an active process where teachers "simultaneously develop all aspects of knowing how to teach" (Cochran et al., p. 263). In their construct of *Pedagogical Content Knowing* (PCKg), Cochran et al. viewed an integrated understanding of pedagogy, subject matter content, student characteristics, and the environmental context of learning. Cochran et al. (1993) also identified

that learning must be *situated* so that the focus is on teaching "*specific* content to *specific* students in *specific* contexts" (p. 266). This situated learning for teachers serves as the premise of this study where teachers can learn in and through their practice.

Craft Knowledge

The general idea of *craft knowledge* is: "the massive collection of experiences and learnings that those who live and work under the roof of the schoolhouse inevitably accrue during their careers" (Barth, 2002, p. 56). Much like PCKg, craft knowledge is action-oriented, however the research in this area takes a grounded approach. Rather than looking at mathematical teacher knowledge as a set of ideas, craft knowledge references space and time because it is used and develops *in action*. In its *Professional Standards for Teaching*, the National Council of Teachers of Mathematics (NCTM, 1991) stated the following:

> Experienced teachers … are better able to anticipate timing, overall organization and management, and student response. Their repertoire of instructional methods has "filled out," and they often can successfully anticipate what works and does not work in the classroom. (p. 68)

In this characterization, the teacher is positioned as observing, varying, and enacting activities for students. The resulting characterization for knowledge, in turn, allows for these processes to improve over time and in practice. Huberman (1992) framed craft knowledge for teachers as:

> Gradually developing a repertoire of instructional skills and strategies, which correspond to a progressively denser, more differentiated and well-integrated set of mental schemata. … They come to read the instructional situation better and faster, and to respond to it with a greater variety of tools. They develop this repertoire through a somewhat

> haphazard process of trial and error, usually when one or another segment of the repertoire does not work repeatedly. (p. 124)

Huberman discussed how teachers experiment with their practices. Based on whether or not something goes well, they will try something new, and if that new way works well, they will experiment with it in other areas.

According to Grimmett and MacKinnon (1992), "craft knowledge represents the construction of situated, learner-focused, procedural and content-related pedagogical knowledge through 'deliberate action'" (p. 393). The deliberate action stems from the ability to reflect on one's own practice. Ruthven (2002) also stated that craft knowledge is created "through processes of reflection and practical problem solving that teachers engage in to carry out the demands of their jobs" (p. 595). Similarly, Fortune (2018) made a case for the importance of integrating this type of knowledge into education research since teachers' craft knowledge "develops in response to problems of practice and their solutions are grounded in the context of day-to-day work" (p. 111). As teachers gain experience in the classroom through their teaching actions and interactions with students and colleagues, they begin to learn what works (or not) and consequently adapt their practice.

Another possibility provided by craft knowledge is that it can assist in converting scholarly knowledge into actionable form (Ruthven, 2002), meaning that findings from research can be applied in a practical setting. If craft knowledge and scholarly knowledge can mutually inform each other, then the gap between research and practice can be bridged (Ruthven, 2002; Hiebert et al., 2002). This supports the critical idea that the practical and scholarly arenas of education must continuously evolve together if there is to be progress towards a professional sense of teaching.

The literature on craft knowledge does not explain what happens through teaching that allows a teacher to learn from their practice, except for the multiple references to the teacher's ability to reflect, which is not specified. For this study, I am focused on teachers learning from their practice with the goal of enacting student-centered teaching that allows for mathematical meaning making. Teachers may build up their craft knowledge through experience and "reflect" on their actions and interactions with students, but this may not be adequate for teachers to more productively learn from their practice in order to accomplish reform-oriented teaching. Further work is required to understand the mechanisms by which teachers *develop* their craft knowledge in practice.

Teachers' Beliefs

As Alba Gonzalez Thompson (1984) articulated, there is something more than knowledge influencing teachers' decision making. She expressed a teacher's "driving forces" (Thompson, p. 105) as consciously or unconsciously held beliefs, intuitions, notions, and preferences that shape behavior. These driving forces act as a filter of knowledge, creating more subjective ways of teachers' knowing. Beliefs and knowledge go hand in hand: a belief system plays a role in driving the knowledge that is acted on. Although two teachers may have the same knowledge, in "hot action" (Pajares, 1992) what knowledge will be called upon? This will be dependent on the "depth of the espoused beliefs: the extent to which they are integrated with other knowledge and beliefs, especially pedagogical knowledge" (Ernest, 1989, p. 25).

For SMK, beliefs about the nature of mathematics influence how the knowledge is formed for a teacher. For example, mathematics may be viewed as a procedural and disconnected system or mathematics may be viewed as a dynamic system. Previously established beliefs can restrict or allow development of mathematical content for a teacher. For PCK, beliefs about

teaching, learning, students, and local context all filter the knowledge that is called upon and acted on for decisions regarding the classroom.

Since belief systems underlie the use of knowledge, it is necessary to analyze them directly alongside knowledge. Although it is possible for new experiences to alter a teacher's beliefs, the stronger and more internally consistent the system is, the harder it is to change. This suggests that pre-service and beginning teachers are likely more amenable to non-traditional methods of teaching. More experienced teachers, however, have gained experiences and learned what works for them, so that their prior ideas influenced ideas later in their career (Kennedy, 2005). Since beginning teachers often work with and learn from experienced teachers within a system that does not recognize or attempt to shift beliefs, this also contributes to the continued struggle for reforms to take hold. Therefore, in order to work with experienced teachers towards reform ideals, teacher educators have to access teachers' belief systems *and* use their current practice as a starting point. Researchers and developers must consider those personal experiences and values that have formed teachers' beliefs since these are brought to the profession and influence how teachers may make decisions in the classroom and how teachers may subsequently gain knowledge or shift beliefs about teaching and learning.

With this understanding of how teachers' belief systems or orientations inform the use of knowledge in the practice of teaching, I will next turn to a theoretical construct that can allow for understanding how a teacher uses knowledge in action.

Teachers' Habitus

In their article on teacher change, Milne et al. (2006) called upon sociocultural theory and a teacher's *habitus* as a way of explaining how dispositions can support or restrict the way teachers go about their practice. "The *habitus* – embodied history, internalized as a second nature

and so forgotten as history – is the active presence of the whole past of which it is the product" (Bourdieu, 1990, p. 56). The phenomenon of teachers learning through their experiences to build their disposition and practice assists in explaining the amalgam of influences on teaching.

Roth (2002) conceptualized teachers' *becoming* in the classroom through development of their *habitus*, which can more specifically be defined for teachers as "a set of dispositions that structure actions, perceptions, and expectations" (Roth et al., 2000, p. 8). Roth and Tobin (2002) explained that the habitus is "not accessible to our consciousness and therefore without reflection, generates the patterned ways we interact with the world" (p. 10). As a result, habitus provides a comprehension of actions in practice "entirely different from the intentional and conscious decoding acts" (Roth & Tobin, 2002, p. 10).

Milne et al. (2006) claimed that categorizing teacher knowledge is artificial and reductionistic, and that a teacher's purposeful and directed actions are more important and necessary to explore. Their theory of teaching as praxis, the act of *doing* in the classroom, allows a deeper and more relevant understanding of how teachers do their job and behave in their everyday work environment. Reflecting on these actions can allow researchers to identify the *schema* (internalized codes) and *resources* (cognitive components) used in particular matters, and thereby pinpoint crucial aspects for teaching within a domain, which can then inform professional development. Using this view of habitus as the set of experiences embodied from a teacher's history into the possibilities for their practice, we can begin to acknowledge the interactions among knowledge, beliefs, and the whole being of the teacher. With a goal of achieving reform-oriented practice, viewing a teacher's actions as being informed by a holistic set of knowledge and beliefs can assist in understanding the current viewpoint of the teacher and possibilities for change.

Thus far, this chapter has addressed what it is that we are attempting to develop in an effort to meet reform-oriented teaching. As a field, we are aware that beliefs and knowledge inform a teacher's decisions in the classroom, but without taking into account the entirety of a teacher's experience (habitus) and the ways in which they may be acquiring knowledge in practice (craft knowledge), we cannot hope to actually effect change. The remainder of this chapter will discuss literature on professional development, teachers' reflection, and a model of coteaching to propose a strategy for creating transitions towards reform-oriented pedagogy in mathematics classrooms. The development of craft knowledge and the altering of teachers' habitus will figure prominently in this model.

Achieving Professional Growth with Teachers in Practice

Studies of important aspects for successful professional development have wide agreement that in order for teachers to change their practices, the intervention must be ongoing and sustained (Darling-Hammond et al., 2009), be focused on the subject matter (Kazemi & Franke, 2004), be integrated into the teacher's daily work (Darling-Hammond & McLaughlin, 1995), and provide teachers with feedback (Elmore, 2002). Professional development as it mostly occurs does not meet these established components to result in teacher growth. Professional development models "lack concreteness, specificity, and intensity" (McLaughlin, 1991, cited in Gersten & Brengelman, 1996, p. 68) when instead, change requires "ongoing observation, feedback, and discussion" (Gersten & Brengelman 1996, p. 68). Most sessions are one-time offerings at conferences or special events where teachers are expected to turnkey their brief experiences into their own practice. We know that this, by and large, is not happening as teachers in the United States are overwhelmed by a number of constraints, including their high

amount of instructional time, their large student loads, and the limited time with which they have to prepare lessons (Kennedy, 2005).

Moreover, "models of teacher professional development have not matched the complexity of the process we seek to promote" (Clarke & Hollingsworth, 2002, p. 947). Based on the limited impact of standards-based reform efforts in mathematics education, there is evidence that the complex process we seek requires a different and more purposeful approach. Instead of professional development existing outside the classroom, how can teachers' practice be impacted *inside* the classroom, without extra (and unsupported) time spent translating professional development experiences into their local setting? To address this question, I next turn to the literature on the *development* of craft knowledge.

Building Craft Knowledge through Reflection in Practice

Beginning teachers tend to share and seek out knowledge (Barth, 2001, p. 58). They can learn by conversing with, teaching with, and being supervised or coached by those who know mathematics and have been successful in exploring new ways to teach (NCTM, 1991). There are also teacher induction programs that may assist beginning teachers to develop their practice from the start. However, over time experienced teachers become isolated and keep their knowledge to themselves. While experienced teachers learn to not distinguish themselves among their colleagues, perhaps teachers can share their knowledge by working together with beginning teachers in their practice. In this way, beginning teachers can find the support required to more efficiently build their craft knowledge.

Through the Research in Teacher Education (RITE) program at The University of Texas at Austin, Griffin (1986) and his colleagues created a framework for clinical teacher education. Framing the teacher as a decision maker, a learner, and someone who is working in context, they

defined seven critical features of an effective professional development program for teachers: it must be context-sensitive, purposeful and articulated, participatory and collaborative, knowledge-based, ongoing, developmental, and analytic and reflective. Using these features, clinical teacher education programs can inform teachers at all levels of experience to learn "at the elbows" from what occurs in their own classrooms. Such programs can provide opportunities for teachers to develop their craft knowledge by building their repertoire and the schemas required to teach mathematics.

Rodgers and Scott (2008) discussed how research "has deepened and complicated our understanding of the role of self and identity in learning to teach" (p. 732). Learning to teach can be viewed as an ongoing process that exists not only in the beginning stages of being in the classroom. Work in the area of teacher's identity has largely subsumed the components of beliefs, past experience, and orientations. Using Gee's (2001) identification of one's "core identity" (p. 99), rather than one's identity in relation to others and contexts, the idea of identity holds stable within oneself and across contexts. This *core identity* is another way to describe the set of beliefs and dispositions that influence a teacher's practice. Much like the use of *habitus* as a set of dispositions from which to work, identity can include the core beliefs that the teacher works from, which inform the habitus. Whereas a habitus is more of an automatic response system in real time, a core identity can include beliefs that are not acted upon or may not be realistic in particular situations.

To become aware of one's own identity, teachers can reflect on their backgrounds and intentions, and to develop one's identity, teachers can acquire experiences and reflect on these experiences to gain understanding of the outcomes of their own actions. Reflection, therefore, can play an essential role in ongoing teacher development. In their review of contemporary

programs that were developed to support teachers' identity formation, Rodgers and Scott (2008) state, "While several programs refer to reflection as a necessary tool for making meaning, questioning external authorities, and constructing identities, many left the actual process of reflection undefined" (p. 748). Reflective processes can take shape differently for teachers and can target various areas of focus.

Regarding student-centered rather than teacher-led teaching, Roza Leikin states: "Any a-didactic situation has great potential for teacher LTT [Learning through Teaching] because it includes a variety of unpredicted situations" (2010, p. 83). When the student is centered in the learning process, there is a greater likelihood for unpredicted situations, and as a result, opportunities for reflection on practice increase. Grimmett and MacKinnon (1992) confirm this proposition in their statement: "When teachers reflect on pedagogical matters from the perspective of the learner, they generate knowledge that represents … craft as transformed experience" (p. 393). As teachers increase their attention to students, the unpredicted responses may be reflected on, thereby creating new craft knowledge for the teacher.

Specifying the Levels, Process, and Content of Reflection

Larrivee (2000) called upon John Dewey to define *reflective thinking* as the "continual evaluation of beliefs, assumptions, and hypotheses against existing data, and against other plausible interpretations of the data" (p. 294). In her examination of various definitions for reflective thinking, Larrivee discussed that this "cognitive problem solving" (p. 295) involves a *stance* (towards inquiry, open to investigation) and a *dance* (of experimentation and taking risks). She also described practitioners engaging in reflective thinking to have a reflexive loop that selects data, adds personal meaning, makes assumptions, draws conclusions, adopts beliefs, all informing actions, as well as challenging the assumptions and practices of the status quo.

Larrivee (2010) presented the three most commonly depicted levels of reflection from her extensive literature review. The initial level, which I will refer to as *surface level reflection*, focuses on "teaching functions, actions, or skills" (Larrivee, p. 139). The second level, referred to as *pedagogical reflection*, is more advanced because it considers theory and rationale for teaching practices. The third level, *critical reflection*, examines the "ethical, social, and political consequences of teaching" (Larrivee, p. 139). This higher order level of reflection examines both professional and personal beliefs, providing teachers a way to examine the full range of the consequences of their actions (Larrivee, 2010).

Many views on reflection focus on "rational" analysis of teacher behaviors (e.g., Kolb & Fry, 1975), where rational analyses are based "on logical and analytical ways of information processing during teaching" (Korthagen, 2001, p. 231), or left-brained thinking. However, more recent perspectives demonstrate that teachers can behave in ways that are less than rational (Korthagen & Vasalos, 2005). When nonrational processes such as emotions and feelings inform teacher decision-making, they cannot be analyzed and influenced as easily as their rational counterparts (Korthagen, 2001). This suggests that constructs are needed to understand these less rational aspects. Fred Korthagen and his colleagues have developed models and constructs to address these aspects based in positive psychology (Csikzentmihalyi, 1990), building up strengths rather than treating what may be "broken."

Korthagen (1985) described a structured process for reflecting and "learning from practice with the aid of five phases: (1) Action, (2) Looking back on that action, (3) Awareness of essential aspects, (4) Creating alternative methods of actions, and (5) Trial" (the *ALACT model*; Korthagen, 2014). The last phase of this structure overlaps with the first phase for a continued cycle of reflection. Phase 3, awareness of essential aspects, differentiates this

reflection cycle from a strictly *action*-oriented reflection to be a *meaning*-oriented reflection (Hoekstra, 2007) so the underlying processes can be understood. Transformational changes, changes in the underlying sources, including beliefs, for behavior (or second-order changes; Korthagen, 2014) can occur when deeper reflection takes place.

Although the ALACT model describes a process of reflection, it does not describe the *content* of reflection, or what deeper reflection would look like. Korthagen and Vasalos (2005) found that teachers commonly focus on the aspects of the environment, behaviors, competencies, and beliefs, and built these into their *onion model*. The onion model incorporates ways that teachers function into layers of an onion as concentric circles where "the inner levels determine the way an individual functions on the outer levels, but that there is also a reverse influence (from outside to inside)" (Korthagen & Vasalos, 2005, p. 53).

Korthagen (2014) defines the outermost concentric circle of environment to be everything a teacher encounters outside of themself such as the classroom, subject matter, and culture. The next level of the concentric circles, behavior, refers to the actions of the teacher, followed by the competencies, or what the teacher is able to do. The fourth layer, beliefs, "refer to assumptions about the outer world, which are often unconscious" (p. 8). Two additional levels were incorporated into the inner core of this model: *professional identity* as the "teachers' assumptions about themselves, their self-concepts and the professional roles they see for themselves" (p. 8), and *mission*, the layer about ideals, inspiration, meaning, and significance in the work of teaching. Rather than describing the *process* of reflection, the levels in the onion model provide a way to identify concrete *content* of a situation that gives rise to reflection because it remained on the teacher's mind, and subsequently, how it may have been reflected upon at the inner levels (Korthagen & Vasalos, 2005).

Korthagen and Vasalos (2005) define *core reflection* as reflection that takes place at the professional identity and/or mission levels (the core of the onion) and focuses on *core qualities*, or strengths, of individuals. A view of an effective teacher through the onion model is: "someone who is strong at aligning all the layers of the onion model, and who thus impacts their environment on the basis of a certain coherence between [their] core qualities, ideals, sense of identity, beliefs, competencies, behaviour, and the characteristics of the environment" (Korthagen, 2017, p. 397). The relations between the layers make the differences between teachers.

When factors such as behaviors, feelings, images, or beliefs that would limit accomplishing an ideal situation, create for the teacher a difference between their ideal situation and what they are able to accomplish, a *core discrepancy* or tension arises, which can help clarify the problem (Korthagen and Vasalos, 2005). "When, through *thinking*, a teacher arrives at the rational conclusion that a certain pattern of thinking and acting is counterproductive, and also gains an insight into more constructive possibilities, this may have some influence on the teacher's future behaviour" (Korthagen & Vasalos, p. 58). Core reflection takes place when "there is less emphasis on an extensive analysis of the problematic situation" (Korthagen & Vasalos, p. 55). This is because the more problem-solving, or quick fixes, the teacher executes, the narrower the set of actions from which to work (the teacher's habitus), leading to "tunnel thinking" and a "'reactive' way of dealing with reality" (Korthagen & Vasalos, 2009, p. 7). As the teacher steps back and considers limiting factors, they can then become aware of their choices and build their personal autonomy, based on their strengths (Korthagen & Vasalos, 2005).

This "stepping back" can be understood with the term *flow*, introduced by Csikszentmihalyi (1990). Flow can be defined as: "a state of being completely in the here-and-now, optimally connecting the demands of the situation with one's inner capacities" (Korthagen & Vasalos, 2009, p. 6). Rather than simply "downloading" the quick fix for that particular problem, a teacher's *presence* allows them to connect with their core potential. Korthagen and Vasalos (2009) refer to Senge et al.'s (2004) "Theory U" in their U-turn model between a person and a problem. A U-shape and flow are created (a longer, curved path representing deeper, more genuine considerations) as opposed to a straight line for downloading (rational, logical) solutions. Additionally, in their U-turn model, Korthagen and Vasalos (2009) incorporated the perceptual, rather than conceptual, awareness (mindfulness) in that reflection with an open mind, open heart, and open will (presence) can lead to deeper reflection. Instead of only *feeling* the feelings, reflection provides an avenue to also *think* about feelings (Damasio, 1999), to process the feelings by looking back and building awareness of where the feelings are originating in order to create and enact alternatives. Korthagen and Vasalos (2009) also cite research in neurobiology that has "yielded strong evidence for the close relations between cognition and emotion" (p. 7), pointing to a personal side of professional work.

Korthagen and Vasalos (2005) used concrete examples of a supervisory setting where the teacher may work through a reflection cycle with their supervisor. This supervision could take the form of a teacher educator-teacher relationship, an administrator-teacher configuration, or a peer-to-peer teacher team. The supervisor, in whatever role that takes shape, can use two tactics in working with a teacher to reflect. The first is that of *self-disclosure*, where the supervisor shares their own struggles or limitations, and models the core reflection process. The second intervention is *confrontation*, where the supervisor engages in sharing areas the teacher may

dispute, which can help the teacher build awareness of core discrepancies (Korthagen and Vasalos, 2005). Confrontations require empathy and the setting should remain supportive, but through these experiences, the teacher can more deeply reflect.

To summarize, providing opportunities and the ability to reflect on their practice, teachers can build their craft knowledge, which in turn will develop their instructional abilities. Reflection can occur through a cyclical process (the ALACT model), it can occur on various levels of content (the onion model), and it can occur at varying depths (Larrivee's surface, pedagogical, and critical levels). Reflecting on teaching is one component for understanding how teacher pedagogy can transition towards a reform orientation. This review will next turn to understanding the nature of such opportunities to build experiences and reflection for improving mathematics teaching practices.

A Mechanism for Shifting Teaching and Learning

To further understand the mechanisms at work in professional learning, building on Korthagen's (2017) models of reflection and Bourdieu's concept of habitus, I incorporate the construct of *internalization.* Vygotsky's (1978) view of internalization:

> Consists of a series of transformations: (a) An operation that initially represents an external activity is reconstructed and begins to occur internally; (b) An interpersonal process is transformed into an intrapersonal one; (c) The transformation of an interpersonal process into an intrapersonal one is the result of a long series of developmental events. … Their transfer inward is linked with changes in the laws governing their activity; they are incorporated into a new system with its own laws. (pp. 56-57)

As teachers work with others, social (interpersonal) interactions can influence them, which in turn are psychologically processed (intrapersonal) and integrated into their embodied history (habitus).

Milne et al. (2006) discussed how through *continuous* interactions, teachers' dispositions can be modified and reinforced, which has important implications for professional development in education. Teacher educators, and also researchers in mathematics teacher education, are limited in their understanding of how to improve teachers' practice to a level of high quality. In accordance with the concept of habitus, without following a premise of constant, continuous development, teachers will be left to their own devices, reverting back to what they are more familiar with. For example, teachers may attend a professional development session, decide to attempt a particular strategy in their classroom, struggle with its implementation, and subsequently remove it from their "toolkit." By adopting a perspective of slow yet constant growth, the education community can begin to restructure teachers' professional development sessions towards a more effective means.

Through their work, Milne et al. (2006) described how teachers can build a capacity for agency (an ability to act). By forming intentions, acting creatively, and setting goals, teachers can move beyond their current ability and capacity for effecting change in their classrooms. By acting purposefully, they can more successfully work with their students. The authors also developed links between the previously mentioned components: professional education can lead to new cultural schema, which can then mobilize resources in new ways, and therefore enact specific schema *in practice*. Although change is variable and dependent on social actions, improvement is possible because teacher educators and researchers can be explicit about what schema are associated with particular resources.

A sociocultural perspective allowed Milne et al. (2006) to take a holistic approach towards what teachers are doing in their classroom and to identify additional necessary components for change. As teachers change their practices, it is hoped that the change is ultimately for the better, that teachers adapt for their students' needs, taking the community of the classroom into consideration, and building skills and knowledge toward understanding of the subject area.

Evidence of teachers and professional developers engaging in learning together can be seen as one benefit of their participation in the Cognitively Guided Instruction professional development program (Carpenter et al., 2000). Teachers' interactions with one another were found to be generative for growth in their practice (Franke & Kazemi, 2001). As teachers participated in their work group communities, their roles transformed and teachers crafted new identities in their classrooms, thereby shifting their practice and improving conditions for student learning. Purposeful engagement in collaborative environments in local contexts provides opportunities for the ongoing disruption, re-evaluation, and alteration of teachers' identities, and therefore can lead to reflective actions that result in meaningful growth for teachers' practices.

Teacher Learning through Coteaching

One way that professional development can regularly and naturally occur *in* the classroom is through *coteaching*. Coteaching can be defined as "a practical situation in which two or more teachers work together in the same classroom at the same time, thereby changing the teacher-student ratios in significant ways" (Roth, 2002, p. 111). Co-teaching (with the hyphen) is typically referred to as a partnership between a general education teacher and a special education teacher, or between teachers of two different subjects, creating a multidisciplinary partnership (Sileo, 2011; Sileo & van Garderen, 2010). The use of the term

with a hyphen (co-teaching) is often applied in inclusive special-general education class situations, whereas coteaching without a hyphen has been established as a "commitment to coplanning, copractice, and coreflection" (Murphy & Martin, 2015, p. 277). Murphy and Martin (2015) summarized that the term coteaching holds special significance and this particular use has an aim that "is threefold: to reduce the theory-practice gap in teacher education, to improve reflective practice in the classroom, and to develop further teachers' pedagogical content knowledge" (p. 277). Aligned with Murphy and Martin (2015), I use the term *coteaching* (purposefully no hyphen) to refer to two teachers of the same subject area as a collaborative, complementary team, through which each teacher has the opportunity to learn from the other both explicitly and implicitly during planning, enactment, and reflective stages of teaching. Throughout this study, two teachers who work together will be considered *co-teachers*, until they are established as a collaborative, complementary team, at which point they will be considered *coteachers*.

> In general, for any co-teaching situation, the basic structures can include:
> (1) one teacher instructing, the other observing; (2) having stations around the class; (3) parallel teaching, in which the teachers would divide the class and teach two groups of students; (4) team teaching in which one teacher assumed responsibility for a section of the curriculum and did the classroom instruction and assessment; (5) alternate teaching, in which teachers assumed single responsibility for instruction on a particular topic within a lesson; (6) one teacher assuming teaching responsibility and the other monitoring students, helping where needed; and (7) complementary teaching. (Murphy & Scantlebury, 2010, p. 1)

The term *team teaching* also has been used to describe when "the teachers share equally in planning and delivering all components of academic instruction" (Sileo & van Garderen, p. 16). However, working as a team can refer to each bringing one's own individual specialization, and professional "learning may actually be impeded in this situation because teachers can rely on working in their specialty areas and are less likely to gain experience in any others" (Roth, 2002, p. 111).

These various structures of co-teaching can be useful in classrooms for students of diverse populations but is not the concern of this current study. Instead, for the goal of professional learning, two teachers of the same content area can work together, thereby working with the same intentions, with all of the students in their classes. A coteaching context in a high school mathematics classroom provides the setting of this study in which the development of instructional practices for creating such opportunities for students to gain mathematical power and proficiency is researched. In the remaining sections of this review, I describe prior research on coteaching in classrooms and a theoretical framework for the study of coteaching.

Becoming in the Classroom Through Coteaching

As Roth (2002) explained, there is a temporality of teaching: decisions are made according to "doing the right thing [for that teacher] at the right time in the here and now" (p. 22). Due to this temporality, without another teacher present, it is challenging to reflect upon one's own practice. Just as in an apprenticeship model (e.g., a co-pilot learns from a pilot), two teachers interacting with students in the same setting provides an "immediacy of the relationship between thought and action to redirect a problematic situation" (Roth, 2002, pp. 59-60). Just as in Schön's (1983) reflection-in-action, which accounts for learning by doing a job, Roth (2002) describes coteaching as *colearning*, provided there is a "common ground that serve[s] as a

communicative basis [that is] so important for mutual understanding" (p. 108). Instead of the traditional fashion of teachers acting and reflecting in professional isolation, coteachers can introduce alternative approaches and views of the classroom, in turn altering each teacher's perspective of the classroom and the act of teaching. Additionally, the microprocesses that take place in a classroom (i.e., norms around use of social, material, physical resources) can be adapted with the coteacher's influence or reflected upon for future reference and alteration (Roth et al., 2005).

With a coteacher present to explicitly recognize issues at hand and provide an alternative, the practitioner may find ways to reconsider their habitus. "*Being-in* situations and *being-with* others … is central to apprenticeship" (Roth & Tobin, 2002, p. 11) because it forms a coparticipation in situations and familiarity of the existing teaching habitus. This "*being-in/with* is the central underpinning of the 'co' in *co*teaching" (Roth & Tobin, p. 11) and is necessary for instantiating change in teachers' dispositions.

Furthermore, the benefits of coteaching are not only for professional learning, but also for students and their achievement. Two teachers working together provide a greater set of possibilities for action as a collective (Roth, 2002, p. 130), meaning that each teacher has their habitus to draw on and the teachers collaborate to create an optimal situation for student learning. There have been studies demonstrating students' increased interest and enjoyment of subjects (e.g., Murphy & Beggs, 2010) and achievement in coteaching situations. "Coteaching capitalizes on the strengths of both teachers, positions both as learners and … offers an enhanced science learning experience for pupils and teachers" (Conaill, 2010, p. 188). With two teachers in one class there are opportunities for students to work with more than one adult resulting in a variety

of teaching styles, reduced behavioral disruptions, and students feeling more connected to school (Bacharach et al., 2010).

Co-teaching is a practice that currently exists in many K-12 classrooms in the United States in a variety of forms. This study takes a closer look at one partnership in an effort to understand how the planning, enactment, and reflection on high school mathematics lessons in a coteaching situation provided opportunities for the teachers to learn from each other and through teaching. In the following section, particular aspects of coteaching are identified to inform how the partnership of this study is analyzed.

Research on Coteaching in Practice

Based on Roth and Tobin's (2002) work in *At the Elbow of Another* and their extensive research on coteaching in high school science classes (Bacharach et al., 2010), many science education researchers have further developed the use of coteaching for preservice teachers (e.g., Gallo-Fox & Scantlebury, 2015; Milne et al., 2011), and in recent years the practice has spread to more areas such as primary school (e.g., Carlisle, 2010) and language arts (e.g., Guise et al., 2017). Moreover, research on the development and benefits of coteaching partnerships primarily takes place with preservice and cooperating teacher partnerships, or preservice teachers and teacher educators (Murphy & Scantlebury, 2010). Alternatively, this study focuses on coteaching in secondary mathematics between two in-service, experienced teachers with the goal of shifting practice towards more reform-oriented teaching. The section that follows analyzes the literature to date regarding components of coteaching in order to frame the current study.

Practice Made Public Through Collaborative Teaching

Traditional norms of practice can be viewed as conservative pedagogies (Gallo-Fox, 2010) as teachers operate in isolation and resort to their habitus. Through coteaching, typically

private thoughts and decisions are made public. In Gallo-Fox's (2010) study, teaching interns shared ideas and participated in teaching and learning in front of one another, moving them to teach in different ways, outside of their habitus. Coteaching facilitates flexibility, spontaneity, and deviation from lesson plans in teaching (Conaill, 2010), providing a platform on which to break away from conservative pedagogies. Furthermore, there is increased "ownership of the teaching process" (Conaill, p. 189), which "enables risk-taking, responsive teaching beneficial to all as teachers and learners" (Conaill, p. 189).

Of course, experiences that break away from the norm are not easy to participate in and require that teachers experience exposure and vulnerability, resulting in *risk-taking* in their teaching practice. Risk is shaped by social interactions, organizational processes, and values, which influence the collective behavior of a group (Ponticell, 2003). Through this risk-taking, within a supportive environment, there is opportunity for change, which can "work in opposition to the traditional structures of isolation" (Gallo-Fox, 2010, p. 122). Coteaching can provide a platform for accessing, opening, and altering the "soul" of a teacher's habitus through the challenge of taking risks in practice.

Reflection via Cogenerative Dialoguing

Crow and Smith (2005) investigated reflective conversations in coteaching partnerships and found that the "joint reflections on shared experiences … add another dimension to the reflection. This provides the opportunity for the deconstruction of those experiences and the reconstruction of a shared meaning in a way that transforms understandings and changes practice" (p. 491). Through such reflections, analysis of "one's implicit values and beliefs about teaching and learning" (Yoo et al., 2019, p. 74), and therefore shifts in practice, can occur.

In much of the coteaching literature, reflective discussions among all participants of the classroom praxis are referred to as *Cogenerative Dialogues* (Roth et al., 2002): "The intent of these sessions is to use current understandings to describe what has happened, identify problems, articulate problems in terms of contradictions (generalization), and frame options that provide us new and increased choices for enacting teaching and learning" (p. 9). The classroom is taken as an object of inquiry, where "participants operate out of their own initial frames using their primary discourses … [and] in so doing, they communicate at a level at which frames can be analysed and altered, and new frames can be generated … [forming] 'locally relevant theory'" (Roth et al., 2002, p. 11). By embracing such a "critical awareness of challenges and a capacity for critical reflection" (Conaill, 2010, p. 183), coteachers can transcend the typical, surface-level concerns of practice towards a reconceptualized version of teaching where one continually learns through praxis.

As coteachers of separate habitus work together and as theory and practice approach each other, contradictions are likely to occur. However, "contradictions should not be viewed negatively. They should be considered as a driving force for change and development because they lead to the 'articulation of actions of change' (Roth, 2002, p. 9)" (Carlisle, 2010, p. 129). Contradictions pinpoint tensions that teachers will likely work to alleviate. Through reflective conversations, cogenerative dialogues, and contradictions, the values and subsequent actions of each teacher can be witnessed in order to characterize changes in practice and professional growth.

Partnerships

The above components of making practice public and cogenerative dialoguing cannot occur unless the partnership is built on mutual *co-respect*: "When teachers lose respect for each

other, co-planning and a shared responsibility for enacting the curriculum decline" (Juck et al., 2010, p. 243). Coteaching is in a sense a form of informal peer mentoring where the investment made by each teacher can payoff as professional growth. Researchers have learned that in order to create "harmonious" (Murphy & Scantlebury, 2010, p. 20) coteaching relationships, there are steps that can be taken in order to strategically partner two teachers together based on their needs. There are also aspects of the dynamic relationship to consider as teachers begin to work with one another. "Co-teaching is analogous to a professional marriage in which teaching partners collaborate to provide instructional services to students" (Sileo, 2011, p. 32). In order for the collaboration to be successful, much like a marriage, a strong relationship has to be built using clear communication and problem-solving skills. Most partnerships will not automatically allow for high levels of collaboration, teachers must continually work with one another to foster such a relationship.

Recent work has gone as far as assessing coteaching partnerships according to their degree of fluency. Carambo and Blasie (2010) adapted Roth and Tobin's (2002) initial list to include the following measures of fluent coteaching: "Creating and using space; creating and using resources; anticipating unspoken needs; seamlessness of interactions; reciprocal coparticipation (playing off one another); complementarity of actions; respect; rapport" (pp. 202-203). The components itemized here can provide a lens for making sense of the interactions between coteachers and the ways in which they can support each other in their collaborative practice.

Coteachers as Equals. Coteaching partnerships exist in such a way that one teacher does not have an upper hand. With true collaboration and co-respect through communication, power is equally distributed, negotiation takes place, and each teacher has an equal voice in decision-

making, thereby increasing teacher confidence in their abilities and decisions (Murphy & Scantlebury, 2010). “Coteaching partnership, which casts both as learners, albeit at different levels, encourages this reconceptualization towards shared inquiry and recognizes the potential contribution of each coteacher” (Conaill, 2010, p. 174). Without a *shared inquiry*, levels of teacher agency would be out of synch, restricting growth in confidence and performance (Murphy & Scantlebury, 2010).

Teacher Orientation. Similar to a growth mindset perspective for students (Boaler, 2016), teachers also have to hold a growth mindset of lifelong learning as a professional in their practice. With such a mindset, professionals would be more likely to take risks to develop their practice and not be paralyzed with concern about making mistakes. Particular to coteaching situations, attempting different instructional practices or use of materials requires an openness and willingness to take risks. This study includes the tracing of both the willingness and the negotiations that exist in the coteaching partnership, and how these actions contribute to the collaboration among the two teachers towards a more developed practice.

From the coteaching research literature, it has been established that student teachers learn more about teaching practices through coteaching than their non-coteaching counterparts. Cooperating teachers can also learn from their student teaching partners in coteaching situations because they bring new ideas from their coursework and remind cooperating teachers of the theory that drives teacher education. In a similar way, two experienced teachers with different histories who come together in a collaborative coteaching situation can provide knowledge and resources to each other: teaching skills can be gained through experiencing the coteacher’s habitus in action. By being shown, rather than told, how to adjust practice or incorporate a new teaching strategy, and by engaging in a shared inquiry, a new or developed habitus can be

formed for each teacher through this coparticipation (Yoo et al., 2019). In such partnerships the two teachers must continually negotiate their copractice throughout the stages of coteaching. This negotiation can provide insight into each teacher's habitus and the emerging collective habitus that is the source of action for enacting a lesson in a cotaught class.

Zones of Proximal Development (ZPD) in Coteaching

As one coteacher experiences the habitus of the other and participates collaboratively in the planning and enactment of lessons and reflection on the lessons, it can be said that "a zone of proximal development is created between the actions of individuals and the new form of collectively generated activity" (Roth, 2002, p. 130). In relation to ZPD for teacher learning (as opposed to the common application for student learning), "it is a two-way learning process with all participants learning through their interactions with each other" (Murphy et al., 2015, p. 285). No matter the professional status of each teacher, partners in a coteaching relationship can come together to create a mutually shared practice. "As a ZPD, coteaching emphasizes the collective and socially mediated nature of teaching and learning and enables (new) collective actions to become part of the ongoing praxis of teaching and part of the repertoire of the individuals' action" (Murphy et al., 2015, p. 285). When a practice is taken up by an individual teacher, it has been taken from the social interactions of the classroom and *reified* into their own habitus. Reification, the use of something that has been learned, is evidence of professional growth, which can occur as a result of coteaching partnerships in a slow-but-meaningful change process.

Many of the science education coteaching researchers call upon Sewell's (1992) dialectic of structure|agency (e.g., Scantlebury et al., 2008; Carambo & Stickney, 2009), which provides a conception of an individual's agency "in relation to their knowledge of their surrounding cultural structures" (Juck et al., 2010, p. 243). The cultural structures consist of both *schema*, an

individual's knowledge of cultural structures, and *resources*, both human and material. As the resources interact with one's schema, the individual's agency is impacted. Through this agency, it seems that reification may occur, and the individual's habitus may be altered. This theoretical stance specifies the ideas of craft knowledge in that the interaction of schema and resources informs action and possible subsequent reification, therefore creating new knowledge while in practice.

In summary of these terms, a theory of how teachers can learn through coteaching is presented. Two teachers who coteach serve as resources for each other in practice, among other resources such as standards, reform goals, or curricula, which interact with each teacher's individual schema. The teachers each relate their schema as "socially held norms" (Juck et al., 2010) to these resources and generate individual actions in response. Collectively, these actions come together to develop a teaching practice, which affects a newly acquired combined habitus that would not have existed without the resources and interactions of these individuals. These can be represented as the ZPD for each teacher's learning. I view this process through a situated, coupled, back and forth action between the teachers that can continually advance their practice.

Building a ZPD Framework for Coteaching

Murphy et al. (2015) developed a framework for ZPD for coteaching by integrating six elements related to coteaching, only some from Vygotsky, with two elements for each phase of coteaching, presented in Table 1.

In coplanning, theoretical thinking can shape an *ideal* plan (Shepel, 1999). However, in reality, the ideal plan cannot possibly be fully achieved due to the nature of human activity. An ideal can provide the actors with motive and focus, which interacts with the *real* form and can lead to development (Veresov, 2010). "Coteaching provides opportunities for coteachers to

theorise and test theory, with the result the new, *local* theory can be produced" (Murphy et al., 2015, p. 287). Teachers' knowledge of the latest theories coupled with contextual knowledge allows for their own practice-based understandings to develop. Also, in coplanning, *buds* or flowers (Vygotsky, 1978) from shared ideas serve as starting points that can be developed in order to learn. With proper co-respect in partnerships, there is "an appreciation of starting buds and an acceptance of ideal models or goals for practice" (Murphy et al., 2015, p. 287).

Table 1

Phases and Elements of Coteaching

Coteaching Phase	Coteaching Element
Coplanning	• Interaction between real and ideal forms • Buds of development
Copracticing	• Vygotskian imitation • Unity of affect and intellect
Coevaluation	• Regression/recursion • Structured reflection

Note. Table adapted from Murphy, Scantlebury, and Milne (2015)

During the coenactment of a lesson, coteachers may *emulate* the other, signifying a maturing function (Chaiklin, 2003), being like the other. "During coteaching, Vygotskian *imitation* can be enacted as one coteacher emulates practice of the other that is nearer to the ideal, thereby expanding his or her agency in relation to creating his or her new practice" (Murphy et al., 2015, p. 287). Additionally, during coenactment, "emotional engagement is required for a learner to maintain attention" (Murphy et al., p. 287). Both negative and positive emotions "can be harnessed to develop better understanding of each other's needs as coteachers, so that, in turn, the collaboration can generate conditions that engage the emotions of their students" (Murphy et al., p. 288).

For the final stage of coevaluation, Murphy et al. (2015) acknowledged that deep learning and real growth occur through both *regression* and *recursion*. The authors called upon Zebroski's (1994) imagery of a tidal wave, which represents development as both progressive and regressive, and Tharp and Gallimore's (1988) recursive loop where learners revert to earlier stages to then progress through and relearn, similar to the work on habitus discussed earlier. In structured coreflection, coteachers can "explore on how their practice was progressing towards the 'ideal'" (Murphy et al., p. 288). In contrast from trial-and-error methods (tinkering), Murphy et al. discussed Dewey's view on structured reflection: a "careful survey … defines and clarifies the problem, and … elaboration of a tentative 'solution' to make it more precise and more consistent" (p. 289). In their work, Murphy et al. adapted tools for teacher reflection based in Shepel's (1999) and Larrivee's (2008) work.

Murphy et al. (2015) concluded from their study of ten preservice teachers partnered with ten teachers that only three of these elements were present and useful for explaining teacher learning through coteaching, one in each phase of coteaching: real versus ideal forms (coplanning); Vygotskian imitation (copractice); and structured reflection (coevaluation). Although all six elements may not have been present in Murphy et al.'s study, they each inform this research study.

Although Murphy et al. (2015) used the terms coplanning, copracticing, and coevaluation to group their theoretical coteaching elements for the phases of coteaching, I will instead use the terms *coplanning*, *coenacting*, and *coreflecting*. Rather than a copractice, which to me encompasses the entirety of the coteachers' teaching practice, I specify this phase to be the *coenactment* of lessons that were coplanned by the coteachers. Additionally, I use the term *coreflecting* rather than coevaluation to represent the continual evaluation of data (Larrivee,

2010) that the coteachers obtain in their practice during coplanning and coenactment of lessons. Coteachers may have regular reflective conversations in which they examine their teaching actions (surface level), teaching practices according to theory and rationales (pedagogical reflection), or the consequences of their actions (critical reflection). These three levels of reflection, along with the phases of the ALACT model and the levels of the onion model, can inform coteachers' learning through coteaching.

Small-Group Development

Although the prior research on coteaching discussed aspects of coteaching that may be present, there has not been a specified path or stages of development for working in collaborative partnerships. In his review of literature and proposal of a model for small-group development, Tuckman (1965) focused on the process of change over time for groups. He differentiated between the *group structure* or the "pattern of interpersonal relationships" (Tuckman, p. 69), and the *task activity* or the "content of interaction" (p. 69), which occur simultaneously in group functioning. Tuckman's general stages of *forming, storming, norming,* and *performing* capture how groups move through orientation, hostility, cohesion, and functioning, respectively, where there may be more stages depending on the setting of the group. The following summary of Tuckman's model provides context through which small-group development can be understood.

While forming, group members are orienting themselves through testing and exploration to determine boundaries. The members are acclimating themselves and there may be "open exchange of relevant interpretations" (Tuckman, 1965, p. 70). In the storming stage, an emotional response is involved, possibly including "anxiety, threat, and resistance" (p. 74). Conflict and polarization between group members can exist regarding particular issues, which can "boil down to the conflict over progression into the 'unknown' of interpersonal relations or

regression to the security of earlier dependence" (p. 69). While norming, a sensitivity develops and the group overcomes resistance, allowing for personal opinions to be expressed about the task activities. More specifically, the group now demonstrates "growth of an interlocking network of friendship; role interdependence; mutual involvement and identification between members with a concomitant increase in harmony and solidarity; and the establishment of group norms" (p. 76). These first three stages describe how a group may develop into a functional team.

Once a group has reached the performing stage, members have gained insight regarding the interpersonal processes, which allows for support among members and mutual acceptance, and as a result, productive collaboration regarding tasks. The "interpersonal structure becomes the tool of task activities. Roles become flexible and functional, and group energy is channeled into the task. Structural issues have been resolved, and structure can now become supportive of task performance" (Tuckman, 1965, p. 78), with a "high degree of work in the absence of affect" (p. 75). This final stage involves "consensual validation in which group interpersonal problems are solved and the group is freed to function as a problem-solving instrument" (p. 75) because they have internalized and generalized their learnings and can now apply them to other situations. After a group becomes cohesive, the "structure is institutionalized by the group and thus becomes rigid … [which] would most apply to groups with a long or indefinite group life" (p. 75). After a group reaches a performing stage, they are able to continue to function in productive ways indefinitely.

The stages of forming, storming, norming, and performing describe the process of change over time for small groups. As coteachers form a small group, their development while coplanning, coenacting, and coreflecting on lessons can be described using these stages.

Theoretical Framework and Research Question

This literature review has served to connect mathematics education research on knowledge and beliefs to the study of craft knowledge of practitioners, to legitimize the work of developing professionals while they teach, to justify and detail the components involved in coteaching, and to begin to develop a sociocultural framework that describes the ways teachers learn through coplanning, coenacting, and coreflecting on lessons.

The coteaching structure can serve as a professional development model where coteachers can work together towards reform-oriented, student-centered teaching with activities that are designed for students to make meaning of mathematics. With the NCTM (2000) *Process Standards*, the NRC (2001) mathematical proficiencies, and the NGA (2010) *Mathematical Practices* in mind, coteachers can transition their pedagogical strategies by reflecting on and in their practice. Korthagen and Vasalos's (2005) *ALACT model phases* for the process of reflection and *onion model levels* for the content of reflection can characterize how the act of coreflection can promote critical reflection at a teacher's core and such a transition in practice.

Additionally, the human and material resources (Juck et al., 2010) at hand can interact with each teacher's schema (knowledge of cultural structures, Juck et al., 2010), building their agency to act within the context. In other words, as each teacher acts as a resource, or brings resources, to their coteacher, the teachers' ranges of abilities to act are altered, creating zones of proximal development for the teachers. From these zones and through critical reflection, each coteacher can take up evolved knowledge as well as behaviors, competencies, beliefs, identities, and missions (Korthagen & Vasalos, 2005) for teaching.

The professional learning within an ongoing, sustained, subject-based coteaching context is situated in the teachers' daily work and provides each coteacher with feedback, and therefore

is authentic and can have lasting effects. The result of the professional development experience is possible genuine and substantive change in each teacher's habitus. Tuckman's (1965) small group development stages of forming, storming, norming, and performing can provide a lens to understand how the coteaching partnership may develop.

As a result of the need for mathematics education to transition classroom pedagogy and the movement in research on coteaching as a structure for expanding teacher agency, this study incorporates these arenas to explore the following research question: *How can secondary mathematics teachers in a coteaching partnership serve as resources for each other's professional growth towards reform-oriented pedagogy?* In particular, this study looks closely at how one coteaching partnership developed over the course of a school year through their communications about and reflections on their teaching practice and instantiation of coplanned lessons.

Chapter 3: Methodology

The driving question of this study is whether and how reform-oriented instruction can be instantiated in an authentic classroom where students can make meaning of mathematical situations and engage in mathematical thinking. Coteaching as a professional development structure provides one way for teachers to work together in a situated environment towards such an instantiation. The following sections describe how this study was designed to address the research question: *How can secondary mathematics teachers in a coteaching partnership serve as resources for each other's professional growth towards reform-oriented pedagogy?*

Research Design

A year-long high school mathematics co-teaching partnership was designated to the researcher in her full-time teaching position. The resulting teaching structure proved to be a "harmonious" (Murphy & Scantlebury, 2010, p. 20) and productive coteaching relationship that continued beyond its initial year. This naturalistic inquiry (Patton, 2002) studies the initial year of this coteaching partnership by analyzing the ways the teachers served as resources for each other for their shared practice to develop. Within a naturalistic study, there is an "*openness* to whatever emerges" (Patton, p. 46): the shared teaching practice was able to develop within its natural context, without any constraints placed on it by the research process. The authenticity of this project is discussed further.

With the goal of understanding the ways in which a coteaching partnership can develop, this study uses a single-case study methodology (Yin, 2018). The single coteaching partnership serves as an *elaborated* and *holistic* case study that aims to "find out what happened, why, and what it means more broadly" (Rubin & Rubin, 2005, p. 6). The case study is "an empirical method that investigates a contemporary phenomenon in depth and within its real-world context,

especially when the boundaries between phenomenon and context may not be clearly evident" (Yin, p. 15). This design allows the conditions involved to rise to the surface so that their boundaries can be specified and connected to the relevant theory.

The design of this project is grounded in studying a genuine classroom situation in which two mathematics teachers are part of a coteaching partnership, enacting lessons within a classroom, which exists within an institution abiding by state level public education policies and standards. This hierarchical structure of the units of analysis, by definition, creates an *embedded* case study design: the coteaching partnership does not exist independently, but in relation to the population of students within the classrooms, and the building-based, district, and state expectations. By looking closely at the particular ways that the teachers in a coteaching partnership negotiate and enact their practice, this study captures the realistic possibilities and constraints of mathematics teaching as related to accomplishing progressive standards.

Setting

The setting of this study is a public high school located in the northeastern United States serving approximately 2,000 students in grades nine through twelve. The school's district is a suburb of a large urban city and has a diverse population. The approximate demographic breakdown of the town's population and the public high school student body can be seen in Table 2. The substantial difference in the population of White students in the town versus the high school can be attributed to the number of private, religious schools in the area.

A difference in achievement between racial subgroups has been prevalent at this high school. For example, during the 2019-2020 school year, as can be seen in Table 2, a disproportionate number of students for each racial subgroup are enrolled in honors level mathematics classes. Additionally, a subgroup of about 80 predominantly non-White students,

out of approximately 500 students in each grade level, struggle to meet the state graduation requirements on standardized tests and have to appeal through a portfolio creation process each year. Initiatives to address mathematics underperformance over the past fifteen years include creating and implementing an integrated Algebra 1 and Geometry three-year course, a learning strategies course for study skills across all subjects, and Saturday morning "boot camps" to prepare students for standardized tests. As the high school restructured their three-tier track of courses (Basic, College Prep, Honors) down to two (College Prep and Honors, along with a new honors cohort-based program), the administration recognized that some students would require supports, particularly at the ninth-grade level as students were entering the high school and taking Algebra 1. In this school district, students could be recommended to take Algebra 1 in Grades 7 or 8, essentially directing them into an honors track, while all other students take Math 7 and Math 8, and are subsequently enrolled in Algebra 1 in their freshman year.

Table 2

Demographic Data for the Context of this Study for School Year 2019-2020

Subgroup	Town	High School	Honors Mathematics Classes
Black	27%	48%	23%
White	43%	19%	42%
Hispanic	20%	27%	11%
Asian	8%	6%	22%
Multi/Other	2%	<1%	2%

As a result, co-teaching (with a hyphen) between two subject area teachers began in 2015 as a district initiative at the high school level to support underachieving students in both Algebra 1 and English 9. Not all students enrolled in Algebra 1 in grade 9 are placed in a co-taught

section, only those who are identified to need extra support. Students are placed according to their previous mathematics teacher or school counselor recommendation, mathematics course grades, and standardized mathematics test scores. The subpopulation of students identified as needing extra support is aligned with the group mentioned above that appeals for graduation requirements in their senior year through portfolio creation since they struggle to meet conventional state-determined criteria. Special education students are not enrolled in these particular co-taught sections because they are legally required to be in a section with a special education teacher. The school district provides professional development on the types of co-teaching (e.g., parallel or team teaching) for newly assigned teachers scheduled to co-teach. Within the mathematics department of twenty mathematics teachers, about one-third have previously been involved in co-teaching with one another. When the initiative began, there were eight sections taught by four co-teaching partnerships. In recent years, to directly address the struggling subpopulation, the number of co-taught Algebra 1 sections was reduced to four, capped at 24 students in each section with two pairs of mathematics teachers each teaching two sections.

Participants

The two teachers, Kadyn (a pseudonym, meaning “companion”) and Jessica (the researcher), who form the partnership for this case study were assigned to co-teach together for the first time in the 2018-2019 school year. Kadyn, who began her teaching career at this high school in 2007, had taught Algebra 1, Algebra 2, Precalculus, and AP Calculus individually and had co-taught Algebra 1 with another mathematics teacher, Matthew (a pseudonym), for two class sections during the 2015-2016 school year. After Kadyn went on maternity leave, Jessica took over her position as co-teacher with Matthew for two class sections for two school years

(2016-2018). As Kadyn was planning to return from maternity leave in 2018, the mathematics department supervisor asked Jessica and Kadyn if they would be willing to co-teach together for all four Algebra 1 co-taught sections during the following school year, to which they agreed: the teaching of these four sections, which became a coteaching partnership (no hyphen) between Kadyn and Jessica, served as the context for this research study.

Kadyn, the primary participant of this research study, was provided with a consent form approved through the Institutional Review Board (IRB) informing her of the various aspects of the study that would impact her, such as being audio and video recorded. She not only signed the informed consent form, but also provided the IRB with a letter in her own words stating that she was "excited about the opportunity" and that she did not "anticipate doing anything extra or different" due to the research project, only what would occur anyway in her day-to-day teaching.

Without our 77 individually unique and wonderful students of the 2018-2019 school year, Kadyn and I would not have been challenged to strive to make our instruction the best it possibly could be. Although the students' learning and actions in the classroom are not the focus of this study, they nevertheless influenced our teaching and are important to understanding how we responded and adapted our lessons accordingly. Since audio recordings of every lesson, and occasional video recordings, for the four co-taught Algebra 1 sections were part of the data set, students' parent/guardians were provided with informed consent and students were provided with informed assent prior to the study. Not all students and guardians in the four sections provided signed assent and consent, as is typical in a real-world research setting. Although the use of the audio recordings has largely been limited to the discussion between Jessica and Kadyn and the teacher voices in the lessons, all possible steps were taken to respect student anonymity in this study.

Researcher's Positionality

Jessica, the second co-teacher, is the author of this study. I, Jessica, take on the role as a participant-researcher, in the data, I am genuinely a practicing co-teacher. I began my teaching career in the same school district as Kadyn one year prior to her (2006). I taught for three years before leaving my position for graduate school full-time for four years, after which I returned to the same high school, and have been teaching full-time in the courses of Algebra 1, Geometry, Precalculus, and Calculus. My historical investment and involvement in this high school demonstrates that my position can be viewed as a teacher, not only a researcher, and also provides me with a wealth of institutional knowledge from which to work.

In addition to teaching mathematics full-time at the high school level, I have also pursued doctoral studies in mathematics education, providing me with knowledge and resources regarding what the field knows about teaching and learning. I therefore hold a perspective that is atypical of most secondary mathematics teachers, allowing me to serve as a resource in this partnership for my coteacher, Kadyn, who was open to learning with me.

As a doctoral student, I have become aware of the alternate possibilities for the mathematics education classroom: student-centered rather than teacher-centered learning situations; conceptually-based activities as precursors to procedurally-based work; reasoning and justifying for meaning making rather than relying on teacher-provided explanations. However, as described in the Personal Significance section at the end of Chapter 1, in my own classroom, I found it nearly impossible to accomplish this type of teaching and learning having no supports, other than my own knowledge, while existing within a traditionally-oriented department and institution that prioritizes "getting through the curriculum" and preparing for tests. Armed with the vision for an ideal mathematics classroom and practicing within the constraints of my full-

time job created an internal tension for me, from which this study stems. Without the support of my doctoral adviser and this opportunity for professional growth through partnership, this "knot" in my teaching practice could have steered me down a very different path, rather than through this enlightening and positive experience.

By partnering with Kadyn and purposefully working to integrate my knowledge of student-centered, conceptual and reasoning-based pedagogy into our everyday practice, both our shared and our individual teaching practices have developed as a result of the relationship that Kadyn and I have fostered. The work that Kadyn and I accomplished during the 2018-2019 school year was entirely collaborative and ended up being a reflective experience for me, from which I know that I have been able to transgress some of my own professional constraints. By having daily support from Kadyn, I gained the ability to begin to bring the ideal and reality closer together by making sense of the ideal in our classroom. This study aims to capture the ways in which this connection of ideals to reality has been possible and the themes of how and in what ways we have professionally grown. The ideals I came with to the coteaching partnership were reform-oriented and so the work of this established coteaching partnership informs the research question of how secondary mathematics teachers can act as resources for each other's professional growth towards reform-oriented pedagogy.

Data Sources

To understand the ways in which this embedded, holistic, single-case study can answer the proposed research question, data was collected in multiple ways to provide different sources that can come together to characterize the development of the coteaching partnership. Multiple sources of data will converge to allow for triangulation (Yin, 2018) of the developed theory during the data analysis. The data was collected over the course of the entire 2018-2019 school

year (September through June), providing a prolonged engagement between the researcher (as a coteaching participant) and the coteacher.

Coplanning/Coreflecting Meetings

The primary source of data for analysis is the audio recordings of the one-on-one meetings between the coteachers. The nature of these meetings was for the teachers to coplan their Algebra 1 lessons together and to also coreflect on lessons that had been coenacted. Within the school day, there was common planning time for the teachers to meet on a regular basis, as well as time during lunch and after school. An audio recorder was used to capture every meeting throughout the school year. These meetings naturally involved both coreflection on already enacted lessons and coplanning for subsequent lessons, and so will hereafter be referred to as *coplanning/coreflecting meetings*.

The content of the coteachers' meetings reflects genuine coplanning and coreflection of the teachers in their natural setting. However, Jessica wore two hats: she not only participated in the meetings as a coteacher, but also as a participant-researcher. As a result, some of the meetings served as *responsive interviews* in which Jessica asked Kadyn deeper questions to further unpack her thoughts and intentions. Throughout these meetings, over the course of the year, Kadyn became a *conversational partner* allowing for a deeper exploration of the work they were doing. Not only was Jessica asking Kadyn probing questions about her view of their teaching practice, but Kadyn was also directing the conversation according to what she valued. This mutual influence on the conversation established a foundation of trust where both Jessica and Kadyn could question their shared practice in a dynamic, flexible, contextualized, and iterative way. Jessica was able to push Kadyn for further understanding, which Kadyn responded to by pushing back with her own perspective and questions, developing a collaborative inquiry of

their shared teaching practice. Both Jessica and Kadyn have benefitted from their individual attempts to make sense of the other's perspective and from this shared reflection on practice.

At the end of the 2018-2019 school year, I asked Kadyn to sit with me for an "End of the Year Interview," which was more of a conversational interview than a coplanning/coreflecting meeting. I prepared some general questions for this scheduled meeting (see Appendix A) that took place after school on June 21, 2019.

Coenacted Classroom Lessons

In alignment with the coplanning/coreflecting meetings for the lessons are audio recordings of the actual lessons enacted in the classroom. The teachers cotaught each lesson to four different sections of Algebra 1 on a rotating-drop schedule (this is detailed in Chapter 4: Results). Each class period lasted 53 minutes and classes met either three or four times a week, depending on the drop schedule. Two of the sections took place in the morning and two in the afternoon, always with a period off in between. The classes included a roster of 14 to 21 students each of the subpopulation identified above with a total of 77 students across the four sections.

The coenacted lessons became the material on which coreflection took place, as well as subsequent coplanning. Often, Kadyn and Jessica would meet in between the enactment of the four sections, reflect on the lesson, and revise accordingly for the remaining sections. This may have occurred from morning to afternoon, or from one day to the next (e.g., reflection on teaching periods 1 and 5 to revise for periods 3 and 7 to be taught the next day).

During the teaching of the lessons, the audio recorder was placed on Jessica's desk, located on one side of the classroom, capturing most of the audible dialogue when one person was speaking in the class (not during small groupwork). At times during lessons, Jessica would pick up the audio recorder to converse with Kadyn about a particular issue or to ask her a

question about something that occurred in the class. Therefore, there are segments of audio during, before, or after the teaching of lessons that are, in essence, short coplanning/coreflecting meetings.

Teaching Artifacts

In addition to the verbal intentions of the coplanning meetings and to the actual enactment of lessons, there are physical and visual materials that were created and used in the classroom that represent the intentions of the coteaching partnerships' instruction. The planning materials include the teachers' handwritten notes or creation of a lesson plan. The teaching materials include worksheets or slides for display that were created before the lessons, and also slides or whiteboard writing that was generated during the lessons. The worksheets and slides have all been archived digitally; the whiteboards were often captured by taking a picture.

Fieldnotes

Throughout meetings with Kadyn and the coteaching of four sections on a regular basis, I took fieldnotes to record observations of activities and behaviors, and to promote reflection on the teaching practice. Notes were taken during or after meetings and classes, and before or after school when reflecting on the current status of the coteaching partnership's work.

One way that fieldnotes were taken was as notes of events of the classroom that would point out what had occurred to refer back to during analysis. The documentation of these events helped direct and focus the analysis process. Another purpose of the fieldnotes was to document any changes in the content or practices of teaching. While describing the occurrences of the classroom, reflection was fostered, which began to make meaning of the classroom situations through beginning emerging themes (Robert Wood Johnson Foundation, n.d.). All fieldnotes

contained the date and the audio recording number associated with the meeting or class and exist in three journal notebooks that span the entire school year.

Data Analysis

The purpose of this qualitative study is to describe, understand, and interpret the unit of study, the coteaching partnership, based on the collected data (Merriam & Tisdell, 2016). This is primarily accomplished through the analysis of coplanning/coreflecting meetings because they hold the regular interactions among the researcher and co-teacher. The goal of this study's data analysis is to elaborate theory where a deep understanding of the phenomena at hand was generated through the construction of a grounded theory. As Yin (2018) stated: "The *analytic generalization* may be based on either (a) corroborating, modifying, rejecting, or otherwise advancing theoretical concepts that you referenced in designing your case study or (b) new concepts that arose upon the completion of your case study" (p. 38).

Through this analysis, I intended to better understand ways to advance teaching towards reform-oriented practices by identifying the resources that the coteachers provided for each other. An analysis of the development of the coteaching partnership, accomplished through identifying and describing stages of the year, revealed the resources the coteachers provided for each other. The elaboration of the coteachers' development, actions, and resources is related to the teachers' craft knowledge, habitus, and the role of reflection. The process of identifying and describing the stages of the year is outlined below.

The overall process detailed below began with indexing the data collected over the course of the 2018-2019 school year, followed by the identification of relevant and valuable coplanning/coreflecting meetings, open coding of these meetings in a cyclical process to generate themes, and the identification of secondary sources of data (fieldnotes, teaching

artifacts, class recordings) that could further extend and support the themes generated from the analysis of the coplanning/coreflecting meetings. These successive levels of analysis (Charmaz, 2006) allowed for description of the development of the coteaching partnership and of the ways in which the teachers were able to serve as resources to support each other's professional growth towards reform-oriented teaching.

Data Organization

Since there was an extensive amount of data collected over the course of a school year, including common meeting times between the two teachers and the enactment of lessons for four sections of Algebra 1 cotaught by the two teachers, it was necessary to review the data to confirm the content of each audio recording. After initially documenting what had been recorded in the fieldnote journals into a spreadsheet, I chronicled every audio recording that I had saved and backed up to my password-secured Google Drive on a regular basis. I then created an index of all recordings, both teacher meetings and class audio recordings, to ensure alignment to the school schedule, my fieldnotes, and the teaching artifacts. The index provided reference and detail for the 305 recordings that are part of my data set.

In order to centralize the resources within the coteaching partnership, my first focus was on organizing the data that had captured the interactions between the coteachers. In total, there were 42 audio recordings of the coplanning/coreflecting meetings that occurred during common periods of the school day or after school between Kadyn and Jessica from September 2018 through June 2019. To give a sense of the existence of where these meetings naturally occurred, the month of September held eleven of these recordings and January had ten, while October, November, and April had five or six, and other months had less than three.

Additionally, during the review of all 305 audio recordings, one-on-one conversations between the teachers that were attached to the beginning or end of class recordings were also identified. As a result, I reviewed the beginning and end of each class recording for any attached coplanning or coreflection segments, and found there were 20 additional shorter recordings, half of which (10) occurred in September or October, the remaining were spread throughout the rest of the school year.

Data Reduction of the Primary Data Source

An initial listen was conducted of the 42 coplanning/coreflecting meeting recordings to get a sense of the content of these meetings. This listening of the data revealed that some recordings were very limited in actual useful data whereas others were rich in conversation between the two teachers. The meetings were categorized as very relevant, moderately relevant, or not relevant, and timestamped segments of the recordings were notated for the portions containing relevant content. The relevance of meeting segments was determined by the alignment to the theoretical lens of the study in that they spoke to the teachers' craft knowledge in practice. Examples of relevant content included reflections on instantiated lessons, negotiations about the content or strategies for teaching particular mathematical topics, or more general discussions about teaching practices. The recordings that were limited in content often had dead air (teachers silently, individually working), conversations of topics outside of teaching or with other teachers, or were very focused on logistics of planning lessons, grading, assessing, or individual student progress, rather than the planning or reflection of mathematical content and teaching practices. The segments for very relevant content totaled approximately 255 minutes of discussion between Kadyn and Jessica. When reviewing the 20 shorter recordings attached to

lessons, applying the same lens as above, six were identified as relevant and two as moderately relevant, totaling approximately 23 minutes of additional audio recording.

From the first listen of the 42 coplanning/coreflecting meetings, the relevant segments naturally grouped into sets of data based on time periods and lessons that were created and enacted. I termed these periods of time to be *stages* of the school year. This natural emergence of stages of the year provided a way for me to begin to isolate and describe separate intervals to trace the progression of the coteaching partnership. During a second listen of the identified relevant meetings, and by taking notes and partially transcribing portions that stood out, more description of the stages across the year began to emerge. Each stage could be represented as a part of the year according to events that happened or the developmental level of the coteaching partnership. Rather than the stages simply representing time periods of the year, they emerged as stages of development of the cohesion between the two teachers. An alignment to the stages of small group development (Tuckman, 1965) began to form, which is described in detail in the following chapter.

In general, September involved the beginning weeks of Kadyn and Jessica working together, along with a lesson cycle representing their beginning copractice. Next followed a period of growth for the copractice that lasted through November, into a December stage of working together by building a model for their practice. January marked a stage with a substantial amount of data in which Kadyn and Jessica had come together to develop a unit of instruction. February and March were relatively quiet due to school closures and teacher absences. At the end of March into April, there was state test discussion and preparation, and in May through the end of the year, there was more room for reflection, development, and preparation for the following school year.

The stages of the year emerged as distinct time periods that each had their own characterization in the content of the conversations that occurred between the coteachers. I continued analysis by choosing a *benchmark recording* for each stage, which was chosen as being the most informative and representative of the set of recordings for that stage. These choices were based on the richness of the data present in each recording segment as well as the occurrence of tensions, negotiations, or altered practices in the partnership. After the benchmark recordings were identified for each stage, the audio files were each trimmed for relevant segments, and transcribed. These recordings only included audio footage of Kadyn and Jessica speaking, there were no other teachers or students involved in these sessions.

Coding of Coplanning/Coreflecting Meeting Transcripts

When reading the transcripts of benchmark coplanning/coreflecting meetings, open coding was accomplished with the use of a lens focused on reform-oriented instruction. After recognizing that a central theme throughout the school year was “application first, procedural second” (detailed below in Chapter 4: Results), allowing students to make meaning by connecting with real-world contexts or their prior knowledge, my next stage of analysis identified how this theme existed throughout the data. Open coding of the transcripts revealed where the theme of “application first” existed, which I integrated into summary statements of the events for each stage.

Following this first-level coding of the benchmark transcripts, I re-listened to the non-transcribed relevant coplanning/coreflecting meetings in order to apply my developed lens of "application first,” accompanied by my understanding of the first round of data I had analyzed. After this round of listening, I identified ten additional transcripts that were connected to the “application first" theme. Additionally, the theme of meaningful professional development was

prominent at this point in the analysis and so I began focusing on the features that made this professional development model meaningful. For example, the coteaching partnership provided a structure through which the teachers could reflect and improve upon lessons between instantiations, dialogue about teaching practices, support each other from reverting to prior practices, wrestle with the constraints of teaching, and extend their teaching practices to support student learning. These various aspects of coteaching as a professional development model were apparent throughout the year of data and were further developed using the secondary sources of data, described next.

Data Reduction of Secondary Data Sources

As the analysis process moved to the secondary sources of fieldnotes, teaching artifacts, and recordings of enacted lessons in classrooms with students, I used the lens of "application first" along with the general characterizations and summaries of the stages of the year I had written to direct the identification of subsequent relevant data. This lens directed me to the teaching artifacts and enacted lessons surrounding six major lessons or groups of lessons of the year (September: Functions; November: Solving Equations; December: Linear Equations Standard Form; January: Systems of Linear Equations; April: Introduction to Quadratic Functions; May: Quadratic Functions Real-World Task) that aligned with the periods of time identified to be the stages of the school year.

A major part of my work process in this respect was to use the transcripts of coplanning/coreflecting meetings and fieldnotes to determine which of the 263 class recordings I should listen to that connected to the six major groups of lessons. Forty-five class periods were identified and as I listened to them, I kept in mind the theme of “application first” and worked to capture the essence of the classes by summarizing what occurred overall in the stage during

those classes. There were marked differences between the distinct stages of the year in the nature of the classes: from one stage to the next, the level of involvement of the teachers varied. This variation, beginning in September with limited involvement from Kadyn during particular lessons, transitioned into a more collaborative involvement in December and into January for coplanned lessons. When listening to April and May classes, however, the interactions of the two teachers in class with the students sounded very similar to January's classes, meaning that by January, the coteaching partnership had reached a collaborative, functional, performing stage.

Structuring the Presentation of the Study's Findings

An open coding of the transcript of the End of Year Interview provided four main areas of content that Kadyn discussed: (a) a focus on applications in teaching, (b) application lessons from the year and transitional points during the school year, (c) her intention to include more investigation and discussion in her teaching, and (d) Jessica's role as a coteacher and what the partnership provided Kadyn. These four areas exist in the data across the school year and provided an organizing structure to present the themes of findings in the following Results chapter. The data from the End of Year Interview reiterated the findings from the analysis of the primary and secondary data sources in that the "application first" lens was appropriate and truly stood out as a key component of the partnership's work, and that the coteaching model had an impact on Kadyn's solo teaching practice and on her as a professional. After data reduction and the identification of the stages of the year, the resulting significant events and lessons of the data set were triangulated by Kadyn's reflection at the end of the school year. Each of these stages, with details of the lesson(s) that were created and enacted, is illustrated in the following chapter. Although the transcripts were transcribed literally, they are conversational in nature, and include

the many stops and starts that characterize the everyday language of teachers working in an authentic context.

Secondary Analysis

After writing a draft of the stages of the year, as well as the two succeeding portions of the Results chapter, a secondary analysis was conducted of the selected data. Whereas the first substantive round of analysis utilized Tuckman's (1965) stages of small group development to define the progression of development of the coteaching partnership along with a lens for the ways in which the teachers prioritized applications first and procedures second, the secondary analysis applied a lens utilizing the theoretical models for reflection. Throughout the year, the coteachers coreflected on their practice in their coplanning/coreflecting meetings, where each teacher shared their evaluations of data from their teaching practice.

Korthagen and Vasalos's (2005) five *ALACT model phases* for the process of reflection and six *onion model levels* for the content of reflection were used to code the selected data within each of the six stages throughout the year. Instances for secondary analysis were chosen from the set of selected data based on the existence of coreflection. Some stages of the year did not have meaningful, deep reflections that were present and thus, not all stages include coding for reflection. Where there were instances of more genuine considerations of the coteachers' work, the ALACT and onion models provided an approach to determine the process phases and content levels of those coreflections.

In addition, in combination with coding for instances of coreflection, cases of one coteacher serving as a resource for the other were identified. The cases were identified by determining where one teacher shared an idea, modeled teaching, proposed an alternative in planning, created a lesson activity, or challenged the other teacher. These instances of teachers

serving as resources align with the process and content of reflection and substantiate the developments of the coteaching partnership's teaching practices.

Chapter 4: Results

The presentation of qualitative data that follows is the result of the grounded analysis process described in the prior chapter. From the indexing of audio recordings and included coplanning/coreflecting meetings, along with the prominent events that were highlighted by Kadyn in the End of Year Interview, pivotal moments of the year emerged. These moments included lessons created by the coteaching partnership, lessons enacted in an iterative and revisionary cycle, and meetings in which the partners dug deep to understand each other, a mathematical topic, or the implication of a teaching practice.

The format of this chapter begins with a selection of data from each prominent stage of the year, over two phases, illustrating the lessons that were created and enacted, and the key events of each stage. The selection of data presented here emerged from the analysis of the development of the coteaching partnership over stages of the year. By identifying and describing these stages, the resources that each coteacher shared in the partnership were revealed and connected to the application or development of craft knowledge and actions stemming from each teacher's habitus. The two phases of the year, detailed below, encompass the first two sections of the chapter. The third section of the chapter presents data regarding the impact on Kadyn's teaching, with a focus on the use of real-world applications and the ways Kadyn and Jessica negotiated and deliberated about particular teaching practices. Finally, the fourth section centers the coteaching model as meaningful professional development, depicting the events of the year as transitions in professional growth.

As described in the Data Analysis section, the data emerged as existing in separate stages of the year, which were naturally chunked during data analysis. Each stage centers on the coteaching of a particular lesson or series of lessons. Data for each stage is presented first for the

reader to understand the scope and progression of the coplanning/coreflecting meetings, and the lesson materials and enactments connected to those meetings. The month and lesson topics are presented in Table 3 along with a reference name for each stage that represents the theme of the work for the teaching partnership during that stage. The two phases of the year provide a structure for grouping the presentation of the chapter's findings. Notice that the months of October, February, March, and June are omitted from the table, not because data was not collected, but because the meetings, lesson materials, and enactments contained neither shifts in the partners' practice, nor moments that emerged as speaking to the resources that the partners provided to each other (the reader may think of October, February, March and June as continuations of prior months).

Table 3

Stages of the Year for 2018-2019 Coteaching of Algebra 1

Phase	Month	Lesson	Stage of the Year
Phase I: Becoming a Coteaching Partnership	September	Functions	*Forming* a Partnership
	November	Solving Equations	*Storming* Towards an Alternative Lesson
	December	Linear Standard Form	*Norming* with the "Application First" Approach
Phase II: Solidifying the Coteaching Partnership	January	Systems of Linear Equations	*Performing*: Building a Practice
	April	Introduction to Quadratic Functions	*Performing*: Structuring for Student Independence
	May	Factored Form of Quadratic Functions	*Performing*: Instantiated Lesson Re-sequences the Teaching of Quadratics

This main portion of the study's results is presented in two sections: the first section is the development of a copractice (from a co-practice, with a hyphen) from September into November and December (Phase I); the second section is the solidification of the copractice in the months of January, April, and May (Phase II). A Standard Form lesson in December stands out as a pivotal point in the school year during which Kadyn articulated that she "was sold" (personal communication, July 30, 2020). For Phase I, this December stage and corresponding lesson is described first to orient the reader toward this turning point. This is followed by descriptions of the Forming (September) and Storming (November) stages leading to this critical goal post. Afterwards, in Phase II, the Performing stages are described in the order they occurred (January, April, May). Throughout, the data are connected to how Kadyn and Jessica served as resources for one another to accomplish the progress made throughout the year.

School Context Details

This section provides some of the institutional details for this study. The reader will notice various abbreviations and institutional references throughout the chapter. These are described for the reader here as reference.

In the data, the teachers use the acronym *PBA* quite often, which stands for Project Based Assessment. In the years prior to this study, the school district at large participated in rewriting curricula to include project-type assignments intended to assess student understanding of content and student ability to independently transfer their knowledge to create their own products. Jessica participated as a curriculum writer for many subjects and was very familiar with the format and intention of the assessments. Kadyn, who had been on maternity leave for two years, was not very familiar with the assessments prior to their implementation during the 2018-2019 school year.

The teachers also refer to *A days*, which are part of the A/B/C/D rotating drop schedule at the high school (meaning the classes rotate in AM/PM cycles and one class drops every four days). In accordance with the scheduling of the four Algebra 1 sections for the study's coteaching partnership, A days included a scheduled Algebra 1 class for all four sections, which the coteachers intentionally established as the beginning of a lesson cycle so that all classes would have a common lesson on the same day. B days included only two of the sections (periods 1 and 5), causing a staggered schedule for the remainder of the cycle: periods 1 and 5 also had class on C days, but not D days; the other two sections (periods 3 and 7) met on C and D days. During the 2018-2019 school year, Kadyn taught the four Algebra 1 cotaught sections along with one section of Advanced Placement Calculus (AP Calc) to seniors. Jessica taught the four Algebra 1 sections along with one section of academic-level Calculus. Jessica and Kadyn's schedules left two class periods in the afternoon (plus lunch and after school) for common meeting times.

Transcript Citations: Format and Reading

In an effort towards clarity for the reader, transcript data are cited in the following format: *(month/day, recording number, timestamp)*. The dates fall within the 2018-2019 school year, September through June. The recording numbers range from 1 through 340 in chronological order and are useful because there may be more than one recording referred to for a single school day (potentially four class lessons plus common meeting times). The timestamps are dependent on each recording, representing either one starting point in time for a single quote, or an interval of time for an exchange between Kadyn and Jessica.

Towards the latter part of this chapter and in the following discussion chapter, these citations are used to refer back to transcript data already shared in a previous portion of this

chapter, hoping the reader is able to highlight and copy the citation, then search the document for the same reference. For example, the citation "(1/25, 207, 2:34)" would represent a citation for January 25, Recording 207, timestamp beginning at 2:34. Afterwards, the citation "(see 1/25, 207, 2:34)" would be used for the reader to refer back to the original quote.

Phase I: Becoming a Coteaching Partnership

During the End of Year Interview, Jessica asked Kadyn: "Do you think there were some turning points during the year for us? Do any standout?" (6/21, 340, 13:57). Kadyn responded: "Standard Form. That's when we kind of thought … started introducing things this way: application first, procedural second" (6/21, 340, 14:14). To Kadyn, this December lesson was the *first* lesson she connected with as instantiating the idea of teaching with an application before a procedure. The phrase "application first, procedural second," referring to a formulation of teaching aligned with the reform-oriented practice of making sense of problems (NGA, 2010), became fundamental in the coteaching partnership and is referred to throughout this chapter.

The origin for Kadyn's observation can be found in a planning document for a September 12 lesson. As Jessica considered how to plan a lesson within her partnership with Kadyn, she wrote to herself, "*Planning: flip everything - start with real-world, then get into specifics." From the beginning of the school year, Jessica had been focused on using real-world applications as a vehicle for inspiring reform-minded teaching. Jessica's intention of including real-world applications first in lessons beginning in September and Kadyn's acknowledgement that this practice was genuinely accomplished in December represents the change and growth of the teaching practice among Jessica and Kadyn between these months. By identifying this difference and describing the stages of September and December, the teachers' actions as resources were revealed and will be specified in the description in the following two sections.

Norming through the "Application First" Approach (December)

The following sections will present data that demonstrates the process through which the lesson on the Standard Form of a Line was created, then present data instantiating the lessons, along with Kadyn's reflections on the partnership's work as captured in her explanation to a teacher visiting their classroom.

Creation of the Standard Form Lesson

During a coplanning meeting on December 7 to prepare for upcoming lessons, the following exchange occurred between Jessica (J) and Kadyn (K):

> J: So here's my thought, you come into the topic by exploring why it's important, applicable, surface level. Here's why we care, right. By doing a task that models a situation, whatever it may be. Okay? And then you get into the meaning of … a typical introduction lesson. Right, based on the context,
>
> K: You're doing, PBA [Project Based Assessment], like project-based learning, essentially, you're showing the task, then we're seeing why we have to have these components, right?
>
> J: Yes. So that, so that you get buy-in, engagement and then you get into procedural fluency. And then you bring it back. (12/7, 150, 0:27-1:15)

Jessica continued:

> J: I'm saying, give them an opportunity to make meaning of it. It could be five minutes. I'm not saying a whole day to, I'm saying, just start there. This is why we care. This is what's going on. Let's think about it for five minutes and then let's dive in. So just setting the stage, something, something that gets them interested, but it's not just, it's not just a hook. There's some, there's some math meaning there. (12/7, 150, 2:10)

Kadyn related to what Jessica proposed: “I feel like it's project-based learning on this mini little scale. Project-based learning is, here's a problem, fix it, and then you have to learn all the components” (12/7, 150, 4:07). Jessica proposed beginning the lesson with a form of real-world application or exploration regarding the topic’s meaning before working on any procedural aspects. Kadyn aligned herself with approaching lesson planning in this way by identifying Jessica’s proposal with a familiar strategy, namely, project-based learning.

In a next coplanning meeting, where the goal was to plan the lesson, Jessica and Kadyn discussed more specifics:

> J: So we want to write equations from two points using Point-Slope Form, and that's going to be after the break. I think we should go from Slope-Intercept to Standard using the intercepts.
>
> K: I think you're right, and we've also focused on intercepts, yeah
>
> J: Cause we've been using intercepts. Right. We started with interpretation of *x* and *y* intercepts. We went to *b* as the *y*-intercept, and then you can kind of nail it down here. So what I'm writing here about Standard Form is like that idea of starting with a real-world example. Why do we care about what we're going to talk about for two days? (12/11, 156, 0:02-0:16)

Jessica proposed moving to Standard Form for Linear Equations in the upcoming lesson, and Kadyn agreed, supporting the rationale that they had previously focused on *x* and *y* intercepts, which could be leveraged in the teaching of Standard Form. This planning solidified connections to students’ prior knowledge, and a topic (intercepts) that was already studied, rather than moving to something completely new and disconnected from prior learning. Additionally, during this meeting, Jessica rationalized using a real-world situation for this lesson because Standard

Form word problems are integrated into the following unit, Systems of Linear Equations. Thus, this planning further reinforced connections among the topics of study for students.

Jessica then moved the discussion into planning more specifically, focusing on starting with a real-world example and providing students with a reason for the study of this topic, which largely reiterates the discussion from above (12/7, 150). Jessica further related other terms that have been studied in class such as "coefficient, constant, intercepts," and realized during this meeting (evidenced by laughter) how a real-world scenario can integrate all of these various ideas for the students.

Materials Created for the Standard Form Lesson. As a result of the coplanning meetings, a student worksheet was generated that included the following problem:

> Problem 1: You have been saving your money because you want to buy your family members some tickets to see a show. Adult tickets cost $20 each and student tickets (under 18 years old) cost $10. You have saved $300. How many of each type of ticket can you buy?

The worksheet, which can be found in Appendix B, was structured for students to write in "important pieces of information," the variables of the situation, expressions for the total cost of each, an equation that models the situation, a table of values, and a graph. Additionally, the problem leaves open the amount of money the saver has, thereby reflecting the variability in the dependent variable of a linear equation. A second, similar problem was also included on the worksheet, with space for summary notes and also more procedural practice problems (non-contextualized) copied onto the back. The prior version of Kadyn's worksheet for teaching this topic included only a space for notes, then space to graph Standard Form equations (procedural skill practice). This was now on the back of the worksheet. Slides were generated from the

worksheet into a SMART Notebook file for instructional purposes. The teachers wrote on the slides during the lesson as ideas were generated during class.

Enactment of the Standard Form Lesson

The Linear Function Standard Form lesson was taught to two classes on December 13, and the remaining two classes the following day. Kadyn was not present for the first instantiation, only Jessica taught it. During the second instantiation, Kadyn was present but Jessica still led the lesson. After this second instantiation, Kadyn encouraged Jessica to reach out to teachers who might be interested in observing this lesson in the classes to take place the following day (this suggestion fit into a school-wide effort to build a culture of collaboration at this time). Kadyn's suggestion demonstrates that after her first experience with this lesson, she believed it to be something different and worthwhile for others to also experience.

During the third instantiation of this lesson, another high school mathematics teacher, Tyler (a pseudonym) visited for the beginning portion of the lesson. While Jessica was leading the instruction, Tyler publicly interjected a thought to Jessica in front of the whole class, asking if Jessica could give an example of what she was asking of the students ("write a mathematical expression to represent the total cost"). Jessica replied:

> J: Not yet, not yet, not yet, not yet, because they can make sense of this, I don't want to give them any hints … [Kadyn] is smiling too because this is how I operate … because when they move to the second problem they won't be able to set it up if I give them hints. (12/14, 161, 7:37)

Jessica's response to the visiting teacher demonstrates her stance for students to think about the situation, and develop their own ideas, without teacher-led examples. Jessica continued the class,

while Kadyn went to speak with Tyler. On the audio recording of the class, Kadyn's response to Tyler can be heard (some portions inaudible):

> K: It's so frustrating, she makes them think. We're trying to find a balance between understanding and drill and kill. So I just wanted to just start graphing, by finding the intercepts. It was like a problem that represented a carnival problem, the front was just notes. On the back, we gave them the actual problem, I mean like the *y*-intercept is this, it represents this, the *x*-intercept is this. … So, Jess is now, before we do anything new we have to give them a rationale as to why … and then we're going to … before the drill and kill…. This [lesson] is perfect, it's like a 20 minute, 25 minute, introduction and then they make sense… it's a good representation of "us" and what we can do to make them think. (12/14, 161, 8:06-10:50)

In this segment, Kadyn expressed her views to a person outside the partnership on what this lesson represented for her: although she wanted the students to "just start graphing," she acknowledged that "we have to give them a rationale as to why," and it is less than half the class period, allowing for us to move from making them think to more procedural practice ("drill and kill"). Following Kadyn's exchange with Tyler, he exited the class, and Kadyn became even more involved in the guided class thinking. She stepped in, navigating and establishing her presence as a coteacher in her partnership with Jessica.

During the fourth and final instantiation of this lesson, Kadyn, by minute 8 of the class, fully took over guiding the student thinking for a majority of the class period. She had observed the lesson twice, after being involved in the creation of the lesson structure, both of which offered her an entry point to fully engage in the teaching of the lesson.

Summary and Secondary Analysis of the Norming Stage (December)

Rather than a procedurally focused lesson where students copy notes, then make sense of the mathematics, Jessica worked with Kadyn to enact a lesson that allowed students opportunities to make sense of new ideas within a contextualized scenario and to create meaning and connections before engaging in the nuts and bolts of the topic. The resulting materials included two real-world problems for a new topic that supported students in problem solving that would otherwise have been an afterthought, or non-existent, in the lesson. Students more openly interpreted and represented their thinking, rather than extending procedural knowledge to application situations, the former of which can be categorized as strategic competence. Kadyn accepted, voiced, and instantiated students making meaning of this mathematical topic for half a period before students learned the procedures for the topic. Kadyn not only agreed to the creation of this lesson, but also understood it and became comfortable enough with it that she took over leading the enactment of the material.

While planning, Jessica proposed providing opportunities to students to make meaning of a topic before diving in (see 12/7, 150, 0:27-1:15). She also led the creation and the enactment of the Standard Form lesson. In these instances, Jessica served as a resource to Kadyn by opposing Kadyn's established point of view, designing a new introductory activity for students, and enacting the lesson for Kadyn to observe. Kadyn also served as a resource for Jessica because 1) Jessica had not created such an activity for this topic in her prior role as a teacher of Algebra 1, and 2) through reflection on the content that had been covered, the pair of teachers established that Standard Form of Linear Equations could be founded on prior student knowledge of the intercepts. With Kadyn's suggestions that other teachers should observe this lesson and her

leading of the fourth instantiation of the lesson, not only do we see that Kadyn's practice has developed, but also, that an aligned coteaching partnership exists.

How was a Norming Stage Reached?

As Kadyn expressed to Tyler, an "us" (12/14, 161, 8:06-10:50) had formed by the instantiation of the Standard Form lesson. The following sections will present data that illustrates the two stages of the year that built up to Norming (December): the Forming stage (September) and the Storming stage (November).

During the summer of 2018, after Jessica and Kadyn accepted the mathematics department supervisor's request to teach the four sections of Algebra 1, the teachers began discussing their perspectives and intentions for this course and the school year. They had a few phone conversations, met in person at the high school twice, and Jessica shared documents in which she outlined her brainstormed ideas and compiled resources for "revamping" the teaching of Algebra 1. Jessica and Kadyn were excited to be working together and communications over the summer began the process of forming their partnership. The conversations and meetings prior to September took place before IRB approval. Once the IRB was approved and Kadyn signed the consent form, meetings began to be recorded.

Forming a Partnership (September)

During the first month, Jessica and Kadyn had many coplanning/coreflecting meetings where they worked to understand each other regarding their intentions for lessons and students, and also regarding how they could work with one another in the co-teaching partnership. Next, I present data from this beginning month of school to describe the initial ways the teachers worked together to begin to serve as resources for one another. The month of September represented a stage of forming, serving as a basis for the partnership's ongoing development.

Kadyn's Discomfort with Giving up Classroom Control

Through the following portion of transcript, Kadyn shared her concerns about a lesson that was enacted on the topic of relations. Kadyn voiced that she "didn't know how to connect their answers to be right answers" (9/12, 5, 4:57), after which she asked Jessica to take over leading the class. Jessica replied to Kadyn by focusing her on how she redirected the lesson. Kadyn's reflection on this instance follows:

> K: No, I said I was uncomfortable. But sometimes they start talking and they lead you in a good direction. And sometimes we're talking and they're like, I didn't think, I wanted them to go a different direction than they were going. This is me trying to figure out how to give up control. But also I don't want them learning the wrong things.
>
> J: Sure. So my point of view is that I want to understand what they're thinking. Right? So when you turned it over to me, I was like, okay, I don't know, I don't know what's in their heads. I don't know why they're talking about two lines.… So they're making stuff up, but they know things to work from. So that's why I'm trying to understand where they're coming from. They know, they know things … ok so I need to, I need to *see* what they're thinking about. So that's why I had different cases go up to the board and as many different cases as they want. Right. Get them up on the board and let's talk about them. Can you describe them?
>
> K: So what I'm scared, what, what makes me nervous, is still guiding, I understand what you're, I understand what you're saying, but then how do I then get them to do what I want? And maybe I don't, how, you know,
>
> J: Right. So, so that's one huge thing. It takes time. It takes time away from moving forward. Right. Is that one of your concerns … or just getting to the right place? Did we

> get to the right place after that? So one of my key questions was which one is different. There were three different pictures. Two of them, two of the three had two points. And so we were able to, to differentiate. (9/12, 5, 5:25-7:13)

In using the ideas students had written on the board, Jessica created a classroom environment where multiple approaches could be taken and connections with student thinking (rather than teacher thinking) could be made. However, this environment made Kadyn uncomfortable. In this passage, Kadyn understood what Jessica was doing and explicitly reflected on this shift. Kadyn reiterated: "I'm uncomfortable leading that because it's not familiar to me" (9/12, 5, 7:50). Jessica responded that yes, "that's part of where we're headed. … the teacher not knowing, right? Oh, I've been here before. Here's what I can say to get them to the next step" (9/12, 5, 7:58). Jessica recognized and supported Kadyn through her discomfort: by opening up the classroom to student thinking, the path is not always clear and may require flexibility. Kadyn let Jessica know: "But when I get there, I still want you to take over it until I see it done right" (9/12, 5, 8:12). At this stage, Kadyn articulated that she was still working to observe, take it in and understand how to go about this style of teaching.

Jessica reiterated that the path is unknown: "I don't know where this is going. I don't know what the students are putting up there" (9/12, 5, 8:16), and Kadyn expanded on the reasons for her discomfort:

> K: And I don't think, I don't think it's not like I can't learn to like it, but right now I don't like it.
>
> J: Can you say why?
>
> K: I don't want to ever give students misinformation and if something starts going, and I also don't want to ever tell students that they're, that, no, you're totally wrong. I mean,

> they try, they're doing something and … you've lost control. I mean, not in a bad way, but like you lost, you've given up control, right? So you have to trust that they're gonna, well not trust, you have to guide them to do the right thing without giving them the answers, which is an art. (9/12, 5, 8:24-9:00)

Kadyn struggled with giving up control as a teacher regarding the information in the classroom: by allowing student thinking to direct the lesson, there were possibilities that incorrect information could be shared and heard. From the beginning, the essential struggle for Kadyn was the shift to how, as a teacher, one could guide students (without giving answers) towards "the right thing."

In a similar way, during a coplanning meeting on September 13, Kadyn voiced concerns about a future lesson not going as planned, saying she's going to have "to be ready to adapt" (9/13, 10, 25:48). Opening up her teaching to being adaptable is something that she was not used to doing. Kadyn further said: "I just have to think these things out in my head, you know what if it's not working out the way I want it to work out? This is, this is different. This is …" (9/13, 10, 26:18). Again, the flexibility in this style of teaching towards which Jessica was guiding the partnership was concerning to Kadyn, who was contemplating how she could adapt and be ready to respond in the moment while enacting lessons.

Classroom Discourse as a Window to the Teaching Practice

During a lesson on September 20, Kadyn was leading the lesson through questioning in a whole class discussion when Jessica interjected (in response to a student's observation) and asked: "Why? … Does anyone want to fight him [a student] on this?" (9/20, 20, 11:25). In the same lesson segment, Jessica used more questions such as "Who wants to add something to this?," "Who can rephrase?," "Who can restate what he just said?," "Anyone changing sides?

Why?," and "Do you want to stick with your answer or revise it?" (9/20, 20, 12:20-14:27). In this lesson, such questions supported the discourse of the classroom for students to argue and justify their reasoning. Jessica offered the lead back to Kadyn, who responded: "Just go, you love this, bring it home" (9/20, 20, 19:30). Kadyn allowed and supported Jessica's instructional moves and observed her enacting these different and more reform-oriented discourse practices, which Kadyn was still not comfortable continuing herself.

During a coreflection meeting on September 20 (after the previously discussed lesson), Jessica brought up an area of Kadyn's practice that she witnessed regarding responding to incorrect answers or thoughts from students. The lesson naturally created a situation where Kadyn was responding to students with behaviors that messaged they were incorrect, whereas Jessica was attempting to build student thinking by asking "Why?" or requesting that students defend their position with reasons. Kadyn responded:

> K: That's fine. So when you see this happening, I wouldn't mind if you jumped in so I could watch it happen a few more times. What questions are you asking? How are you getting this. … I'm not comfortable with it, so I'm going to let you … just take over because, because I, I want to watch, I want to learn. (9/20, 21, 17:41)

Building on Kadyn's initial discomfort, she was still not ready to lead the discussion when she cannot anticipate where it is headed. But in this reflection, she was more specifically asking questions that could be asked which would direct the class conversation towards a learning goal. Additionally, she shared: "Well, I think … sometimes I'm not aware and sometimes I'm not comfortable. And I also have this internal clock that I want to finish things" (9/20, 21, 18:12). Kadyn's years of experience in the classroom informed her teaching because

she implemented what she was comfortable with and knew that she had to meet particular objectives so kept lessons moving forward toward those objectives.

In contrast, Jessica found value for students in providing students opportunities for them to engage and think, as Kadyn described next:

> K: You had a great… I watched you lead the discussion with the, and at first I was like, Oh, this is taking a really long time. But then I mean and they were into it. Right. And that's like … And I was like, just fricking go with it because it was, it was working. Right. It was turned on. But I feel like sometimes when it's not working and then I get frustrated, I'm like, I feel like we're just wasting time … I don't know when you turn the off switch, when you just say …
>
> J: Cut it off and let's … so I think that's between the two of us as well. Right? That's the beauty of this. Like you got anything else? No. Do you have anything else? (9/20, 21, 19:11-19:45)

Kadyn reflected that she was excited that Jessica had led a class discussion that students "were into" and she supported Jessica taking the time out of the lesson as long as "it was working." However, Kadyn recognized her struggle with what to do when it's not working and "just wasting time."

Kadyn Engages in Inquiry: Co-teaching Becomes Coteaching

During class on September 21, Kadyn was heard asking, "What's your evidence?" (9/21, 25, 25:16) and stated, "I just heard 'how do you know?', which is better than an answer!" (9/21, 25, 42:20). After days of this type of discourse in the multiple lessons that had been enacted focusing on reasoning, this use of language in the realm of justification from Kadyn is a first

piece of classroom-based evidence of her integration of reasoning, a reform-oriented instructional practice, into the shared teaching practice.

At the beginning of a coplanning/coreflecting meeting on September 24 where Kadyn and Jessica were planning for an upcoming lesson, Kadyn shared:

> K: Yeah, I'm gonna make this come next and then I'll do the other one. Is that okay with you? It's better, right? It's more in tune with …
>
> J: Yeah, it's great.
>
> K: And I kind of love it, even though, it's gonna slow me down.
>
> J: Even though what? Can you say that a little louder?
>
> K: I kind of love it, even though it's going to make me go slower, make us go slower.
>
> J: Why do you love it?
>
> K: Cause it's awesome. It's like real-world stuff. It's multi-representations of the same thing. I think that hopefully it will sink in. I mean, that's the, what we want. (9/24, 33, 29:12-29:44)

Kadyn reached a point in the shared teaching practice where she had chosen lesson materials that she knew were more aligned with what Jessica wanted to incorporate into their classroom. Kadyn knew it would take longer but had resolved that it was worth the integration of multi-representations and the real-world connections and hoped "it will sink in" for the students.

Kadyn continued to connect with the shared teaching practice as she reflected on the specifics of what she had observed. On September 26, during a coplanning/coreflecting meeting, she said to Jessica: "There's a balance, right? You want everything to be … student driven, right? But I don't think it can be, with this set of children, right, with these freshmen Algebra 1" (9/26,

42, 0:55). Kadyn recognized that Jessica wanted "everything to be … student driven," but at this time she was considering whether this was even possible with this population of students.

In the same session, Kadyn reflected on integrating contexts to connect with definitions, continuing her analysis of what Jessica had been bringing to the practice the past few weeks:

> K: One piece has to be there, right, so either we have to be giving them a formal teacher definition, not a textbook definition and then they're interpreting it with some kind of contextual problem or vice versa, give them the contextual problem and ask them to pull out something important. Do you know what I'm saying by that? Like the miles and the minutes, that helped them to pull out the independent and dependent and then have a discussion about what that means and function and non-function, right so that one, we used a real-world example with real-world data, to talk about these concepts. But in this problem, I feel like you're doing both things and it didn't work. (9/26, 42, 3:07)

Kadyn gave a suggested model for either moving from context to definition, or vice versa, arguing that she felt the students must be given the important piece: the takeaway has to be crystal clear for students. She was sharing with Jessica that she did not think it was effective to have contexts throughout, that the meaningful understanding for students was hidden and not explicit for students due to the contexts. Kadyn continued regarding this area of thought:

> K: Yeah, but I still, I still am not convinced that in the end of this, I wanna make sure I have certain things written down even if they didn't give it to me. Otherwise how can I ensure that you can then go home and do your homework? (9/26, 42, 3:58)

In the lesson segment Kadyn was referring to, she shared with Jessica that the important pieces were not formally written down with and by the students, and that this could restrict their acquisition of the knowledge needed to be able to do subsequent assignments for this topic.

Summary and Secondary Analysis of the Forming Stage (September)

After this beginning month of interactions with Jessica, Kadyn gave her opinion about the teaching practice rather than saying "you do it" in order for her to watch and see; she has seen enough to judge for herself and reflect on it. Kadyn has become more comfortable after exposure to practices in Jessica's teaching repertoire and has an understanding that served as a basis for making sense of how to go about creating situations such as these in the classroom. The discourse of the classroom activities provided an open platform for the co-teachers to honestly discuss particular practices. Each teacher shared their stance regarding the educational aims that were priorities for them. Jessica and Kadyn have each been informing the other of aspects of their dispositions, working through negotiations towards forming a partnership. In particular, what stands out in September is Kadyn's acceptance of Jessica's focus on student thinking and her integration of this focus into the coplanning of lessons.

During the month of September, Jessica served as a resource to Kadyn through her modeling of teaching practices. In a coplanning/coreflecting meeting, after Jessica led a class discussion, she explained to Kadyn (see 9/12, 5, 5:25-7:13) how she did not know what the students were thinking, but that she was working to understand their thinking in the lesson. While reflecting on what had transpired in class where Kadyn did not know how to connect student answers to her intentions (see 9/12, 5, 4:57), Kadyn was looking back on these actions of the classroom. In the second phase in the ALACT model of reflection (Korthagen & Vasalos, 2005), while looking back, Kadyn was assessing her competencies to be able to facilitate the class as Jessica had, participating in a pedagogical level of reflection. Kadyn shared that she was "trying to figure out" and was "scared" and "nervous" (see 9/12, 5, 5:25-7:13) about this style of leading the class and recognized that "it's not familiar" (see 9/12, 5, 7:50). This reflection led to

Kadyn sharing her belief in the practice of not ever giving students misinformation (see 9/12, 5, 8:24-9:00). Using the onion model for reflection (Korthagen & Vasalos, 2005) as a lens, Kadyn has evaluated her competencies (level 3), and stated a belief (level 4) based on the environmental experiences (level 1) with her coteacher and her own behaviors (level 2) in the classroom. These pedagogical level reflections provided rationales regarding teaching practices to give genuine consideration of the teachers' experiences.

In a meeting where Jessica discussed with Kadyn her reactions to students who provided incorrect answers (see 9/20, 21, 17:41), a *confrontation* occurred. Jessica felt the need to bring this issue up with Kadyn because it was a tension that Jessica saw as detrimental to the goal of building student thinking and reasoning. Kadyn responded that she was not comfortable because she did not know what questions to ask or how to go about leading such discussions, opting instead to continue watching and learning (see 9/20, 21, 17:41). Additionally, Kadyn referred to her "internal clock" (see 9/20, 21, 18:12) that moved her forward constantly, a part of her identity (level 5) as a teacher. Although she knew the discussion was taking a long time, Kadyn shared that the students "were into it" and "it was working … it was turned on" (see 9/20, 21, 19:11-19:45), thereby simultaneously recognizing the value in this practice. Here, Jessica again served as a resource to Kadyn in that she was modeling teaching that was unfamiliar and not part of Kadyn's "competencies," or habitus, and so through this observation and reflection, Kadyn began to gain understanding of Jessica's mission.

A few days later, in her planning of a lesson (see 9/24, 33, 29:12-29:44), Kadyn chose an activity that was "more in tune with" what the beginning shared goal was and that was "awesome" because it was real-world and multi-representational. Kadyn was now creating alternatives (ALACT model, phase 4) based on prior pedagogical level reflections of the shared

practice. This beginning month of coteaching included variation in the ways that Kadyn and Jessica worked with one another. Kadyn's adoption of a new view that she was excited about was a shift from the initial weeks where Kadyn only observed, and did not participate in, their classroom practice.

Storming Towards an Alternative Lesson (November)

After the coteachers formed their partnership in September, the next stage of the year that emerged as relevant data regarding a shift in teaching practices was in November. This stage involved disagreements and tensions that provided opportunities for the partnership to establish new norms, resulting in an evolved partnership. Through the identification and description of this stage, additional teacher actions as resources for professional growth were revealed.

Kadyn and Jessica had moved to a unit on solving equations and had students work with equations of increasing complexity. In the following exchange on November 7, Kadyn and Jessica were working to figure out how they would teach the next type of equation that involved the distributive property and combining like terms.

> J: It's an A day and you're not going to want to do it four times. So what can we do? How can we strategically use pairs or groups?
>
> K: We're at a point where we have to get through material
>
> J: No, no, I, I will *not* accept that! You come up with things on the fly that are beautiful. Can we try to plan for one, instead of having a Monday, times four, killing ourselves. Let's build up. I mean, this is just an idea. Let's build up. Here's the problems that we did. You can do them. We're gonna look at the homework somehow, I don't know, bring that in. What's the difference between this problem and the problem you wrote on the paper

there, right? What's the added step? So we're working on talking about what work has to be shown. You know what I'm saying? And then build up again.

K: Yeah. But this is direct instruction.

J: No, no, no, no, no. No!

K: Yes!

J: I'm not doing it, but the students are doing the math. I give them, we give them the problems. [pause]

K: I often buy in. I'm not buying it today. I feel like I'm going to teach this by DCS: distribute, combine, solve.

J: Can *they* come up with that?

K: Do you want to get to consecutive integer problems still Monday? You're going to tell me what I'm doing is a waste of time cause they're not going to remember it anyway so why am I in a rush? I don't know. I don't know.

J: You said that, I didn't say it. I never said those words to you.

K: I know I just said that, I'm thinking, I can think both sides of this, but I also, I, I don't. My main philosophy with freshmen is that I don't like to rush things into tests and quizzes. So if I feel like I can move through faster at the end of the marking period, trying to squeeze in too many things. I don't see a whole lot of difference between what you're saying and what I'm saying. You just want them to come up with it on their own. But …

J: And I want them working, which, which is what you do, but there's a, there's a, like a default that happens like a fallback, that they don't want to do the work cause it's Monday or whatever reason, and we end up doing all the work for them. (11/7, 102P, 16:17-19:17)

Jessica suggested ideas to move this lesson from very direct instruction towards something more active for student involvement. Kadyn, from the start of the coplanning meeting, wanted to move through the material, meaning that she just wanted to get it done. She wanted to resort to her traditional strategy of writing a mnemonic on the board (DCS) and solving each part accordingly. At two locations in this passage, Kadyn referred to the timeline and the fact that it was the end of the marking period and she did not want to rush or squeeze in too much. The coteaching partnership was storming in this instance because there are emotional responses about what each teacher believed they should be doing in the classroom: responding to school constraints and continue moving forward by any means necessary (reverting to teacher-directed notes) or navigating the constraints and continue working to develop a meaning-making activity that could take time.

Based on Kadyn's response and her knowledge of two months worth of their shared practice, Jessica forecasted that due to the factors involved in the lesson they were planning, the students would not do the work Kadyn has proposed and instead rely on the teachers to do it, lowering the level of students' cognitive activity and engagement. As this meeting continued, Jessica shared her concern regarding teachers doing the "heavy lifting" for students. Kadyn responded that the one or two strong students in the class answer on behalf of everyone else, so most of the class would not do the work on their own anyway. Jessica persisted: "My argument is that they have the tools and the know-how to do those problems. It just has to be called out. And then they have to try them on their own…" (11/7, 102P, 21:58).

Kadyn and Jessica have spent almost six minutes storming when Kadyn offered a possible alternative: "Okay, how about this, how about we just give them a problem where it's all

the way solved out and we ask them what they did? Justify their reasoning" (11/7, 102P, 22:12). This idea from Kadyn prompted the creation of the coplanned lesson that follows.

The DCS Alternative Lesson

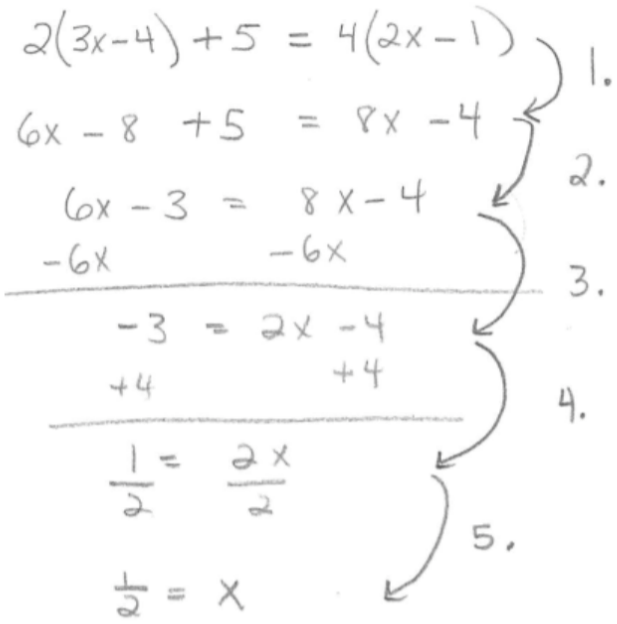
Based on Kadyn's idea to have students justify their reasoning for how a problem is solved, a worksheet was created that had multiple problems correctly worked out with numbers for each line of work and arrows for movement through the solving process. Students were asked to "Justify, in words, how the student moved from one line to the next" (see Figure 1).

Figure 1

Teaching Artifact Created November 7

Look at each problem below. Justify, in words, how the student moved from one line to the next.

1)



After this lesson plan and activity worksheet were created, Kadyn reflected on the progress the partnership has made. She asked Jessica: "Do you feel happy? I feel, I feel like this is a wonderful lesson, but maybe you feel sometimes I feel like we have to compromise. Do you feel compromised?" (11/7, 102P, 28:33). Kadyn was checking in with Jessica to see how she was

assessing the new version. When Jessica asked why she was happy, Kadyn responded, “Because this is better analysis and it's prepping them for the PBA [Project Based Assessment]” (11/7, 102P, 39:22). Although Kadyn was resistant in the beginning of this meeting, she had progressed to this understanding of having students engage in analysis. After an initial storming, the partnership had resolved the issue by creating a new option that both moved the knowledge forward and allowed for meaning making through justification.

During the enactment of this lesson, Kadyn was conducting one-on-one conferences with students in the hallway, so she was in and out of the classroom. While Jessica guided the lesson, various instantiations of reasoning manifested. For example, Jessica said in response to students, “beautiful explanations” (11/12, 105, 22:10) and the class applauded. Jessica also asked, “What did you hear in [student’s] explanation that you really liked?” (11/12, 105, 37: 23). In reviewing a problem that students had completed in groups, Jessica stated to the class: “I heard somebody say…” (11/12, 105, 38:31). Jessica was able to build on student thinking during this class because of the more open nature of the activity that students were engaging in. Rather than a teacher-centered lesson, the coteaching partnership had opened up interactions among students and teachers through the use of justification in this activity.

By the third enactment (11/12, 105), Jessica’s guidance in the whole class discussion for student explanation and justification was strong. For Jessica, it took working with Kadyn as a coteacher to develop a lesson that could viably function, and a third enactment of it (and possibly this class dynamic) to find in-roads to develop the argumentation. In this sequence, it was as though Kadyn’s idea in the planning session had supported the creation of a more practical lesson, providing the coteachers an understanding of how to enact it and providing the students opportunities to consider the justification in their thinking.

Summary and Secondary Analysis of the Storming Stage (November)

The creation and instantiation of this justification lesson for solving equations was initiated by the disagreement between Jessica and Kadyn regarding how to teach the distributive property and combining like terms to solve algebraic equations. This stage represents an example of the connection between the teachers' dispositions, the materials they use, and how lessons are instantiated: without alignment and understanding of the materials to the teacher's habitus, reform-oriented practices such as justification would not be enacted in intended ways. In the coteaching partnership, Jessica adamantly continued to challenge Kadyn to pre-emptively create materials rather than flying by the seat of her pants while teaching this lesson. The eventual creation of new material restricted Kadyn from reverting back to her traditional method of doing example problems and giving a mnemonic at the lesson's outset. Instead, the new material provided opportunities for students to reason and justify their understandings.

In the exchange regarding planning the distributive property and combining like terms (see 11/7, 102P, 16:17-19:17), Kadyn's focus on getting through material and her reflection on being in a rush sheds light on her identity as a teacher. She referred to her "main philosophy" where she does not want to assess students if she has not properly covered material, so she has to "rush" to fit everything in that should be on the end of marking period test. Jessica served as a resource to Kadyn here to drive her to adjust their teaching practices (to plan ahead) and to pedagogically reflect on how they were going about teaching this topic; Jessica's stance highlighted the essential aspects of what needed to be accomplished (ALACT model, phase 3). After Jessica's pressure on Kadyn to change this lesson, Kadyn created an alternative (phase 4) that is then enacted (phase 5, trial and phase 1, action). Through this exchange, a reflection that might have been surface level was elevated to the pedagogical level because the teachers

provided rationales to each other about their prior and transformed teaching practices. The reflection that occurred here among the coteachers created seeds for change in practice.

Phase I Review: Becoming a Coteaching Partnership

Before moving forward into the next stage, I now chronologically trace the main events of the three stages presented thus far. This first phase of the partnership's growth moved from the two teachers forming a partnership, into an instance of storming, all the while norming towards a shared practice. The features of norming are seen throughout the stages, particularly when the teachers evaluate each other's practices, appreciate or negotiate with the other's perspective, and resolve any differences.

In September, Kadyn observed Jessica modeling practices and voiced concerns about giving up control. She recognized the need to be able to adapt in the moment when lessons are more student-centered but rose to the occasion by participating in the inquiry of the classroom and choosing materials for lessons that incorporated real-world applications and multi-representations. During this forming stage, Kadyn reflected on where the partnership's work was headed and also voiced her priorities regarding what she believed to be important in teaching. Through this month of teaching and reflecting, Kadyn began by looking back and assessing her competencies, sharing how she felt and what she was not comfortable with. As the month progressed, Kadyn shared that part of her professional identity included a need to continue moving forward. Then, she entered the fourth phase of the ALACT model by creating alternatives based on the reflections that had transpired.

The creation of November's alternative lesson for solving equations that incorporated justification represents the outcome of the negotiation process between Jessica and Kadyn. With Jessica's focus on providing students opportunities to make meaning, and Kadyn's focus on

having the work clearly documented for students, this lesson combined the teachers' intentions and resulted in "a wonderful lesson" that "is better analysis" (see 11/7, 102P, 28:33). This case of storming provided an opening for the teachers to each share from respective features of their habitus and to merge their views into a combined and improved lesson. After two months of working together, the partnership was progressing in their work by becoming aware of the essential aspects that they wanted to focus on. The storming stage of their development included confrontations that challenged the other and revealed each individual's beliefs.

The December Standard Form lesson was the first of its kind in that it began with a problem scenario for students to interpret and model. The creation and enactment of this lesson provided opportunities for students to build their strategic competence and "sold" Kadyn on using real-world scenarios and meaning-making situations before moving to procedural practice. Kadyn's explanation to the visiting teacher of "a good representation of 'us'" described her current perception of the partnership's work. The teachers served as resources for each other to develop the Standard Form lesson, which *both* teachers enacted in different class periods, demonstrating their aligned copractice.

The upcoming second phase of the school year builds on much of what already transpired. The months of January, April, and May can be viewed as existing at the performing level of small group development. However, the characteristics of storming and norming still prevailed throughout the following stages as the coteachers continued to collaborate and cultivate aspects of their coteaching practice, further revealing their actions as resources for professional growth.

Phase II: Solidifying a Coteaching Partnership

Building a Copractice (January)

After the successful Standard Form lesson (December), a holiday break followed. Upon returning to school in January, Kadyn and Jessica embarked upon a new topic: Point-Slope Form for linear equations, which then led into writing equations of lines given two points. The month of January proved to hold a series of interactions between the coteachers that led to further instantiation of contextualizing mathematical topics for students to reason and make connections.

Instantiating the "Application First" Approach

On January 8, Kadyn reflected, "today was like a good, a good day" (1/8, 181, 3:11). After students worked on writing equations of lines given two points, which was introduced with real-world contexts, then integrated with practice, Kadyn had a positive reflection of the lesson because the cycle had been completed all the way through. As an extension of reflecting on the lesson, Kadyn and Jessica continued to reference this cycle of introducing a topic with a real-world context to make sense of it, then moving through to the procedural side. Kadyn summarized:

> K: So maybe this should be more of our plan. Right? We should plan for this every time. Right? Because I feel like every time we do something we like, miss one piece. Right? Either we miss an introduction that's meaningful or we miss the procedural practice. (1/8, 181, 3:56)

Kadyn has come to own the "application first, procedural second" strategy and has integrated it into a planning model for herself of a lesson cycle that incorporated both of these components. She was concerned about including both components in every lesson and wanted to strategically plan for this.

As January continued and the partnership planned to teach systems of linear equations, Kadyn and Jessica created a set of problems that represented different forms of linear equations in systems. An example of a problem that applied Slope-Intercept Form to systems follows:

> Will is tired of students not returning their pencils to the math class pencil box. He finds a subscription service that will deliver boxes of pencils for $3 a box after a $4 sign-up fee. Instead he could buy boxes of pencils at the store for $5 each. Should Will sign up for the subscription service or just shop at the store? Why?

A problem that applied Standard Form to systems was as follows:

> Jacob and Kevin love going to Burger King after school. They also buy food for their friends. They buy $3 burgers and $2 fries with the $18 they have. They want a total of 8 items for themselves and their friends. How many burgers and how many fries should they buy?

The full set of problems on the systems worksheet for solving by graphing and interpreting solutions can be found in Appendix C.

On January 23, the planned intention was to work with students on the meaning of solutions of systems of equations graphically using the "Will" problem above. During the first instantiation, Jessica captured in her fieldnote a "Quick switch in middle of lesson," which included Kadyn saying, "This has no meaning!" Jessica was prompted to write a "new systems sequence" in her fieldnote and during a subsequent coplanning/coreflecting meeting, Kadyn shared:

> K: I think that we should have started with a Standard Form problem, because then the answer I think makes more sense. Like here we're really, we've been doing with these two problems is comparing when the cost when something becomes cheaper, the point of

> intersection is not that meaningful. I mean, it's meaningful cause it's changing. But the actual point (2, 10) is not … revolutionary. (1/23, 202, 0:19)

Kadyn recognized that Slope-Intercept Form does not have a contextual meaning that naturally exists for the intersection of two lines: the idea of a break-even point for when the cost became cheaper had to be imposed. Instead, she reflected that "we should have started with a Standard Form problem" because the intersection represents a meaningful solution. In this exchange, Kadyn reasoned through how to make the real-world connections even *more real* and described that reasoning's impact on revising the lesson.

In the following passage, Kadyn referred to the cycle of application first, then "bridging" the ideas of the contexts to procedural practice:

> K: Well, I don't think … what we've been doing has been bad, but I don't think it bridged … it's not bridging, it's not bridging what the solution means, cause now we started talking about what solutions mean or what the solution is, right? But that doesn't, we didn't, we didn't bridge it. We didn't bridge it, but I don't know that we could have bridged it unless we started with Standard Form. (1/23, 202, 0:50)

Kadyn introduced the language and idea of "bridging" to mean students connecting ideas from the real-world contexts to the learning goal, in this case to the meaning of a solution to a system of equations. For her, Slope-Intercept Form applications do not lend as easily to bringing meaning to the intersection point itself.

In considering planning for next lessons, Jessica proposed graphing examples, to which Kadyn replied: "But they don't make sense. Doesn't have a context…. Yeah, I think you have to do it with a context first" (1/23, 202, 2:47). Kadyn was now advocating for all new topics to be

contextualized in order for the meaning of the context to then be bridged to procedural understandings.

A Revision After a Reversion

Moving into the topic of solving systems of equations by substitution, Kadyn admitted to reverting and teaching substitution in the first enactment (period 1) the way she always had: procedurally, setting one expression equal to the other, without connecting to the intended word problem. On January 28, Kadyn and Jessica reflected on not just the teaching, but on the mathematics of the lesson (Anthony, another mathematics teacher, sitting nearby listening to the conversation, interjected his assent to the ideas Jessica shared):

> K: I don't know what I was doing, right after, but yeah, go ahead.
>
> J: But why, why did we think, there was a reason we thought, or an intuition at least that we could go to substitution by *y* equals, cause they've been graphing *y* equals?
>
> K: You can see it, graphically.
>
> J: Okay. How can you see that whatever it is, *x* plus four is equal to negative
>
> K: So I think we should start it like this now. I think we should say
>
> J: Cause that's what I thought we were doing. You just kind of jumped into this same old substi-, your substitution and they can't see it.
>
> K: Well we would have let them see it if they solved it,
>
> J: On the graph, we're trying to get a backwards way in. Right.
>
> K: But, I think we should start with like a dollar, right? So one dollar is equal to, um, how many pennies? A hundred pennies. Right. And, one dollar is equal to 10 dimes, right, this should be four quarters, right? (1/28, 212P, 0:07-0:55)

The exchange continued:

J: So I, I look at the graph and I see a point of intersection and so if *y* equals that and *y* equals this, I'm trying to connect the graphic, visual.

K: You … Good. I don't know how to

J: By setting the equations equal to each other

Anthony: mm hmm.

J: So if you have *y* equals this *mx* plus *b* and that *mx* plus *b*, it's where those two lines … we're using *mx* plus *b* as a representation of the line

Anthony: mm hmm (1/28, 212P, 2:12)

In this exchange, Kadyn was not clear on the connection to the graph. Jessica and Kadyn referred to teaching so the students could "see it," which refers to this connection of $y=mx+b$ to the graph, specifically using the substitution method that $mx+b=mx+b$. In the beginning portion of this passage, Kadyn believed she had intended for students to "see it, graphically," but in the latter part of the passage, Kadyn responded to Jessica that she didn't know how to connect the graphs. After Jessica pushed Kadyn on the understanding that was underlying this lesson, it can be seen that the connection had not been clearly identified. Subsequently, Kadyn chose a real-world scenario to reason about the situation that Jessica had been challenging her on. The scenario she chose was on a graded classwork assignment that had been created for students to work on that morning. The problem that Kadyn chose was as follows:

Mora and Rena are both saving up their money. Mora has $30 and can save $20 per week. Rena already has $50 and can save $10 per week. After how many weeks will they have the same amount of money?

Referring to this problem, the following exchange occurred:

> K: Okay. So let's go back to, let's go back to our, our scenarios, our word problems, right? So even if we took the *y* equals … 20 *x* plus 30, and *y* equals 10 *x* plus 50. So this represents the um,
>
> J: Total money that Mora has
>
> K: And this is the total money that, and if we want to know when they have the same amount of money, we want to know when the amount of money that Rena has or Mora has, is the same as the amount of money that, so that to me makes more sense and then it's equal to each other. Where the total cost is the same. (1/28, 212P, 5:21)

Kadyn oriented herself regarding the algebraic substitution within the real-world context, concluding that the total cost would be the same. Kadyn continued: "The total cost of Mora equal to the total cost of Rena. But I don't think that's really teaching substitution. It's not showing them you can replace it" (1/28, 212P, 6:06). Kadyn has made sense of the situation and was working to reconcile her new orientation with her established teaching practice of procedurally replacing a variable in one equation with an expression from the second equation.

Ultimately, Kadyn reasoned the scenario through to her understanding, and confidently reported: "Cost of the subscription and the store are the same" (1/28, 212P, 8:46). She interwove her knowledge of the algebraic procedure of substitution with her understanding of the real-world application. Kadyn used a real-world context to reason through this mathematical concept, demonstrating that connecting to scenarios is a useful approach for both student and teacher learning. The reasoning that took place in this meeting provides a lens through which Kadyn's reversion in teaching substitution and acknowledgement of absent meaning in a lesson can be understood as an advancement.

After the coplanning/coreflecting meeting just described, Jessica and Kadyn chose a problem to use in the second instantiation of the teaching of substitution: a collaborative revision to the lesson. The enactment in period 5 (1/28, 213) seemed to resolve the issue that existed in period 1. A fieldnote indicated that Jessica and Kadyn were both involved in asking questions to guide the thinking of the lesson. A fieldnote also reported that students were "lighting up" and saying, "that's crazy!": they were really "seeing it" and understanding. Afterward, Kadyn reflected on the lesson, "we're presenting it in a way that has meaning, yeah. I thought it was really nice" (1/28/19, 213, 0:19).

According to a January 28 fieldnote, Kadyn said, "we can't just do elimination the old way," indicating that she was considering how to incorporate a meaning-making context and providing students with a lesson that was not teacher-directed notes with limited reasoning and connections. The January lessons demonstrate Kadyn's growing appreciation of providing students opportunities to connect meaning of applications to each new topic.

A Shift Towards Posing Meaningful Questions, Away from Procedural Directions

On January 30, Jessica and Kadyn planned a lesson on solving systems by the elimination method. In this episode, Jessica tried to convince Kadyn that in their modification of an old worksheet, the step-by-step procedural directions should be removed:

> K: But I don't think this is harmful.
>
> J: If I am following directions versus looking at the problem and paying attention to what I see here, and I'm talking, I'm referring to this idea of, I want to eliminate something. So how do I eliminate?
>
> K: Okay, alright alright alright.

J: I eliminate six and negative six. What can I eliminate? I see a, I see a *y* and a negative *y*. Okay I can eliminate that. So, let's write a sentence. Let's use space, to write …

K: I just don't, in this case I agree with you but I don't think it's bad to have extra things on the worksheet, like extra steps or instruction,

J: Do you understand what I'm saying that procedural directions versus what I want them to be noticing in the math?

K: I do. But I still think you can do both,

J: As we get down into the more involved examples. I just literally cut it [the directions] out. So this, so this is a category: use addition to eliminate. Right? And now example two is the same. Right. But now example three, you can, you can give them a, a heads up. Hey, use the opposite. Right. This is a different category of problem. So let's look at it. What do we note … Okay. Notice the *x* terms will cancel out

K: Yeah. But I just … part of me sees what you're saying, but part of me doesn't agree, you want them to notice it on their own … Just to go back to period three, right. So if they notice it on their own, [a student's] going to shout it out in about thirty seconds. (1/30, 221, 2:30-4:14)

Jessica provided a rationale for removing step-by-step algebraic directions for students to be able to consider strategies for solving. Kadyn did not initially buy-in to the removal of instructions on the worksheet. Kadyn then realized Jessica had proposed for students to notice mathematical ideas on their own and voiced her concern about students calling out when they have an idea or answer. She rationalized that there is no difference between the same one or two students articulating the idea for the class and the teacher articulating the idea. Jessica followed up that she would like to preempt students from doing that.

Kadyn then proposed analysis questions to be on the worksheet in place of the procedural directions:

> K: But you can delete them. It's fine.
>
> J: So is there a middle ground?
>
> K: No, and you're right, that, it's shit there. That doesn't, it's not really valuable. Yeah, I think that, I think that the shift is that we, we, um, let's do this. Let's do this. Take this worksheet. Uh, we, no, no, no, no. Hold on. Let me just, what do you notice about these equations? Give the actual, give the actual question. They have to then write something. We can write the answer on the board. Then they solve the pa, you have them solve onto this paper? (1/30, 221, 5:24)

Kadyn expanded on her thought:

> K: So like, I like put the, put the question that you're going to ask here. So that they can write a response. So that we can at least, and we should write the response on the board that we want them to have, so they can talk about it. Right? Then we can write it. So then if they were to look back, they at least know,
>
> J: We asked a question here
>
> K: Yeah and we noticed the difference between example one and example two, or whatever's gonna come next. Am I making sense or no? (1/30, 221, 7:35)

This shift to include analysis questions allowed Kadyn to have something written and organized for all students and allowed Jessica to have a meaning-making opportunity for students. Jessica and Kadyn worked through what possible responses to the questions would be. Kadyn tried again to push back against Jessica: "But I think you have to just show, I don't think they're really

going to discover this" (1/30, 221, 12:25). In response, Jessica continued to create questions as Kadyn worked alongside her.

Summary and Secondary Analysis of Building a Copractice (January)

This stage is the first of the performing level stages because within it, a number of occurrences rose beyond Kadyn and Jessica's evaluation of each other's practices. The language of "completing the cycle" and "bridging" was used to represent Kadyn's understanding of what the partnership wanted to accomplish: a shared goal with specific components to plan for. The set of linear systems word problems included different forms and served as a basis for much of the unit. The natural connection (or lack of) between the real-world problem and the meaning of the solutions when solving by graphing caused a switch during the first enactment and a shift in teaching in the remaining classes. Kadyn's ambiguous comprehension of the partnership's plan for teaching the systems substitution method caused a reversion in class and in a meeting discussion. Through the partnership's discussion, Kadyn's connection between the real-world scenario and the meaning of equivalence was developed, which was apparent in the following class. During the creation of the lesson for the elimination method for systems, the partnership negotiated to remove step-by-step directions, which were replaced with analysis questions. These instantiations of various types of reasoning and problem-solving strategies transitioned the partnership towards more reform-oriented practices.

When Kadyn reflected that "we should plan for this every time" (see 1/8, 181, 3:56), she was developing both her beliefs and the identity of the coteaching partnership. Based in the work Kadyn and Jessica had done from September through December, Kadyn recognized the essential aspects (ALACT model, phase 3) of the work in order to then assess the behaviors and competencies (onion model, levels 2 and 3, respectively) of what they were or were not

accomplishing. The areas in which the partnership was lacking, namely missing a piece, either the meaningful or the procedural lesson components, had to be targeted in order to accomplish what they were setting out to do. Kadyn, at this time, was acting as a resource by drawing the partnership's attention to what was missing and what the goal was to be moving forward. These actions represent pedagogical level reflections since the partners gave rationales for their teaching practices.

During the reflection on the "quick switch" where there was "no meaning" (see 1/23, 202, 0:19), Kadyn proposed an alternative (ALACT model, phase 4), which promoted further conversation regarding how to bridge the ideas or complete the cycle with connections. She had a solution to address this "incompetence" of the partnership not instantiating an effective lesson, which was to begin with Standard Form rather than Slope-Intercept Form. The clarification of this behavior (what the partnership *should do*) built the competence of the partnership's actions and informed each teacher's beliefs about their work. Furthermore, in the same meeting, Kadyn advocated for using a context for the upcoming lesson (see 1/23, 202, 2:47), demonstrating that she believed this is the behavior the partnership should be enacting by creating these types of actions. In these ways, Kadyn moved between phases of reflection (process, ALACT model) in order to inform the levels of reflection (content, onion model), considering the teaching of this mathematical topic with her new knowledge through pedagogical reflection, and developing new knowledge and beliefs about their teaching practices.

In the reflection on her teaching of solving systems of equations by substitution (see 1/28, 212P, 0:07-0:55), Kadyn looked back (ALACT model, phase 2) and was aware of her behavior and competency (onion model, levels 2 and 3, respectively) in enacting this lesson. Through the exchange with Jessica, Kadyn recognized she did not know how to connect her algebraic

representation to the graphic representation and called upon a word problem scenario to reason. By creating an alternative way of accessing the meaning of substitution graphically, rather than algebraically, Kadyn enhanced her knowledge of this topic for teaching. Empowered with this knowledge and shared understanding, Kadyn and Jessica revised the lesson (creating an alternative, ALACT model, phase 4) and enacted it (phases 5 and 1), which proved to be successful and was a result of pedagogical reflection.

Further evidence of the use of contexts in teaching and progressing teaching practices arises in Kadyn's statement that "we can't just do elimination the old way" (see 1/28 fieldnote). The confidence in Kadyn's statement represents her belief in the partnership's teaching. When discussing the removal of step-by-step directions for teaching elimination (see 1/30, 221, 2:30-4:14), Jessica *confronted* Kadyn with the perspective of providing students chances to think about and notice aspects of the problems. After an exchange, Kadyn suggested an alternative (see 1/30, 221, 5:24) where the students could write answers to the questions the teachers would ask to guide thinking. This adjustment in the activity illustrates how Kadyn has been informed by the evolving identity of the coteaching partnership. The role of reflection, in these cases, not only developed the practitioner's knowledge, but also influenced the competencies and beliefs of their teaching practice. The coteachers, working at a performance level, were now serving as resources for each other on an almost continuous basis. Their work is closely aligned with each other through shared goals and practices, also providing alignment among the layers of the onion (behavior, competencies, beliefs) towards a shared coteaching identity and state of flow.

Structuring for Student Independence (April)

As the school year progressed, April was the next stage that included meaningful conversations between the partners regarding their teaching practice. The following data

describes a divergence in teaching intentions between Kadyn and Jessica. Although the teachers had reached a level of performing, there were still differences in their priorities for students. The description and analysis below further revealed the ways in which the teachers served as resources for each other in professional growth.

In a meeting on April 3, Kadyn and Jessica coplanned the enactment of a worksheet Kadyn had created as an introduction to quadratic functions. The worksheet asked students to graph $y=x^2$ and $y=-x^2$ by completing provided tables with x and y headers (no values given), followed by the question "What can you conclude about the graphs?" (intended for students to compare the positive and negative coefficients and the direction the graph opens). The worksheet also included a function in Factored Form with the question, "What do you know about the function?" and contained space for students to use a table to determine the intercepts and sketch a graph. On the back of this worksheet, Kadyn had compiled three sample state test questions from the set of samples provided by the state on quadratics.

In the following passages, Jessica and Kadyn discussed how the material would be implemented in the classroom. Here Jessica advocated, much like she did in November, for students to actively do the work while the teachers would support, rather than lead, them:

> J: Okay, with these problems. Number one, from everything I know, they need independence, right? The more we scaffold, the less they're ever going to be able to be independent. They have to be able to think and do these things. So if we guide their thinking in any way, then it's lessening the cognitive demand. Right? If you think they can do these, then we have to let them do them in class. (4/3, 278, 0:47)

The exchange continued:

J: Doing them together, once they know that we're going to be up there and doing it, they're just going to sit back and relax and wait until it's on the board and they'll copy it down, which is not helping them with anything.

K: I don't agree with that statement, I mean you're the one who does the research, but I don't agree with that statement. What about someone who just doesn't have any idea what to do? (4/3, 278, 1:46-1:58)

Jessica tried to conceive of how students could work towards their independence, but Kadyn was concerned about students who might not have any idea of what to do. Jessica reiterated here that if teachers are leading, then students will not work independently but just copy down what is on the board. Kadyn and Jessica continued this conversation to consider what students can do on their own:

K: Well, I think, so we should talk about this [with the students]. If you're stuck, start making a table of values. Almost every single question. Start putting some stuff on the table.

J: Yeah, so general, general strategies that we can list separately. Right? But not doing that problem for them,

K: But, but I guess what I'm concerned about is I want to go ov-, when you say doing the problem for them, is that different from going over the answers to the problem? I don't, I don't understand …

J: (Sigh) It is different. What I was just saying that there has to be some motivator and accountability measure for them to really put effort into it.

K: So then do we just try on the, on Friday we just give out the packet and we say, get as much done as you can get done. If you're stuck on something go to the next question.

> Work with a partner. You have one person to work with. You do your best. We can give little hints here and there, we collect it, but then we go over it the next day. Okay. So is that a plan? (4/3, 278, 4:24-5:07)

Kadyn provided the strategy of "making a table of values" as one way that students who are stuck can move forward on their own and Jessica began thinking about other possible general strategies. Kadyn continued by discussing what she imagined would follow in the lesson: after students attempted the problems by making tables of values, she would want to review them. Kadyn asked about the difference between going over answers to problems and providing the answers to problems. Jessica had pushed Kadyn to make this distinction and to consider how students can become conditioned to knowing they can "sit back" and get the answers eventually. Jessica wanted students actively doing the problems, knowing that they have to put in the work and the teacher will not eventually just do it for them.

After the exchange above, in the data, Jessica reiterated that she did not want the teachers to do all the problems for students. In response, Kadyn shared how she learned mathematics by using the answer key and working backwards. Jessica then attempted to solidify the agreement between herself and Kadyn that students need to build independence:

> J: These kids are missing some of those, not just skills, but habits, of thinking.
>
> K: It's a, it's a habit.
>
> J: Habits of thinking.
>
> K: Yes I don't even think that you're strong at math or not strong at … But you're asking them to do it without …
>
> J: Without anything. They need some, some resources, but, but we're assuming that they have learned this, this, content and so it's a transfer. It's taking what they know and

> transferring it into a new situation that they have to interpret. It's the interpretation that is like the barrier for them. They don't have access. And that's, that's what we have to scaffold. Yes. There's, that's the first level.
>
> K: It's also having the skill, yeah. A kid like [student] can do the transfer, but can't do the skill. (4/3, 278, 13:48-14:28)

Jessica believed that the barrier for students was in the interpretation of the situation, identifying the problem and classifying what they have to do, and if they were able to do this, they could then bring in the needed skill(s). Kadyn believed, as seen through her repeated defense, that students lacked the skills to be able to be successful in mathematics. From this conversation, Jessica and Kadyn had the idea of partnering students to work with one other student for support, which carried forward into their shared practice the following school year.

Summary and Secondary Analysis of Structuring for Student Independence (April)

The divergence of perspectives among Kadyn and Jessica regarding the balance of teacher to student work during learning demonstrates the differences in priorities for the teachers. Kadyn suggested making tables of values as a strategy for students to access questions, along with working with a partner to move through a set of questions. These alternatives as a response to Jessica's wish for the teachers to not actively do the problems for the students became common practice for the partnership.

As Jessica pushed Kadyn on this issue of structuring for student independence, even if students do not have ideas about what to do (see 4/3, 278, 1:45-1:58), their beliefs as teachers (onion model, level 4) were called into question. As Kadyn brought up these essential aspects of teaching (ALACT model, phase 3), that students require teacher support to some extent, she served as a resource to push back on Jessica's challenge. Kadyn suggested the strategy of making

a table of values if a student is stuck (see 4/3, 278, 4:24-5:07), an alternative that became prominent in their practice after this moment. Jessica and Kadyn's conversation next moved to weighing the factors of why students cannot work independently (see 4/3, 278, 13:48-14:28). Each of these issues served as pedagogical level reflections since the teachers were providing rationales to each other regarding their teaching intentions. With the coteaching partnership's identity fairly solid at this stage, they were able to engage in evaluating these beliefs, and to create alternatives in their teaching.

Instantiated Lesson Re-sequences the Teaching of Quadratics (May)

The final stage of the school year that included meaningful discussion between Jessica and Kadyn regarding their teaching practice was in May where they focused on quadratic functions. As a third stage at the performing level, the data presented below describes additional ways that the teachers acted to serve as resources for each other's professional growth.

Jessica proposed to Kadyn that they use a task she had been developing over many years to introduce quadratic functions through a real-world scenario that builds on student understanding of linear functions. Jessica had developed the task during graduate school and had refined it through a few enactments in her own previous classes. The task posed the following scenario:

> Four hundred people came to last year's winter play. The ticket price was $5. This year, the Drama Club is hoping to earn enough money to take a trip to a Broadway play. They estimate that for each $0.50 increase in the price, 10 fewer people will attend their play.
>
> How much should the tickets cost in order to maximize the income from this year's play?

The worksheet, included in Appendix D, then asked questions about the scenario to ensure student understanding of particular aspects of the problem, followed by a table of values that

provided an incremental change of fifty cent intervals and columns to complete the resulting ticket price, the number of people attended, and the income. After students analyze the situation numerically, the questions ask students to model the scenario algebraically and graphically, while incorporating analysis questions.

In this study, the task was first instantiated on April 29 for all classes, which Jessica led for students to initially analyze the scenario and begin the table of values. On April 30 and May 1, a second lesson of working through additional table values was led by Jessica, with Kadyn's additional input in certain places for students to be supported as needed. At one point, Kadyn looked for patterns in the table with the students, an activity that was *not* part of the specified question sequence, but that demonstrated Kadyn's engagement with the problem scenario. In a fieldnote on April 30, it was noted that Jessica and Kadyn had not recently met, and Jessica wanted to get a sense of Kadyn's view of the task and the implementation of the task across the four classes. Also, in this fieldnote, it was documented that Kadyn asked what would come the following day, after which "we agreed [to] move into graphing using Factored Form." It was not yet clear what the sequence following this would be, but graphing in Factored Form was a natural next step after engaging in this quadratics task.

The extension of this quadratic task allowed for many Performing level aspects in the coteaching partnership. On the third day of enactment, in the final of four instantiations (5/2, 307), Kadyn said to Jessica: "I pulled a 'you' right there!" because she kept asking "Why?" (Fieldnote, 5/2, period 7). Also, during this class, Kadyn said, "I'm not going over this, ok? If you're not getting it come for extra help" (5/2, 307, 17:20). In a fieldnote after Kadyn says this, Jessica wonders: "Is this ok for her? Did I influence this at all re: we can't do everything for them?" Jessica's intention to promote student activity and engagement, and not leading the class

by reviewing every step of every problem was, in this instance, part of *Kadyn's* practice. Kadyn stated to the class: "Tell your partner how he figured that out. This is a really big conceptual idea" (5/2, 307, 25:05). Kadyn focused on the big idea at hand rather than the procedural steps required to achieve an answer. Each of these actions by Kadyn exhibits her progress towards integrating reform-oriented instructional practices in her teaching. The quadratics task provided an opportunity for Kadyn to connect prior practices of the coteaching partnership to this new scenario.

On May 3, the fourth day of enactment of this lesson, Kadyn and Jessica held a coplanning/coreflecting meeting on the task enactment so far, the sequence of topics in quadratics stemming from this task, and where exactly they would head next in their teaching. In this post state-testing atmosphere, the teachers had a freedom and comfort level that allowed them to prepare students for Algebra 2 and to explore the possibilities that this quadratics task raised.

> J: Yeah. I think, I think I feel good about this. Um, I've never used this approach, you know, Factored Form. Um, so do you feel like you want to go to fact, we're going to graph in standard or we're going to go to … factoring? We're gonna give them Standard Form and say, what's the problem here, we don't have factors.
>
> K: Yes. How can we do this?
>
> J: And then learn to factor. And then graph in Standard Form by factoring.
>
> K: Yes, by doing them, I'd like to do five steps on that. Like *x*-intercepts, *y*-intercepts, vertex
>
> J: By factoring, all factorable, but still using, trying to connect to the ideas of Standard Form with the *y*-intercept.

K: I think perhaps after we really did a nice job of that, then we can give them one that's not factorable, what do we do with this.

J: And then we go to negative B over two A. (5/3, 309, 1:37)

Jessica posed either graphing in Standard Form or factoring to Kadyn, and they agreed that a move to a presentation of Standard Form could motivate the use of factoring to find intercepts in the same way they had been doing recently. Jessica and Kadyn were cothinking in this exchange in their resequencing of the subtopics of teaching quadratic functions. They were aligned in the way they were logically progressing to each next topic, informed by the prior steps that had been accomplished, unlike in earlier stages when one teacher led the progress, or the two teachers were storming.

The types of questions that Kadyn posed, such as, “How can we do this?” and “What do we do with this?” are more open ended and are evidence of Kadyn’s shift towards more analysis-level questioning. Furthermore, Kadyn integrated her prior knowledge into this new sequence by referring to the “five steps” she wanted to do. In her work with Jessica, Kadyn had held on to her own practices, and incorporated them where she saw fit into the copractice.

During the End of Year Interview, Kadyn reflected back on this series of lessons as different and impacting her view of teaching quadratic functions:

K: So first of all, I never really taught Factored Form. So this was … I never started with Factored Form, I never emphasized Factored Form, I always started with Standard Form and went to Factored Form, but I liked it and even in the PBA you can see it explaining what each piece of the Factored Form represents, this is my initial price. This is my change in the price. That was great. (6/21, 340, 12:30)

Kadyn reiterated and reflected on the discussion from May 3 in that the quadratics function task disrupted the sequence of teaching quadratics that she had been developing over the past ten years. She connected the relevance of Factored Form to the meaning in the real-world applications and to student opportunities to understand this meaning in their Project Based Assessment.

Summary and Secondary Analysis of Resequencing Quadratics Topics (May)

The Drama Club task for quadratic functions provided a rich scenario in which students engaged in various reasoning strategies. As Kadyn worked with students to engage with the concepts within the task, many instances were captured of her instantiating reform-oriented practices. The task also activated a new, naturally occurring sequence for moving forward in specific quadratics topics. The coteachers were able to easily coplan their new sequence based on their common knowledge and shared practice.

In this stage, there were no instances of storming within the coteaching partnership. This absence of confrontations and discrepancies between real and ideal forms of their teaching highlights that the teachers were able to meet situational demands of their context using their inner capacities, that is, they had experienced *flow*. The task applied and connected preceding and upcoming material, was a scenario for teachers and students to connect interpretations to understanding, and created a situation in which the teachers dialogued to evolve their craft knowledge regarding the teaching of quadratics. As the onion levels became aligned, the partnership was able to connect demands of the environment with their behaviors and competencies, which also aligned with each teacher's beliefs and professional identity as they had been established in the coteaching partnership.

Phase II Review: Solidifying the Coteaching Partnership

While Phase I described the beginning months of the school year, which included stages of forming, storming, and norming, the events of Phase II were primarily at the performing level. This section will retrace the significant findings of each month of this phase as an extension of the Phase I review to provide a concise overview of the coteaching partnership's teaching practice and reflections over the school year.

The month of January held a number of opportunities for Kadyn and Jessica to serve as resources for each other and to build their craft knowledge. When solving by graphing, the partnership learned the importance of the form of the linear equation for connecting to the real-world context. At this time, Kadyn also reflected on the importance of planning for both the meaning-making and procedural components of a lesson to "complete the cycle" and eventually "bridge" the components. This reflection process on the partnership's competencies and behaviors promoted the development of their knowledge for teaching this topic. When teaching solving systems by substitution, the initial lesson plan lacked clarity, which prompted Kadyn to revert to her prior knowledge for teaching this topic. In her reflection on this reversion, Kadyn called on a real-world scenario to reason about the meaning of substitution and eventually used her new knowledge to enact an improved lesson. When creating material for the teaching of solving systems by elimination, Jessica and Kadyn reflected on the inclusion of step-by-step directions on worksheets, from which Kadyn provided the alternative of replacing them with analysis questions for students to make mathematical meaning on their own. The increasing alignment of behaviors, competencies, and beliefs in this stage created the higher level of performance of the coteachers.

In April (pre-state test), Jessica and Kadyn discussed building student independence while also strategically supporting students. Jessica advocated multiple times for the teachers to not work out the problems nor do the heavy (cognitive) lifting for students, and Kadyn struggled to align with Jessica's position. This exchange brought the teachers' beliefs to light within their practice by citing specific instances of teaching and evaluating the consequences.

During May (post-state test), the enacted quadratics task initiated a new progression for teaching subsequent unit topics. The task combined components of previously established lessons (December's Standard Form task structure; April's make a table of values strategy), which led Kadyn to easily appreciate the value of instantiating it over multiple days. As a result, Kadyn focused on conceptual aspects of the scenario with students and voiced how each subtopic that followed was motivated by these aspects. In this later stage of the school year, without constraints such as the timing pressures of the state test, Kadyn was open to and flexible with the implementation of the quadratics task and the topics that followed.

During Phase II, Kadyn no longer solely observed Jessica leading the class but participated regularly in the creation and enactment of lessons that included opportunities for students to interpret, analyze, and model. Kadyn was no longer unsure of where lessons were headed, becoming confident in reasoning through applications and connecting conceptual and procedural understandings. Although fewer in number in Phase II, the coplanning/coreflecting meetings more often generated meaningful, productive discussions regarding the lessons that had been or were going to be enacted. The coteachers had successfully negotiated their way to a truly collaborative practice where new issues for consideration arose in practice.

Shifts in the Coteachers' Practice

Thus far, this chapter has presented the main events of the stages of the school year revolving around pivotal moments of the year, which mostly centered on the creation and enactment of lessons that were different from each coteacher's prior practice. The events of the stages of the year described the evolution of the coteaching partnership to reveal how the teachers' actions served as resources for professional growth. Based in the events of the year, this section will focus on three shifts that emerged regarding the partnership's teaching: (1) a resequencing of mathematical topics within units, altered from sequences based in Kadyn's prior eight years and Jessica's prior five years of teaching Algebra 1, (2) the development of the thinking regarding a planning model for interweaving applications and procedural practice, and (3) Kadyn's view of teaching practices connected with experience-based realities of teaching freshmen Algebra 1, particularly with the identified population of this study.

The structure of this section and the data presented below derive from the End of Year Interview of Kadyn, and are largely in Kadyn's words, but represent the work of the partnership as a unit. Throughout the year, Jessica and Kadyn worked as partners, listening to each other's concerns and intentions regarding teaching, and taking each other's ideas seriously. In addition, rather than responding with advice or direct instruction as a coach or supervisor would, Jessica built on Kadyn's ideas within the respectful partnership's teaching practice.

Real-World Contexts Influence a Resequencing of Topics

In December, when Jessica and Kadyn agreed to leverage student knowledge of intercepts to teach Standard Form of Linear Equations (see 12/11, 156), it was a purposeful move that generated a resulting resequencing of topics. The lesson connected to additional student prior knowledge, given the context, such as coefficients and constants. This order of topics was

different compared to the way either Kadyn or Jessica had previously, individually taught Algebra 1.

In a similar way, in January during the teaching of Systems of Linear Equations, Kadyn recognized that using Slope-Intercept Form contexts "has no meaning" (1/23, 202, 0:19). She further concluded that the meaning had no possibility of connecting for students unless Standard Form came first, since it naturally provided a meaning for the point of intersection.

The Quadratics Function task in May, which was taught over multiple days, utilized Factored Form so that students could build a quadratic equation from two linear components. Since students began graphing quadratic equations in Factored Form due to this real-world context, it was a logical progression to motivate factoring of Standard Form next. In the May 3 exchange (see 5/3, 309, 1:37), Kadyn and Jessica discussed the topics that would follow and easily reorganized their thinking around the sequence they would pursue as coteachers.

During the End of Year Interview (6/21, 340), Kadyn discussed how they had recognized a shift in the (PARCC-like, reform-oriented) state test questions for mathematics:

> K: It's that every question on the test was an application problem, nothing was "Graph a line," nothing was, and we had to, we have to do more of that. If you're going to do more of that [application problems], you might as well start it out like that, like lots of comparison of linear and quadratic and exponential right. So we should be talking about this every day. (6/21, 340, 16:16)

Influenced by state test scenarios, Kadyn shared that the partnership had resolved that comparing the three families of linear, quadratic, and exponential functions should be a high priority in Algebra 1. Kadyn also connected these underlying areas of focus to how she and Jessica would begin the following school year:

> K: Because we kind of talked about how we want to start the year with the introduction of three different kinds of functions and try to develop deeper understanding of why these things happen. Why they behave this way and what they can be used to model. (Kadyn, 6/21, 340, 15:38)

Not only did Kadyn share that they aimed to continually focus on the function families throughout the year, but also, they intended to attempt to help students build understanding of how the families behave from the very start of the year, serving as a foundation to the course.

The instances of topic resequencing described here, for units of study and also for the year as a whole, illustrate one impact of the real-world contexts on the coteaching partnership's teaching practice. Kadyn shared how the types of state test questions influenced the partnership's work and what the coteachers talked about to reframe the course during the following year. Jessica participated alongside Kadyn in these discussions and the coteachers, together, developed new ideas about teaching Algebra 1. The two instances of data presented here were both pedagogical level reflections because Kadyn reflected on the *reasons* they should re-sequence the topics in their teaching. Although Kadyn and Jessica had been teaching high school mathematics for over a decade, their experiences and pedagogical reflections such as these reshaped their thinking about how content can be organized and sequenced for students in a more meaningful way.

Applications First, Interwoven with Procedural Practice

During the year, and discussed in the End of Year Interview, the cycle of leading with an application, then connecting the concepts to more procedural skills, was a prominent theme in the partnership's work. At the end of the year, in response to a question that asked her what she thought was different during this year from what she had previously done, Kadyn said:

> K: Oh well obviously the application stuff, I really put a focus on that … I do think, we kind of found our groove, like start, start with an application problem. … I don't think we've had success marrying them yet. We have the intention of marrying the application with the skill but it's still, we gotta work on that next year. (6/21, 340, 1:11)

Kadyn reported that she thought as coteachers, she and Jessica had figured out a good place from which to work, their "groove," but still had to figure out the integration of application and skill. Additionally, as discussed in the resequencing of the year (see 6/21, 340, 16:16), Kadyn recognized that the state test had shifted to include many application problems, not only skill-based questions. The reference to this shift supported the work that Jessica and Kadyn were doing to interweave different strands of mathematical proficiency (NRC, 2001). The instances of the generation of this "groove" and the partnership's attempts to "marry" application and skill are presented in this section.

As described in the December stage, in the creation of the Standard Form lesson, Jessica introduced the idea of first exploring a topic to understand some level of meaning for it, then moving to procedural fluency (see 12/7, 150, 0:27). In this meeting, Kadyn identified with Jessica's idea by referring to project-based learning on a small scale. In the same meeting, Kadyn followed up by saying, "I feel like we have to do a better job balancing both of these, because I feel like the way far to be exploration where like five kids get it," to which Jessica responded, "Marrying them. Right. Right. So they have to be integrated. So that's our new model" (12/7, 150, 3:17). Kadyn wanted a balance and was concerned about "the way far" exploratory activity that most students will not understand or be able to make meaning of. Kadyn identified that her priority is to make sure all students understand the mathematical work and can be successful, she does not want only five students participating if the activity is overly exploratory. Kadyn

recognized that the application and the procedural practice have to be "balanced," while Jessica used the phrasing of "marrying" and "integrating" them. Together, Jessica and Kadyn began making sense of how to integrate exploration for conceptual understanding and practice of procedures. This first conversation about targeting both application and procedure in their lessons began the partnership's evolution towards creating a balanced approach of these two areas as a negotiation of the two teachers' priorities.

In January, Jessica and Kadyn reflected on a series of lessons that provided students experiences with real-world connections, then practice for writing equations of lines given two points. Kadyn voiced that she would like to plan for each of the pieces to be integrated every time (see 1/8, 181, 3:56). Additionally, in January, Kadyn referred to "bridging what the solution means" (see 1/23, 202, 0:50) from a given real-world scenario to what the solution of a system of equations is. Now that Jessica and Kadyn were sharing a practice of purposefully incorporating both applications and skills for each mathematical topic, they worked together to strategically connect these different perspectives for the students.

In May, Jessica and Kadyn reflected on the enactment of the Quadratics Function task, the extension they had just taught for graphing in Factored Form without a context, and how to connect it more for students:

J: Today, like the most important thing, because it's the bridge, right.

K: Yeah. But we didn't connect it to the other day either. Uhh, yeah.

J: Right. So I'm trying to tread,

K: I think the bridge is going to be two days.

J: Oh, it's multiple days, this takes time.

> K: Like we need to do a whole 'nother day of just this. And maybe put in the analysis, I didn't look at the questions that are on the back. (5/3, 309, 0:01-0:20)

Jessica and Kadyn were aware that they were progressing into the bridging stage of this topic, after exploring a real-world scenario, wanting to provide students the ability to connect the meaning of the scenario to the decontextualized problems they were now working on. Both teachers recognized that it would take time for students to make these connections and that they would have to "tread" lightly to carefully make such connections. Kadyn also thought to integrate some analysis level questions to support the connections. The coteachers worked together to continue to make sense of how to bridge the conceptual and procedural aspects of lessons.

At the end of the year, Kadyn reflected on their instructional practices after the Factored Form graphing lesson:

> K: This last PBA [Project Based Assessment] would have been a perfect trans[ition], if we had showed them the income problem and then showed them graphing in Factored Form, which we did, before we taught factoring and then given the PBA, I think that would have married it, maybe just a little bit better, but instead we kept going … So I do think that that's the piece … that does, requires you to think about a scenario and requires you to do the skill … I just feel like you [Jessica] have focused more on the app, on the application, on the making them think. And I can't wrap my head around getting rid of the skill yet, and I want them to have the skill and. We, I don't think we succeeded. Right. Like but I think we have the pieces in place, to be successful next year. (6/21, 340, 1:56)

Kadyn attributed the poor timing of the PBA assignment to students not having the opportunity to make connections between the quadratics task and the decontextualized graphing. Her

proposal for moving forward into the following school year was to assign students the PBA that would do the work of integrating application and skill in a timely manner, which she believed would be successful. Furthermore, Kadyn voiced that she aimed to do more of these types of assignments, but on a smaller scale, to provide students the opportunities to make connections. In the End of Year Interview, Kadyn stated: "Kind of developing, like, I don't want to call them all PBAs, but more of these things, it could just be graded classwork assignments, that are bridging right, they are bridging the things together" (6/21, 340, 4:28). In working with Jessica, Kadyn interpreted for herself the types of activities Jessica had integrated by stating intentions to integrate them more regularly into practice. Jessica developed in her understanding of how non-procedurally focused activities could be utilized with students.

In her quote above (6/21, 340, 1:56), Kadyn also attributed Jessica's goal to be only about making students think. Kadyn struggled and was uncomfortable with the idea of not teaching the skill. Kadyn had been considering this issue on her own: she reported having a phone conversation with her brother, who is an engineer. She asked him:

> K: Do you need more … thinking and analyzing or do you need more of these skills? And he's like, I analyze things every day. I don't do math every day, but at the same time … Cause I was like, I want you to really think about this. You've taken all of these calculus courses, you've done all of this stuff and you know, what do you need? And it's not having strong skills, math base, gonna inhibit you from being able to think analytically about some of the stuff that you do. … He's like … you can't do this until colleges do this because otherwise you're providing a disservice to your students. And that's, I think still how I feel and that … I would feel like I was providing such a disservice if my kids were not prepped for SAT math. (6/21, 340, 4:54)

Based on this conversation with her brother, and her work with Jessica, Kadyn considered her personal position on the goal(s) of her job: she recognized that her work exists within the larger realm of society, in which there are assessments that are gatekeepers for college and career. Her aims in her daily work regarding the integration of different perspectives presented in real-world contexts, analysis questions, and procedural decontextualized problems, and ensuring that students have the skills needed (for success on the SAT), reflect this articulated position. Kadyn's experiences, such as this conversation with her brother, illustrate the ways in which she viewed her interactions with Jessica and their work in teaching. As Kadyn and Jessica spent the school year integrating analysis questions as a reform-oriented practice, Kadyn also considered the consequences of their work in additional ways such as conversations with her brother.

Kadyn's views of the types of mathematical proficiencies students should have, as well as her views of the benefits of learning mathematics, informed her dispositions as a teacher and her approach in working with Jessica. Kadyn spoke about being able to do skill-based problems, and the value of the ability to learn mathematics in the End of Year Interview:

> K: Well, I still very much believe in the fact that if you can learn how to do math, that you are very well respected and it opens up some doors for you and you can do, you can learn anything. And, um, I think there's something to be said for teaching them about taking notes and staying organized and looking back at your notes and referring back to your notes and then thinking about removing all of the applications. I know it's kind of dry. I know people don't like it, but the ability, I do think that [student] has come a long way this year and she does, she does a problem. She goes back, she tries it again. She finds her mistake. Like, just thinking, just trying, just trying to be organized, just thinking out how is this problem different from the other problem and how am I supposed to go

> about getting into this problem…. I do think it's a skill, that does transfer to other elements of your life. Like the ability to just, I almost want to say persevere, look at something, learn about it and then try it again…. I still think if a kid can learn how to learn math, you can learn lots of things. (6/21, 340, 9:14)

Kadyn referred to learning situations detached from real-world contexts ("removing all of the applications"), recognizing how these can be difficult and "dry." But she valued the thinking, organization, and learning from mistakes that come along with doing these types of problems. She also recognized problem comparison, which is more of an analysis level of the structure of problems, as one way to view decontextualized problems. In this passage, Kadyn connected qualities of doing mathematics with the ways students work, such as referring back to notes or persevering, for which I will present more related data next. Kadyn's views of mathematics and of learning mathematics inform her teaching practices and the dispositions that she brought to the coteaching partnership. This analysis of data from the End of Year Interview continues to inform Kadyn's dispositions that influenced the actions of the coteachers.

Kadyn's Teaching Focus on Students

As a teacher, Kadyn believed in the value of learning mathematics and wanted her students to learn *how to learn* mathematics, as in the previous passage (see 6/21, 340, 9:14). Also, as a teacher, Kadyn wanted to support all her students in learning mathematics in the best ways she knew how. This year of working with Jessica as a coteacher disrupted Kadyn's prior status quo teaching practices and opened her to alternatives that required consideration and evaluation. As a result, the new alternatives that were created impacted the partnership's teaching.

During the End of Year Interview, Kadyn was asked if she thought anything that they did during the year translated into her teaching of calculus, her solo teaching, to which she responded: “Yeah. I am going to try to do more, like, I don't know, don't want to call it investigative, but some more thinking and not so much shoving” (6/21, 340, 18:40). Kadyn was asked to say more about how she might accomplish a focus on student thinking and less teacher-given explanations:

> K: Having some better activities or even just having some better discussion questions…. This year when I did Riemann sums I think … I butchered it, but how do you know if it's an overestimate or under, this is a very simple scenario. How do we know it? Does concave matter? Does increasing, decreasing matter? … So just trying to let them feel it out and come up with a conclusion, come up with a justification, um, you know, like skills once again, the skills that transfer to other areas … here's what I think, here's why. Agree or disagree. And the AP, the AP stuff that's on here … it has some nice questions already about when it's like … and not in a context, like not, I don't have to go into a context. I just have to come up with questions that are a little bit more open, not quite so “Find the limit,” but “What do we know about the limit?” So more open ended, I guess. (6/21, 340, 19:31)

Kadyn seems to have shifted her perspective toward one that engages students in thinking about mathematics. This has become a more central principle in her teaching. She claimed that she “butchered” teaching Riemann sums, which should have been more focused on how the students may know something to be true. Kadyn shared her future intentions for allowing students to “feel it out, … come up with a justification” and “agree or disagree,” providing them opportunities to analyze and think, not *just do*. This move to more open-ended questions in her solo teaching of

AP Calculus represents her desire to transition from her students being able to answer a question to being able to describe what they know about a topic of study.

Next in the interview, without being prompted, Kadyn compared the populations that she was teaching, namely freshmen in Algebra 1 and seniors in Advanced Placement (AP) Calculus:

> K: I think, so, in my mind, I feel like I'm working with two very different populations. … But with Algebra 1, we have kids that are not into math and don't, don't care and they need a con[text], they need it. They need to relate. They need to be able to relate it to something that might be important to them to have any chance of retaining it. Whereas AP Calc, they're going to retain what I teach them because they are our academics. They're, it's not the right word. They want to succeed academically. They're going to retain, they're going to learn. But I have to challenge the thinking, like if I challenged the Algebra 1 thinking without context, I'd have [one student] and no one answering the questions. (6/21, 340, 20:54)

Kadyn's experiences in working with each population of students had informed her thinking regarding who would respond to the types of questions she intended to integrate in her teaching (of AP Calculus). Many students, particularly the Algebra 1 subpopulation, may not have had prior experiences engaging in this type of thinking: previous mathematics classes may very likely have asked students to passively take notes so that this more active interaction could be a new experience for many. The norms to engage in reasoning had to have been developed for students. Kadyn found that contexts were relatable for the Algebra 1 population, for whom she previously had seemingly not found an inroad to extend their thinking. Kadyn's acceptance of using real-world contexts with Algebra 1 students informed the coteaching partnership's subsequent actions and preparation of teaching materials. The following sections build more on

this idea of Kadyn's understanding of students and ways to support students in Algebra 1 with their success in mind.

Structuring Teaching to Support Student Engagement

Kadyn's work with Jessica throughout the year was cultivated by her dispositions that existed as a result of her craft knowledge and habitus. The following portions of data focus on Kadyn's sharing of her views of teaching Algebra 1 students, which she brought to the coteaching partnership and utilized in her work with Jessica to act as a resource.

In November, when considering ideas to create the alternative worksheet, Jessica proposed using incorrectly worked out problems, to which Kadyn replied:

> K: It's too hard for them, because then they're going to be arguing about 17 million different things and like period seven and period five, they're all going to be arguing the wrong thing is right. I don't, I don't, I'm not comfortable with that. You know this already about me because then I feel like the people that are not listening all the time are hearing these things that are not right and that's what they're remembering. And they should be listening all the time. (11/7, 102P, 23:25)

Kadyn was very concerned about students who may not be paying close attention, hearing or copying down incorrect information. This passage speaks to Kadyn's discomfort from earlier in the school year (see 9/12, 5) about incorrect information in the mathematics classroom and a rationale for not engaging in discussion about incorrect ideas. Additionally, echoing the end of September's need to have the important aspects of a lesson written down (see 9/26, 42, 3:58), Kadyn stated: "I want them all to have perfect justifications" (11/7, 102P, 39:55).

When teaching systems of equations (January), Kadyn considered how students think and work:

> K: But I also think maybe we have to teach them how to, how they should be doing this outside of class. Like a focus on like you were doing. I was just giving it to them. Look back at your notes. Like, what did you do here? Why did you do it? So this is another skill that we have to be integrating in, which we have kind of been doing here and there. But like, I almost feel like my latest revelation has been, they cannot do it because they cannot focus. (1/8, 181, 0:39)

Kadyn recognized that the students may need to hear an explicit articulation of the strategy of looking back at notes as reference. Much like the previous passage in which Kadyn said students have to be listening all the time, here she is saying that students are not able to remain focused: this ability to stay on task is a concern that stands out in Kadyn's teaching of this Algebra 1 population. Based in this concern, the way that Kadyn prioritized the structuring of materials for teaching with Jessica informed the coteaching partnership's work throughout the year.

As an extension of the previously shared passage (1/30, 221, 2:30-4:14) in which Jessica advocated for removing step-by-step directions, Kadyn shared:

> K: Sometimes though I think it's nice for them to have things written down for them, it's almost like reading a textbook, but not reading a textbook, because it's also a different skill as to looking back and then like showing you something and then seeing it as opposed to coming up with it yourself and seeing it. So in my mind, the top kids are going to see this and they're going to answer the question when we ask them to answer the question, the mid to low and low are not, they're just going to sit there and wait. They, maybe some of them are going to engage. But I guess, I guess, I guess where I'm feeling now is that I'm frustrated by these kids that have little to no skill and are continually failing. So I think that, maybe, maybe I do want to have a second worksheet, but, that like

> helps them. But I think that for them, the skill might be like, when I looked back into my notes, which are disastrous, at least I can see that this is what she was talking about because I was not paying attention. I was tracing my protractor on my paper into a smiley face. (1/30, 221, 5:24)

Kadyn differentiated between the “top kids" in the class who would understand and be able to answer versus the mid to low students who would not; she conveyed frustration for these and was trying to figure out how to support these students. Some students may be able to discover ideas on their own, but Kadyn argued here that to reach all the students in the class, she had to “show the idea” so each student can work to see it for themself. She then argued that it is worthwhile for this subpopulation of students to have “things written down for them” since they may not have been paying attention, so they may refer back and work to understand using the notes. Kadyn was concerned that students who do not pay attention and may also have “little to no skill” are at high risk of not learning the material because it does not come easily to them and they would not accept the supports provided to them. Therefore, a focus in coteaching is to strategically structure lessons and ideas for students to engage in learning the material.

During this meeting on January 30, Kadyn advocated for the examples and explanations to be written down for students so that they could then make sense of them rather than “coming up with it” (see 1/30, 221, 5:24). Kadyn continued to advocate for this in the creation of the lesson:

> K: How can we turn this into two equations with two unknowns into one equation with only one unknown? But I think you have to just show, I don't think they're really going to discover this, or they're going to know from last year. (1/30, 221, 11:58)

Rather than an exploration or set of analysis questions, Kadyn felt that the teachers just had to demonstrate how to eliminate a variable by adding two equations with a pair of opposite coefficients for one variable.

Jessica and Kadyn discussed a similar issue in April regarding how to build student independence (see 4/3, 278). Jessica encouraged Kadyn to not just do problems together as a class, perhaps instead students could do them with a partner because otherwise they would "sit back and relax and wait until it's on the board" (Jessica, 4/3, 278, 1:46-1:58). Kadyn questioned what they would do if students did not know what to do: this issue of not doing the work versus not knowing how to do the work treads a fine line regarding student engagement. Following this discussion, Kadyn and Jessica continued:

J: Like the explanations today. That was great. Right?

K: Not period seven.

J: Why?

K: I had two kids that knew what they were doing. Nobody else had a good explanation. (4/3, 278, 2:20-2:43)

Jessica then worked to convince Kadyn that there were good explanations, citing particular students, and helped Kadyn to understand that it is not necessary for every single student to engage every single day. Instead, providing instances where students could engage in more advanced thinking could eventually build to the expectation that every student should be paying attention and should be doing the work on their own or with a partner and not relying on the teachers to do the heavy lifting. This time in April held underlying pressure while the teachers were trying to prepare students for the state test. As a result, Kadyn likely wanted everyone on

task, giving good explanations, and understanding the material she believed they needed to be successful on the test.

In this section, Kadyn referred to her wish for students to have the classwork well-documented for themselves so they may refer back to completed examples and justifications. Although Kadyn had transitioned to a practice where she wanted students to engage in more advanced thinking, she still wanted to support their engagement and learning by requiring organization of their work. Kadyn also resolved that students would not create mathematical processes on their own (such as the elimination method for solving systems), and it was worthwhile for students to write down the process, then work to make sense of it themselves. This debate between Jessica and Kadyn points to the balance of the different types of reasoning required to realistically support students in learning mathematics and to the constant negotiation that the coteachers participated in, which informed the development of their partnership.

Summary and Secondary Analysis of Shifts in the Coteachers' Practice

By working together, Jessica and Kadyn modified their individual teaching practices in order to integrate into a shared practice. In turn, the shared practice reflexively influenced the teachers' individual practices. The data presented in the previous section were primarily about Kadyn's focus on students throughout the year, which provide insight regarding her dispositions that informed the coteaching partnership. Kadyn and Jessica's individual perspectives and priorities directed their work with each other, creating new possibilities in their collaboration. The coteachers engaged in creating and applying the "applications first, procedural second" model, reflected on the creation of lesson materials and their instantiated lessons, created alternatives for teaching, and re-evaluated the constraints in which they worked.

One consequence of Jessica and Kadyn's partnership and "applications first" approach was a resequencing of topics in the various units of the year. From the sharing of the teachers' resources with one another, their craft knowledge regarding these topics was advanced because the teaching and reflection that took place was situated, relevant, and informed by multiple sources. The state test served as an environmental constraint that impacted Kadyn's focus on the sequencing of teaching Algebra 1. As Kadyn stated in the End of Year Interview (see 6/21, 340, 16:16), "every question on the test was an application problem," in turn influencing how she wanted to approach the course. Through this reflection, Kadyn reoriented herself among the partnership's work, the state test, and the foundational topics for teaching Algebra 1.

Kadyn discussed her beliefs about the benefits and strategies for students to learn how to learn mathematics. She stated that there are benefits for students' futures in knowing how to learn mathematics regarding problem solving, persevering, and looking back and using resources. These discussions spoke to her belief that her job as a teacher was to provide clear, thorough resources to students for their understanding and reference. She saw these as requirements of her role, and therefore, these ideas were part of her professional identity.

While reflecting on her teaching of AP Calculus and Riemann sums (see 6/21, 340, 19:31), Kadyn stated she believed that she "butchered" her teaching of it, speaking to her competence. During the End of Year Interview, in her response during this instance she created alternatives for how to do "not so much shoving" (see 6/21, 340, 18:40). The central component of her alternatives was using more open-ended questions that generally asked about the concept rather than how to accomplish a procedure.

Kadyn voiced concerns about students throughout the school year because her concerns represented her need to structure her teaching to support them. Some of the ways Kadyn was

concerned about students included their lack of skills, their inability to stay focused or organized, their failure to get all the notes written, or their incapacity to discover an idea on their own. Kadyn was forced to teach in particular ways in order to address the ways that students responded in mathematics class. As part of her professional identity, Kadyn needed to support each student to the best of her ability, and that meant that her energy was directed towards ensuring clear and thorough notes for all.

Students spent nine years of formal schooling in other classrooms, with other teachers before entering the Algebra 1 course that is the focus of this study. Students become accustomed to the practices of prior classrooms and when they encounter something different, they challenge the teacher to change their practices to be more like what they are accustomed to. If students never develop norms of argumentation or strategies to persevere through a mathematical problem, then behaviors such as these would be difficult for them. In response to students, teachers who attempt new or different ways of teaching and learning in their mathematics classroom are often met with defiance (Jessica, prior to this study). These realities of the educational system constrain teachers' abilities to focus on and develop alternate teaching methods that would actually be enhancements for student learning.

The Coteaching Partnership as Professional Development

Throughout the data presented thus far, the support the partners provided to each other is evident. This section will highlight moments of the year that presented as directly connected to coteaching as a means towards professional growth. Kadyn reflected on the year-long partnership as an experience that was productive, which was accomplished through the teachers observation of each other in practice, through open communication with each other, and through stages of storming and norming.

Observation and Communication

Kadyn took opportunities to observe Jessica lead the class in the beginning of the school year. Kadyn voiced discomfort with allowing student thinking to guide the direction of the class, as Jessica was allowing, and so she asked Jessica to take over so she could see it (see 9/12, 5, 3:27). On another occasion, Kadyn remarked that she observed Jessica leading a class discussion that was "turned on" and students were "into" (see 9/20, 21, 19:11-19:45), which permitted Kadyn to share her frustration regarding where the discussion might go and when to "turn the off switch." Such opportunities for Kadyn to observe Jessica and evaluate teaching practices provided pathways for the partnership to begin forming around a common understanding of what their shared practice could be.

Similarly, Jessica observed Kadyn teaching, from which Jessica found an opening to discuss responses to incorrect student answers (see 9/20, 21, 17:41). Kadyn responded to Jessica: "This is what I need help with. This is what I'm not comfortable with" (9/20, 21, 14:53). Kadyn was open to the feedback and again requested that Jessica take over because she wanted to watch and learn (see 9/20, 21, 17:41).

After the first few weeks of Kadyn and Jessica's co-teaching partnership, Kadyn shared her honest thoughts about working together:

> K: You said you felt like a backseat driver kind of. Right? And I think I actually feel the same way, right? Because we're not doing what we're used to be doing. And I think that we both like to be in control and like I knew a little, it's like self-analysis too. Like I like I do like to be in control of my classroom and my life at home…. I started thinking, well, are we both supposed to feel this way? Is this how and not in a bad way … is this how it's kind of supposed to feel because I don't want to be doing what other pairs have

> unsuccessfully done. Right. And that does mean that I have to give, right? But I also like, I kind of … I felt like I was already giving. You know what I'm saying by that. And then I feel like you feel like you're also giving right. So I think I'm okay with this as long as we feel like our class is going in the direction that we want it to go. (9/20, 21, 0:06)

Kadyn recognized the new roles that she and Jessica were each adopting in their partnership, that they had to *both* "give" to build a successful partnership. Kadyn called on the previous partnerships that had co-taught Algebra 1 and stated that she understood she had to be open to moving outside of her habitus in order to move forward in a common direction. The discomfort that Kadyn repeatedly cited came from unfamiliarity and new practices being integrated into the shared teaching. In this reflection, Kadyn evaluated the consequences of her and Jessica's actions within the partnership. Kadyn had been deeply considering how she felt within the coteaching partnership and *thought* about her feelings. By stating, "that does mean that I have to give," Kadyn recognized the moral requirement of giving in the partnership for their teaching practice to be successful, unlike prior pairs of unsuccessful co-teachers. Implied in the importance given to the success of the partnership is the mathematical learning of their shared students. Since Kadyn examined the moral principles as an ethical concern of working with Jessica, this reflection rises to the level of critical.

After creating the alternative lesson for solving equations in November, Kadyn thought they had created a "wonderful lesson" (see 11/7, 102P, 28:33). Kadyn was concerned that Jessica felt "compromised," she wanted to make sure that Jessica was happy with the lesson. Connected to the passage above, Kadyn knew there was a level of negotiation and compromise in most everything they did because they were coming from two separate practices, merging as one.

To create a functional partnership that did not meet the fate of the previous co-teaching pairs, Jessica and Kadyn found ways to effectively communicate in order to understand each other. Early in the year, Kadyn wanted to understand why Jessica was making certain decisions or teaching moves, and asked Jessica to help her understand:

> K: So sometimes I think it's helpful for me for you to tell me why you do things…. I don't always agree, but once you give, tell me why then I'm like, oh it's [expletive] awesome. But without the "Why?" I want to revert to what I've been doing for 10 years. Right? I mean, that's like my natural inclination, that's all. (9/24, 33, 1:21)

Kadyn was open to listening to Jessica's rationales for decision making in teaching and knew that she often needed more information in order to acquire a full understanding. Kadyn also recognized that understanding and moving forward with Jessica was in opposition to reverting to her previously established practices: without the rationale for Jessica's decisions, Kadyn was likely to return to what she had learned to do, the actions of her habitus, over her past decade of teaching.

In the same way, Kadyn shared in the End of Year Interview that Jessica had encouraged her to provide rationales about her practice by asking her why she made those decisions:

> K: All these things I didn't think about previously. And that you always ask me why I'm doing things…. I don't know. I have to think about it. And I think, well, maybe this is a good thing. Maybe isn't a good thing. Maybe I should do something different. (6/21, 340, 24:17)

By asking Kadyn about her decisions and moves in teaching, Jessica had influenced her to pedagogically reflect on her practice and consider whether there were aspects of her practice that she could justify, or aspects she would consider altering. Kadyn continued:

> K: But it's, it's great. It's not like you're like challenging me. You're just like, asking me why I think something's important. And then I have to like, think well why is this important to me? Why do I think this is? And then either I come up with something that I, I'm like, "Oh yeah, I have to do more of this." Or I'm like, "Oh, I don't know what it means. Why am I really giving homework? I don't know, but I can't stop." (6/21, 340, 24:56)

Through Jessica's questioning of Kadyn to reflect on and defend particular practices, Kadyn was able to step back and consider the reasons for elements of her teaching. In this passage, Kadyn used homework as an example of something she felt she must assign, but then had trouble validating the practice. Here, Kadyn questioned and analyzed beliefs that were part of her habitus.

During the End of Year Interview, Kadyn also reflected on the nature of the partnership's communication that had taken place throughout the school year:

> K: I think that that's why we have worked so well is because we're both flexible. And I think that, I very much respect your knowledge and the way you carry yourself and the way that you have treated me all year, right? Like you're not going to come up to me and be like, this is bullshit…. If you don't agree, tell me you don't agree, and you work it out. (6/21, 340, 27:04A)

Kadyn felt that the partnership held a level of respect: Jessica may have pushed Kadyn to question and consider alternatives to her practice, but Kadyn did not view it as an imposition nor a challenge against her as a professional.

Summary and Secondary Analysis of Observation and Communication

In looking back on actions in planning and teaching, coreflections became opportunities for Kadyn to identify the essential aspects for the coteaching partnership to work well. The coteachers did not go into the partnership by defining their roles preemptively, instead the teachers each brought their own craft knowledge and habitus to the partnership, which they reflected upon in the actions of planning and teaching.

In September, Kadyn realized that she had to "give" (see 9/20, 21, 0:06), which she accepted as long as the class was headed in the right direction. Kadyn also knew that sometimes she required explanations for the decisions Jessica made (see 9/24, 33, 1:21). In this instance of data, Kadyn voiced that an essential aspect of the partnership for her to move forward, rather than revert, was in understanding Jessica's rationales. This communication was essential in building shared competencies, and therefore behaviors, in the classroom.

In the End of Year Interview, Kadyn shared that in their work Jessica pushed her to consider what aspects of teaching were important to her. Kadyn's beliefs and identity as a teacher were integrated into her reflection, creating a way for her to more deeply consider what actions or behaviors she wanted to continue using in her teaching. The coteaching structure provided an environment in which behaviors of each teacher were evaluated by the other, providing opportunities for competencies and beliefs to be assessed (and self-assessed), evolving each of their identities as teachers.

As coteachers, Kadyn and Jessica identified essential aspects of partnering to teach, which were accomplished through communication. In their communications, through the process of reflection, the content of their reflections considered the environment in which they worked, their individual behaviors and competencies, and how these areas aligned (or did not align) to

their beliefs and individual professional identities. In turn, the coteachers' craft knowledge and habitus transitioned to consider, support, and align the behaviors and competencies with their newly evolved beliefs and identities.

Storming as Part of the Process

At points during the year, the partnership may have been categorized as storming, but the tensions were resolved through communicating about the issues at hand. For example, before creating the alternative solving equations worksheet, Jessica and Kadyn had a disagreement regarding how to plan the upcoming lesson (see 11/7, 102P, 16:17-19:17). Jessica challenged Kadyn to not just move through the material and teach by direct instruction, and offered suggestions, after which Kadyn proposed a feasible alternative that satisfied both teachers.

An additional example of a disagreement occurred when Jessica advocated to remove procedural directions from a worksheet for solving systems by the elimination method (see 1/30, 221, 2:30-4:14). Kadyn did not agree that students could figure out ideas on their own and she believed that one or two students would shout out the answers for the whole class anyway. Jessica persisted by refocusing the conversation on what a "middle ground" could be that would advance the level of thinking and learning for the students. As a result, Kadyn took another look at what was on the previous version of the worksheet and offered a revision that was a negotiation between the partners because it combined the analysis questions Jessica was asking for and the written documentation Kadyn wanted for the students.

Through these examples, it can be seen that not only respect, but also patience, listening, and compromise is required for the team to move through storming phases, and communicate to come together to create new norms and perform. Communication is necessary to advance any work and productive disagreements are a natural part of the process of becoming coteachers.

Looking Back to Move Forward

In addition to observing, communicating, storming, and other qualities already mentioned, reflection played a large role in this coteaching partnership's success. The meetings held between Kadyn and Jessica were occasions for recounting the events of enacted lessons, critiquing the events or teaching moves, and revising for future lessons. Throughout the year, as particular events emerged, the coteachers addressed the underlying issues.

In April, when planning to move forward into a new topic (the introduction to quadratic functions), Kadyn and Jessica reflected on their partnership. Kadyn shared: "We have to rethink how we're going to start next year" (4/3, 278, 15:09), referring to their teaching practices. Kadyn specified that she "hate[s] having wrong answers up on the board" (4/3, 278, 15:09), it "makes [her] tick." In general, Kadyn was considering the impact of two different teachers' practices on the students:

> K: So whatever we decide we want to do next year, I want more consistency. With everything. But you're right. We've been like, we've learned a shit ton this year. We have, like, I've like - about everything. Yeah.
>
> J: Like what?
>
> K: I don't know. What do I want, like even like changing what I want, what do I want them to be able to do when they leave here? And what's working. (4/3, 278, 16:38)

Kadyn wanted consistency in the established shared practice for students so that there was clarity regarding the class's expectations and norms. Kadyn also stated that as a result of the combined practices, the partnership had learned and grown since September. Here, Kadyn had acknowledged the shift in her stance: she was questioning what she believed to be the purpose in her work with the students and what she felt was effective in the teaching practice. As Jessica

questioned Kadyn to specify what she learned during the year, Kadyn's response began to reach the level of critical reflection as she considered the ethical impact of their teaching on their students and what she wanted them "to be able to do when they leave" their classroom.

Related to this questioning of purpose and practice, Kadyn shared the following during the End of Year Interview:

> K: So I feel like we've kind of evolved at the end of the year, you're … guiding me, making me think. All year actually. I feel like you've challenged me to think a lot about what's the purpose of my job. Like in a good way, not in a bad way, in a good way. Like, what's the purpose of, of being a math teacher? Like what do I, what do we want these kids to get out of, out of class? And, I feel like you brought a lot of ideas to the table and applications to the table, which … I feel like you've made me a way better teacher this year. (6/21, 340, 23:16)

Kadyn found her partnership with Jessica to be fruitful in the respect that she was challenged to reconsider her purpose as a mathematics teacher. Through this critical reflection, Kadyn was challenged to deeply consider her mission as a teacher and the ethical implications of her work for her students. From the day-to-day work Kadyn and Jessica accomplished, and the issues they deliberated on together, these more general reflections on practice arose. Rather than quick fixes of their practice generated from surface level reflections, pedagogical and critical level reflections created awareness of the purpose of their work, which Kadyn shared as making her "a way better teacher."

Through the partnership, Kadyn had witnessed growth in her own teaching, and therefore her craft knowledge and habitus, which she attributed to the challenges that Jessica had provided throughout the year for her to justify and explain her reasoning for making decisions for, and in,

the classroom. In addition, Kadyn saw Jessica's partnership as strengthening her teaching: "I feel like I, like, I have like a teacher, a teacher for me in the room, like helping me be better" (6/21, 340, 27:04B). Not only was Jessica able to bring ideas and applications to their teaching, but Kadyn also felt the support of a peer on a constant basis, working with her to advance instruction.

Chapter 5: Discussion

This study began by considering whether and how a transition can occur in an everyday secondary mathematics classroom so that a teaching practice could move from procedural towards more student-centered instruction. If mathematics teachers can instantiate the goals of the reforms, then the sense-making, reasoning-based intentions of the reforms can reach the students and provide them opportunities to engage in the kinds of mathematical thinking encouraged by the reforms. In this study, the instantiation of the reform ideals inside a classroom required the collaborative work of the two participating teachers.

The results chapter presented data to address the research question: *How can secondary mathematics teachers in a coteaching partnership serve as resources for each other's professional growth towards reform-oriented pedagogy?* By describing the relevant stages of the year, the development of the coteachers' partnership revealed how the teachers' actions in coplanning/coreflecting meetings and in coenacting lessons served as resources for each other toward reform-oriented professional growth. This chapter will first retrace the different ways the coteachers' actions in practice served as resources for each other and discuss how these resources led to professional growth. The resources among the coteachers developed from interactions between the teachers and their actions for and in the classroom, enabling the partnership to move towards a more reform-oriented practice. The constructs of craft knowledge and habitus are connected to this discussion with a focus on craft knowledge as "the construction of situated, learner-focused, procedural, and content-related pedagogical knowledge through deliberate action" (Grimmett & MacKinnon, 1992, p. 393) and habitus as "a set of dispositions that structure actions, perceptions, and expectations" (Roth et al., 2000, p. 8). After characterizing the coteachers' actions as resources for professional growth, this chapter will discuss how the

coteaching structure in this context relates to Murphy et al.'s (2015) six coteaching framework elements and the role of reflection in coteaching. The chapter will conclude with implications, limitations, and next steps for this work.

Coteachers' Actions as Resources for Each Other's Professional Growth

The six stages over two main phases of the school year illuminated the pivotal moments for Kadyn and Jessica to progress through their stages of development as coteachers. The coteachers' actions served as resources for each other in a range of ways as they first formed, then stormed and normed, and ultimately performed. The ways in which the coteachers acted, serving as resources for their partner, included (a) modeling and observing teaching, (b) sharing ideas for teaching, (c) communicating with and challenging each other, and (d) creating alternatives for teaching. Each of these actions as resources are discussed next in relation to the data previously presented and their connection to the teachers' craft knowledge and habitus.

Modeling and Observing as Resources for Professional Growth

In the beginning of their partnership, Jessica modeled the type of teaching she hoped to instill in their shared classroom while Kadyn observed. During their coreflections, the teachers discussed the actions Jessica took in class and the discomforts Kadyn felt putting those actions into practice (see 9/12, 5, 5:25-7:13 and 8:24-9:00). In September, Kadyn frequently voiced that the discourse Jessica was leading, with open student responses, was unfamiliar to her. As a result, she asked Jessica to continue leading because she wanted to observe and learn (see 9/20, 21, 17:41). Kadyn's discomfort informed Jessica by moving her to reflect on why Kadyn was uncomfortable and what Jessica was doing that may have caused such feelings. In response, Jessica continued to model for Kadyn to continue to observe and learn.

As Jessica led the new types of lessons during September and modeled reform-minded teaching (see 9/20, 20, 11:25 and 12:20-14:27), Kadyn was observing and actively processing Jessica's actions. In her reflection on her discomfort, Kadyn's struggle between her own intended lesson path and the possibility of students leading the discourse in a "good direction" created a tension between her perceived ideal path and the realities of the classroom (Murphy et al., 2015). Kadyn continued to wonder how she would get the students to do what she had planned for, representing a teacher-centered transmission of knowledge.

In September, Kadyn held a view of teaching as a solo act (Tekkumru Kisa & Stein, 2015). Jessica's modeled teaching challenged this solo approach, being more "interactionary" (Tekkumru Kisa & Stein) among the teachers and students through the content. The interactions that Jessica worked to create brought attention to student thinking and prioritized the building of understanding by varying classroom discourse rather than invoking one teacher-led path. As Kadyn observed this feature of Jessica's teaching, she gradually gained an appreciation for how students were "into it" (see 9/20, 21, 19:11-19:45). Jessica's modeling of eliciting student thinking was different from Kadyn's prior practice. As Kadyn observed, she developed comfort from her continued exposure and active processing to eventually lead portions of the classes on her own within the newly planned lessons (see 9/21, 25, 25:16 and 42:20). This shift originated from Kadyn's request to observe and study Jessica's teaching, and Jessica's regard for this request and her understanding that this was what Kadyn needed at that time. Thus, Jessica's modeling of teaching in an interactional way and Kadyn's simultaneous active observation of Jessica and participation with their shared students became a resource. As Kadyn shared what she was processing during observation, Jessica learned how and why her practice was "different." This dynamic generated coreflection and Kadyn's eventual enactment of this feature. As the year

continued, Kadyn's concerns and discomfort with the interactional practice decreased, demonstrating her acceptance and willingness to put into practice what she observed. Through the modeling and observing resource, Jessica and Kadyn gained craft knowledge that supported the instantiation of more student-centered thinking in the mathematics classroom.

Further evidence of the modeling and observing resource's impact manifests in Kadyn's defense and explanation to the visiting teacher (Tyler) of what "Jess is now" doing (see 12/14, 161, 8:06-10:50). In this lesson instantiation, Jessica purposefully limited her responses to students' questions and withheld examples, creating a space for students to respond to each other and generate examples for themselves. Jessica modeled how the Standard Form lesson could be enacted by maintaining higher levels of cognitive demand (Henningsen & Stein, 1997) and by building strategic competence, adaptive reasoning, and productive dispositions (NRC, 2001). In this instance, Kadyn's "observation" of Jessica's modeling becomes *public* in her response to Tyler. In particular, Kadyn demonstrated (and celebrated) that she had recognized the difference between what Jessica was working towards versus a more traditionally-oriented, teacher-led discourse. By December, the coteachers had transitioned into providing students opportunities to interpret and model on their own without as much teacher direction.

In the fourth instantiation of this Standard Form lesson, Kadyn was able to fully engage in reform-minded teaching. Her practiced alignment and lack of discomfort or concern demonstrates the development of her craft knowledge, and a shift in the actions and dispositions of her habitus. Kadyn was able to interact with students in the classroom through a problem-based lesson that gave students opportunities to mathematically reason and make connections for themselves.

As summarized in the review of Phase II, there were no instances during the latter part of the school year where Kadyn solely observed Jessica. In effect, Kadyn was no longer unsure or uncomfortable with the classroom discourse Jessica instantiated. This resource of modeling and observing existed within the forming stage of the partnership's development and was situated *within* the classroom, with students, in action. The remaining resources were situated in the one-on-one meetings where Kadyn and Jessica coplanned lessons and/or coreflected on enacted lessons.

Sharing Ideas as a Resource for Professional Growth

At the beginning of the school year, Jessica decided to focus on developing, in her students, a sense of why they should care about the mathematical topics of study in the course. Lesson planning sessions incorporated Jessica's attempts to connect to real-world situations or to include a lesson component where students could make mathematical meaning of topics. Although this mission was not yet taken-as-shared (having a common, normative understanding) among the coteachers in the beginning of the year, it was a set of ideas that Jessica brought to the partnership.

As the partners continued to work with one another, Kadyn began bringing ideas to the lesson planning sessions. She found lessons that she believed to be "more in tune with" (see 9/24, 33, 29:12-29:44) what she believed Jessica wanted to accomplish in their classroom, meaning the interactionary and reasoning-based goals of their teaching. After engaging in the beginning weeks of teaching with Jessica, Kadyn was constructing and applying craft knowledge to develop context-relevant lessons focused on supporting a classroom discourse that promoted reasoning and interactions.

Through conversation, Kadyn and Jessica worked together to establish how they would teach the Standard Form of linear equations by leveraging student knowledge of intercepts (see 12/11, 156, 0:02-0:16). The creation and enactment of this lesson was a turning point for the taken-as-shared understanding of the partnership. That is, this lesson established that Kadyn "was sold" and had "bought in" to Jessica's professional mission in their practice (see 12/7, 150, 0:27-1:15). Moreover, Jessica realized that working with Kadyn to instantiate reform-oriented teaching was attainable. In this series of data segments for the Standard Form lesson creation, enactment, and sharing with other teachers, Jessica and Kadyn collaborated to instantiate their craft knowledge in relation to their reform-oriented instructional goals.

After the Standard Form lesson, Kadyn was able to leverage her understanding of this model example in subsequent work with Jessica. When meeting with Jessica to plan for the teaching of systems of equations, Kadyn wanted to strategically plan for the lesson cycle of a meaningful activity bridged to procedural practice (see 1/8, 181, 3:56), since she had witnessed the instantiation of both components in December. Kadyn recognized the essential aspects of the December stage and intended to incorporate them in the planning of lessons as new ideas to share. In this stage, not only did Jessica's mission become realized, but also Kadyn's connection to bridging activities strengthened the work of the partnership as a whole because Kadyn joined in on the effort that was now shared among the coteachers.

Furthermore, after Kadyn created and shared her introduction to quadratic functions worksheet, the coteachers discussed how to support student learning in this activity (see 4/3, 278, 4:24-5:07). Kadyn shared the strategy of making a table of values, which she believed to be a worthwhile path for students to take when engaging in this work. Jessica also shared strategies for students to engage intellectually during this activity. Without sharing the activity and

discussing its enactment, Kadyn and Jessica may not have extended their plans and supports for students to the same extent as they did in this case.

The creations of these lessons were the results of the coteachers bringing ideas or resources to each other, and their combined reflection upon the materials, which often resulted in revisions towards more reform-oriented lessons. This resource of sharing ideas was largely connected to the coteachers' coplanning of lessons but was also informed by their coreflection on and coenactment of lessons. The partners' sharing of ideas as a resource took place during norming developmental stages, moving beyond the forming stage, and aligning the coteachers' craft knowledge and actions of their habitus.

Communicating and Challenging Each Other as Resources for Professional Growth

Communication for the teachers, as a resource, was necessary for them to understand each other and proceed through their stages of development. For example, Kadyn explained it was helpful for her to know why Jessica made certain decisions and further admitted that she was aware she would likely revert to her practice without understanding Jessica's rationales (see 9/24, 33, 1:21). In response, Jessica regularly shared her rationales with Kadyn. Without clear communication, Kadyn and Jessica would not have addressed events that occurred during enactment of lessons, nor critiqued or revised their lessons. In addition, this communication was integral to the teachers' ability to reflect on their teaching practices.

One recurring theme in Kadyn's communications at the beginning of the year centered on her concerns about shifting her teaching practices and the consequences of this shift. She considered areas that could be impacted if she were to change her practice in the way Jessica was modeling. An example of a concern was giving up classroom control and possibly giving students misinformation (see 9/12, 5, 8:24-9:00). Kadyn also shared concerns about providing

some form of definitions in lessons for students (see 9/26, 42, 3:07). Jessica's recurring concern, which Kadyn assisted in working out, was building student independence into lesson plans. The sharing of concerns served as seeds for growth in the partnership's teaching, as the coteachers worked to address them and create alternative practices. As a resource, communicating rationales and concerns provided additional occasions for the coteachers to develop in norming stages, creating the possibility for further alignment of their craft knowledge and habitus.

In addition to communicating concerns, Kadyn and Jessica also communicated by challenging each other. The partners respectfully opposed each other throughout the year on issues where they did not agree. For example, Jessica confronted Kadyn regarding her responses to students' incorrect answers and thoughts, which Kadyn was open to hearing and wanted to work on (see 9/20, 21, 17:41). Storming stages included both coteachers pushing and responding to each other to shift their thinking about a topic of discussion. One example was Jessica's insistence to plan for student engagement (see 11/7, 102P, 16:17-19:17) where Kadyn learned to negotiate with Jessica by sharing her genuine ideas, and Jessica learned that Kadyn was able to create an alternative that could satisfy both parties.

In their exchange regarding Kadyn's reversion in teaching solving systems of equations by substitution, Jessica challenged Kadyn to extend her understanding of the algebraic procedure of substitution toward a graphical interpretation (see 1/28, 212P, 5:21). As a result, Kadyn reasoned through a real-world scenario and interwove her reasoning with her content knowledge for systems of equations, thereby influencing her craft knowledge. In this instance, Jessica learned that their plan to teach this topic had not been clear and that just as real-world scenarios were useful for students to engage in reasoning, so can teachers use contexts to clarify the thinking behind a lesson. The revised lesson that followed provided an opportunity for the

instantiation of craft knowledge, which in turn developed Kadyn's habitus for teaching systems of equations. This development additionally influenced the subsequent teaching of solving systems by elimination because Kadyn had voiced an intention for "presenting [material] in a way that has meaning" (see 1/28/19, 213, 0:19).

Additional examples of the coteachers challenging each other and serving as resources towards professional growth included: Jessica challenging Kadyn to remove step-by-step procedural directions from earlier student worksheets (see 1/30, 221, 2:30-4:14) and to believe in learning that could be student-centered, with students discovering on their own (see 4/3, 278, 1:46-1:58). Both of these examples illustrate negotiations that took place as a result of the challenges to each other's practices and opened discussions regarding the coteachers' beliefs. In turn, challenges and discussions on their beliefs often led to the creation and implementation of alternative lesson plans.

Creating Alternatives as a Resource for Professional Growth

After the coteachers challenged each other on a topic, they were able to effectively communicate and resolve their differences by negotiating alternative solutions. There are two notable cases of substantial shifts in the planning of lessons that followed situations of tension between the coteachers.

In one example, Kadyn offered an alternative to her procedural lesson for solving equations with distribution and combining like terms after Jessica was adamant about planning the lesson ahead of time (see 11/7, 102P, 22:12). During this exchange, Jessica argued that students should be able to work on their own to increase their cognitive engagement, yet she did not have the answer to what the activity would include. Kadyn's response of an alternate justification activity provided a pathway for the creation of a more cognitively rich classroom

environment (Tekkumru Kisa & Stein, 2015). Kadyn felt that the resulting creation was "a wonderful lesson" because it was "better analysis" (see 11/7, 102P, 39:22). The outcome of this first storming stage involved a shift in the partnership's understanding of the possibilities for their teaching.

A second example of a challenge, followed by a proposed alternative for a lesson, arose during Jessica and Kadyn's discussion of removing step-by-step procedural directions for solving systems by the elimination method (see 1/30, 221, 2:30-4:14). After first defending her position, Kadyn ultimately agreed that the procedural directions for students were not very valuable for learning, perhaps realizing that the directions did not support students' cognitive engagement. She proposed instead to give the students the analysis questions the teachers would ask, an alternative that could promote interactions between the teachers, students, and the content, and reduce the centrality of the teacher in the lesson. Again, Jessica challenged Kadyn to alter the lesson without knowing of an alternative herself. Kadyn was able to interpret Jessica's mission and create something new to include in the lesson that matched the ideals of the mission. Ultimately, it was important to Kadyn that students have proper documentation of the lesson's ideas, which could now include reasoning and justification of mathematical ideas, and not just procedural steps for solving.

These two examples of storming stages involved high tensions between the partners, yet the outcomes included lessons that had evolved towards reform-oriented practices. Although discrepancies arose in the beliefs and habitus of the coteachers, after challenging each other and sharing their ideas regarding the situations, they created negotiated, aligned lessons, and therefore transitioned into norming stages of development.

Alternatives were also created as new versions of lessons that Kadyn or Jessica had previously taught. Often Kadyn and Jessica referred to their prior materials to clarify learning goals or to work from activities they had instantiated in previous school years (independently, pre coteaching). Although Kadyn had been developing these materials throughout her years of teaching Algebra 1, and Jessica had utilized some of Kadyn's materials as well as materials of her own creation for an honors-track course of Algebra 1, practically all of their activities were altered as a result of the coteaching partnership's work. Kadyn and Jessica sought out and chose new activities (e.g., see 9/24, 33, 29:12-29:44; Appendix C: Teaching Artifact for Systems of Linear Functions) or integrated a meaning-making situation first before moving to a previously instantiated activity (see Appendix B: Teaching Artifact for Linear Functions in Standard Form). The alterations of the teachers' materials used in class represent changes in their respective craft knowledge stemming from beginning shifts in habitus.

Conclusions Regarding Resources for Professional Growth

The stages of development during the year of forming, storming, and norming align with the identified types of resources. The coteachers demonstrated the usefulness of modeling and observing during their beginning forming stage, after which they entered norming stages by sharing ideas and communicating rationales and concerns. When challenging each other, the coteachers were in storming stages, after which they were able to resolve discrepancies by creating alternatives in norming stages. As the teachers reached the performing stages of the year, they continued to observe each other in teaching, to share ideas, communicate, and challenge each other in planning processes in order to continue to create alternatives in their teaching. The alignment between the stages of development and the resources is outlined in Table 4.

Table 4

Secondary Mathematics Coteaching Resources for Professional Growth

Stage of Development	Teacher Resource
Forming	● Modeling and Observing Teaching
Norming	● Sharing Ideas for Teaching ● Communicating Rationales and Concerns ● Creating Alternatives
Storming	● Challenging Each Other
Performing	● All of the Above

As the teachers acted as resources for one another, they progressed through the various stages of development, but these stages were not always mutually exclusive, nor did the progress through each stage occur in a linear path. The coteachers began by forming, but during their forming stage, they were norming in order to create understandings of each other and common understandings between them. During instances of storming, the teachers worked through tensions that resulted in new norms (norming stage). Once the teachers reached a performing level, during Phase II of the school year, they continued to summon the resources of the storming and norming stages. The continuation of storming and norming integrated into performing demonstrates that small group development is not a step-by-step (linear) process. The coteachers had reached a level of alignment and productivity but continued to build combined understandings by negotiating discrepancies between their beliefs and teaching identities.

By partnering in their work, Kadyn and Jessica created taken-as-shared understandings as craft knowledge in their practice, which resulted in evolved abilities to create lessons for the topics they had previously taught. As the coteachers collaborated, their actions as resources influenced their craft knowledge and habitus to align with their newly shared practice, thereby

transitioning the interactions between teachers, students, and content through the stages of the year, and ultimately shifting Kadyn's and Jessica's views of teaching. As Kadyn "bought into" Jessica's discourse and creation of interactive materials, she also developed reform-oriented lessons and participated in the open interactions of the classroom. Kadyn transitioned from holding a view of teaching as a solo act to a view of teaching as interactions (Tekkumru Kisa & Stein, 2015). Simultaneously, as Kadyn shifted towards Jessica's interactionary pedagogy, Jessica learned from Kadyn's creation of alternatives various ways to integrate reasoning into structured materials for students.

Based on the results discussed thus far in this chapter, the process by which the teachers' decisions were influenced by the resources they provided for each other can be described as: the resources developed the coteachers' craft knowledge, which altered their habitus, impacting the decisions made for teaching in lesson planning and in the act of teaching. Each of these components influenced the others; an interconnectedness existed among resources, craft knowledge, and habitus. The specific actions of the resources interacted with each individual's existing *knowledge* (of all types) and *dispositions*, both of which construct the individual's *schema*. As the individual's existing schema interacted with resources, the teachers may have converted this "capital" into their practice (Milne et al., 2006). Resources as actions by the coteachers informed the craft knowledge and habitus of each coteacher, theoretically becoming internalized to each teacher's schema, thereby effecting subsequent actions and more permanent pedagogical strategies by each coteacher.

The movement among and through the craft knowledge, habitus, and decision-making components can be further explained using zones of proximal development (ZPD). The ZPD can be viewed generally as a space being created for assisted performance and more specifically as

the interactions between individuals and the environment around them, which could include others and objects (e.g., materials or tools; Murphy et al., 2015). As Milne et al. (2006) explained, similar to the explanation above, as teachers explore new resources that are not part of their habitus, their options for action expand, and teachers' agency increases, thereby creating opportunities for change in practice. This discussion will next revisit Murphy et al.'s (2015) framework for coteaching in relation to the results of this study.

ZPD Conceptual Framework for Coteaching

The results of this study can be compared to the six elements in Murphy et al.'s (2015) study (in which three of their six elements presented) in order to gain a deeper understanding of the dynamics involved in the coteaching partnership working towards reform-oriented teaching. In the current study, all six elements presented and are detailed in this current section. Murphy et al. did not provide an apparent reason for aligning each of their six elements into their coteaching phases of coplanning (interaction between real and ideal forms; buds of development), copracticing (Vygotskian imitation; unity of affect and intellect), and coevaluating (regression/recursion; structured reflection), originally presented in Table 1. In the following discussion of their six elements, I relate the data of this study to a variation in their three phases: instead of copracticing, I more specifically have termed this phase *coenacting*; and, instead of coevaluating, I defined the partnership's evaluation of data to be *coreflection*. The findings of this study also re-align the six elements among the three phases of coteaching, presented in Table 5 and discussed here.

Table 5

Revised Phases and Elements of Coteaching

Coteaching Phase	Coteaching Element
Coplanning	● Interaction between real and ideal forms ● Unity of affect and intellect
Coenacting	● Vygotskian imitation
Coenacting and Coreflecting	● Buds of development ● Regression/recursion
Coreflecting	● Structured reflection

Note. See original phases and elements in Table 1, adapted from Murphy et al. (2015)

The first element, interaction between ideal and real forms, was present in Jessica and Kadyn's coplanning sessions. The coteachers of this study frequently voiced their ideal components or features of lessons to the other when sharing ideas, communicating, and creating alternatives. These ideals were formed from their beliefs, identity, and mission as teachers: their core dispositions. Many times, when one coteacher voiced an ideal, the other coteacher responded with the realistic conditions of the classroom. The sharing of ideal and real forms occurred bi-directionally between the coteachers. As Jessica challenged the partners to center student thinking, Kadyn stated that the Algebra 1 population they worked with would not share thoughts or answers (see 6/21, 340, 20:54). As Kadyn requested to do direct instruction by giving examples as notes for students, Jessica voiced concern about not preparing lessons purposefully for student interaction and engagement (see 11/7, 102P, 16:17-19:17). This "testing" of ideas against the coteacher made teaching practices explicit among the partners, allowing for evaluation of whether the other teacher considered them to be feasible and worthwhile for their shared classroom. The discrepancies between ideal and real forms of lessons align to the

storming and norming stages of development because differences caused tensions that required negotiation. But as the ideal form more closely represented the real form, as the partnership created a shared vision and practice, there were less tensions, creating a zone for the partnership to reach a performing level and a state of flow.

The second element is buds of development, which Murphy et al. (2015) interpreted as stages close to the next level of development. In the forming stage of September, Kadyn voiced her openness and willingness to watch and learn during the modeling and observing portion of the year, from which she did adapt her practice. After Kadyn and Jessica reflected upon particular practices, they adjusted accordingly in practice. As a result, these buds of development are connected to both the forming and norming stages of small group development, as well as both the coreflection and coenactment phases of coteaching.

The third element of Vygotskian imitation, which is "not copying but *emulation* of an activity" (Murphy et al., 2015, p. 287), was present in the coenactment of lessons. After Jessica modeled more student-centered discourse moves, Kadyn did begin to utilize them on her own. Once examples of the behavior are present, the coteacher is able to observe them, reflect on them in conversation, communicate concerns, and then implement the behaviors themselves.

The fourth element, the unity of affect and intellect, has the most drastic change from Murphy et al.'s (2015) proposed framework. This element refers to the interdependency of emotions and learning, Murphy et al. report that their pre-service teachers did not refer to emotions in their reflections, thus they focused more on Vygotskian imitation in their findings report, since they were both placed in the copractice phase. I was able to identify instances of teachers' affect because of the many coplanning meetings in which Jessica and Kadyn worked

with, and sometimes challenged, each other. The recorded meetings proved to be a methodological advantage that captured various emotions.

In the findings from Kadyn and Jessica's partnership, affect impacted how the teachers progressed and the ways they were able to learn. Early in the school year, Kadyn voiced uncertainty (see 9/12, 5, 5:25-7:13) when she was unsure of how to get students to do what she wanted within Jessica's new discourse style. Also in September, Kadyn voiced excitement (see 9/24, 33, 29:12-29:44) over finding a new lesson that she loved. Kadyn moved from being unsure into true excitement in the same stage, representing the shift in her affect and her learning. In the storming stage, due to Jessica's adamant demands (see 11/7, 102P, 16:17-19:17) for the partnership to plan ahead, a new lesson was created that included justification, which Kadyn expressed happiness over because it turned out to be a "wonderful lesson" and was "better analysis" (see 11/7, 102P, 28:33). A final example of expressed emotion rests in Kadyn's engagement in valuing Jessica's rationales for why she makes decisions (see 9/24, 33, 1:21).

The fifth element, regression and recursion, is cited as the "key to deep learning" (Murphy et al., 2015, p. 288). Regression occurs when learners (the teachers) revert to earlier stages whereas recursion retraces back to where the learner was by re-learning in a new situation. Murphy et al. did not find much evidence of this element in their data. In this study, Kadyn reverted to her prior methods when planning for solving equations (see 11/7, 102P, 16:17-19:17) and when teaching substitution (see 1/28, 212P, 0:07-0:55). Kadyn's reversion in teaching substitution demonstrates the power and consequence of a coteacher who was present for a lesson who can reflect with her on the content of the lesson. In this way, the teachers acted much like a supervisor interacts with teachers in an evaluation cycle, which typically only occurs a few times per year. In this structure, the teachers were able to *constantly* evaluate and provide

feedback on each other's practices. Kadyn also explicitly recognized the possibility of reverting to her previous practices if she had not had an understanding of Jessica's rationales for the differences in their practices (see 9/24, 33, 1:21). This element made old and new practices transparent by contrasting when decisions were aligned or not with the normative practice the partners had established. By regressing, norms were called into question or re-established and further transformed with new experiences and actions informing the teachers' decisions.

The first five elements of the ZPD framework all incorporated some form of reflection. As their sixth element, Murphy et al. (2015) referred specifically to structured reflection: in their study, Murphy et al. had pre-service teachers write a reflective essay. In contrast, during this study Kadyn and Jessica were regularly challenged to reflect on their individual and partnered teaching practices through conversational interactions. Murphy et al. reported that their reflective essays provided structured reflection for deeper engagement than interviews. I argue that the reflection in the partnership established in this study occurred at the critical level. This is discussed in the following section.

In this section, I have presented connections between Murphy et al.'s (2015) six elements of a conceptual framework for coteaching to revised phases, as well as to the small group developmental stages. The alignment of the phases, elements, and stages is outlined in Table 6. Next, I will turn to discussing the role of reflection in professional growth and the connections among the theoretical models for reflection made during coding.

Table 6

Revised Phases and Elements of Coteaching with Development Stages

Coteaching Phase	Coteaching Element	Development Stage
Coplanning	● Interaction between real and ideal forms ● Unity of affect and intellect	Storming, Norming All Stages
Coenacting	● Vygotskian imitation	Forming, Norming
Coenacting and Coreflecting	● Buds of development ● Regression/recursion	Forming, Norming Norming, Performing
Coreflecting	● Structured reflection	All Stages

Note. See original phases and elements in Table 1, adapted from Murphy et al. (2015)

The Role of Reflection in Transitioning Pedagogy

The complexity of teaching lies in the combination of factors influencing and constraining the knowledge and actions of teachers. The archetypes that are created by students observing their teachers lead to a system that would perpetually run counter to reforms (Foss, 2010), unless powerful agents of change are introduced to the system. One of these change agents is the power of critical reflection as a social problem-solving and decision-making process (Larrivee, 2008), which can interrupt the habitual ways of knowing (Foss).

Critical reflection can allow for examination of both professional and personal beliefs, as well as the consequences of teaching actions (Larrivee, 2010). In these ways, critical reflection analyzes content of the onion model at the belief, identity, and mission levels; that is, at the core. The practice of teaching in the field "becomes a site of negotiation and contestation" (Milne et al., 2006, p. 327), constantly creating interactions that can be reflected upon. The extent to which the reflection occurs, based on *content* (onion model levels), is the mechanism by which change in teaching practices may occur. Milne et al. (2006) discussed teaching as praxis, which "means

that teacher action is purposeful and directed, and leads us to recognize the tensions that teachers feel when they critically reflect on their actions" (Milne et al., 2006, p. 331).

This study worked to shift practice through coteaching, and the role of reflection was found to be prominent in most stages of the year. In this study, reflection was pushed further than either participant could conduct on their own through the challenging of the other and supporting one another. Research on reflection often structures reflection purposely for participants, utilizing portfolios or writing prompts. In this study, reflection was promoted naturally in the harmonious coteaching partnership of the coteachers' work. The negotiations, contestations, and tensions that represent reflection in this study's data will next be discussed.

In September, when Kadyn was unsure about shifting her practice in the way Jessica was modeling (see 9/12, 5, 5:25-7:13), reflecting on the situation provided Kadyn the space to process her concerns and identify why she was uncomfortable. By reflecting at a pedagogical level, she worked to negotiate her way to find the actions she could next take in order to address her discomfort and uncertainty. By looking back and becoming aware of the behaviors and competencies aligned (or not aligned) with her beliefs, Kadyn eventually alleviated her concerns. By the end of this stage (see 9/24, 33, 29:12-29:44), Kadyn began participating in a new form of teaching practice. In these examples, the ALACT phases did not occur in one coplanning/coreflecting meeting, but across multiple sessions throughout the month.

In subsequent stages of the year, Kadyn's reflections produced alternatives. In the meeting to plan solving equations (see 11/7, 102P, 16:17-19:17), tension built between Kadyn and Jessica until negotiations and reflections provided an opening for an adjustment to the lesson materials. In the teaching of systems of equations, Kadyn reflected on the sum total of experiences of the year when she stated that they should "plan for this every time" (see 1/8, 181,

3:56), referring to the components of applications and procedural practice. She also connected prior and new craft knowledge to develop their teaching of solving systems by substitution (see 1/23, 202, 0:19) and elimination (see 1/30, 221, 2:30-4:14). Kadyn looked back on her work with Jessica in the first phase of the year, assessed their behaviors and competencies, and, in combination with the resources of the coteaching partnership, created alternatives. The alternatives provided new resources as actions to enact in class or to reflect upon. Again, in the examples provided here, not all ALACT phases occurred in each data segment, but their identification assists in recognizing the interplay between reflection and resources.

In this section's examples, thus far, the movement between the layers of the onion model can be seen. Kadyn constantly considered elements of the environment reflected on her (and Jessica's; and their partnership's) behaviors and competencies. Most of these instances were considered pedagogical level reflection, since they focused on the rationales for actions of teaching.

In April, however, the conversation shifted to beliefs about what the students were able to accomplish. Kadyn referred to her prior experiences with students and what she believed to be true about the limit of student abilities. Jessica, armed with her belief that students must work independently to learn, challenged Kadyn to develop more strategies, motivators, and accountability measures as student supports (see 4/3, 278, 4:24-5:07). This tension (storming) between the beliefs of each teacher created opportunities for the teachers to consider more core qualities of their work. In the same session, Kadyn began voicing her intentions for the next school year by reflecting on their experiences thus far. These instances examined the consequences of the coteachers' actions and thus are considered to be critical reflections. As the

teachers reflected on core qualities of their pedagogy, they confronted frustrations and more objectively addressed their behaviors by articulating their associated beliefs.

During the End of Year Interview, Kadyn reflected on a conversation with her brother on the topic of teaching procedural understanding (skill) or strategic competence (see 6/21, 340, 4:54) in combination with the work she had done with Jessica during the year. Kadyn felt strongly about her position regarding continuing to teach skills and gave multiple reasons to do so. She was very concerned about teaching for conceptual understanding *overcoming* teaching procedural skills and had not yet connected her stance to the bigger idea of mathematical proficiency, which intertwines five different strands (NRC, 2001). This critical reflection again demonstrates Kadyn confronting issues of the coteaching partnership's practice where she felt a dilemma between what she understood Jessica's mission to be (which was *not* to eliminate the teaching of procedures) and what she believed herself to be necessary for students.

Also during the End of Year Interview, when discussing the influence of the state test as application-based problems (see 6/21, 340, 16:16) and her contention towards her teaching of Riemann sums (see 6/21, 340, 19:31), Kadyn reflected on her practice to identify the components that she felt needed improvement moving forward. These critical reflections illustrate how Kadyn was evaluating teaching situations and problem solving with the intention to improve her practice, during which she had integrated information and discovered new meaning for teaching (Larrivee, 2000). By considering the state test as an environmental factor (onion model, level 1), Kadyn identified teaching behaviors (onion model, level 2) according to her current beliefs, professional identity, and mission (onion model, levels 4, 5, and 6), all which had transformed.

As Larrivee (2000) discussed, as teachers deliberately consider and articulate rationales for their teaching decisions, they are able to identify and confront the issues of daily teaching.

"Becoming a perpetual problem-solver involves synthesizing experiences, integrating information and feedback, uncovering underlying reasons, and discovering new meaning" (Larrivee, 2000, p. 297); teachers participate in shifting their *stance* and doing a *dance* to "continually forge new ground" (p. 297). For the coteaching partnership of this study, perpetual problem solving and forging new ground occurred in alignment with their performing developmental stage and became normative in their continued work.

The Use of Models for Reflection in Research and Practice

This study used the ALACT model for reflection processes, the onion model for content levels of reflection, and Larrivee's (2010) levels of surface, pedagogical, and critical reflection to analyze how Kadyn and Jessica reflected on their practice. Each model provided a variation on how to view the role of reflection and taken in combination, the three models can inform both future research and transitioning teaching practices.

The ALACT model's five circular phases of action, looking back, awareness of essential aspects, creating alternatives, and trial (which overlaps action), provide a way to capture the process of reflection. In their coplanning/coreflecting meetings, Kadyn and Jessica often looked back (phase 2) on their instantiation of lessons (actions, phase 1) that they had coplannned and evaluated their satisfaction with the features of the lesson. Kadyn and Jessica would identify what was effective or what may have needed adjustment, bringing to light the essential aspects (phase 3) of the lesson plan. Often when the coteaching partnership was not satisfied with an aspect, either Jessica or Kadyn would contest it, and after an exchange, an alternative (phase 4) would be offered. Once this alternative was established, agreed upon, and developed, it was trialed (phase 5) in a subsequent class.

The onion model's six layers begin with the external layer of environment, which informs and impacts each individual's layers. The next layer of behavior (level 2) consists of observable actions that occur, coupled with the competencies (level 3) of that actor. In the case of the coteachers, Kadyn and Jessica reflected on their work with students within the institutional context (environment) as well as what they knew and what they could do while enacting lessons (behaviors, competence). The layers that people do not see, that are closer to the core, are beliefs, identity, and mission. Although these are not observable, through the reflections captured on audio recordings, references to the coteachers' beliefs, identities, and missions could be considered.

Larrivee (2000) stated that one of the essential practices for becoming a reflective practitioner, who reflects at the critical or core level, is to have a solo reflective practice. Larrivee (2010) referred to pressures on teachers for students to reach imposed standards, which "come at the expense of ongoing reflection … However, the best antidote for feeling like mere pawns in the system is for teachers to take control of their teaching lives by developing the habit of engaging in systematic reflection" (p. 138). This study demonstrates that reflection is vital to develop shifts in teaching, but solo reflection may not be the only antidote: coteaching partnerships can provide a continual, structured way for teachers to cognitively problem-solve issues of practice.

Transitioning Towards Reform-Oriented Practice Through Coteaching

The standards developed by NCTM (2000), NRC (2001), and NGA (2010) described mathematical learning environments that are cognitively rich and that provide students opportunities to engage in mathematical thinking. Learning environments that are aligned with a reform orientation centralize the students, rather than the teacher, and promote learners' problem

solving, reasoning, and building of strategies. This study's coteachers accomplished professional growth towards such learning environments through their sharing of resources as actions in their coplanning and coenacting of lessons in Algebra 1.

The resources of modeling and observing teaching, sharing ideas for teaching, communicating concerns and rationales, challenging each other, creating alternatives for teaching, and pedagogically and critically reflecting provided the coteachers ways to collectively act to combine the ideals of the reforms with the realities of day-to-day teaching. By "harmoniously" partnering, the coteachers opened up their teaching practices to each other, taking risks that could expose their personal practices or result in setbacks, but stayed the path on which their aligned collaboration directed them. Through their actions throughout the year, their copractice developed to become a meaningful resource itself for the teachers to continually question and negotiate their way towards reform orientations in their classrooms.

Coteaching as a Structure to Reduce Constraints that Undermine Reform Efforts

I next refer back to Kennedy's (2005) hypotheses regarding why reforms may not take hold due to teachers' competing intentions of avoiding distractions and keeping momentum, of covering content, and of increasing student participation and fostering learning. This study addressed each of Kennedy's hypotheses of lack of knowledge, differing beliefs, constraining circumstances of teaching, and the realistic (un)attainability of reform ideals.

Regarding knowledge, Kadyn and Jessica were both experienced teachers who had participated in graduate-level schoolwork and regular professional development. Although they may have each had knowledge of reform practices, until they partnered, their competence to truly instantiate mathematics education reforms at the secondary level was limited. Through their collaboration, they were able to isolate components of the student-centered reforms and work

together to develop lessons or teaching moves that would center the learners and promote mathematical meaning making. Kennedy (2005) shared that her findings appeared to show that "teachers lack the capacity to manage high intellectual engagement with ideas" (p. 227). She explained that in order to maintain any level of student engagement, teachers want to keep their own line of thinking, not using students' unexpected ideas or questions, in order to keep the class as a whole on track to finish the intended lesson on time. Kadyn experienced this in September, and Jessica was able to model the management of time and of their classroom in order to increase intellectual engagement. Conversely, throughout the year Kadyn shared many competing concerns with Jessica so that as a pair, they could plan to mitigate the impact of each concern. Jessica's prior practice may have taken into consideration ideals of high intellectual engagement, but not the necessary reality of engaging participation of all students in the classroom. The coteaching model provided the teachers resources to address and balance the various competing concerns of teaching in order to learn new ways to manage unexpected occurrences of the classroom while also encouraging intellectual engagement.

Regarding beliefs, values, and dispositions that differ from reform orientations, Kadyn and Jessica worked together to investigate how ideals of the reforms could fit into their evolving practice. Kennedy (2005) argued that teachers conceive of the reform ideas differently than the reformers. Additionally, she argued that due to multiple competing intentions, teachers focus on content set by institutional guidelines rather than content that is inherently important. Kadyn and Jessica addressed these issues in two ways. First, through their coordinated resequencing of topics, they demonstrated their focus on, and responsiveness to, student thinking and the appropriate presentation of content based on what was important for learning. Second, as prompted by Kadyn, they considered ways that all students can participate in lessons in order to

engage at some intellectual level. Although belief systems are complex, internally consistent, and resistant to change, the coteachers' daily practice and sharing of resources continually acted on their individual belief systems to create closer alignment to reform ideals. The teachers were able to learn ways to instantiate lessons that fit within their local context while simultaneously shifting their previously established ideas.

Kennedy's (2005) hypothesis on the circumstances of teaching referred to teachers' inability to rise above the constraints imposed by the daily details of teaching and the job structure itself. However, firstly, with the constant presence of a second teacher, the disruptions and distractions in a classroom are mitigated in the coteaching model. With an aligned practice, while one coteacher addresses the disruption, the second coteacher is able to continue the intended lesson and maintain the intellectual engagement for students, thereby supporting momentum towards continuing to implement reform ideals instead of discouraging their possible instantiation. Secondly, against the common constraint of limited time for lesson preparation, common planning sessions for coteachers are more productive: rather than pulling from only one set of possibilities, the coteachers' interactions more than double the possibilities for creating and detailing lesson plans. Thirdly, Kennedy stated that the "content gets lost in the commotion" (p. 232) in the instantiation of reform-oriented lessons since they can involve activities that indirectly address content and increase unpredicted situations. The availability of both coteachers to think and respond throughout a lesson provides greater opportunity for the lesson to be instantiated as intended and for the coteachers to continue to learn and develop their practice regarding instantiating such lessons. With ideal components of a harmonious partnership at hand, the coteachers effect a greater impact on student learning *together* than they do as individual practitioners.

The final hypothesis Kennedy (2005) considered was whether reform ideals are realistically attainable because teachers are constantly forced to choose among competing intentions. Kennedy discussed how reformers do not adapt to the current realities of teacher work overload involving distractions and creation of materials, nor to the new issues that arise when implementing reform ideals such as orchestrating classroom discussions and building into the larger curriculum for novice learners to relate new ideas. In response, Kennedy suggests building sustainable practices regarding creating classroom communities, creating engaging lessons, and completing intended lessons that move students through the curriculum.

I believe that this study has demonstrated that reform ideals are in fact attainable, but it takes purposeful structures, such as harmonious coteaching partnerships, to budge the heavy load that is "traditional teaching."

In summary, in order to transition the teaching of secondary mathematics towards a more reform-oriented stance, the coteaching structure can provide teachers with resources to develop knowledge and opportunities for action, with continual engagement in coreflecting and coenacting lessons for problem solving, and with supports for each other in the instantiation of intellectually engaging lessons.

Coteaching as Meaningful Professional Development

In the field of professional development, we know that "the complexity of the process we seek to promote" (Clarke & Hollingsworth, 2002, p. 947) requires a different and more purposeful approach in working with teachers. This purposefulness can be found in coteaching partnerships where teachers are provided resources and supports on a regular basis. Although professional development may arm teachers with knowledge of reforms or forward-thinking ideals, it does not necessarily mean that the ideals can be instantiated. In this study, the tensions,

negotiations, and creation of alternatives among the coteachers illustrate the complex process of working to instantiate reform ideals in authentic classrooms. The coteachers achieved professional growth through the "mediating processes of reflection and enactment" (Clarke & Peter, 1993, p. 167).

In reference to the critical features of effective professional development programs by Griffin (1986) and others, coteaching partnerships are inherently ongoing and sustained, integrated into daily work, and purposeful and articulated for the shared classroom around particular subject matter. As teachers integrate their established knowledge, beliefs, and teaching habitus with new experiences involving "*specific* content to *specific* students in *specific* contexts" (Cochran et al., 1993, p. 266), they work within a context-sensitive atmosphere providing opportunities to develop local theory (Murphy et al., 2015). The qualities of the teaching partnership (e.g., beginning orientations, power dynamics) determine the extent to which the coteaching structure can be participatory, collaborative, analytic, and reflective. Harmonious and productive partnerships have the ability to incorporate continuous feedback and reflexivity of resources, and to support growth towards reform orientations.

In relation to the act of reflection, which has potential to create sustainable change, most professional development takes place at the surface level. Sessions for teachers typically address environmental factors, teacher behaviors, or competencies. The type of professional development that research demonstrates as being effective (sustained, local, etc.) must be able to effect changes by reaching the belief and identity layers of the onion model. Lasting change is accomplished through these layers plus the inner core, that of the teacher's mission. As teachers reflect on the goals of their work, they are "driven" to align this transformed mission to their

identity, beliefs, competencies, and behaviors, continuously moving inward and outward on the onion.

In his approach for *professional development 3.0*, Korthagen (2017) proposed integrating practice and theory "namely by giving the person of the teacher a more central place" (p. 389). The personal side of professional development connects everyday work to personal qualities, ideals, and affect of teachers (Korthagen, 2017) thereby addressing the complexity and interconnectedness of knowledge, beliefs, and dispositions of teachers. Using this study's approach, coteachers have the ability to influence each other through socialization (Lortie, 1975), and when focused on reform-oriented ideals, pedagogy can be transformed.

Studying the Complexity of Teaching for Professional Change

For genuine, lasting change to occur in mathematics teaching, we know that teachers must learn in their situated context. Therefore, the contexts of teaching must be further understood, beginning with the multitude of constraints and the complex situations that teachers work with on a daily basis. The development of craft knowledge and teacher habitus within these situations is possible through intensive collaborative work among professionals, such as in coteaching.

Teachers are bounded to the environment of their instructional systems, which can provide sources of justification for teachers' actions. Herbst and Chazan (2012) termed this phenomenon *practical rationality*. Although various stakeholders were involved in the creation of the NCTM (2000) and NGA (2010) documents, the reform ideals have not taken hold in a widespread fashion due to competing intentions (Kennedy, 2005) and teachers' practical rationalities of their work.

As a result, the complexity of teaching cannot be captured by focusing on any one of the complexity's components at a time. For example, studies on reflection with pre-service teachers at the university cannot possibly take into account the realities of day-to-day teaching in a public K-12 setting where the pre-service teachers are headed. The dreams of students who are studying to become teachers often are not realistic within the system's constraints. Teacher education and professional development has to be situated in authentic classrooms in intensive ways for teachers to break down their images of the teacher archetypes they have created during their time as a student and teacher.

In this way, the field of education research requires more foxes rather than hedgehogs (Kilpatrick, 2013). Jeremy Kilpatrick applied these terms to describe how "we in mathematics education have an oversupply of hedgehogs, or at least of hedgehog ideas. We have many people who have one idea that governs everything they do" (p. 177). Instead of staying focused on one idea (or having only one strategy for survival), foxes know many things and are eclectic, "fascinated by the world's variety" (p. 175). Foxes, with multiple (survival) strategies, can study the multitude of factors impacting teaching to further understand the complexity of what impacts teachers' decision making and why reforms cannot reach students. Foxes can also research how various areas of study (e.g., classroom discourse practices and cognitive demand) can combine to strengthen both understanding and developing teaching.

Limitations and Next Steps

Capturing Beliefs and Levels of Reflection

The analysis of the data in this study revealed the prominent role of reflection in the coteachers' work and growth. Integrated within the reflections were the teachers' beliefs and dispositions. However, this study did not premeditatively plan to incorporate tools that would

capture data to measure or describe these constructs. For example, a beliefs survey regarding student learning could have been administered before and after the study's school year to further inform analysis. Additionally, purposeful probing questions for Jessica to ask Kadyn during coplanning/coreflecting meetings could have added understanding to the levels of reflection and the impact of pedagogical and critical reflection on the teachers' growth. Tools such as these could have provided more detailed qualitative data to formulate possible interconnections among knowledge, beliefs, and dispositions.

Participant-Researcher and Self-Study

As the researcher of this study who was also a teacher participant, I *lived* the experience of conducting this study. Prior to establishing the study in the 2018-2019 school year to the time of this writing, I have been informed by both my teaching career and my graduate school work, which is atypical of any co-teachers or of researchers. This duality has allowed me to gain deeper understanding of Kadyn as a teacher and our coteaching partnership that would just not be possible for most researchers. One impact of this interconnectedness of my roles was the ongoing analysis, intentionally and not, of my and our current status. I conducted ongoing analysis while teaching during the 2018-2019 school year as well as being informed during the analysis and writing process of this dissertation, as I continued to coteach with Kadyn in the 2019-2020 and 2020-2021 school years. Additionally, I was able to reflect on our coteaching practices and self-reflect on my personal teaching ideals while taking field notes and documenting our teaching practices, extending my own understanding along the way.

In relation to the significance of my continual involvement and analysis, I did not foresee the importance of incorporating tools to understand my own professional growth within the partnership. A useful methodological addition to accomplish this would have been self-study

methodology (Whitehead, 1993), but I did not design the study to conduct such an analysis. Therefore, the data presented is slightly skewed, sharing Kadyn's concerns and responses to Jessica more so than Jessica's responses to Kadyn, particularly due to Jessica's End of Year Interview of Kadyn. However, the resulting study does present the planning, products, and teaching of the coteaching *partnership*, and this *does* include input from both Kadyn and Jessica. One way to interpret this scenario is that Jessica essentially served as an interventionist, sharing ideas with Kadyn with a reform-oriented mindset, challenging her to plan for and enact different lessons. As Jessica acted in this role, she did not react in the same way Kadyn did, drawing the attention of the findings of the study to the teacher who was experiencing an apparent transition in her teaching.

Uncommon Coteaching Structure

Typically, teachers are paired as co-teachers to support special education students in inclusion classrooms where one teacher is a general education subject area teacher and the second teacher is a special education teacher. Instead, this study was a naturalistic inquiry into a pair of high school mathematics teachers as co-teachers, neither of whom was certified in special education. This uncommon teaching structure is a limitation of this study because of the narrow applicability to other contexts. However, the possibilities for the transformation of teaching apparent in this study can inform and support the creation of more pairs of same-subject area general teachers.

Suggestions for Future Work

In Practice and Professional Development

As Lortie (1975) found, teacher learning occurs through working with colleagues: on the job, during the in-between moments of teaching activities. Therefore, the optimal method for

achieving professional growth is composition of small groups that include teachers who can possibly build towards harmonious relationships through support in forming, storming, and norming stages. Common subject area teachers as collaborators, especially around common classes of students, with regular coreflecting and coplanning meetings, have the potential to greatly increase their creation and enactment of lessons to incorporate progressive practices. In particular, secondary mathematics teachers often work in isolation, but this study has demonstrated the power of coteaching as a structure that can support not only student learning, but also teachers as professionals.

For teachers to successfully migrate their practices to include intellectual engagement of students and interactions with and among students, the competing concerns (Kennedy, 2005) that exist for teachers must be reduced so they can focus on chosen intentions of their work (e.g., reform-oriented practice). Practitioners who work with teachers and policymakers need to listen to and account for the concerns that teachers voice if our society would prefer more impactful educational experiences for students. Additionally, teachers need to be supported in their work to learn about and enact various ways of centering students in the learning process. By listening to student thinking during classroom discourse or utilizing formative assessment during learning activities, teachers decentralize themselves and can subsequently adapt instruction.

In addition, beyond those who work directly with teachers, professional development experiences should allocate time and money to meaningful experiences where teachers engage in reflecting on their own practices and circumstances. Without active participation and rationales provided for professional development, teachers will likely not integrate the intentions of professional development sessions into their daily work. Korthagen's (2017) "professional development 3.0" shifts working with teachers to a more personal level, not only taking into

consideration the local context, but also the qualities of the person. To this end, teacher educators should develop strategies for structuring effective partnerships, professional learning communities, and collaborative relationships among teachers and their administrators.

In Research and Curriculum Development

Case Studies of Coteaching. The single-case study methodology provided a means to "find out what happened, why, and what it means more broadly" (Rubin & Rubin, 2005, p. 6). As is typical in qualitative research, this study deeply investigated the research question on how coteachers served as resources for each other for professional growth. This study described, understood, and interpreted the collected data (Merriam & Tisdell, 2016) to advance theoretical concepts (Yin, 2018). Additionally, working within and defining the context of any professional development is critical to understanding how and why phenomena occur in that particular case. Studies that utilize similar constructs in new contexts can inform the theoretical framework and types of resources that can be made available in coteaching partnerships.

The *trustworthiness* of a study (Lincoln & Guba, 1985) provides a way to evaluate its worth for use in subsequent studies. In this study, many steps were taken to establish credibility such as prolonged engagement and observation over the course of an entire school year, triangulation of data sources through multiple modes and stages of analysis, and member-checking with confirmations of data with Kadyn. Transferability of this study was established through the thick description provided in the chapter of results where detailed accounts of each stage of the year were provided. In particular, this study laid out, for a coteaching partnership aimed at instantiating reform-oriented teaching, "one detailed example of how it was organized, developed, and pursued" (Shulman, 2004, p. 148), providing the richness of portrayal that case study methodology avails in order to impact decision making. As a result, relevant practitioners

working at the classroom and policy levels can be trusted to use this rich portrayal to apply the results to their own situations.

Model Examples for High School Mathematics. To accomplish the vision of teaching and learning in secondary mathematics education as described in the NCTM (2000), NRC (2001), and NGA (2010) documents, teachers and teacher educators require model examples. The field requires examples in various formats of intellectually engaging activities that promote student learning. As models become readily available, and a transition of expectations towards student-centered learning occurs, reform-oriented practices can become more widespread in the field. Additionally, given the disconnects between research and practice, researchers have to both consider the constraints and intentions of teacher practitioners and study the realities of teaching in combination with their areas of focus.

One strategy to support the transition of secondary mathematics teaching towards reform-oriented practices is to supply highly engaging tasks for teachers to implement. At the middle school level, curricula have become widely available that engage learners in conceptual thinking, problem solving, and modeling. This is not true at the high school level, or at least the curricula are not widely promoted to or accepted by practitioners. Within curricular materials, supports (and associated professional development) can provide opportunities for teachers to shift their views of teaching to more interactionary events, rather than solo acts of teacher-centered lessons, with specific support for working with unexpected responses in classroom discourse.

At the Crossroads: Rediscovering Dreams for Teaching

As a high school mathematics teacher, my goal of instantiating research-based, reform-oriented instruction was the driving factor for me to conduct this study. In my partnership with

Kadyn, we were able to instantiate lessons that accomplished the modeling of real-world scenarios and justification of mathematical thinking that the standards (NCTM, 2000) call for.

Before partnering with Kadyn, prior to the launch of this study, I found myself at a crossroads that prompted professional action. It is fitting then to end this dissertation by revisiting that crossroads. My frustrations from not being able to instantiate more engaging and connected lessons within the constraints of the position in which I worked were growing. This crossroads was also caused by the loss of the dreams that inspired me when becoming a mathematics teacher. Often, teachers who reach such crossroads have two directions: continue teaching and slowly forsake their initial dream, or, leave teaching. The former represents many teachers who may, over time, lose their inspiration because they are unable to connect with this initial dream due to the constraints and competing intentions of the job. Of course, the latter has great implications for teaching as a field, for those who were once inspired are able to realize that their potential as a teacher has been restrained by the system in which they work. I have been fortunate to be able to take a third option at my crossroads: I did not have to turn off the road that includes my dreams for teaching. This option has been to partner with Kadyn, a partnership through which we each evolved as teachers and developed our beliefs about teaching and learning, our professional identities as teachers, and our missions in our work with students.

Kadyn, as a partner, served as a resource by challenging me to consider the realities of the classroom and to create, alongside her, alternatives to the status quo of traditional teaching and to my less than stellar attempts of integrating intellectual engagement in my own classroom. By partnering with Kadyn, I found a pathway that allowed me to move forward, evolving in my practice rather than retreating from an inspired vision for teaching.

References

Bacharach, N., Heck, T. W., & Dahlberg, K. (2010). Changing the face of student teaching through coteaching. *Action in Teacher Education, 32*(1), 3-14.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education, 59*(5), 389-407.

Barth, R. S. (2002). *Learning by heart*. John Wiley & Sons.

Blomhøj, M. (2009). Different perspectives in research on the teaching and learning mathematical modelling. In M. Blomhøj & S. Carreira (Eds.), *Mathematical applications and modelling in the teaching and learning of mathematics: Proceedings from Topic Study Group 21 at the 11th International Congress on Mathematical Education in Monterrey, Mexico, July 6-13, 2008.* Roskilde Universitet.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Bourdieu, P. (1990). *The logic of practice*. Stanford University Press.

Brodie, K. (2009). *Teaching mathematical reasoning in secondary school classrooms*. Springer Science & Business Media.

Brodie, K. (2010). Pressing dilemmas: Meaning-making and justification in mathematics teaching. *Journal of Curriculum Studies*, *42*(1), 27-50.

Carambo, C., & Blasie, C. (2010). Coteaching in the Penn STI: Evolution of fluent praxis. In C. Murphy & K. Scantlebury (Eds.), *Coteaching in international contexts: Research and practice* (pp. 199-222). Springer Science & Business Media.

Carambo, C., & Stickney, C. T. (2009). Coteaching praxis and professional service: Facilitating the transition of beliefs and practices. *Cultural Studies of Science Education, 4*(2), 433-441.

Carlisle, K. (2010). Enactment of coteaching in primary schools: Moving towards a shared responsibility. In C. Murphy & K. Scantlebury (Eds.), *Coteaching in international contexts: Research and practice* (pp. 129-150). Springer Science & Business Media.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2000). *Cognitively guided instruction: A research-based teacher professional development program for elementary school mathematics.* National Center for Improving Student Learning and Achievement in Mathematics and Science.

Chaiklin, S. (2003). The zone of proximal development in Vygotsky's analysis of learning and instruction. In A. Kozulin, B. Gindis, V. Ageyev, & S. Miller (Eds.), *Vygotsky's educational theory in cultural context* (pp. 39-64). Cambridge University Press.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis.* Sage.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*(8), 947-967.

Clarke, D., & Peter, A. (1993). *Modelling teacher change.* [Paper presentation]. Contexts in Mathematics Education Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia (MERGA), Brisbane.

Cochran, K. F., DeRuiter, J. A., & King, R. A. (1993). Pedagogical content knowing: An integrative model for teacher preparation. *Journal of Teacher Education*, *44*(4), 263-272.

Cohen, D. K., & Mehta, J. D. (2017). Why reform sometimes succeeds: Understanding the conditions that produce reforms that last. *American Educational Research Journal, 54*(4), 644-690.

Conaill, N. Ó. (2010). A learning space: Student teachers' experience of coteaching science. In C. Murphy & K. Scantlebury (Eds.), *Coteaching in international contexts: Research and practice* (pp. 173-197). Springer Science & Business Media.

Crow, J., & Smith, L. (2005). Co-teaching in higher education: Reflective conversation on shared experience as continued professional development for lecturers and health and social care students. *Reflective Practice, 6*(4), 491-506.

Csikzentmihalyi, M. (1990). *Flow: The psychology of optimal experience*. Harper & Row.

Damasio, A. R. (1999). *The feeling of what happens: Body and emotion in the making of consciousness*. Houghton Mifflin Harcourt.

Danielson, C., & McGreal, T. L. (2000). *Teacher evaluation to enhance professional practice*. ASCD.

Darling-Hammond, L. (2000). Teacher quality and student achievement. *Education Policy Analysis Archives, 8*(1), 1-44.

Darling-Hammond, L., & McLaughlin, M. W. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan, 76*(8), 597-604.

Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N., & Orphanos, S. (2009). Professional learning in the learning profession. *National Staff Development Council, 12*.

Elmore, R. F. (2002). *Bridging the gap between standards and achievement: The imperative for professional development in education.* Albert Shanker Institute.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*(1), 13-33.

Fortune, N. C. (2018). *Supporting a mathematician's instructional change in undergraduate mathematics through faculty collaboration.* North Carolina State University.

Foss, D. H. (2010). Using reflective journals to close the gap between a vision of reform and teaching practice. In E. G. Pultorak (Ed.), *The purposes, practices, and professionalism of teacher reflectivity: Insights for twenty-first-century teachers and students* (pp. 301-323). Rowman & Littlefield Education.

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into Practice, 40*(2), 102-109.

Gallo-Fox, J. (2010). Risk-taking as practice in a coteaching professional learning community. In C. Murphy & K. Scantlebury (Eds.), *Coteaching in international contexts: Research and practice* (pp. 109-128). Springer Science & Business Media.

Gallo-Fox, J., & Scantlebury, K. (2015). "It isn't necessarily sunshine and daisies every time": coplanning opportunities and challenges when student teaching. *Asia-Pacific Journal of Teacher Education, 43*(4), 324-337.

Gee, J. P. (2000). Identity as an analytic lens for research in education. *Review of Research in Education, 25*(1), 99-125.

Gersten, R., & Brengelman, S. U. (1996). The quest to translate research into classroom practice: The emerging knowledge base. *Remedial and Special Education, 17*(2), 67-74.

Griffin, G. A. (1986). Clinical teacher education. In J. V. Hoffman & S. A. Edwards (Eds.), *Reality and reform in clinical teacher education* (pp. 1-24). Random House.

Grimmett, P. P., & Mackinnon, A. M. (1992). Chapter 9: Craft knowledge and the education of teachers. *Review of Research in Education*, *18*(1), 385-456.

Guise, M., Habib, M., Thiessen, K., & Robbins, A. (2017). Continuum of co-teaching implementation: Moving from traditional student teaching to co-teaching. *Teaching and Teacher Education, 66*, 370-382.

Hekimoglu, S., & Sloan, M. (2005). A compendium of views on the NCTM Standards. *The Mathematics Educator, 15*(1), 35-43.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM*, *44*(5), 601-612.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher, 31*(5), 3-15.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hoekstra, A. (2007). *Experienced teachers' informal learning in the workplace*. IVLOS, Universiteit Utrecht.

Huberman, M. (1983). Recipes for busy kitchens: A situational analysis of routine knowledge use in schools. *Knowledge, 4*(4), 478-510.

Hunter, M. C. (1982). *Mastery teaching*. Corwin Press.

Juck, M., Scantlebury, K., & Gallo-Fox, J. (2010). Now it's time to go solo. In C. Murphy & K. Scantlebury (Eds.), *Coteaching in international contexts: Research and practice* (pp. 241-262). Springer Science & Business Media.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*(3), 203-235.

Kennedy, M. M. (2005). *Inside teaching*. Harvard University Press.

Kilpatrick, J. (2013). Needed: Critical foxes. In K. R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 173-187). Springer.

Kolb, D., & Fry, R. (1975). Towards a theory of applied experiential learning. *Theories of Group Processes.* Addison Wesley.

Korthagen, F. A. (1985). Reflective teaching and preservice teacher education in the Netherlands. *Journal of Teacher Education*, *36*(5), 11-15.

Korthagen, F. A. (2001). *Linking practice and theory: The pedagogy of realistic teacher education*. Lawrence Erlbaum Associates Publishers.

Korthagen, F. A. (2014). Promoting core reflection in teacher education: Deepening professional growth. In L. Orland-Barak & C. J. Craig (Eds.), *International teacher education: Promising pedagogies (Part A)* (pp. 73-89). Emerald Group Publishing Limited.

Korthagen, F. A. (2017). Inconvenient truths about teacher learning: Towards professional development 3.0. *Teachers and Teaching*, *23*(4), 387-405.

Korthagen, F. A., & Vasalos, A. (2005). Levels in reflection: Core reflection as a means to enhance professional growth. *Teachers and Teaching*, *11*(1), 47-71.

Korthagen, F. A., & Vasalos, A. (2009, August). *From reflection to presence and mindfulness: 30 years of developments concerning the concept of reflection in teacher education.* [Paper presentation]. EARLI Conference, Amsterdam.

Larrivee, B. (2000). Transforming teaching practice: Becoming the critically reflective teacher. *Reflective Practice*, *1*(3), 293-307.

Larrivee, B. (2008). Development of a tool to assess teachers' level of reflective practice. *Reflective Practice, 9*(3), 341-360.

Larrivee, B. (2010). What we know and don't know about teacher reflection. In E. G. Pultorak (Ed.), *The purposes, practices, and professionalism of teacher reflectivity: Insights for twenty-first-century teachers and students* (pp. 137-161). Rowman & Littlefield Education.

Leikin, R. (2010). Learning through teaching through the lens of multiple solution tasks. In R. Leikin & R. Zazkis (Eds.), *Learning through teaching mathematics: Development of teachers' knowledge and expertise in practice* (Vol. 5, pp. 69-90). Springer Science & Business Media.

Liljedahl, P. (2016). Building thinking classrooms: Conditions for problem-solving. In P. Felmer, E. Pehkonen, J. Kilpatrick (Eds.), *Posing and solving mathematical problems.* Research in Mathematics Education. Springer.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage.

Lortie, D. C. (1975). *Schoolteacher: A sociological study* (Vol. 21). University of Chicago Press.

Merriam, S., & Tisdell, E. (2016). Designing your study and selecting a sample. In S. Merriam (Ed.) *Qualitative research: A guide to design and implementation* (pp. 73-104). John Wiley & Sons.

Milne, C., Scantlebury, K., Blonstein, J., & Gleason, S. (2011). Coteaching and disturbances: Building a better system for learning to teach science. *Research in Science Education, 41*(3), 413-440.

Milne, C., Scantlebury, K., & Otieno, T. (2006). Using sociocultural theory to understand the relationship between teacher change and a science-based professional education program. *Cultural Studies of Science Education, 1*(2), 325-352.

Murphy, C., & Beggs, J. (2010). A five-year systematic study of coteaching science in 120 primary schools. In C. Murphy & K. Scantlebury (Eds.), *Coteaching in international contexts: Research and practice* (pp. 11-34). Springer Science & Business Media.

Murphy, C., & Martin, S. N. (2015). Coteaching in teacher education: Research and practice. *Asia-Pacific Journal of Teacher Education, 43*(4), 277-280.

Murphy, C., & Scantlebury, K. (2010). *Coteaching in international contexts: Research and practice*. Springer Science & Business Media.

Murphy, C., Scantlebury, K., & Milne, C. (2015). Using Vygotsky's zone of proximal development to propose and test an explanatory model for conceptualising coteaching in pre-service science teacher education. *Asia-Pacific Journal of Teacher Education, 43*(4), 281-295.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the l980s*. NCTM.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. NCTM.

National Governors Association. (2010). Common core state standards. *Washington, DC*.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Nuzzi, J., Murray, E., & Golnabi, A. (2020). *Working towards a unifying framework for knowledge for teaching mathematics.* [Paper presentation]. Research on Undergraduate Mathematics Education, Boston, MA.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Patton, M. Q. (2002). *Qualitative research and evaluation methods*. Sage.

Ponticell, J. A. (2003). Enhancers and inhibitors of teacher risk taking: A case study. *Peabody Journal of Education, 78*(3), 5-24.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*(1), 175-189. doi:10.1037/0022-0663.91.1.175

Robert Wood Johnson Foundation. (n.d.). Qualitative Research Guidelines Project. http://www.qualres.org/HomeFiel-3650.html

Rodgers, C. R., & Scott, K. H. (2008). The development of the personal self and professional identity in learning to teach. In M. Cochran-Smith, S. Feiman-Nemser, D. J. McIntyre, & K. E. Demers (Eds.) *Handbook of research on teacher education* (pp. 732-755). Routledge.

Roth, W.-M. (2002). *Being and becoming in the classroom*. Greenwood Publishing Group.

Roth, W.-M., Lawless, D., & Tobin, K. (2000). Towards a praxeology of teaching. *Canadian Journal of Education, 25*(1), 1-15.

Roth, W.-M., & Tobin, K. G. (2002). *At the elbow of another: Learning to teach by coteaching*. Peter Lang Pub Incorporated.

Roth, W.-M., Tobin, K., Carambo, C., & Dalland, C. (2005). Coordination in coteaching: Producing alignment in real time. *Science Education, 89*(4), 675-702.

Roth, W.-M., Tobin, K., & Zimmermann, A. (2002). Coteaching/cogenerative dialoguing: Learning environments research as classroom praxis. *Learning Environments Research*, *5*(1), 1-28.

Rubin, H. J., & Rubin, I. S. (2011). *Qualitative interviewing: The art of hearing data*. Sage.

Ruthven, K. (2002). Linking researching with teaching: Towards synergy of scholarly and craft knowledge. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 581-598). Lawrence Erlbaum.

Scantlebury, K., Gallo-Fox, J., & Wassell, B. (2008). Coteaching as a model for preservice secondary science teacher education. *Teaching and Teacher Education, 24*(4), 967-981.

Schoenfeld, A. H. (2014). *Mathematical problem solving*. Elsevier.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*. Routledge.

Senge, P. M., Scharmer, C. O., Jaworski, J., & Flowers, B. S. (2005). *Presence: An exploration of profound change in people, organizations, and society*. Currency.

Sewell Jr., W. H. (1992). A theory of structure: Duality, agency, and transformation. *American Journal of Sociology, 98*(1), 1-29.

Shepel, E. L. (1999). Reflective thinking in educational praxis: Analysis of multiple perspectives. *Educational Foundations, 13*(3), 69-88.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (2004). Autonomy and obligation. In L. S. Shulman & S. M. Wilson (Eds.), *The wisdom of practice: Essays on teaching, learning, and learning to teach* (pp. 133-162). Jossey-Bass.

Sileo, J. M. (2011). Co-teaching: Getting to know your partner. *Teaching Exceptional Children, 43*(5), 32-38.

Sileo, J. M., & van Garderen, D. (2010). Creating optimal opportunities to learn mathematics: Blending co-teaching structures with research-based practices. *Teaching Exceptional Children, 42*(3), 14-21.

Tekkumru Kisa, M., & Stein, M. K. (2015). Learning to see teaching in new ways: A foundation for maintaining cognitive demand. *American Educational Research Journal*, *52*(1), 105-136.

Tharp, R., & Gallimore, R. (1998). A theory of teaching as assisted performance. In D. Faulkner, K. Littleton, & M. Woodhead (Eds.), *Learning relationships in the classroom* (pp. 93-110). Routledge.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), 105-127.

Tuckman, B. W. (1965). Developmental sequence in small groups. *Psychological Bulletin*, *63*(6), 384.

Veresov, N. (2010). Introducing cultural historical theory: Main concepts and principles of genetic research methodology. *Cultural-Historical Psychology, 2010*(4), 83-90.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Harvard University Press.

Wagner, A. C. (1987). “Knots” in teachers' thinking. In J. Calderhead (Ed.), *Exploring teachers' thinking* (pp. 161-178). Cassell.

Whitehead, J. (1993). *The growth of educational knowledge*. Hyde Publications.

Wilson, S., Shulman, L., & Richert, A. (1987). 150 different ways of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teachers’ thinking* (pp. 104-124). Cassell.

Yin, R. K. (2018). *Case study research and applications: Design and methods*. Sage.

Yoo, J., Heggart, K., & Burridge, N. (2019). Collaborative coteaching (CCT): Practitioner learning through shared praxis. *Australian Journal of Teacher Education*, *44*(4), 5.

Zebroski, J. T. (1994). *Thinking through theory: Vygotskian perspectives on the teaching of writing*. Boynton/Cook.

Appendix A: End of Year Interview Questions

1) Thinking back on the school year, what stands out for you?
   Follow up questions:
   a) What lessons were most different from what you previously taught?
   b) What were some of turning points during the year? (referring to possible shifts in practice)

2) How do you feel that your practice has changed after this year of (co)teaching?
   Follow up questions:
   a) In what ways did you teach differently based on what I brought to you

3) Do you think the state test influenced our teaching and our plans for next year? (standards?)

4) What do you see as the value for our students to learn math?
   a) Does this apply to *all* students or just this population?

5) Anything else you'd like to share?

Appendix B: Teaching Artifact for Linear Functions in Standard Form (December)

Graphing Linear Functions in Standard Form

Problem 1: You have been saving your money because you want to buy your family members some tickets to see a show. Adult tickets cost $20 each and student tickets (under 18 years old) cost $10. You have saved $300. How many of each type of ticket can you buy?

A) Write three important pieces of information that are in this problem:

A1)

A2)

A3)

B) Define the two **variables** in the situation:

B1)

B2)

C) What will the total cost be for:

C1) the adult tickets:

C2) the student tickets:

D) Write an equation that models this situation:

E) Create a **table of values** with possible combinations of tickets

F) Graph the values using appropriate **scales** on the axes

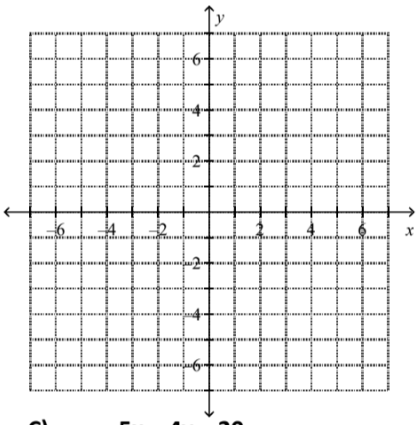
Problem 2: Jason wants to buy an Xbox One X, which costs $400. His mom gives him a weekly allowance of $20 for doing his chores and during some weekends, he works at the Funplex earning $40 per week. How many weeks will he have to do his chores and work at Funplex to have enough money to buy the Xbox?

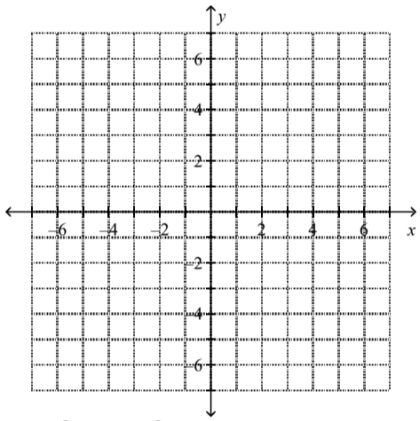
Standard Form for Linear Functions:

For each example below, find the x and y intercepts then graph the line

A) **3x + 4y = 12**

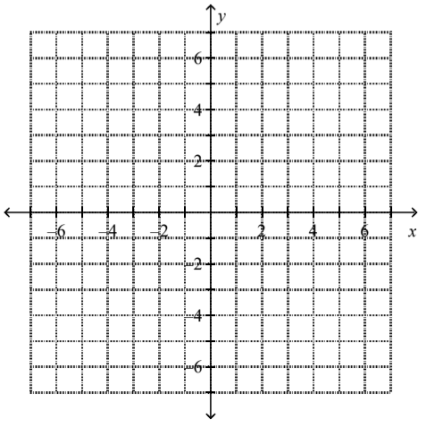
B) **2x – 4y = 8**

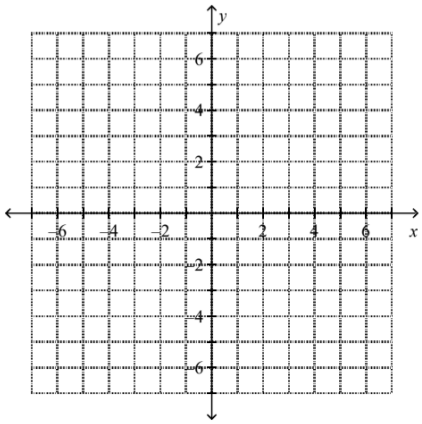




C) **-5x - 4y = 20**

D) **- 3x + y = 6**





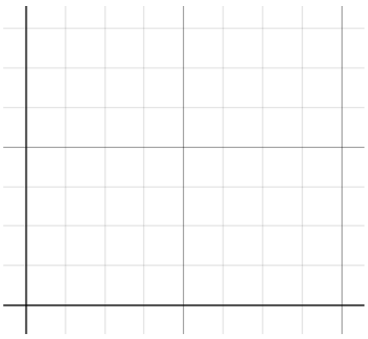
Appendix C: Teaching Artifact for Systems of Linear Functions (January)

Algebra 1: Systems of Linear Functions

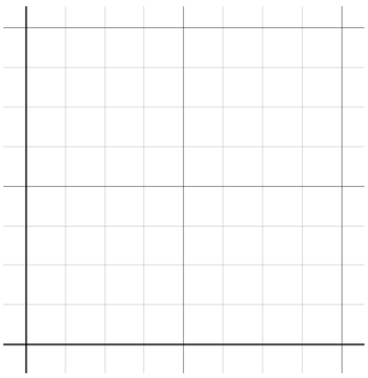
Model each situation by defining variables, writing two equations, and sketching a graph with labeled axes.

1) Will is tired of students not returning their pencils to the math class pencil box. He finds a subscription service that will deliver boxes of pencils for $3 a box after a $4 sign-up fee. Instead, he could buy boxes of pencils at the store for $5 each.

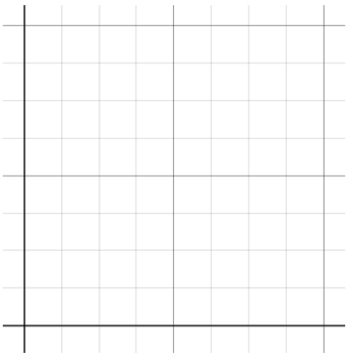
Should Will sign up for the subscription service or just shop at the store? Why?



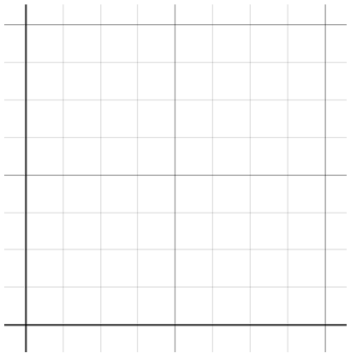
2) Mrs. N's daughter enjoys going ice skating. They pay $25 to rent a pair of ice skates for a season and pay $15 each time she goes skating. Mrs. P's son enjoys going to SkyZone Trampoline Park where they have to buy special jump socks one time, which cost $5 and they pay $15 each time he goes for a jumping session. Who is spending more money?



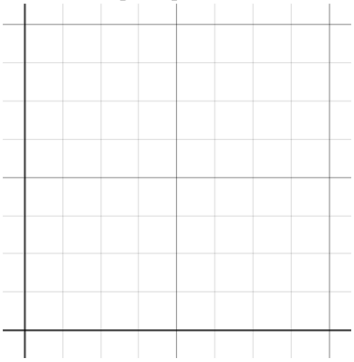
3) A restaurant is considering starting a loyalty program where if customers pay $40 to join, a certain menu of items will be $10 each. Without the membership, the menu items would be $18 each.



4) Jennifer wants to buy a Great Adventure season pass. One pass is $50 just to get in the park for the season, but she would also have to pay $30 each time she goes for parking and food. A different pass is $100 which includes park admission and food, she would have to pay $12 each time for parking.



5) Jacob and Kevin love going to Burger King after school. They also buy food for their friends. They buy $3 burgers and $2 fries with the $18 they have. They want a total of 8 items for themselves and their friends. How many burgers and how many fries should they buy?



Appendix D: Teaching Artifact for Quadratic Functions Task (May)

Four hundred people came to last year's winter play. The ticket price was $5.
This year, the Drama Club is hoping to earn enough money to take a trip to a Broadway play.
They estimate that for each $0.50 increase in the price, 10 fewer people will attend their play.
How much should the tickets cost in order to maximize the income from this year's play?

1. How much did the Drama Club make last year? Write a sentence describing how you know.

2. a. If the Drama Club increases the ticket price one time, how much will the ticket be?

   b. How many people will come?

   c. How much will they make?

3. Complete the table of values

# of $0.50 increases	Ticket price	# people attend	Income
0			
1			
2			
3			
4			
5			
8			
15			

4. a. Write an expression for the ticket price where *n* represents the number of $0.50 increases

   b. Write an expression for the number of people that attend (also using *n*)

   c. Write an equation that represents the amount of money the Drama Club will make, depending on the number of $0.50 increases they decide on.

   Income =

5. Can the Drama Club make the ticket price very high? Explain why or why not.

   Give an example that illustrates your answer: use the value for *n* to evaluate your example using the equation from 4c.

6. Draw a sketch of the graph that represents the equation you wrote in 4c (use the table from part 3 and your example from part 5). Choose appropriate values for your axes.