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Differences Between Concrete and Virtual Manipulatives in Preparing Tenth Grade Math Students for Standardized Tests

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Differences between concrete and virtual manipulatives in preparing tenth grade math students for standardized tests

by

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ABSTRACT

Research has suggested that the use of manipulatives may enhance students' grasp of mathematical concepts. Manipulatives may be concrete (physical) objects or virtual computer-based images of objects similar to their three-dimensional counterparts. Studies with elementary and middle-school students generally have found virtual manipulatives to outperform concrete manipulatives in enhancing students' conceptual understanding of mathematics. However, very little research on manipulative use has been conducted with secondary school students. Furthermore, few studies have investigated instruction using virtual or concrete manipulatives across more than one content area.

This study assessed the impact of virtual and concrete manipulatives in a group of 164 tenth grade math students across four different content areas (number sense, geometry, algebra, and discrete math). Students were enrolled in the Geometry lab (Geolab) curriculum, which is designed to reinforce mastery of core content standards and to prepare students for standardized tests. A crossover design was employed whereby seven classes of Geolab students were randomized to receive instruction in the first content area using only concrete manipulatives while seven other classes were taught the same topics using only virtual manipulatives. Each week a new content area was presented, and the type of manipulatives alternated between the classes each week. The study duration was four weeks.

Pre and post-tests were conducted for each content area. Difference scores between post-test and pre-test in each area served as the primary measures of achievement. Dependent variables were either the unit difference scores or subset scores. Manipulative subset scores were created by combining the unit difference scores across

weeks where the same type of manipulative was used. Repeated-measures Analysis of Variance (ANOVA) was employed to determine any differences according to the type of manipulatives or the content area. Students' sex and ethnicity were also included in the analyses as control variables.

Overall, student improvement was highest for algebra followed by number sense, geometry, data analysis. However, an interaction was observed between group and unit score. An attitudinal survey was also administered to students upon completion of the study to determine their opinions on the use of manipulatives. Both groups appeared to exhibit a slight preference for using virtual manipulatives, and stated that they learned better using such methods. This is despite the analysis findings showing a slight superiority for concrete manipulatives in terms of achievement scores.

Both types of manipulatives assisted instruction more so for the unit content areas of number sense and geometry, than for the areas of algebra and data analysis. Concrete manipulatives appeared to slightly outperform virtual manipulatives, although this difference emerged significantly only for the second subset (algebra & data analysis). This finding suggests that concrete manipulatives may outperform virtual ones only when the material is more difficult for students. Survey results of students' preferences were not related to achievement scores.

This research was IRB approved.

DIFFERENCES BETWEEN CONCRETE AND VIRTUAL MANIPULATIVES IN PREPARING TENTH GRADE MATH STUDENTS FOR STANDARDIZED TESTS

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1: Introduction

Concrete manipulatives are three-dimensional objects which can be used in hands-on activities. Virtual manipulatives are interactive computer-based visual representations of dynamic objects. Both can be used for presenting opportunities to construct mathematical knowledge. An example of a concrete manipulative is interlocking-cubes. Its corresponding virtual manipulative is computer-based images of cubes which can be manipulated on the screen by pointing, clicking, and dragging.

Many studies conducted on concrete and virtual manipulatives have shown that these tools have considerably enhanced students' grasps of mathematical concepts and situations. Additionally, virtual manipulatives have been found to be more effective than concrete manipulatives. In spite of the research, there have not been sufficient studies conducted with high school students in the use of manipulatives.

This thesis study was undertaken to determine the value of manipulatives in preparing tenth grade math students for standardized tests and more specifically to examine the difference between concrete and virtual manipulatives in achieving the objective of test preparation. The results of this study should provide useful guidance to mathematics teachers in preparing high school students for standardized tests by the use of manipulatives.

The instructional topics used for the study were aligned with objectives defined by the New Jersey Core Curriculum Content Standards and included problems typical of those found on standardized tests such as the New Jersey High School Proficiency Assessment (HSPA). In addition, because of the socio-economic cross section of the students involved, an attempt was made to investigate the dependence on success that ethnicity and gender might have with regards to successful performance using manipulatives. These conclusions are more controversial because other factors are involved in students' performance. For instance, learning style also may affect the effectiveness of manipulatives (Curtain-Phillips, n.d.). Curtain-Phillips (n.d.) suggested that a certain dependence exists as to the type of learner the student is. Those students who are kinetically preferential may respond more positively to manipulatives than the visual learner. According to Curtain-Phillips, "manipulatives appeal to the learning style of kinesthetic learners because they actually touch the objects. Pictures appeal visually for visual/spatial learners" (para. 3).

A correlation between a specific unit's content and the use of physical/virtual manipulatives has also been considered (Taylor, 2001). The approach to each unit has been shown to have greater successes for concepts other than statistics and algebra. Content area where success was supported by the use of manipulatives was in learning fractions. In 2000, Maccini and Hughes (2000) used the STAR algebra problem-solving strategy, in which words are translated into images, and image equation is created. STAR is an acronym for search the word problem, translate the words into picture or equation, answer the problem, and review the solution. The problem then is represented using concrete manipulatives to which an answer is found. The students were asked to create an iconic representation of the equation as part of the learning process. Problems involved addition, subtraction, multiplication, and division of integers. The icons represented were the concrete manipulatives inclusive of pictorial displays. A student survey showed that almost all participants commented on the positive aid that manipulatives provided in solving problems involving integer numbers. The students also suggested the use of manipulatives for future lessons.

Potential candidates for the study were students enrolled in a Geometry Lab (GeoLab) class at Hackensack High School. As part of the Geolab curriculum, some students were taught topics using only concrete manipulatives and others were taught the same topics using only virtual manipulatives. Four units were covered with each unit representing one of the following four content standards:

- number and numerical operations
- geometry and measurement
- patterns and algebra
- probability and discrete math

Each unit extended over five days. Students took the same pre-test on the first day, participated in student-centered activities using their respective manipulatives during the next three days, and took the same post-test on the fifth day. The study lasted approximately four weeks. At the end of the four weeks, students were given attitudinal surveys to gather their opinions of the experience. Students and teachers were also interviewed; however, the data from the student and teacher interviews were not included in the analysis because the questions asked in the interviews were beyond the scope of the research questions.

The purpose of the thesis study was to determine when and if a virtual manipulative offers distinct advantages over using a concrete manipulative. The information will be useful for lesson planning in Geolab and similar courses that are supplemental to the general math courses and courses that are designed to reinforce mastery of the core content standards.

2: Literature Review

Durmas and Karakirik (2006) posited that inspirational educational activities and cognitive tools might develop students' active involvement in the teaching-learning process and promote their reflection on the concepts and relations under investigation. The authors also argued that the use of manipulatives not only enhances students' conceptual understanding and problem solving skills but also encourages the development of positive attitudes towards mathematics since manipulatives evidently provide "concrete experiences" that focus concentration and increase motivation (p. 117). According to Durmas and Karakirik, physical or real-world features do not define a concrete experience in a mathematical context; a concrete experience is determined by how significant the connection is to the mathematical ideas and situations. For example, a student might create the meaning of the concept "four" by building a representation of the number and connecting it with either real or pictured blocks. Virtual manipulatives, also called computer manipulatives, appear to offer interactive environments where students can manipulate computer objects to create and solve problems. An example of a virtual manipulative are blocks which can be manipulated by pointing, clicking, and dragging a mouse that allow the user to move the blocks to create a figure composed of multiple blocks. The software can then provide information such as the surface area or volume of the user-created composite figure. Durmas and Karakirik suggested that it might be because students receive instant feedback about their actions when working in a virtual environment that they form connections between mathematical concepts and operations. However, Durmas and Karakirik stressed that whether using physical or virtual manipulatives, it is necessary to connect the use of a specific manipulative to the mathematical concepts or procedures being studied (p. 119).

Use of models and/or manipulatives gives assessment of mathematical learning a cohesive connection to mathematical instruction (Kelly, 2006). Kelly examined the relationship between mathematical assessment and the use of manipulatives. Kelly also noted that teachers who consistently and effectively model the use of manipulatives in front of all students were demonstrating their belief that using concrete objects to understand abstract concepts was acceptable and expected of all students. Furthermore, developing rubric-based assessments for manipulative-based activities with students and colleagues helps assure that the assessments actually measure what is being taught and practiced. According to Kelly, the use of such assessments in combination with the use of manipulatives should build strong student investment in the teaching-learning process while developing deeper mathematical learning.

Physical Manipulatives

Relative to the teaching and learning of mathematics, physical, or concrete, manipulatives are three-dimensional objects used to help students bridge their understanding of the concrete environment with the symbolic representations of mathematics (Clements, 1999; Hynes, 1986; Moyer, 2001; Terry, 1996). There has been historical documentation of the use of manipulatives such as the abacus, counting sticks, and of course fingers, prior to the Roman Empire (Fuys & Tischler, 1979). Concrete manipulatives come in a variety of physical forms, ranging from grains of rice to models of the solar system. Manipulatives can be simple or sophisticated, purchased or teacher/student-made. The appearance of commercially made manipulatives in the United

States increased during the 1960s after the work of Zoltan Dienes and Jerome Bruner was published (Thompson & Lambdin, 1994). Examples of teacher-made manipulatives include those that use materials such as beans, buttons, popsicle-sticks, and straws (Fuys & Tischler, 1979). Today's teachers have access to a wide variety of commercially available manipulatives designed to aid in the teaching of most elementary mathematical concepts. Examples include Algebra tiles, attribute blocks, Base-10 materials, color tiles, Cuisenaire rods, fraction strips, geoboards, geometric solids, pattern blocks and Unifix cubes.

Many educators continue to view manipulatives as teaching tools that involve physical objects that teachers use to engage their students in practical and hands-on learning of mathematics. These manipulatives continue to be instrumental to introduce, practice, or remediate mathematical concepts and procedures. The manipulatives support the teaching and learning of mathematics from lessons that address number and operations, algebraic concepts and procedures, geometry, probability and even with college level courses such as calculus and linear algebra.

In summary, physical manipulative materials are objects that relate to mathematical concepts or procedures because they are touchable and movable by learners and appeal to multiple senses (Heddens, 2005). According to Reys (1971), manipulatives should be available for students to "feel, touch, handle, and move" (p. 551). The use of manipulatives can help make abstract ideas and symbols more meaningful and understandable to students and widely support mathematics education, and educators have long recommended them (NCTM, 1989, p. 17).

Research Related to the Use of Physical Manipulatives

Just as small group instruction benefits students only if the teacher knows when and how to use this teaching practice, the value of using manipulative materials to investigate a concept depends upon how they are used. Hynes (1986) outlined four pedagogical criteria to consider when selecting a manipulative to support the teaching or leaning of a concept or procedure: (a) the manipulative needs to be a clear representation of the mathematical idea, (b) be appropriate for the student's cognitive development and motor development levels, (c) be of interest to the child, and (d) be versatile. Without proper selection, manipulatives can become set tools that students use to go through the motions of a lesson without understanding the related mathematical topics (Hynes, 1986).

Although kinesthetic experience, which involves using physical manipulatives, can enhance perception and thinking, "understanding does not travel through the fingertips and up the arm" (Ball, 1992, p. 15). Stein and Bovalino (2001) argued, "simply using manipulatives does not guarantee a good mathematics lesson" (p. 356). However, many teachers do not understand how to help students make the connection between concrete representations and the symbolic representations (Moyer, 2001). Without guidance, students may make incorrect or misleading connections (Holt, 1982). For example, it is easy for adults who already understand how to interpret the numerator and denominator in a fraction to see the relationship between Cuisenaire rods and fractions. However, for young students this relationship may not be obvious (Holt, 1982). Thus, without proper guidance from a trained teacher, students may make connections that are not beneficial to their understanding (Clements, 1999). Therefore, students need to learn how to reflect on their actions while using manipulatives (Clements, 1999).

Bohan and Shawaker (1994) claimed concrete manipulatives are effective, but that "transfer of learning" (p. 1) must take place if students are to reap the full benefits of concrete manipulative use. They defined transfer of learning as, "a situation in which studying topic A will help in understanding topic B" (p. 1). Furthermore, Bohan and Shawaker recommended that the transfer of learning should occur during the concrete, bridging (also referred to as iconic), and symbolic stages of learning. The concrete stage of learning is where mathematical situations are solved using manipulatives without the use of any symbols. Simultaneous manipulation of objects and symbols occurs at the bridging stage. At the symbolic stage, students begin working with symbols alone (Bohan & Shawaker, 1994, p. 1-2).

Clements (1999) and colleagues (Clements & McMillen, 1996; Clements & Sarama, 2005) have focused on rethinking what it means to use a "concrete" material in the mathematics classroom and have attempted to redefine the concept of "concrete" manipulatives. Clements and colleagues' concern revolved around the assumption that using physical manipulatives ensures conceptual understanding. For example, Clements and McMillen (1996) asked, "does concrete mean something students can grasp with their hands? Does this sensory character itself make manipulatives helpful?" (p. 270). Clements (1999) and Clements and McMillen (1996) concluded that the sensory character of manipulatives was not the determining factor; instead, "integrated-concrete knowledge" (Clements, 1999) is the result of a combination of "… many separate ideas in an interconnected structure of knowledge" (Clements, 1999 p. 48). More specifically, physical manipulatives may or may not be one of the materials that will help students achieve integrated-concrete knowledge (Clements, 1999).

Taylor (2001), however, questioned the use of concrete manipulatives as an effective tool in teaching all mathematics concepts. Taylor's focused on teaching,probability, and the study results validated the null hypothesis, which was, "There will be no significant difference between students who use concrete manipulatives and students who do not use concrete manipulatives regarding students' learning skills and concepts in experimental probability". Taylor suggested that concrete manipulatives are more useful in teaching certain math concepts than they are in teaching others. While Taylor refuted the effectiveness of utilizing concrete manipulatives in teaching probability, the manipulatives did help students learn incidental fraction concepts. Taylor also noted that students were comfortable with using concrete manipulatives to help them learn about probability concepts.

The application of concrete manipulatives in teaching algebra has not been methodically studied and researched outside of relational word problems (Maccini & Hughes, 2000). Maccini and Hughes used the STAR algebra problem-solving strategy, in which, (a) problems are explored, (b) words are translated into an equation in image form, (c) the problem is represented using concrete manipulatives, (d) the answer is found, (e) and finally the answer is reviewed. Six secondary school students with learning disabilities were involved in the study. The students studied how to represent and solve problems that involved the addition, subtraction, multiplication, and division of integers. Maccini and Hughes asked the students to generate an iconic (pictorial) representation of the problem as part of the learning process as well as apply concrete manipulatives. On a social validation form, almost all participants suggested that the manipulatives helped them understand what it means to solve problems involving integer numbers and recommended its use with other students.

Similarly, Thompson and Lambdin (1994) used concrete materials for two major purposes:

1. Allow teachers and students to engage in and discuss something concrete. Discussions included how to think about materials and to interpret the meanings of several actions.
2. Provide something upon which students can act. Thompson and Lambdin (1994) stressed that the focus should be mainly on what teachers want students to learn and not what teachers want students to do. Thompson and Lambdin argued, "concrete materials can be an effective aid to students' thinking and to successful teaching. But effectiveness is contingent on what one is trying to achieve" (p. 556-558).

Some teachers are likely to view mathematics as isolated rules for manipulating symbols instead of as a unified whole. This leads to students' misconceptions that stem from incorrectly applying concrete materials and as a weakness of the materials by teachers (Hall, 1998).

According to Suydam and Higgins (1977), physical manipulatives can have a significant positive effect on student achievement. In fact, a meta-analysis of 60 studies conducted by Sowell (1989) concluded that mathematics achievement increased when physical manipulatives were used over an extended period, such as a school year or longer. Students who used physical manipulatives outperformed students who did not have access to those manipulatives (Clements, 1999). Another meta-analysis of 64 studies conducted between 1960-1982 found that students using manipulatives scored at the 85^{th} percentile on standardized tests as compared to students not using manipulatives, who scored at the 50th percentile (Parham, 1983).

Hiebert and Wearne (1992) established that students could use manipulatives in a rote learning behavior, having limited or no understanding of the mathematical concepts related to the procedures. In their study, nine students in the fourth grade, 10 students in the fifth grade, and 10 students in the sixth grade learned decimal concepts with the aid of Base-10 blocks. Hiebert and Wearne found that nearly all students "established connections between the blocks and symbols that generalized approximately to extended notation" (p .99-122). Along similar lines, Hall (1998) concluded that concrete materials, because of the ease of relating actions on physical objects to mathematical procedures or concepts, could be a useful pedagogical tool that enables learners to move to applying the same operations to icons, characters and symbols.

Clements and McMillen (1996) proposed that using manipulatives does not always guarantee conceptual understanding. Jackson (1979) identified several common misconceptions about using manipulative materials including the beliefs that

- manipulatives always simplify the learning of mathematical concepts
- manipulatives are more useful in primary grades than in intermediate and secondary grades, and
- manipulatives are mostly applicable for low-ability students and not for high-ability students.

In short, employing manipulatives in a class is not straightforward. Good use of manipulatives requires carefully defining the role of the teacher, the aims of the lesson, and the potential that the use of manipulatives has to assist with the tasks involved. Clements and McMillen (1996) also claimed that students often fail to link their action with manipulatives to describe the actions.

Heddens (2005), however, argued differently. According to Heddens, using manipulative materials in teaching mathematics might help students learn to

- link real world situations to mathematical symbols and concepts
- work as a team and in cooperation for solving problems
- discuss mathematical ideas and concepts
- verbalize their mathematical thinking
- make presentations in front of a large group
- realize that there are many different ways to solve problems
- understand that mathematical problems can be symbolized in many different ways
- understand that they can solve mathematical problems in different ways as from those demonstrated by their teachers

Gravemeijer (1990) stated that although results differ depending on what and how manipulatives work in learning situations, learning with manipulatives positively correlates with later development of mental mathematics. An example of Gravemeijer's point is the use of Base-10 blocks. Base-10 blocks is a widely known mathematical manipulative wherein ones are represented by small square tiles, tens by thin rods, hundreds by ten-by-ten flats, and thousands by a large ten-by-ten-by-ten block. The use of Base-10 blocks was demonstrated to enhance students' conceptual understanding of arithmetic operations (Fuson & Briars, 1990). Similarly, Chassapis (1998) showed that the use of a compass in Geometry to support students' learning about circles developed a better understanding of center and radius concepts in comparison to the understanding created by the use of conventional circle tracings and templates.

In conclusion, physical manipulatives continue to work as important tools that allow students to reach higher levels of thinking. Students can use them to solve problems in non-routine ways. When used properly by knowledgeable teachers, manipulatives can help students make connections between a concrete understanding of mathematical concepts and the corresponding abstract mathematical ideas (Stein & Bovalino, 2001).

Virtual Manipulatives

Advances in technology have given birth to a new generation of manipulatives—virtual manipulatives. Virtual manipulatives are computer-based images of objects that are similar to their three-dimensional counterparts. Virtual manipulatives are generally "interactive, web-based visual representations of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard & Spikell, 2002, p. 373). Additionally, "computer based renditions of common mathematics manipulatives and tools" are virtual manipulatives (Dorward, 2002, p. 329). A user in a computer environment can use and move these virtual objects (Skylar, 2009). Virtual manipulatives are hands-on models that students use in a virtual environment to model mathematical objects by entering, clicking, dragging and dropping computer objects into appropriate locations.

With the evolution and creation of Java software, virtual manipulatives have become easier to create and place on the Internet. As a result, many Internet sites exist and new virtual manipulatives increase regularly. Virtual manipulatives exist on the Internet as applets, or smaller versions of application programs. Students can manipulate these dynamic, pictorial objects by moving the computer mouse. Moreover, some applets are now available on hand-held devices such as iPads and smart phones. Schools and professional organizations have been experimenting with the use of digital tools in the classroom. For instance, interactive whiteboards, such as SmartBoards® enable teachers and students to interact with virtual manipulatives in a natural way within classrooms.

Steen, Brooks and Lyon (2006) advocated that virtual manipulatives be viewed as more than just electronic replications of their physical counterparts. The authors argued that virtual manipulatives typically include features that expand on what a physical manipulative offers. Steen et al. stated, "some virtual manipulatives are able to present a representation that would not be easily made or even possible with physical manipulatives, an attribute shared with types of computer simulations" (p. 375).

A large number and variety of virtual manipulatives have become accessible on the Internet, including those found on the websites of the National Council of Teachers of Mathematics (http://illuminations.nctm.org/), the National Library of Virtual Manipulatives (http://nlvm.usu.edu/), and the Shodor Education Foundation (http://www.shodor.org/interactivate/). Mayer and Anderson (1992) pointed out that, "the general design structure for virtual manipulative applets is to include verbal codes (i.e., letters, numbers, and words) and visual codes (i.e., pictures, movable 2-D and 3-D objects)" (p. 444) presented simultaneously, thereby increasing the effectiveness of multimedia instruction. The assumption is that mathematical environments that provide multiple, active systems of codes have a greater potential for enhancing students' learning capabilities by allowing at least two mental representations instead of just one. Therefore, virtual manipulatives are an exclusive externalized representational form.

Goldin (2003) defined illustrations as arrangements of symbols, signs, visual characters, icons, or objects that are a representation of something else. Virtual manipulatives may be considered an exclusive form of representation or a combination of a number of representations. Goldin and Shteingold (2001) stated that students' ability to convert multiple representational systems determines their abilities to model and grasp mathematical constructs. Thus, virtual manipulatives may be an essential element of mathematics learning and teaching as constituents of representational systems in that representations are essential to students' understanding of mathematical concepts and relationships. Representations enable students to respond to mathematical concepts and arguments and to share understanding with each other. Additionally, representations help students familiarize themselves with connections between related concepts and apply those connections to practical mathematical problems.

Visual representations of concepts and relations help learners to gain insight in mathematics. Furthermore, virtual manipulatives, such as Tangrams and Geoboards enable as much engagement as physical manipulatives do because they are actual models of physical manipulatives (Dorward & Heal, 1999). According to Durmas and Karakirik (2006), virtual manipulatives are interchangeable with physical manipulatives in mathematical explorations because even though virtual manipulatives do not make mathematical concepts "touchable" virtual manipulatives do outline the prominent features of the task under study. Durmas and Karakirik also suggested that virtual manipulatives might offer additional benefits over those provided by physical manipulatives by discarding some of the limitations that physical manipulatives may place on a task. Additionally, Skylar (2009) noted that virtual manipulatives allow students to gain a deeper understanding of complex mathematical concepts, thereby facilitating retention of those concepts.

Haistings (2009) recommended that when using virtual manipulatives for the first time, students be given sufficient time to become familiar with using the computer-based manipulatives. Haistings noted how important it was for teachers to model the proper use of the virtual manipulatives for all of their students. For instance, it was not until the third or fourth day of Haisting's study that all students were comfortable and independently used the virtual manipulatives with ease. Moreover, all of the participating teachers in Haistings' study commented on the need for more time in the computer lab in order to keep this type of practice consistent.

As an assistive technology virtual manipulatives can address a range of learner difficulties. Many manipulative Internet-based environments direct students to engage with the material, provide guiding questions, and create multiple opportunities for success. Students with a history of struggling with mathematics can use the virtual manipulatives to verify thinking and see immediate success. Furthermore, students who struggling with a concept can request a model demonstration, obtain immediate feedback on incorrect answers, or request additional instruction or explanations.

Researchers suggested that development of virtual manipulatives will enhance the environment of learning mathematics. Moyer-Packenham, Salkind, and Bolyard (2008) argued that the skills required to manipulate virtual manipulatives highlight the teacher's role as a guide, rather than as a transmitter of facts. Furthermore, as with any technology, teachers need to plan for the effective use of virtual manipulatives. For example, teachers must consider how they will support students before, during, and after the instructional activity as well as the different types of support needed for introducing a topic, practicing or applying a skill, or remediating a skill or concept (Zorfass, Follansbee, & Weagle, 2006). Depending upon the instructional goal, teachers can determine how the virtual manipulative is introduced, monitored, and supported.

Based on the results from student questionnaire and attitude surveys, Reimer and Moyer (2005) argued that virtual manipulatives

- enabled students to better understand and learn about fractions by offering prompt and specific reviews and feedback
- enhanced students' enjoyment while learning mathematics, and
- were much easier and faster to employ as compared to paper-and-pencil methods

Reimer and Moyer also argued that virtual manipulatives are more effective than physical manipulatives in classroom teaching because physical manipulatives are dependent on the teacher's ability to make these concrete concepts to abstract symbols connections explicit.

Virtual manipulatives are much more than games integrated into the mathematics curriculum because they help students connect abstract ideas with concrete models. Teachers have been incorporating physical manipulatives into their classroom instruction for years, but the greatest barriers to the broader use of physical manipulatives have been having adequate numbers of manipulatives, and the time involved with getting them out, setting them up, and putting them away. Another barrier to the full use of physical manipulatives is the difficulty—or in most cases the impossibility—of sending manipulatives home with students for out of school use. Virtual manipulatives can address or minimize the impact of these barriers. In particular, many virtual manipulatives may be accessible by students from their homes and therefore can be used as a home/school connection for math homework (Lindroth, 2005).

Using virtual manipulatives in connection with instruction is a part of the increasing use of technology in connection with mathematics teaching and learning. Although teachers and researchers are still measuring the impact the use of virtual manipulatives have on students' learning, there has been evidence that showed virtual manipulatives offer unique advantages over the use of physical manipulatives and can be effective in supporting the learning of mathematics. (Crawford & Brown, 2003; Lee & Chen, 2008a; Reimer & Moyer, 2005; Steen et al., 2006; Stellingwerf & Van Lieshout, 1999; Suh & Heo, 2005). Specifically, while using virtual manipulatives, children can apply mathematical concepts and explore processes for representing those concepts (Clements & Sarama, 2005; Moyer-Packenham et al., 2008).

Theories of learning that include the use of media may provide insight into why researchers are finding constructive initial results in studies involving the application and possible benefits of using virtual manipulatives in classrooms (Moyer-Packenham et al., 2008). Despite limitations to the research on virtual manipulatives, it is important to note that studies have shown the unique features of these tools relative to the teaching of mathematics. For example, Zbiek, Heid, Blume, and Dick, (2007) argued that virtual manipulatives are cognitive technological tools because the tools enable users to act as representations of different objects on the virtual manipulatives, with the consequences of the user's activities resulting in visual on-screen response from the virtual tool. Even though virtual manipulatives to some extent resemble their relevant physical manipulative, as cognitive tools, virtual manipulatives have inimitable characteristics that go beyond the capabilities of physical manipulatives. In this aspect, Moyer-Packenham et al. (2008) said that the potential for learning is thus improved "for mathematically meaningful actions by users and influences the user's learning" (p. 203).

However, despite the potential of virtual manipulatives for supporting the teaching and learning of mathematical concepts, the research indicated that, if possible, physical manipulatives should be introduced before using virtual manipulatives. Moyer found that students and teachers believed they were successful when using the virtual Base Blocks Addition because similar exercises had been practiced in the classroom with physical Base-10 blocks. As such, the concept of Base-10 blocks to model numbers was not new when students began to use the virtual version. Thus the students were able to easily build numbers with virtual Base-10 blocks and focus on the action of combining numbers for multi-digit addition. Teachers in a more recent study (Moyer-Packenham et al. (2008) also expressed a preference for using physical manipulatives prior to using virtual manipulatives.

Research Related to Using both Physical and Virtual Manipulatives

Zacharia, Olympiou, and Papaevripidou (2008) investigated the comparative value of experimenting with physical manipulatives in a sequential combination with virtual manipulatives. The researchers had students use physical manipulatives prior to the use of virtual manipulatives or experiment with each alone, and documented changes in students' conceptual understanding in the domain of heat and temperature. The researchers used a pre–post comparison study design that involved 62 undergraduate pupils who completed a basic course in physics. Each participant was assigned to the control or the experimental group. All students used the same inquiry-oriented study materials. The results indicated that experimenting with a combination of physical manipulatives and virtual manipulatives increased and enhanced students' conceptual understanding more than experimenting with just physical manipulatives. The use of virtual manipulatives was identified as the cause of this differentiation.

Terry (1996) examined 102 students in grades 2 through 5 using Base-10 blocks and attribute blocks and discovered that when learners applied a combination of both physical as well as virtual manipulatives, they showed remarkable improvements during the pre-test and post-test when compared to learners who applied either physical manipulatives or virtual manipulatives alone. Overall, the results pointed out that when learners apply virtual manipulatives, either in combination with physical manipulatives or alone, they showed gains in mathematics achievement and understanding and appeared to be more occupied and on task as also described by Moyer-Packenham et al. (2008).

However, the application of virtual manipulatives throughout the core of a mathematics lesson appeared to be different from the way other scholars had described teachers' uses of physical manipulatives. Moyer's (2001) reported observing teachers associating use of physical manipulatives with "having fun" and not putting emphasis on the "real mathematics" in the lesson. Even though Moyer's classroom observations showed that 30% of the lessons in the curriculum employed physical manipulatives for games, observations also indicated that games were only two out of 95 lessons involved the use of virtual manipulatives. This drastic difference showed a remarkable gap between the way teachers in Moyer-Packenham et al.'s study (2008) chose to apply virtual manipulatives and the way the tutors in Moyer's (2001) study employed physical manipulatives.

Of nine studies conducted before 1999, three provided evidence to suggest that students who used virtual manipulatives experienced higher achievement and conceptual understanding in mathematics than those who used physical manipulatives or no manipulatives (Kieran & Hillel, 1990; Smith, 1995; Thompson, 1992). Two studies provided evidence that suggested students who used both virtual and physical manipulatives showed an increase in their conceptual understanding of mathematics (Ball, 1988; Terry, 1996). Four of the nine studies found no statistically significant difference in achievement between students who used physical manipulatives alone, virtual manipulatives alone, a combination of both physical as well as virtual manipulatives, or no manipulatives (Berlin & White, 1986; Kim, 1993; Nute, 1997; Pleet, 1990).

Another study compared the use of virtual manipulatives versus concrete manipulatives among third graders students learning about fractions. The researchers found that the group using virtual manipulatives significantly outperformed the group using concrete manipulatives (Suh, 2005). However, another study of third graders learning about algebraic relationships found no significant difference in student achievement between those that used either virtual or physical manipulatives (Suh & Moyer-Packenham, 2007).

Brown (2007) studied the impact of applying virtual manipulatives and concrete manipulatives on elementary students' achievement and understanding of concepts about equivalent fractions. The primary focus of the study was to determine whether students who used virtual manipulatives would out-perform those who used concrete manipulatives across the researcher/teacher-produced post-test. A secondary interest of the study was to examine students' attitudes concerning the use of manipulatives during mathematics lessons. The study, which involved 48 sixth graders studying fractions, found the group that used concrete manipulatives significantly outperformed the group that used virtual manipulatives (Brown, 2007).

Summary

In addition to analyzing differences between student performance on pre and post-tests, many studies looked at qualitative differences between the uses of the two methods. A common theme found among the studies was that students seemed to stay on task more frequently when using virtual manipulatives over other methods (Hunt, Nipper & Nash, 2011).

Most of the studies that investigated the use of physical and virtual manipulatives involved elementary school students. Only a few involved middle school students, and Howard, Perry and Lindsey (1996) presented baseline data on the use of manipulatives in secondary school mathematics classrooms. Howard et al. showed that the use of manipulatives in the selected secondary classrooms was low in comparison to such use in primary school mathematics lessons.

Generally, there has been an insufficient number of studies conducted with high school students and the use of manipulatives in connection with mathematics courses. In addition, those studies conducted at the secondary level concentrated on one content area such as geometry or only one topic, such as equivalent fractions. Only a few studies investigated instruction using virtual or concrete manipulatives across more than one content area. Thus there appears to be a gap in the available research into these types of comparisons.

Although most initial studies involving virtual manipulatives focused on the difference between physical and virtual manipulatives, as additional research was developed and published, the need for more controlled classroom research became evident. For example, after completing a two-classroom study using one virtual manipulative, Moyer, Niezgoda, and Stanley (2005) claimed, "it is important to investigate problems and questions using different technologies and forms of representation in real classrooms and to explore effective ways to use these technologies in teaching mathematics to all." This thesis research is, in part, a response to Moyer et al.'s call for additional research.

Research Questions

For this thesis study, there are two main research questions:

Research Question 1. What impact do the virtual and concrete manipulatives have on students' achievement when learning concepts in number and numerical operations, geometry and measurement, patterns and algebra, and data analysis, probability, and discrete math?

Research Question 2. What learning preferences exist between the virtual environment and physical environment in teaching concepts in number and numerical operations, geometry and measurement, patterns and algebra, and data analysis, probability, and discrete math?

3: Methodology

Participants

Potential candidates for the thesis study came from a pool of approximately 160 tenth grade students enrolled in a Geolab course at Hackensack High School which is located in Hackensack, New Jersey. Students were enrolled in the course based on their Grade Eight Proficiency Assessment (GEPA) performance in mathematics. The GEPA is used to identify eighth grade students who may need additional instruction in language arts literacy, mathematics, and science (State of New Jersey, 2010a). Students scoring Partially Proficient on the GEPA are considered to be below the state minimum level of proficiency (State of New Jersey, 2010a). Thus, most of the students who participated in the thesis study were of lower ability in core math skills. The student population of the school is culturally diverse with a fairly equal representation of male and female as well as White, African-American, and Hispanic ethnicities. Hackensack High School is located in an urban setting and is approximately 10 miles west of New York City.

Study Design

The thesis study applied a crossover two-sequence four-period (2x4) design, with sequence assignment occurring at the class level. Each Geolab class was randomly assigned to either a CVCV (1^{st} week using concrete manipulatives, 2^{nd} week using virtual manipulatives, etc.) or VCVC sequence of manipulative presentation (concrete or virtual), and all students within that class received the same allocation. The duration of each sequence was four weeks, with 1-week intervals for each period. The study design and variables are depicted schematically in Figure 3.1. A detailed description of the procedures and variables used in this study are presented in the sections that follow.

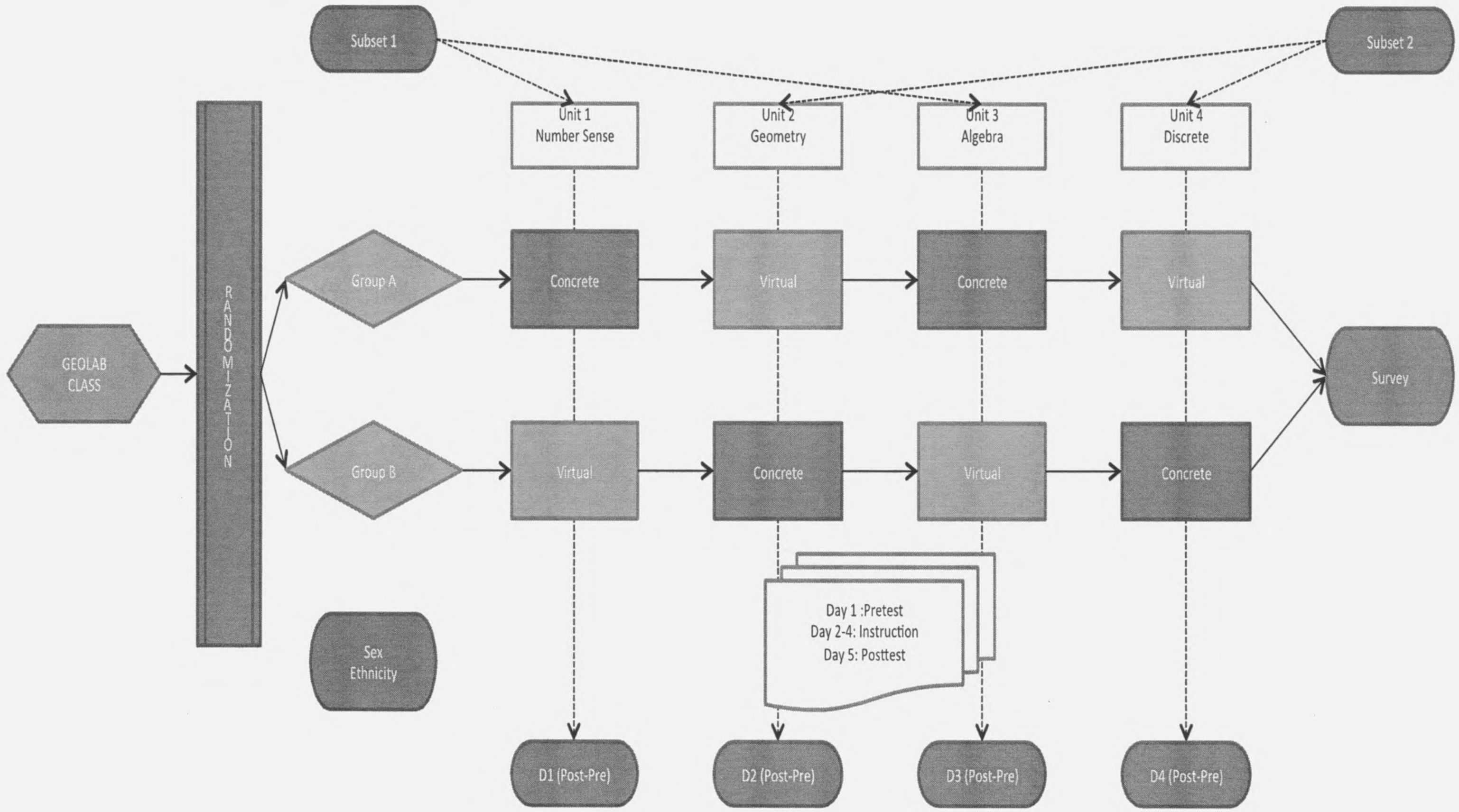

Figure 3.1. Visual algorithm depicting study design and variables.

Procedure

The thesis study occurred during a four-week time period during regular school hours at Hackensack High School. Students participated in the study during their regularly scheduled mathematics class sessions. Four unit topics were presented, one per week:

- Unit 1: Number sense (Week 1)
- Unit 2: Geometry (Week 2)
- Unit 3: Algebra (Week 3)
- Unit 4: Data Analysis, Probability, and Discrete Math (Week 4)

Students were given a pre-test on the first day of each unit. The second, third, and fourth day of each unit consisted of instruction of a topic using a different virtual or concrete manipulative each day. Students were given a post-test on the fifth day. The same routine was repeated for the remaining three units. Difference scores were computed by subtracting the pre-test from the post-test scores in each unit, and these difference scores served as the measure of achievement.

Seven different math teachers taught the Geolab classes. Each teacher taught anywhere from one to three different sections. A Geolab class consisted of material that reinforced topics being concurrently taught in a Geometry class as well as topics found on the mathematics portion of the High School Proficiency Assessment (HSPA). The first four mathematics standards of the HSPA, also called content clusters, correspond to the four unit topics in the study (State of New Jersey, 2010b). The classes were randomly assigned, and the first unit taught used either concrete (Group A) or virtual (Group B) manipulatives. There were a total of 14 Geolab classes that participated in the study:

seven classes randomized to Group A and seven classes randomized to Group B. The classes alternated methods for the second, third, and fourth units. Group A experienced concrete manipulatives during instruction of units one and 3, and virtual manipulatives for units two and 4. Group B experienced the opposite pattern; virtual manipulatives in units one and 3, and concrete manipulatives in units two and 4. Thus, all participants received both treatments thereby experiencing use of concrete manipulatives for two units and virtual manipulatives for the other two units.

Each unit contained topics from its respective content area. Virtual and concrete manipulatives used during instruction included Cuisennaire bars, interlocking cubes, geoboards, algebra tiles, spinners, and dice.

At the end of the final unit, an attitudinal survey was given to students. The survey was created by the researcher and was designed to determine students' opinions on the subject areas that they had trouble with, and the degree to which they liked using manipulatives for each cluster/unit of instruction. The survey (Appendix D) contained 35 items, and each required a response on a 4-point Likert scale (Strongly Disagree = 1, Disagree = 2, Agree = 3, Strongly Agree = 4).

Variables

This section provides operational definitions for the variables used in the thesis study.

Group. This variable referred to which medium of manipulatives classes were initially randomized. Classes were randomly assigned to either Group A (concrete manipulatives in units one and 3, virtual manipulatives in units two and 4) or Group B (virtual manipulatives in units one and 3, concrete manipulatives in units two and 4). This was a categorical variable with two categories.

Sex. Students were coded as male or female. This was a categorical variable with two categories.

Ethnicity. The ethnicity of students was categorized as (1) White, (2) Hispanic, (3) Black/African American, or (4) Asian. This was a categorical variable with four categories.

Week. Each week of the thesis study corresponded to a particular unit of instruction. This categorical variable refers to the four units of mathematics instruction. For each unit, a pre-test was administered on day 1, instruction and use of manipulatives occurred on days 2-4, and a posttest was administered on day 5.

Subset. Subset 1 referred to weeks 1 and 3. Subset 2 referred to weeks 2 and 4. This was a categorical variable with two categories.

Unit Difference Scores. In order to account for differences in students' pre-test abilities, difference (D) scores were created by subtracting the pre-test from post-test scores within each unit. Therefore D1 = Post-test Unit 1 – Pre-test Unit 1. The values D2, D3, and D4 were created in a similar manner for the subsequent units. The difference scores were the primary dependent variable in this study.

Subset Difference Scores. This variable referred to the combined unit difference scores for a particular subset. Subset Difference Score 1 (S1) combined the unit difference scores for units 1 and 3. Subset Difference Score 2 (S2) combined the unit difference scores for units 2 and 4. By combining the unit difference scores for alternate units this dependent variable represents the unit difference scores across weeks wherein the subjects received the same manipulative. Therefore, for Group A – S1 reflects difference scores when using concrete manipulatives , and S2 reflects difference scores when using virtual manipulatives. For Group B – S1 reflects difference scores when using virtual manipulatives and S2 reflects difference scores when using concrete manipulatives.

Statistical Analysis

Two research questions guided the design and analysis of this study. In this section, the research questions are presented and the analyses designed to address each research question are described.

First, descriptive statistics (frequency distributions, mean, standard deviation (SD)) were calculated for each variable as appropriate for the variable measurement level.

Research question 1 (RQ1). What impact do the virtual and concrete manipulatives have on students' achievement when learning concepts in number and numerical operations, geometry and measurement, patterns and algebra, and data analysis, probability, and discrete math?

A number of analyses were conducted to address RQ1 and assess whether virtual and concrete manipulatives resulted in different levels of student achievement. Because each participant was exposed to every condition, the study constituted a repeated-measures design. All analyses for RQ1 were conducted using JMP Statistical Software, v.9 (SAS Institute Inc., 2000).

The model used for the formal analysis of RQ1 was a Repeated Measures Analysis of Variance (RM ANOVA). This has the form:

$Y_1 \ldots Y_k = A + B + A*B + \ldots$

Where k = the number of repeated measures on each student. The factors (A, B, C, etc.) and their interaction (A*B, etc.) determined the actual RM ANOVA model that was fitted.

RM ANOVA was conducted on the unit scores. The repeated measure was week/unit and the difference scores calculated for each unit were the dependent variable. The factors (between-subjects variables) were the variables of group, sex, and ethnicity. Group was the primary factor of interest, whereas sex and ethnicity were included as control variables. Interaction terms with group were also entered in the model to determine whether there were any systematic group differences according to these demographic variables.

To summarize, the RM ANOVA model used was as follows:

$Y_1 \ldots Y_4$ = Group + Sex + Ethnicity + Sex*Group + Ethnicity*Group

Where $Y_1 \ldots Y_4$ equal the four unit difference scores (Y_1 = D1, Y_2 = D2, Y_3 = D3, Y_4 = D4.) The factors were Group, Sex, and Ethnicity. The factor interactions included in the model were Sex*Group and Ethnicity*Group.

The list of questions addressed by this RM ANOVA analysis and the corresponding analysis components are summarized in Table 3.1. The key effect of interest was the interaction term between group and week. Differential scores for one or more weeks according to group suggested that this was due to the type of manipulative, as this was designed to be the only variation between the groups each week.

Table 3.1. *Analysis Questions for RM ANOVA of Unit Difference Scores*

Research Question	RM ANOVA Analysis Component
Did overall scores vary according to respondents' sex, ethnicity, or group?	Between-subjects main effects
Was there variation in overall scores for groups A and B according to respondents' sex, or ethnicity?	Between-subjects interaction effects
Were there significant differences in the scores across the four weeks?	Within-subjects main effect
Was there variation in the pattern of scores across weeks according to respondents' group, sex, or ethnicity?	Within-subjects interaction effects

A second RM ANOVA was conducted on the responses with the subset scores S1 and S2 serving as the dependent variable. The subset scores reflected the manipulative medium. Thus, subset one was the medium that each group started with (combined across weeks). Subset two was the medium that each group switched to (combined across weeks). The subset analysis was collapsed across the four separate weeks/units in an attempt to more clearly define whether the manipulative medium had any effect on achievement.

The primary factor in the model was group. Sex and ethnicity were also included as factors to control for any differences due to these demographics. Of principal interest was whether there was a significant interaction between group and subset. A significant interaction would indicate a differential pattern of responding for groups A and B that could be attributed to the type of manipulatives they used. The questions that this RM ANOVA addressed and the corresponding analysis terms examined are presented in Table 3.2.

The model for the RM ANOVA analysis took the following form:

$Y_1, Y_2 = \text{Group} + \text{Sex} + \text{Ethnicity}$

Where $Y_1 = S1$ and $Y_2 = S2$, and the factors were Group, Sex, and Ethnicity. No factor interactions were specified in the model.

Table 3.2. *Analysis Questions for RM ANOVA of Subset Scores*

Research Question	RM ANOVA Analysis Component
Did overall scores vary according to respondents' sex, ethnicity, or group?	Between-subjects main effects
Did scores in weeks 1 and 3 (S1) differ from scores in weeks 2 and 4 (S2)?	Within-subjects main effects
Was the pattern of scores between subsets different according to respondents' group, sex, or ethnicity?	Within-subjects interaction effects

Finally, independent samples t-tests were conducted on the subset scores. The between-subjects variable was group (A or B) and the dependent variables were S1 and S2. Analyses were conducted on S1 and S2 separately. The goal of this analysis was to clarify whether the type of manipulative had a significant effect on the scores.

Research question 2 (RQ2). What learning preferences exist between the virtual environment and physical environment in teaching concepts in number and numerical operations, geometry and measurement, patterns and algebra, and data analysis, probability, and discrete math?

The dependent variable for RQ2 was the responses to the attitudinal survey presented at the end of the four-week sequence. The primary independent variable was the group/sequence of the respondent.

The responses to the 35 survey items were coded as follows: Strongly Disagree = 1, Disagree = 2, Agree = 3, Strongly Agree = 4. Five survey items (items 20-24) asked respondents about their opinions on manipulatives and the use of concrete versus virtual manipulatives. Frequency distributions were created for these items separately for participants in groups A and B. Mann-Whitney U tests were calculated to determine whether there were any differences in the pattern of responding on these items according to group.

Survey items 9 to 19 asked participants to rate how much difficulty they had with various mathematics subject areas ("I have trouble…"). Summary scores were created by averaging across the items pertaining to each unit. The items corresponding to each unit are shown in Table 3.3. Items 25 to 35 on the survey required participants to rate the degree to which they enjoyed using manipulatives in each subject area ("I liked using manipulatives…"). Items were similarly averaged to provide a summary score for each unit (Table 3.3).

Means and standard deviations were calculated for each summated unit score and separately for groups A and B. Independent samples t-tests were computed to determine whether any of the scores differed significantly between the groups.

Table 3.3. *Summated Survey Items Corresponding to Each Unit*

	Difficulty with Unit	Enjoyed Use of Manipulatives in Unit
Unit 1	9-11	25-27
Unit 2	12-15	31-33
Unit 3	16-17	28-30
Unit 4	18-19	34-35

4: Results

This chapter presents the results of the analyses used to address the research questions of the thesis study as detailed in previous chapters. First the variables are defined, and the demographic characteristics and grouping (between-subjects) variables of the sample are presented. Next follows a presentation of the descriptive statistics of the dependent variables. Then, each research question is addressed in turn. The analysis methods used to address each research question are briefly summarized, the results of the statistical analyses are presented, and decisions regarding the statistical significance of findings are made. Analytic conclusions with respect to the research questions are presented in chapter 5.

Statistical Methods

Chapter 3 presented a detailed discussion of the methodological procedures used in the thesis study. Analyses were conducted with JMP Statistical Software, v.9 for research question one (RQ1) and SPSS v.20 for research question two (RQ2). The alpha level was set at .05 as the decision point for statistical significance. Missing data were excluded on a case-wise basis; thus, the number of valid cases differed according to each analysis.

In the repeated-measures ANOVA analyses conducted for RQ1, least square means were used in the analyses. Least square means (LSM or LS means) differ from observed (arithmetic) means in an unbalanced (unequal-N) design with more than one effect. In these circumstances, the arithmetic mean for a group may not reflect the "typical" group response as it does not account for other effects in the model. LS means are estimates of the marginal means for a balanced population. They are within-group means adjusted for the other effects in the model (SAS Institute Inc., 1999).

Definition of Variables

Table 4.1 provides a definition of variables used in the analyses (Chapter 3 contains a detailed description of all variables). There were five independent variables: group, sex, and ethnicity, week, and subset. The repeated-measures dependent variables were either the unit difference scores or subset difference scores. Subset difference scores were created by summing the unit difference scores across units in which the participants were exposed to the same type of manipulatives.

Description of the Sample

There were a total of 164 participants in the thesis study. Seventy-six students (46.3%) were in classes randomized to Group A (concrete manipulatives in units one and three, virtual manipulatives in units two and four) and 88 students (53.7%) were randomized to Group B (virtual manipulatives in units one and 3, concrete manipulatives in units two and 4). Group was the primary between-subjects variable used in the analyses.

The sex and ethnicity of the participants by group and overall are shown in Table 4.2. These two demographic variables were also used as between-subjects variables in the statistical analyses. There were approximately even numbers of females (52.4%) and males (47.6%) in the sample, and both sexes were fairly evenly represented in each group. The largest ethnic group in the sample identified as Hispanic (43.3%) followed by Black (35.4%). There were comparatively few White individuals (17.1%) and only seven Asian individuals (4.3%) in the total sample. Due to the statistical limitations of using an independent variable with few members in some categories, the Asian and White cases were combined for some analyses and denoted as the AW ethnicity. When combined, the AW ethnicity grouping comprised 21.3% of the total sample.

Table 4.1. *Definition of Variables*

Type	Variable	Type	Values
IV	Group	Between-subjects, 2 categories	Group A; Group B
IV	Sex	Between-subjects, 2 categories	Male; Female
IV	Ethnicity	Between-subjects, 3 or 4 categories	Hispanic; Black; White; Asian -or- Hispanic; Black; Asian/White
IV	Week	Between-subjects, 4 categories	Unit 1; Unit 2; Unit 3; Unit 4
IV	Subset	Between-subjects, 2 categories	Subset 1; Subset 2
DV	Unit Difference Scores	Within-subjects, 4 levels	D1 = Unit 1Post-test – Pre-test Scores D2 = Unit 2 Post-test – Pre-test Scores D3 = Unit 3 Post-test – Pre-test Scores D4 = Unit 4 Post-test – Pre-test Scores
DV	Subset Difference Scores	Within-subjects, 2 levels	S1 = Unit 1 + Unit 3 Difference scores (D1 + D3) S2 = Unit 2 + Unit 4 Difference scores (D2 + D4)

Note. IV = Independent Variable, DV = Dependent Variable

Table 4.2. *Demographic Characteristics of the Sample*

		Group A ($n = 76$)	Group B ($n = 88$)	Overall ($n = 164$)
Sex	Female (F)	37 (48.7%)	49 (55.7%)	86 (52.4%)
	Male (M)	39 (51.3%)	39 (44.3%)	78 (47.6%)
Ethnicity	Asian (A)	2 (2.6%)	5 (5.7%)	7 (4.3%)
	Black (B)	23 (30.3%)	35 (39.8%)	58 (35.4%)
	Hispanic (H)	42 (55.3%)	29 (33.0%)	71 (43.3%)
	White (W)	9 (11.8%)	19 (21.6%)	28 (17.1%)
	Asian/White (AW)	11 (14.5%)	24 (27.3%)	35 (21.3%)

Note. Letters in parentheses denote abbreviations used in following tables and figures.

Description of the Variables

As described in chapter 3, the measures of achievement in the thesis study were calculated as difference scores in order to account for baseline differences in pre-test ability. Difference scores were calculated by subtracting the post-test and pre-test scores for each of the four units. Each pre-test and post-test consisted of 18 questions with a total maximum score of 30 points. The test scores out of 30 were converted to percentages (out of 100) before calculating difference scores.

The descriptive statistics for these scores are presented in Table 4.3. As shown by the minimum and maximum values, there was considerable variability in the scores with negative and positive values present in each unit. A negative value indicates that the score decreased from pre- to post-test, while a positive score indicates improvement pre- to post-test. The means were generally in the positive direction indicating that as a whole, the participants' scores improved following the week of instruction. Scores were highest for unit three (D3) and unit one (D1) indicating the greatest improvement in these units. The smallest difference score was observed for unit four and, in fact, the mean was negative for this unit in Group A. This suggests that there was little improvement in students' scores within this unit.

Table 4.3. *Descriptive Statistics of Unit Difference Scores by Group and Overall*

Group	Unit	N	Mean	SD	Min	Max
A	D1	74	14.94	18.32	-33.33	57.41
	D2	72	5.94	16.44	-31.49	46.30
	D3	68	18.35	18.32	-57.41	51.85
	D4	68	-1.06	14.37	-37.04	37.04
B	D1	83	11.02	16.78	-24.07	51.86
	D2	81	5.19	15.75	-33.33	37.03
	D3	84	19.71	19.18	-24.08	70.37
	D4	84	6.83	13.58	-20.37	38.89
Overall	D1	157	12.87	17.58	-33.33	57.41
	D2	153	5.54	16.03	-33.33	46.30
	D3	152	19.10	18.75	-57.41	70.37
	D4	152	3.30	14.44	-37.04	38.89

Note. D scores refer to difference scores (post-test – pre-test) for each unit. D1 = Number sense, D2 = Geometry, D3 = Algebra, and D4 = Probability.

In the second set of analyses, subset scores rather than difference scores were used as the dependent variable. Subset scores were created by combining across scores in weeks wherein the subjects received the same manipulative. Since each manipulative was alternated between weeks in each group, this equated to summing the difference scores for units one and three, and summing the scores for units two and four. For Group A – subset one (S1) was concrete, and subset two (S2) was virtual. For Group B – S1 was virtual and S2 was concrete.

The descriptive statistics for the subset scores are presented in Table 4.4. As would be expected from the unit scores, the S1 scores (D1 + D3) were considerably higher for both groups than the S2 scores (D2 + D4).

Table 4.4. *Descriptive Statistics of Subset Difference Scores by Group and Overall*

Group	Subset	N	Mean	SD	Min	Max
A	S1	67	32.48	29.43	-61.12	100.00
	S2	66	4.40	24.07	-53.71	74.07
B	S1	82	30.58	24.88	-22.21	96.30
	S2	79	11.98	19.38	-33.33	55.55
Overall	S1	149	31.43	26.95	-61.12	100.00
	S2	145	8.53	21.89	-53.71	74.07

Note. Subset one (S1) created by summing unit difference scores D1 and D3. Subset two (S2) created by summing unit difference scores D2 and D4.

Research Question 1

Research question one (RQ1) asked, what impact do the virtual and concrete manipulatives have on students' achievement when learning concepts in number and numerical operations, geometry and measurement, patterns and algebra, and data analysis, probability, and discrete math?

Analyses were conducted to address RQ1 and determine whether virtual and concrete manipulatives resulted in different levels of student achievement. The results of these analyses are presented in the next sub-section.

Repeated measures analysis of unit difference scores. A Repeated Measures Analysis of Variance (RM ANOVA) was conducted on the unit difference scores. The repeated measure was unit/week and the difference scores calculated for each unit were the dependent variable. The factors (between-subjects variables) were the variables of group, sex, and ethnicity. Group was the primary factor of interest, whereas sex and ethnicity were included as control variables. Interaction terms of sex and ethnicity with group were also entered in the model to determine whether there were any systematic group differences according to these demographic variables. The full analysis effects are shown in Table 4.5. First, the between-subjects effects are presented. Then, the within-subjects effects and interactions are discussed.

Table 4.5. *RM ANOVA Effects of Unit Difference Scores*

	F-value	df	p-value
Between-Subjects			
Group	2.37	1,127	.13
Sex	0.0001	1,127	.99
Ethnicity	0.75	3,127	.52
Sex * Group	9.71	1,127	.002
Ethnicity * Group	1.31	3,127	.27
Within Subjects			
Week	12.91	3,125	< .0001
Week * Group	3.92	3,125	.01
Week * Sex	3.05	3,125	.03
Week * Ethnicity	1.12[a]	9,304.37	.35
	(Wilks' Lambda = .92)		
Week * Sex * Group	1.03	3,125	.38
Week * Ethnicity * Group	2.64[a]	9,304.37	.006
	(Wilks' Lambda = .83)		

Note. [a]Approximate F-value.

Between-subjects effects. Between-subjects main effects showed no overall effect of group ($p = .13$), sex ($p = .99$), or ethnicity ($p = .52$). Thus, the overall scores, when combined across weeks, did not differ according to these factors. However, there was a significant interaction between sex and group, $F(1, 127) = 9.71$, $p = .002$ This indicates that the pattern of scores by group differed for males and females. Inspection of the marginal means shows that the overall mean for females was higher in group A (12.28) than group B (9.23). For males, there was the opposite pattern, and the group A mean (7.25) was lower than the group B mean (12.79). There was no interaction between ethnicity and group ($p = .27$).

Within-subjects effects. There was a significant within-subjects main effect of week, $F(3,125) = 12.91$, $p < .001$ (Figure 4.1). Thus, the improvement observed between pre-test and post-test was not consistent for each unit of instruction. The overall LSM for D3 was highest (19.37) indicating the most improvement from pre- to post-test in unit three (D1-algebra). This was followed by D1–number sense (12.56), D2–geometry (4.97) and D4–probability (4.07).

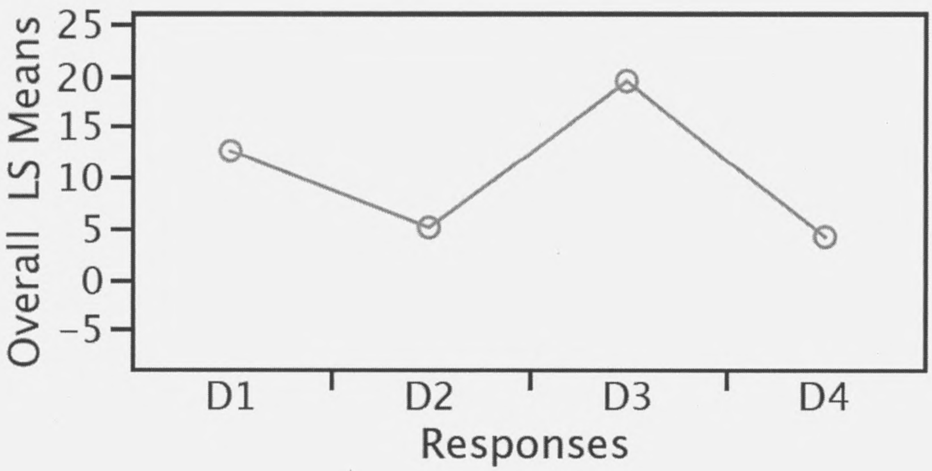

Figure 4.1. Overall least square (LS) means for unit difference scores.

Significant interaction effects were present between week and the group and sex between-subjects factors (Figure 4.2) and the least square means are provided in Table 4.6, $F(3,125) = 3.92$, $p = .01$. Both groups had relatively parallel profiles for units one and two. However, in units three and four, there were significantly higher scores for group B than for group A. In unit three, the LSM for group B (21.73) was approximately nine points higher than the group A mean (12.95). Similarly, in unit four the mean for group B was positive (10.74) and considerably higher than the mean shown for group A (-1.76).

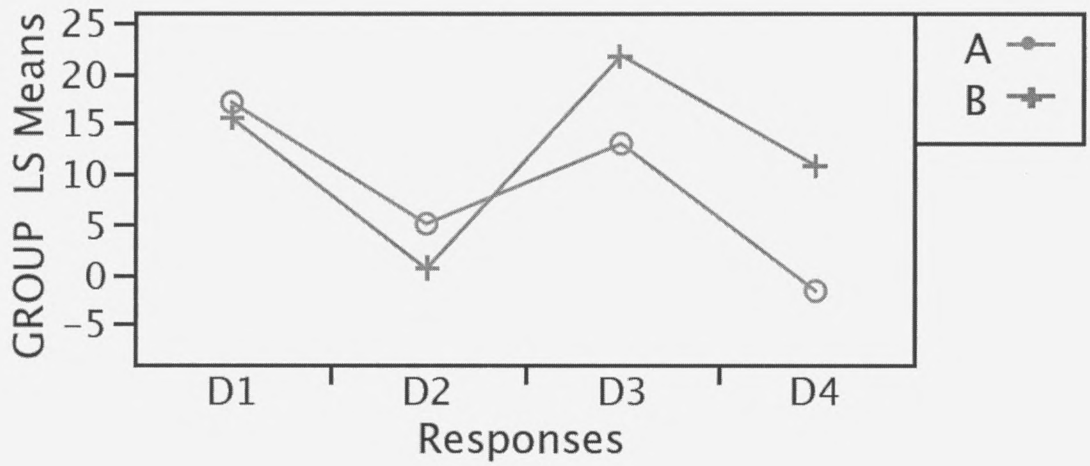

Figure 4.2. Least square (LS) means of unit different scores for groups A and B.

Table 4. 6. *Least Square Means for Unit Difference Scores by Group*

Group	D1	D2	D3	D4
A	17.03	4.94	12.95	-1.76
B	15.44	0.43	21.73	10.74

The significant interaction effect between week and sex is shown in Figure 4.3, and the LSM are in Table 4.7, $F(3,125) = 3.05$, $p = .03$. Females (19.61) outperformed males (12.86) in unit one. However, males (4.77) outperformed females (0.59) in unit two. The scores for males and females in units three and four were relatively similar.

The three-way interaction of week by sex by group was not statistically significant (p = 0.38).

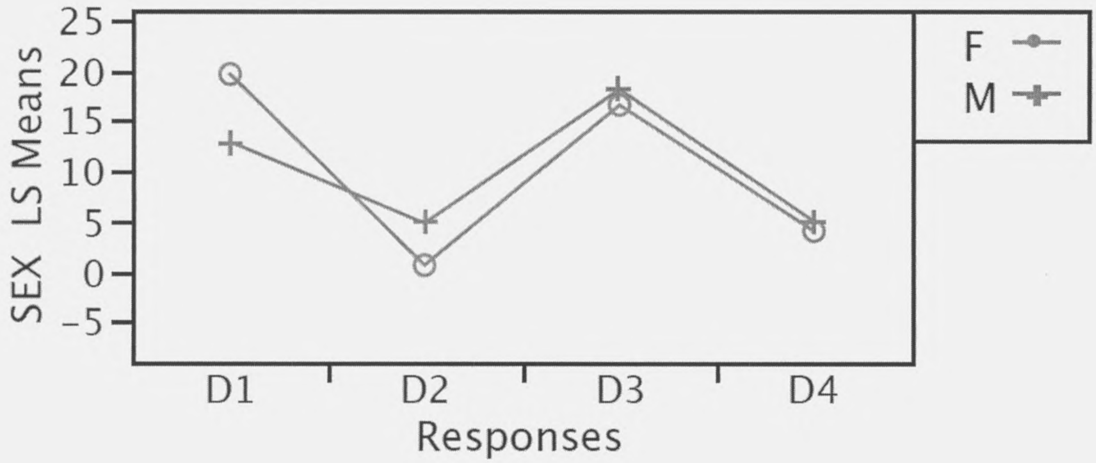

Figure 4.3. Least square (LS) means of unit different scores for males and females.

Table 4.7. *Least Square Means for Unit Difference Scores by Sex*

Sex	D1	D2	D3	D4
Females	19.61	0.59	16.56	4.01
Males	12.86	4.78	18.12	4.97

The unit difference scores by ethnicity are presented in Figure 4.4 and Table 4.8. The week by ethnicity interaction term was not significant ($p = 0.35$). As seen in Figure 4.4, the Black, Hispanic, and White participants each showed a similar pattern of responding. The Asian participants showed a differential pattern, with higher scores in units one and four, and lower scores in units two and three than the other three ethnic groups. However, this did not cause a significant interaction term. The three way interaction term of week by ethnicity by group was statistically significant, Wilks' Lambda = 0.83, $F(9,304.37) = 2.64$, $p = .006$. However, this result is of dubious value as there were only seven Asian participants (2 in group A and 5 in group B). There were also relatively few White participants (9 in group A and 19 in group B). Thus, for the remaining analyses, the White and Asian participants were combined.

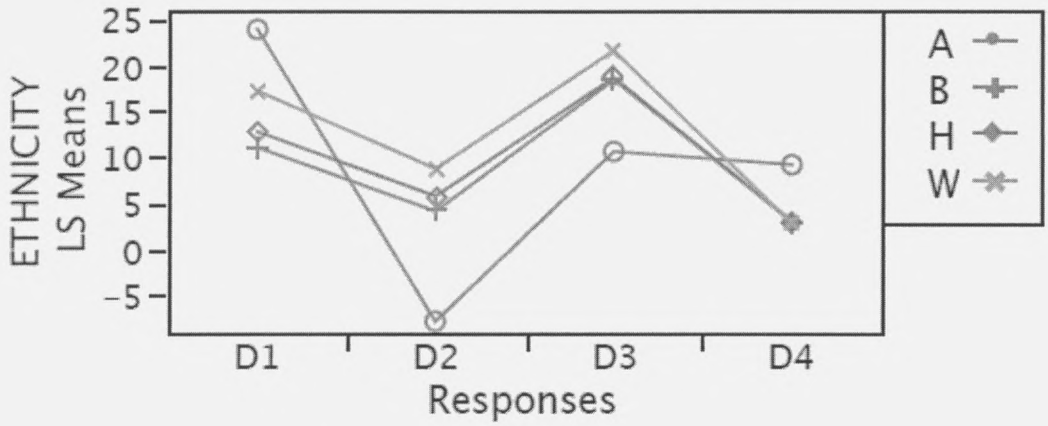

Figure 4.4. Least square (LS) means of unit different scores by ethnicity.

Table 4.8. *Least Square Means for Unit Difference Scores by Ethnicity*

Ethnicity	D1	D2	D3	D4
Asian	23.92	-7.86	10.60	9.17
Black	11.05	4.11	18.43	2.98
Hispanic	12.75	5.75	18.70	3.01
White	17.21	8.74	21.62	2.79

Repeated measures analysis of subset scores. In the second RM ANOVA, the subset scores were the dependent variables. Subset scores combined across weeks in which the same manipulative medium was used. Subset one (S1) was the medium that each group started with (combined across weeks), Subset two (S2) was the medium that each group switched to (combined across weeks). Between-subjects factors were group, sex, and ethnicity (White and Asian groups were combined). No between-subjects interactions were specified in the model. The between-subjects and within-subjects effects for this analysis are presented in this section. The full model results from this analysis are summarized in Table 4.9.

Table 4.9. *RM ANOVA Effects of Subset Scores*

	F-value	df	p-value
Between-Subjects			
Group	0.14	1,132	.71
Sex	0.005	1,132	.94
Ethnicity	1.13	2,132	.33
Within Subjects			
Subset	74.02	1,132	< .0001
Subset * Group	4.11	1,132	.045
Subset * Sex	3.49	1,132	.06
Subset * Ethnicity	0.72	2,132	.49

Note. Ethnicity variable consisted of Black, Hispanic, and a combined White/Asian ethnic group.

Between-subjects effects. There was no significant effect of group ($p = .71$), sex ($p = .94$), or ethnicity ($p = .33$). Thus, overall scores (collapsed across subsets) did not differ according to any of these variables.

Within-subjects effects. There was a significant within-subjects effect of subset, $F(1,132) = 74.02, p < .001$. The LS means for each subset are shown in Figure 4.5. The mean for S1 was 31.93 and considerably higher than the mean for S2 of 9.04. Thus, when collapsed across group, participants performed better on the units presented in weeks one and three than on those in weeks two and four.

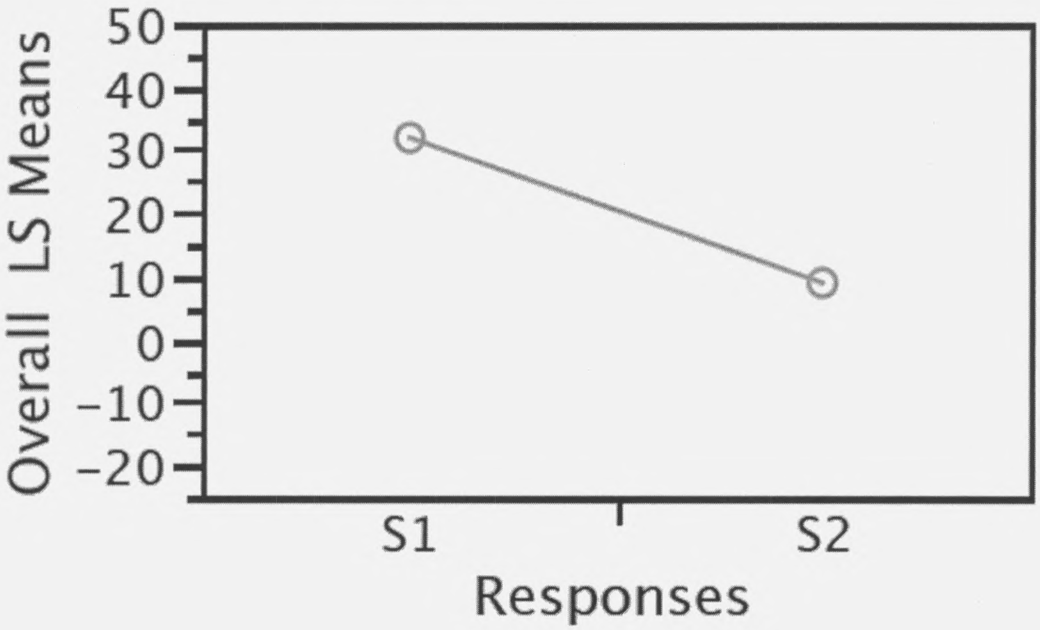

Figure 4.5. Overall least square (LS) means of subset scores (S1 weeks one and three; S2 is weeks two and four).

There was a significant interaction effect between subset and group, $F(1,132) = 4.11, p = .045$. This is the main analysis of interest to the research question. The LS means are shown in Figure 4.6 and in Table 4.10. For S1, the mean for group A (35.04) was higher than group B (30.67). In S1, group A used concrete manipulatives and group B used virtual manipulatives. As such, the group using concrete manipulatives (A) outperformed the group using virtual manipulatives (B). In S2, the pattern was reversed and the mean for group B (12.27) was higher than the group A mean (5.41). As S2 represented the medium that participants switched to, group A used virtual manipulatives in S2 and group B used concrete manipulatives. Therefore, the group using concrete manipulatives (B) once again outperformed the group with virtual manipulatives (A), although the actual groups were reversed. The significant group mean reversal according to which manipulative medium was used provides convincing evidence of the superiority of concrete manipulatives over virtual manipulatives, although this was a small effect.

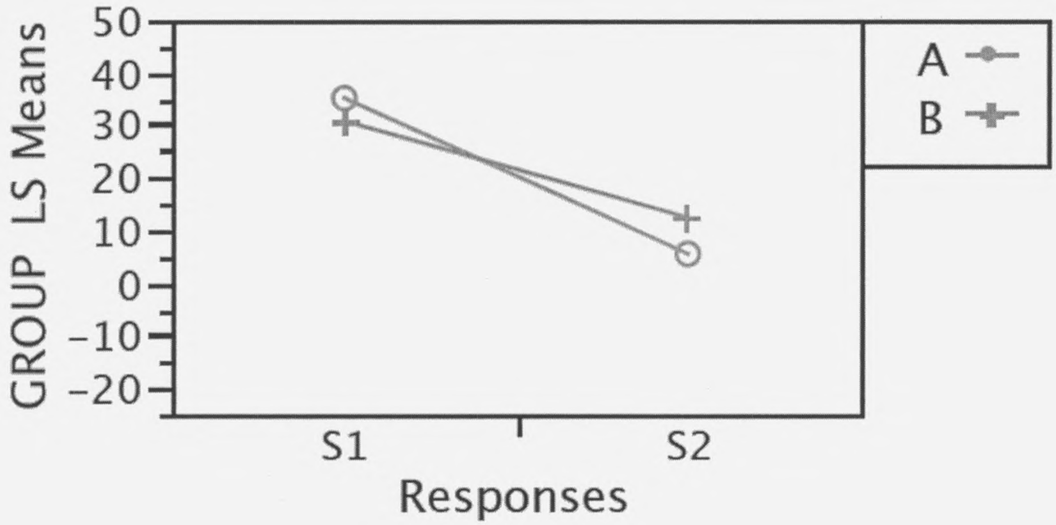

Figure 4.6. Least square (LS) means of subset scores by group (S1 = Weeks one and three, S2 = Weeks two and four).

Table 4.10. *Least Square Means for Unit Difference Scores by Ethnicity*

Group	S1	S2
A	35.04	5.41
B	30.67	12.27

The interaction terms between subset and sex (p = .06) and subset and ethnicity (p = .49) were not significant. The LS means according to sex are shown in Figure 4.7 and the means by ethnicity are presented in Figure 4.8.

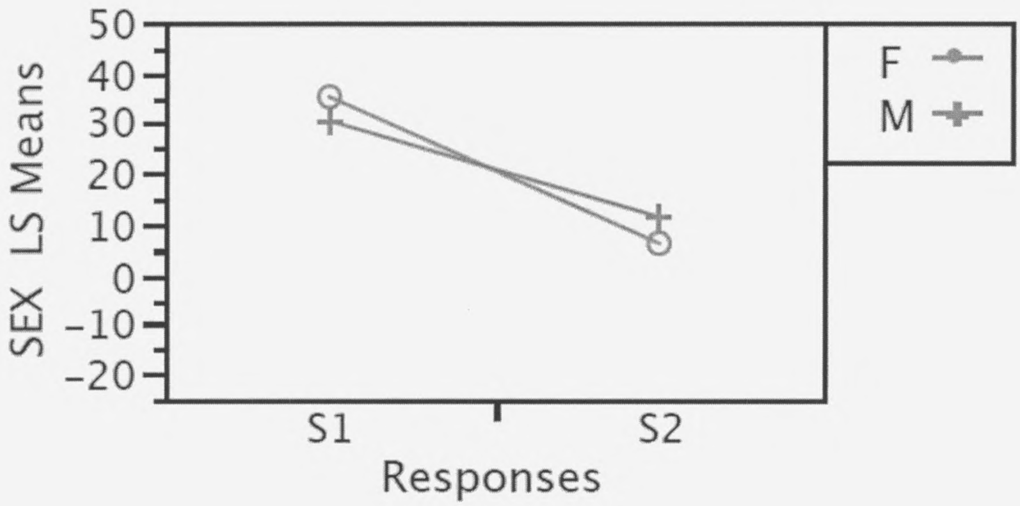

Figure 4.7. Least square (LS) means of subset scores by sex (S1 = Weeks one and three, S2 = Weeks two and four).

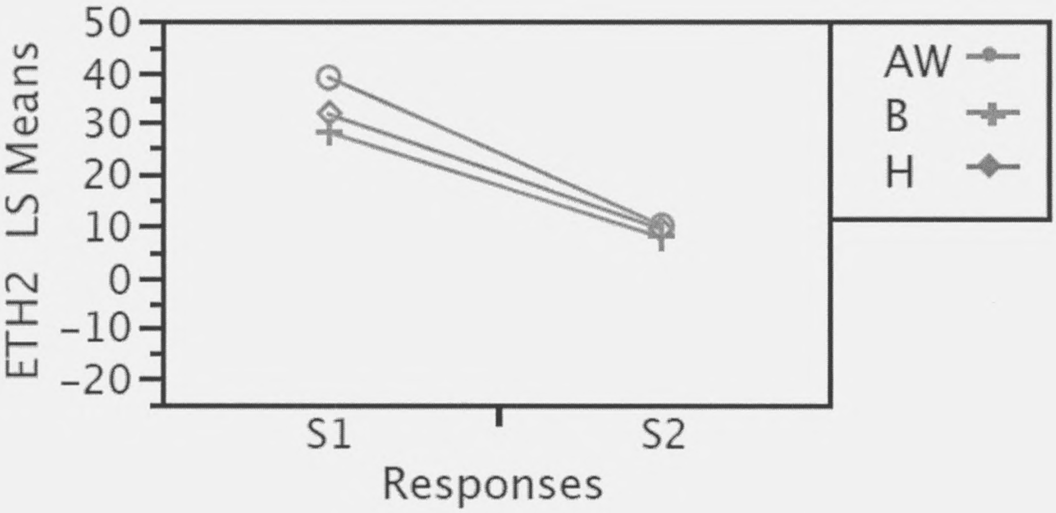

Figure 4.8. Least square (LS) means of subset scores by ethnicity (S1 = Weeks one and three, S2 = Weeks two and four).

Separate analysis of subset scores – observed means. In the final analysis for RQ1, independent samples t-tests were used to investigate mean differences in subset scores by group. These analyses differ from the RM ANOVA as they are based on observed means rather than LSM. Furthermore, the analyses by group were conducted on the S1 and S2 scores separately. The group means are reported in Table 4.4.

For S1, the mean for group A was 32.48 (SD = 29.43) and group B was 30.58 (SD = 24.88). The distribution of scores by group is presented in Figure 4.9. There was no statistically significant difference between the group means, $t(129.59) = 0.42$, $p = .68$.

The score distributions by group for S2 are shown in Figure 4.10. The mean for group A was 4.40 (SD = 24.07), and the mean for group B was 11.98 (SD = 19.38). The group B mean was significantly higher than the group A mean, $t(124.13) = 2.06$, $p = .04$.

Thus, when using the observed means, the group with concrete manipulatives (B) outperformed the group using virtual manipulatives (A) in weeks two and four combined, but not in weeks one and three combined (in which there were no differences according to manipulative type).

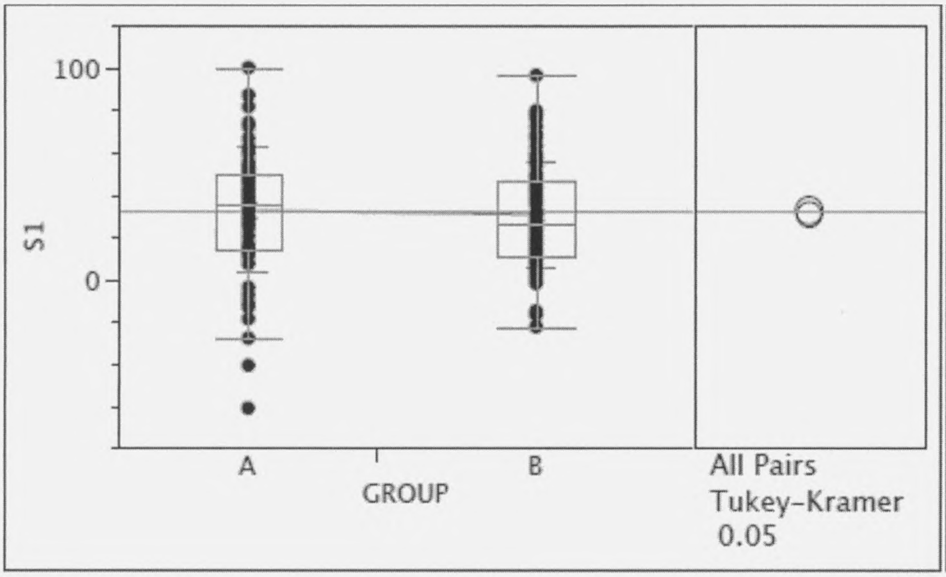

Figure 4.9. Distribution of S1 scores according to group.

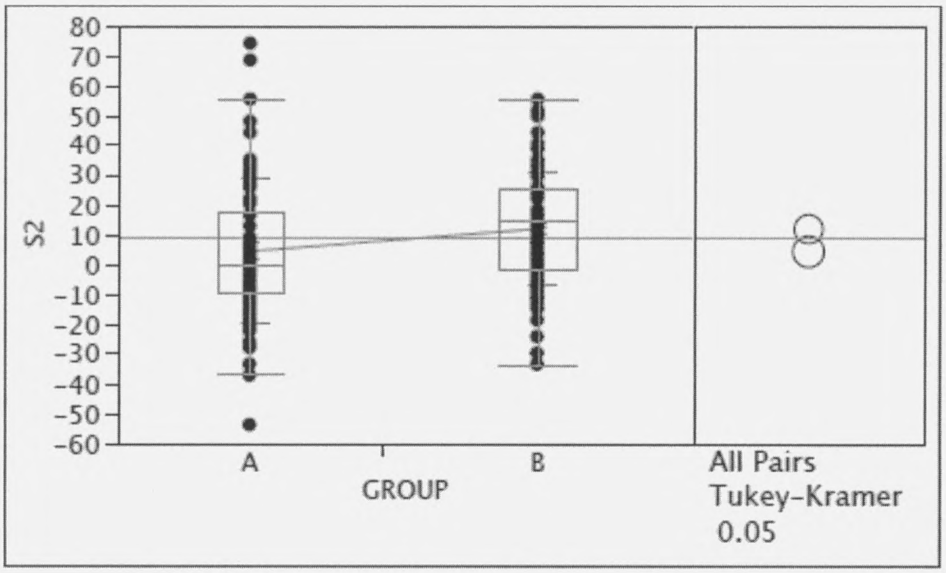

Figure 4.10. Distribution of S2 scores according to group.

Research Question 2

The second research question of this study was as follows:

> What learning preferences exist between the virtual environment and physical environment in teaching concepts in number and numerical operations, geometry and measurement, patterns and algebra, and data analysis, probability, and discrete math?

The research question was addressed by examining the responses on the attitudinal survey presented at the end of the four-week learning sequence. The survey asked respondents to report their agreement on a number of items related to the use of manipulatives. Responses on the survey were coded as follows: 1 = Strongly Disagree, 2 = Disagree, 3 = Agree, 4 = Strongly Agree. Thus, higher scores indicated more agreement with the items. Invalid or blank responses were ignored.

Five survey items (items 20-24) asked respondents about their opinions on manipulatives and the use of concrete versus virtual manipulatives. Frequency distributions were created for these items separately for participants in groups A and B and are shown in Table 4.11.

For both groups, the modal response was "Disagree" across most items. Furthermore, both groups seemed to enjoy using virtual (computer-based) manipulatives more than concrete manipulatives. The students also stated that they learned better using virtual methods. It can also be observed that the responses for group B were generally more negative (showing more disagreement) than those for group A. This was apparent regardless of the manipulative medium.

Mann-Whitney U tests were calculated to determine whether there were differences in the distributions of responding on these items by group. The results are shown in Table 4.12. The responses were significantly higher (showing more agreement) for group A than for group B on the first three items. Thus, group A reported enjoying using both types of manipulatives more than group B. Groups did not differ statistically on the items regarding "learning better" with manipulatives, although there was also a trend for the group A responses to be higher than the group B responses.

Survey items 9 to 19 asked participants to rate how much difficulty they had with various mathematics subject areas ("I have trouble…"). Summary scores were created by averaging across the items pertaining to each unit. Items 25 to 35 on the survey required participants to rate the degree to which they enjoyed using manipulatives in each subject area ("I liked using manipulatives…"). Items were similarly averaged to provide a summary score for each unit.

The means and standard deviations for the summary scores are shown in Table 4.13. Independent samples t-tests were computed to determine whether any of the scores differed significantly between the groups (Table 4.13). The group B means were significantly lower than the group A means regarding the enjoyment of the use of manipulatives for all the units. This indicates that group B did not enjoy using manipulatives as much as group A, and this was not dependent on the type of manipulatives used. Group B also reported significantly more difficulty with unit one than did group A. The groups did not differ on their ratings of unit difficulty across the other three units.

Table 4.11. *Distribution of Responses to Attitudinal Survey Items Regarding Enjoying Use of Manipulatives*

			Distribution of Responses			
Item	Group	N	1 Strongly Disagree	2 Disagree	3 Agree	4 Strongly Agree
I enjoy using manipulatives	A	61	7 (11.5%)	29 (47.5%)	22 (36.1%)	3 (4.9%)
	B	76	26 (34.2%)	32 (42.1%)	16 (21.1%)	2 (2.6%)
I enjoy using concrete (physical) manipulatives	A	61	8 (13.1%)	25 (41.0%)	23 (37.7%)	5 (8.2%)
	B	76	23 (30.3%)	27 (35.5%)	20 (26.3%)	6 (7.9%)
I enjoy using virtual (computer-based) manipulatives	A	60	5 (8.3%)	17 (28.3%)	34 (56.7%)	4 (6.7%)
	B	76	18 (23.7%)	25 (32.9%)	25 (32.9%)	8 (10.5%)
I learn better with concrete (physical) manipulatives	A	61	3 (4.9%)	26 (42.6%)	24 (39.3%)	8 (13.1%)
	B	76	15 (19.7%)	28 (36.8%)	24 (31.6%)	9 (11.8%)
I learn better with virtual (computer-based manipulatives)	A	61	4 (6.6%)	15 (24.6%)	35 (57.4%)	7 (11.5%)
	B	77	16 (20.8%)	20 (26.0%)	33 (42.9%)	8 (10.4%)

Note. Mode for each group is in bold font.

Table 4.12. *Results Of Mann-Whitney Tests Comparing Mean Ranks of Groups A and B on Survey Items Regarding Use of Manipulatives*

Item	Mean Ranks		Standardized	p-value
	Group A	Group B	Test Statistic	
I enjoy using manipulatives	79.95	60.21	3.09	.002
I enjoy using concrete (physical) manipulatives	76.07	63.32	1.97	.049
I enjoy using virtual (computer-based) manipulatives	76.14	62.47	2.14	.033
I learn better with concrete (physical) manipulatives	74.75	64.39	1.61	.108
I learn better with virtual (computer-based manipulatives)	76.42	64.02	1.95	.051

Note. Standardized test statistic (Z) calculated using Mann-Whitney U tests.

Table 4.13. *Independent Samples T-Tests Comparing Summated Survey Scores for Groups A and B*

Score	Means (SD)		t-value	p-value
	Group A	Group B	(df)	
Unit 1 Difficulty	2.11	1.89	2.30	.02
	(0.63)	(0.51)	(142)	
Unit 2 Difficulty	2.02	1.93	0.84	.40
	(0.61)	(0.60)	(139)	
Unit 3 Difficulty	2.16	2.19	-0.25	.81
	(0.65)	(0.67)	(143)	
Unit 4 Difficulty	1.89	1.88	0.17	.87
	(0.63)	(0.63)	(135)	
Unit 1 Enjoyment	2.56	2.21	3.16	.002
	(0.59)	(0.70)	(135)	
Unit 2 Enjoyment	2.61	2.27	3.10	.002
	(0.57)	(0.68)	(133)	
Unit 3 Enjoyment	2.47	2.21	2.29	.02
	(0.62)	(0.69)	(133)	
Unit 4 Enjoyment	2.71	2.33	2.77	.01
	(0.68)	(0.86)	(131)	

Ns for Group A ranged from 58 to 64. N's for Group B ranged from 75 to 81.

5: Conclusion

This chapter provides a summary of the statistical analyses presented in Chapter 4 with respect to the research questions of the thesis study.

The first research question investigated the impact of virtual and concrete manipulatives on students' achievement in four units of instruction. First, differences in achievement were observed between the units regardless of which group of manipulatives the classes were randomized to. The greatest improvements, as assessed by differences in pre-test and post-test scores, were seen in the third unit (algebra) and the first unit (number sense). Considerably less improvement was observed in unit two (geometry) and unit four (statistics and probability). This suggests two possibilities. Firstly, it may be that the instruction topics presented in units two and four were considerably more difficult for students to comprehend than those in units one and three. As such, little improvement in achievement was evident after a few days of instruction, regardless of the method of instruction and the manipulatives used. Another consideration is that each group switched manipulative mediums from week to week. Thus the manipulatives used in the second and fourth units (with the lower achievement scores) were a different type of manipulative than that first presented in units one and three. Thus, it may be that the type of medium used was less important but that switching the type of manipulative used after introducing one method was detrimental to achievement.

Analyses shows that group B had higher achievement scores than group A in units three and four. A comparable performance was observed between groups in units one and two. Group B used virtual manipulatives in unit three (group A used concrete), and in unit four the pattern was reversed (group A used virtual and group B used concrete).

Because the type of manipulative medium used differed in units three and four, yet group B attained higher scores than group A across the units, the results do not suggest the superiority of one manipulative type in achievement. Rather, the result implies that group B may simply be more mathematically skilled when compared to group A regardless of the type of medium used. It is not clear to what extent the differences are attributable to: the manipulative type, the actual unit material, or the group.

In order to clarify the differences observed in the analysis of unit scores, a second RM ANOVA was conducted using subset scores as the dependent variables. Subset scores were created by combining the unit scores in weeks for which the same manipulative type was used. Therefore, S1 was the manipulative each group started with (units one and three, combined across weeks), and S2 was the manipulative each group switched to (units two and four, combined across weeks). As would be expected from the unit difference scores, the S1 scores were considerably higher than the S2 scores, regardless of group. It is unclear whether this result was due to having more difficult material in units two and four, a detrimental effect of switching mediums, or a combination of the two.

There was a significant interaction effect between the subset scores and group, although the effect was relatively small. Group A outperformed group B in S1. The opposite pattern was seen in S2, with group B performing better than group A. It is necessary to consider the manipulatives used by each group in each subset in order to interpret the meaning of these differences. In S1, group A used concrete and group B used virtual manipulatives. Thus, the higher performance of group A suggests that concrete manipulatives were more beneficial to achievement. However, it cannot be determined whether the superiority observed was due to group A being more able, or due to manipulative type, without also examining the S2 effects. In S2, concrete manipulatives also outperformed virtual ones, but in S2 it was Group B that outperformed group A. The fact that the groups switched positions, and in each case the group using concrete manipulatives came out with higher post-test grades, provides convincing evidence that manipulative type was responsible for the interaction. Concrete manipulatives appear to result in slightly higher achievement levels than do virtual manipulatives.

A final analysis was conducted to examine the subset scores separately. This analysis examined whether the groups differed significantly in each subset. In other words, was the superiority of group A over B in S1 a statistically significant difference? Similarly, was the superiority seen of group B over A in S2 significant? These analyses were also conducted on the observed means.

In S1 the mean for group A was *not* significantly higher than group B. However, in S2 the mean for group B was significantly higher than group A. Thus, concrete manipulatives outperformed virtual ones, but only in the second subset. Since subset two was comprised of the more difficult material in units two and four, one possibility is that concrete manipulatives had an edge over virtual ones only when the material was more difficult for students.

The second research question of the thesis study investigated students' learning preferences with respect to concrete and virtual manipulatives. Both groups appeared to exhibit a slight preference for using virtual manipulatives and also reported learning better with such methods. Thus, the findings in RQ1, which suggested a slight edge for concrete manipulatives, were not borne out in the students' opinions or preferences. Group B reported less enjoyment than group A of both types of manipulatives.

Summary scores were created to examine students' opinions on difficulty and enjoyment with the various units. Group B reported having more difficulty with the concepts in unit one than group A. Indeed, group B's mean scores in unit one were somewhat lower than the group A mean score. However, the groups did not differ in their ratings of difficulty for the other units despite mean differences being observed. Furthermore, group B rated their enjoyment of all units as being lower than that of group A. This is despite group B outperforming group A on some units, and the groups switching manipulatives from week to week. These findings indicate little correlation between students' achievement and enjoyment of the various units of instruction. Additionally, there is little correlation between the type of manipulative used and ratings of difficulty of enjoyment.

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Appendices

Appendix A

IRB Approval for Thesis Study

MONTCLAIR STATE UNIVERSITY

Institutional Review Board

Voice: 973-655-4128
Fax: 973-655-5150

December 14, 2009

Mr. John Pappas
712 Main Street
Little Falls, NJ 07424

Re: IRB Number #000785
Project Title: **Differences between Concrete and Virtual Manipulatives in Preparing 10th Grade Math Students for Standardized Tests**

Dear Mr. Pappas:

After a full 1 & 2 review, Montclair State University's Institutional Review Board (IRB) approved this protocol on **November 18, 2009.** The study is valid for one year and will expire on **November 17, 2010.**

Before requesting amendments, extensions, or project closure, please reference MSU's IRB website and download the current forms.

Should you wish to make changes to the IRB-approved procedures, prior to the expiration of your approval, submit your requests using the Amendment form.

For Continuing Review, it is advised that you submit your form 60 days before the month of the expiration date above. If you have not received MSU's IRB approval by your study's expiration date, ALL research activities must STOP, including data analysis. If your research continues without MSU's IRB approval, you will be in violation of Federal and other regulations.

After your study is completed, submit your Project Completion form.

If you have any questions regarding the IRB requirements, please contact me at 973-655-4327, reviewboard@mail.montclair.edu,] or the Institutional Review Board.

Sincerely yours,

Dr. Debra Zellner
Interim IRB Chair

cc: Dr. Kenneth Wolf, Faculty Sponsor
Ms. Amy Aiello, Graduate School

Appendix B

Geolab Curriculum

Objective: to strengthen the students' skills in doing HSPA related work, especially in the application of algebra to solve problems in geometry. Coordinate geometry will be related to theorems early in the year due to its importance in the HSPA. Newer terminology will also be taught.

Workbook: NJ HSPA Mathematics Comprehensive Review [Prentice Hall]

Cluster 1: Number Sense
Students will study types of numbers, estimating, exponents, roots, Order of Operations, factors, and multiples.

Cluster 2: Spatial Sense and Geometry
Students will study triangles and polygons, parallel and perpendicular lines, congruence vs. similarity, coordinate system, perimeter, area, volume, and the basic trigonometry ratios.

Cluster 3: Patterns, Functions and Algebra
Students will study patterns, sequences, series, relations, functions, slope, linear equations, variable expressions, and inequalities.

Cluster 4: Data Analysis, Probability, Statistics and Discrete Mathematics
Students will study probability, scatterplots, measures of central tendency, graphs, counting, sorting, and networks.

Appendix C

Pre and Post Test Examinations

Pre-Test Cluster 1: Number Sense and Numerical Operations

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce you answers.

1. What is the sum of $\frac{4}{7}+\frac{1}{7}$.

a. $\frac{5}{14}$

b. $\frac{5}{7}$

c. $\frac{4}{49}$

d. $\frac{1}{2}$

2. What is the sum of $\frac{1}{8}+\frac{1}{3}$.

a. $\frac{1}{24}$

b. $\frac{12}{24}$

c. $\frac{2}{11}$

d. $\frac{11}{24}$

3. What is the sum of $\frac{2}{11}+\frac{1}{2}$.

a. $\frac{8}{11}$

b. $\frac{3}{13}$

c. $\frac{17}{22}$

d. $\frac{15}{22}$

4. What is the sum of $\frac{3}{12}+\frac{3}{8}+\frac{1}{3}$.

a. $\frac{5}{6}$

b. $\frac{7}{23}$

c. $\frac{23}{24}$

d. $\frac{7}{24}$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

5. A recipe calls for $\frac{1}{4}$ of a cup of water and $\frac{1}{3}$ of a cup of milk. How much liquid in total does the recipe call for?

6. If the recipe in #5 also calls for one egg, which measures $\frac{1}{8}$ of a cup, what is the total measure of the ingredients?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

7. What is the prime factorization of 45?
a. $1 \cdot 45$
b. $5 \cdot 9$
c. $3 \cdot 3 \cdot 5$
d. $1 \cdot 5 \cdot 9$

8. What is the prime factorization of 110?
a. $11 \cdot 10$
b. $2 \cdot 55$
c. $1 \cdot 110$
d. $2 \cdot 5 \cdot 11$

9. Find the GCF (greatest common factor) of 54 and 24?
a. 2
b. 6
c. 3
d. 4

10. Find the LCM (least common multiple) of 18 and 24?
a. 6
b. 72
c. 48
d. 2

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

11. Jack and Diane are running for class president. Jack decides to hang campaign posters at 12-foot intervals. Diane decides she is going to hang her posters closer together. Diane hangs her posters every 10 feet. If they both start measuring from the same point on the wall, after how many feet will Jack and Diane hang their posters in the same spot?

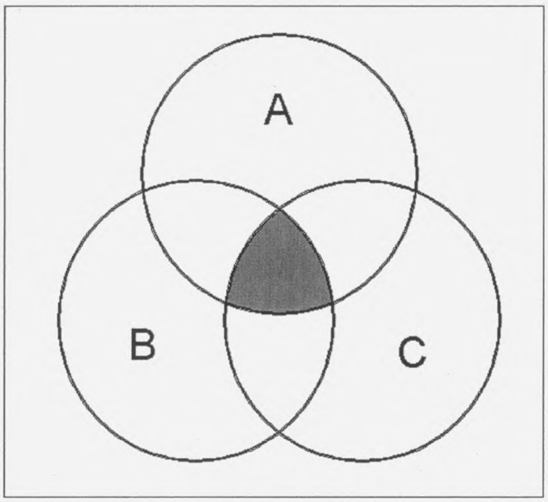
12. Jessica is putting together goodie bags for her birthday party. She has lollipops and candy bars. If there are 48 lollipops and 36 candy bars, how many friends can Jessica invite if each friend receives the same number of lollipops and the same number of candy bars. How many candy bars and lollipops will each friend get?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

For questions 13 – 16, select the statement that describes the region that is shaded in gray.

13.



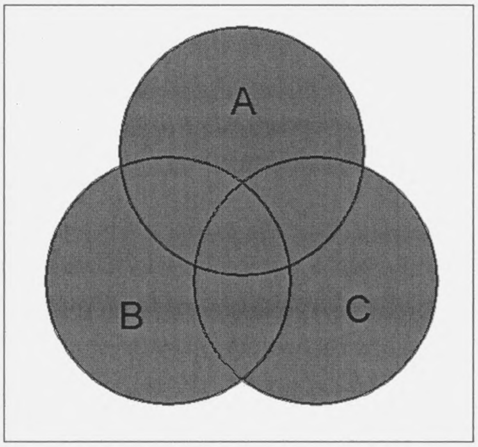
a. $A \cap B$

b. $(A \cap B) \cap C$

c. $A \cap B \cup C$

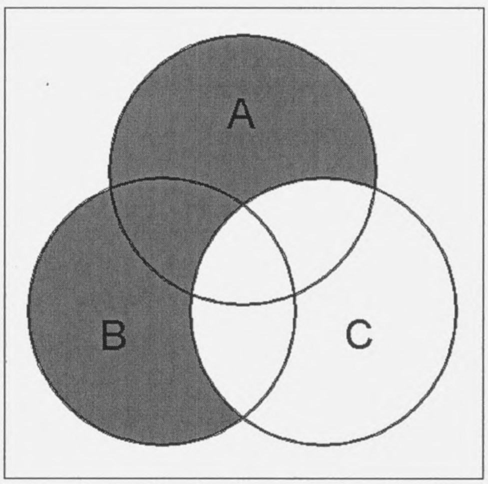
d. $A \cup (B \cup C)$

14.



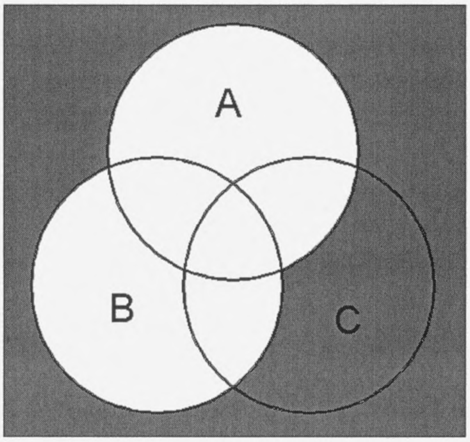
a. $A \cup B$
b. $B \cup A \cap C$
c. $A \cup B \cap C$
d. $(A \cup B) \cup C$

15.



a. $A \cap B$
b. $(A \cap B) \cap \overline{C}$
c. $A \cup B$
d. $(A \cup B) \cap \overline{C}$

16.



a. $\overline{A} \cap \overline{B}$

b. $(\overline{A} \cap \overline{B}) \cap \overline{C}$

c. $\overline{A \cap B}$

d. $(A \cup B)$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

Al, Bob and Craig each have a collection of baseball cards.

17. Draw a Venn diagram representing the cards that all three boys have in common.

18. Draw a Venn diagram representing the cards Al and Bob have in common.

Post-Test Cluster 1: Number Sense and Numerical Operations

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce you answers.

1. What is the sum of $\frac{5}{9}+\frac{1}{9}$.

    a. $\frac{6}{18}$

    b. $\frac{5}{81}$

    c. $\frac{2}{3}$

    d. $\frac{5}{9}$

2. What is the sum of $\frac{5}{10}+\frac{1}{3}$.

    a. $\frac{1}{24}$

    b. $\frac{24}{30}$

    c. $\frac{5}{6}$

    d. $\frac{4}{5}$

3. What is the sum of $\frac{3}{5}+\frac{2}{6}$.

   a. $\frac{9}{10}$

   b. $\frac{5}{11}$

   c. $\frac{17}{22}$

   d. $\frac{14}{15}$

4. What is the sum of $\frac{2}{9}+\frac{1}{7}+\frac{1}{3}$.

   a. $\frac{4}{19}$

   b. $\frac{44}{63}$

   c. $\frac{4}{63}$

   d. $\frac{51}{63}$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

5. Fred is building a volcano for the school science fair. He is making a mixture of water and vinegar. He must mix $\frac{1}{2}$ of a cup of water and $\frac{3}{8}$ of a cup of vinegar. How much liquid in total does he need?

6. If in question 5, Fred also needs to add $\frac{1}{16}$ of a cup of red food coloring to make the lava red. How much liquid in total will Fred pour into the volcano to make it erupt?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce you answers.

7. What is the prime factorization of 54?
   a. $1 \cdot 54$
   b. $6 \cdot 9$
   c. $2 \cdot 3 \cdot 3 \cdot 3$
   d. $2 \cdot 2 \cdot 9$

8. What is the prime factorization of 105?
   a. $1 \cdot 105$
   b. $2 \cdot 55$
   c. $5 \cdot 21$
   d. $3 \cdot 5 \cdot 7$

9. Find the GCF (greatest common factor) of 90 and 36?
   a. 18
   b. 9
   c. 12
   d. 6

10. Find the LCM (least common multiple) of 16 and 24?
   a. 8
   b. 72
   c. 48
   d. 2

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

11. Kristen, Edward and Alex were riding their bicycles around a circular path. They all started out together. If it took Kristen 12 minutes, Edward 15 minutes and Alex 10 minutes to ride once around the path, after how many minutes will they all meet at the beginning of the path?

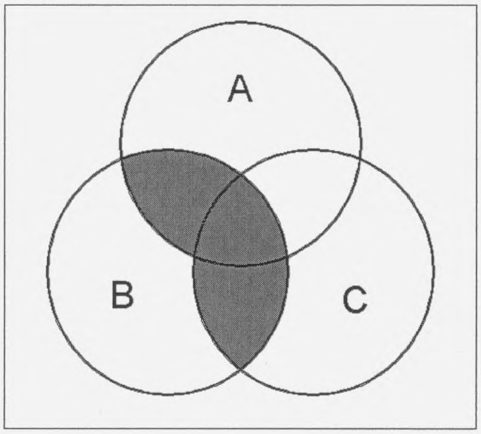
12. Jen and Kenny are volunteering to put together packages of pencils and erasers to give to children in a kindergarten class. They have 96 pencils and 72 erasers. How many packages can they make if each package must contain the same number of pencils and the same number of erasers? How many pencils and erasers will each child get?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce you answers.

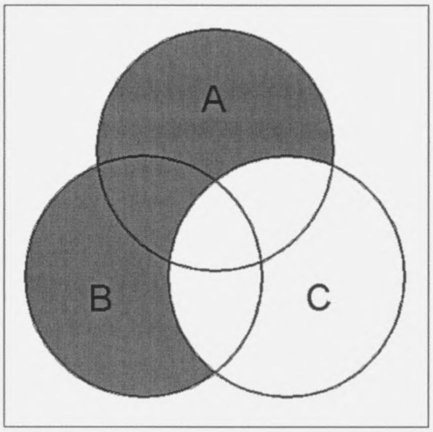
For questions 13 – 16, select the statement that describes the region shaded in gray.

13.



a. $A \cup B \cup C$
b. $B \cap A \cap C$
c. $(A \cup B) \cap C$
d. $(A \cup C) \cap B$

14.



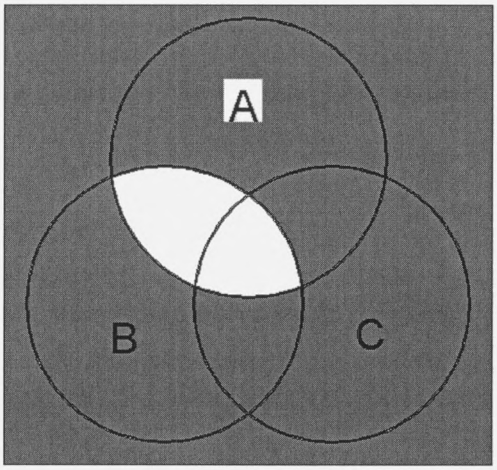
a. $(A \cup B) - C$

b. $(A \cap B) \cap \overline{C}$

c. $A \cap B \cup C$

d. $A \cup (B \cup C)$

15.



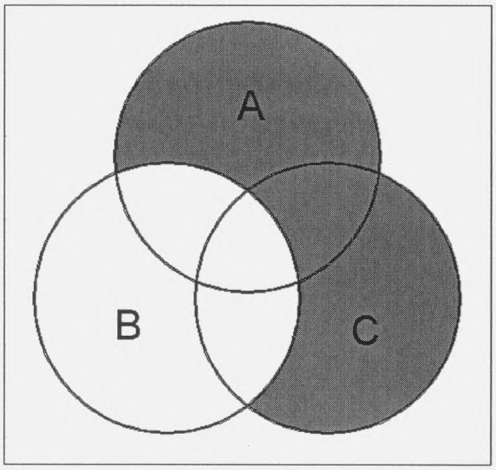
a. $\overline{(A \cap B)}$

b. $\overline{A} \cap \overline{B}$

c. $A \cap B \cap C$

d. $(A \cup B) \cap \overline{C}$

16.



a. $\overline{B}$

b. $(A-B)\cup(C-B)$

c. $(A-B)\cap(C-B)$

d. $(A\cup C)\cap\overline{B}$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

You have collected data about the pets your classmates own. Your classmates own dogs, cats and hamsters.

17. Draw a Venn diagram representing the students who own a cat, a dog and a hamster.

18. Draw a Venn diagram representing the students that own a cat and a dog.

Pre-Test Cluster 2: Geometry and Measurement

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

1. What is the surface area of a cube with side length of 5 inches?

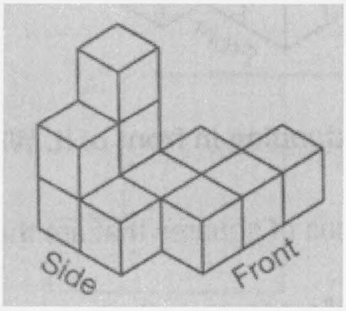
a. 25 in^2
b. 100 in^2
c. 125 in^2
d. 150 in^2

2. What is the volume of two cubes with a combined width of 6 cm.

a. 216 cm^3
b. 27 cm^3
c. 54 cm^3
d. 18 cm^3

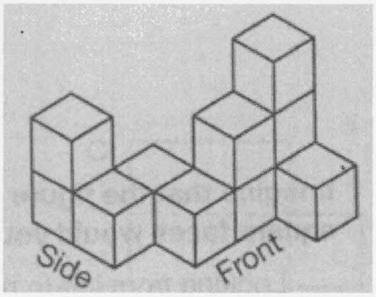
3. If each cube in the figure at the right has a side length of 2 units, what is the volume?

a. 80 $units^3$
b. 88 $units^3$
c. 72 $units^3$
d. 96 $units^3$



4. If each cube in the figure at the right has a side length of 1 unit, what is the surface area, excluding the base of the figure?

a. 30 $units^2$
b. 37 $units^2$
c. 43 $units^2$
d. 49 $units^2$



Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

5. You are building steps with cube shaped cinder blocks. Your steps are going to be 3 cinderblocks wide and have a total of 5 steps. Draw a side view of your steps showing the outline of each cinderblock.

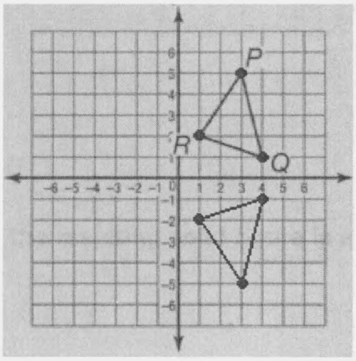
6. How many cinderblocks will you need to create the step design in question 5? If each cinderblock measures 5 inches on one side, what is the volume of your steps? What is the surface area of the part of your steps that you will walk on?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

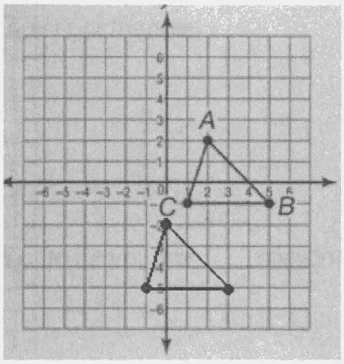
7. The diagram at the right shows a reflection over the:

a. x-axis
b. y-axis
c. y=x
d. x=2



8. The diagram at the right shows a translation:

a. 5 units left, 2 units up
b. 7 units right, 1 unit up
c. 3 units right, 2 units down
d. 2 units left, 4 units down



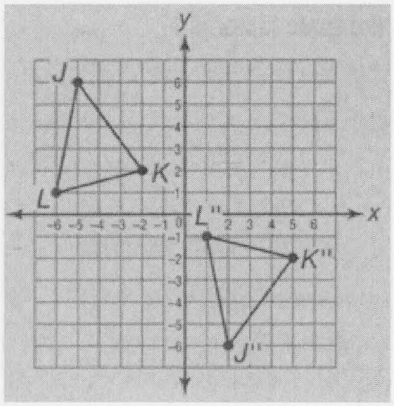
9. If $\Delta J''K''L''$ is the image of ΔJKL after two transformation what were the transformations?

a. A reflection over the y-axis and a reflection over the x-axis.

b. A reflection over the x-axis and a translation of 7 units to the right.

c. A reflection over the y-axis and a translation of 7 units down.

d. A 180° clockwise rotation and a translation of 1 unit to the left.



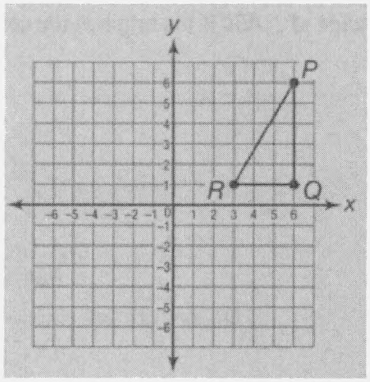
10. ΔPQR is reflected over the y-axis to form image $\Delta P'Q'R'$, which is reflected over the x-axis to form image $\Delta P''Q''R''$. What are the coordinates of point R"?

a. (3,-1)

b. (-3,1)

c. (-3,-2)

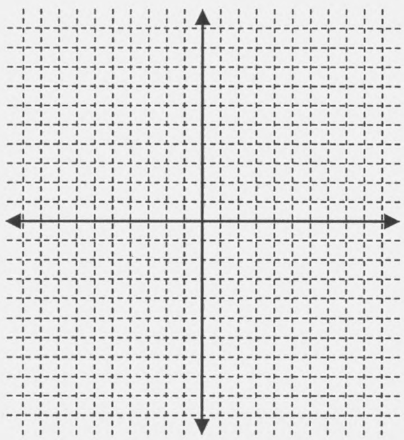
d. (-3, -1)



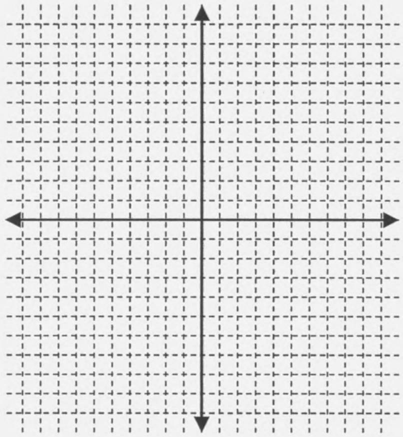
Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

11. Jane is a fabric designer. She has a pattern that she wants to create by reflecting a triangle. Her triangle has coordinates (0, 0), (4, 0), and (3, 3). Draw the shape on the coordinate plane below. Then reflect the shape over the x-axis, y-axis and x-axis again to reveal Jane's pattern.



12. Jane has decided to repeat her pattern from question 11 by translating it. On the coordinate plane below, copy the pattern from question 11 and then translate the figure 3 units to the right and 3 units up. What are the coordinates of the new figure?

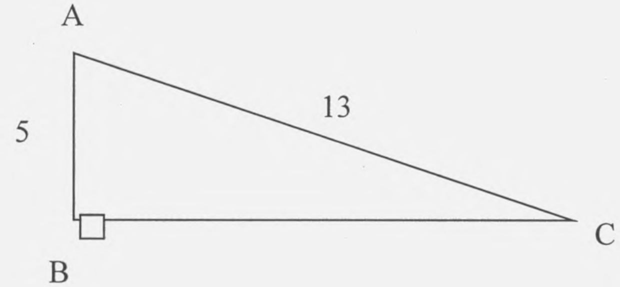


Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

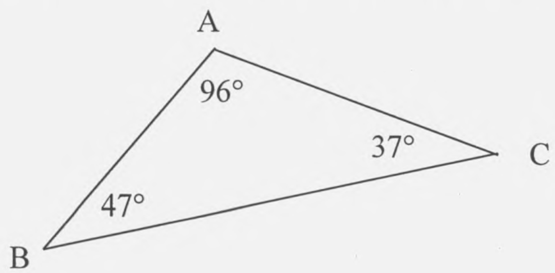
13. Using the diagram to the right, list the angles in order from largest to smallest.

a. A, B, C
b. B, A, C
c. A, C, B
d. C, B, A



14. Using the diagram to the right, list the sides in order of shortest to longest length.

a. $\overline{AB}$; $\overline{BC}$; $\overline{CA}$
b. $\overline{BC}$; $\overline{AB}$; $\overline{CA}$
c. $\overline{CA}$; $\overline{BC}$; $\overline{AB}$
d. $\overline{AB}$; $\overline{AC}$; $\overline{BC}$



15. Which of these sets of numbers could be the length of the sides of a triangle?

a. 5 ft, 12 ft, 13 ft
b. 9 m, 2 m, 6 m
c. 6 cm, 6 cm,15 cm
d. 17 in., 3 in., 14 in.

16. Which of these sets of numbers could NOT be the lengths of the sides of a triangle?

a. 4.5 m, 6 m, 2.5 m
b. 1.25 m, 1.75 m, 3 m
c. 8.4 m, 7.4 m, 6.4 m
d. 0.6 m, 0.9 m, 1.2 m

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

17. Donna is planning to fence in her triangular shaped garden. She gets three pieces of fencing from the home and garden store. The pieces measure 4 ft, 6 ft and 11 ft. Will these three pieces be able to enclose her garden? Explain why or why not.

18. In order for Donna to enclose her garden, what must she do to the longest piece of fencing? How long can the longest piece of fencing be?

Post Test Cluster 2: Geometry and Measurement

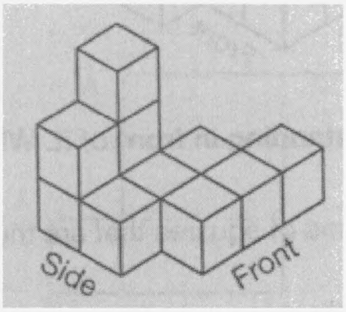
Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce you answers.

1. What is the surface area of a cube with side length of 3 inches?
a. 9 in^2
b. 81 in^2
c. 27 in^2
d. 54 in^2

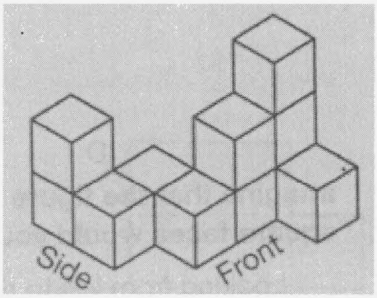
2. What is the volume of two cubes with a combined width of 8 cm.
a. 512 cm^3
b. 64 cm^3
c. 128 cm^3
d. 32 cm^3

3. If each cube in the figure at the right has a side length of 3 units, what is the volume?
a. 540 $units^3$
b. 243 $units^3$
c. 270 $units^3$
d. 297 $units^3$



4. If each cube in the figure at the right has a side length of 2 units, what is the surface area, excluding the base of the figure?

a. 128 $units^2$
b. 88 $units^2$
c. 148 $units^2$
d. 136 $units^2$



Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

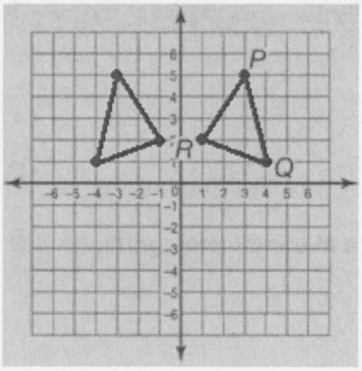
5. You are building steps with cube shaped cinder blocks. Your steps are going to be 4 cinderblocks wide and have a total of 4 steps. Draw a side view of your steps showing the outline of each cinderblock.

6. How many cinderblocks will you need to create the step design in question 5? If each cinderblock measures 3 inches on one side, what is the volume of your steps? What is the surface area of the part of your steps that you will walk on?

Multiple Choice

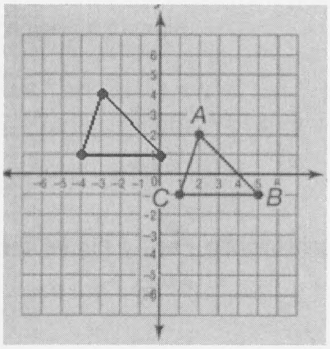
Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

7. The diagram at the right shows a reflection over the:



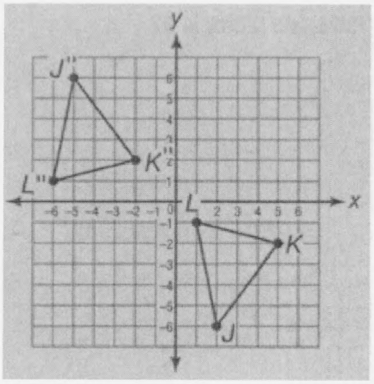
a. x-axis
b. y-axis
c. y=x
d. x=2

8. The diagram at the right shows a translation:



a. 5 units left, 2 units up
b. 7 units right, 1 unit up
c. 3 units right, 2 units down
d. 2 units left, 4 units down

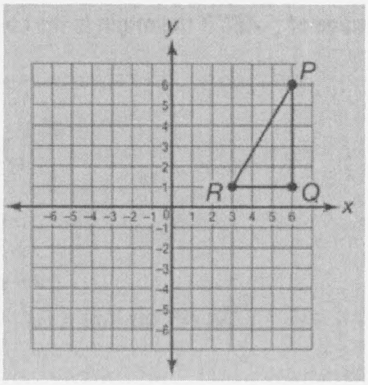
9. If $\Delta J''K''L''$ is the image of ΔJKL after two transformation what were the transformations?



a. A reflection over the x-axis and a translation of 7 units to the left.
b. A reflection over the x-axis and a reflection over the y-axis.
c. A reflection over the y-axis and a translation of 7 units left.
d. A 180° clockwise rotation and a translation of 1 unit to the left.

10. ΔPQR is reflected over the x-axis to form image $\Delta P'Q'R'$, which is reflected over the y-axis to form image $\Delta P''Q''R''$. What are the coordinates of point P”?

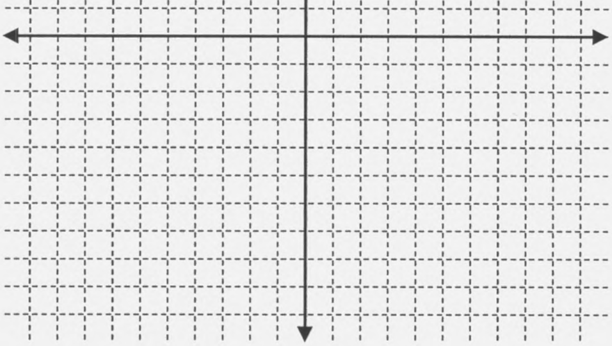
a. (6,6)
b. (6,-6)
c. (-6,6)
d. (-6, -6)



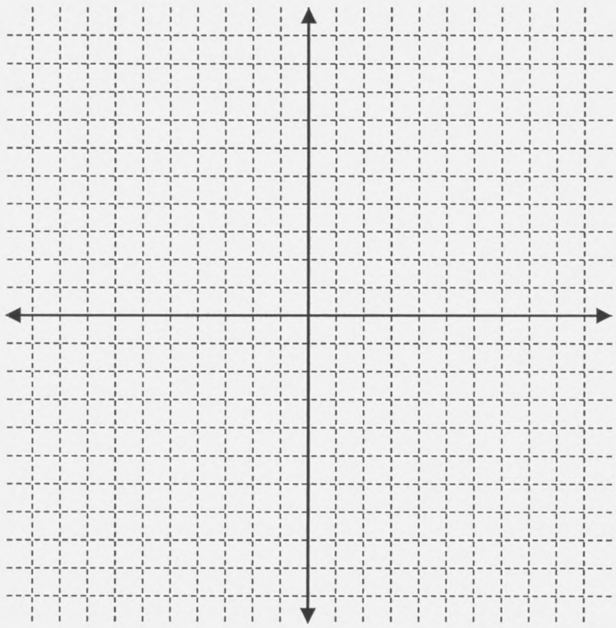
Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

11. Brenda is a fabric designer. She has a pattern that she wants to create by reflecting a quadrilateral. Her quadrilateral has coordinates (0, 0), (5, 0), (0, 5), and (2, 2). Draw the shape on the coordinate plane below. Then reflect the shape over the x-axis, y-axis and x-axis again to reveal Brenda’s pattern.



12. Brenda has decided to repeat her pattern from question 11 by translating it. On the coordinate plane below, copy the pattern from question 11 and then translate the figure 10 units up. What are the coordinates of the new figure?

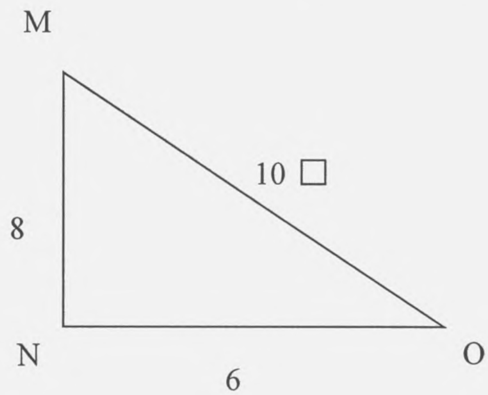


Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

13. Using the diagram to the right, list the angles in order from largest to smallest.

a. M, N, O
b. N, O, M
c. M, O, N
d. O, N, M



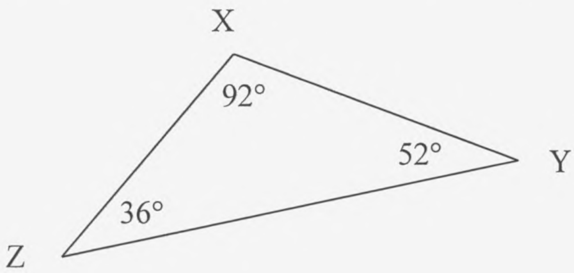
14. Using the diagram to the right, list the sides in order of shortest to longest length.

a. $\overline{XY}$; $\overline{YZ}$; $\overline{ZX}$

e. $\overline{YZ}$; $\overline{XY}$; $\overline{ZX}$

f. $\overline{ZX}$; $\overline{YZ}$; $\overline{XY}$

g. $\overline{XY}$; $\overline{XZ}$; $\overline{YZ}$



15. Which of these sets of numbers could be the length of the sides of a triangle?

a. 10 ft, 1 ft, 13 ft
b. 9 m, 2 m, 8 m
c. 2 cm, 7 cm,15 cm
d. 21 in., 13 in., 7 in.

16. Which of these sets of numbers could NOT be the lengths of the sides of a triangle?

a. 5.7 m, 10 m, 5.7 m
b. 2.6 m, 5 m, 4.4 m
c. 9.1 m, .9 m, 14 m
d. 1.3 m, 2.7 m, 3 m

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

17. Mary is planning to fence in her triangular shaped yard for her kids. She gets three pieces of fencing from The Home and Garden Center. The pieces measure 20 ft, 25 ft and 55 ft. Will these three pieces be able to enclose her yard? Explain why or why not.

18. In order for Mary to enclose her yard, what must she do to the longest piece of fencing? How long can the longest piece of fencing be?

Pre-Test Cluster 3: Patterns and Algebra

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

1. What is the product of $(x+4)(x+6)$?
   a. $x^2+10x+10$
   b. $x^2+24x+10$
   c. $x^2+10x+24$
   d. $x^2+2x+10$

2. Which of the following represents the product of factors of x^2+4x+4?
   a. $(x+4)(x+6)$
   b. $(x+4)(x+1)$
   c. $(x+4)(x-1)$
   d. $(x+2)(x+2)$

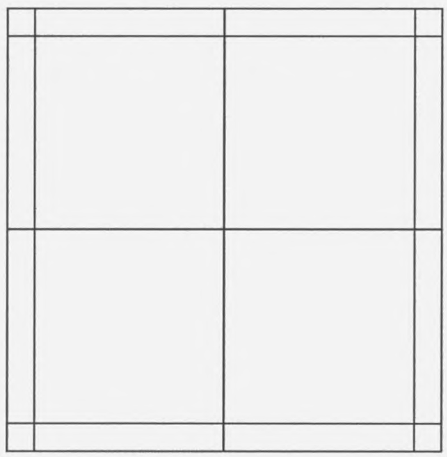
3. Which of the following represents the product of factors of $8x^2+10x+3$?
   a. $(8x+1)(x+2)$
   b. $(4x+3)(2x+1)$
   c. $(4x+4)(2x-1)$
   d. $(4x+1)(2x+3)$

4. Which of the following represents the product of factors of $6x^2+22x+12$?
   a. $(6x+6)(x+2)$
   b. $(2x+6)(3x+2)$
   c. $(2x+2)(3x-6)$
   d. $(2x+2)(3x+6)$

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

5. Sally's grandmother is making a quilt that will look like the one below. Each small square in the corners has a length of 1 inch. Each long rectangle has a length of x and a height of 1 inch. Label the pieces that sum up to be the length and height of the quilt.



6. Does the expression $4x^2 + 8x + 4$ accurately represent the area of the quilt in question 5? If so, prove the factors of $4x^2 + 8x + 4$ are equal to the side lengths of the quilt.

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

7. What is the slope of a line passing through points (2,0) and (5,3)?
   a. $\frac{7}{3}$
   b. $\frac{3}{5}$
   c. 1
   d. 3

8. What is the slope of a line passing through points (-1,-2) and (3,-5)?
   a. $-\frac{3}{4}$
   b. $-\frac{4}{3}$
   c. $-\frac{2}{7}$
   d. $-\frac{2}{3}$

9. What is the slope of a line passing through points (3,0) and (3,4)?
   a. $\frac{4}{6}$
   b. 0
   c. $\frac{2}{3}$
   d. *undefined*

10. What is the slope of a line passing through points (2,4) and (5,4)?
   a. $\frac{7}{8}$
   b. 0
   c. *undefined*
   d. $\frac{8}{7}$

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

11. Ann enjoys going to the track to exercise. She begins by running 1 mile in 10 minutes. She then continues around the track jogging for 2 miles in 30 minutes and finishes her exercise by walking 1 mile in 20 minutes. Draw a graph representing Ann's time (t) vs. miles(m).

12. What was the average rate of change (slope) for each of the three parts of Ann's workout?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

For questions 13 – 16, use Pascal's Triangle at the right.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

13. The sum of the 4th row in Pascal's triangle is:

e. 4
f. 8
g. 16
h. 32

14. The 7th row of Pascal's Triangle looks like:

a. 1 6 15 20 15 6
b. 1 7 21 35 35 21 7 1
c. 1 5 10 10 5 1
d. 1 6 15 20 15 6 1

15. The binomial expansion (product) of $(x+1)^2 = x^2 + 2x + 1$. Which row of Pascal's Triangle matches with the coefficients of this expansion?

a. 0
b. 1
c. 2
d. 3

16. In Pascal's Triangle, the numbers in row 1 place 0, row 2 place 1, row 3 place 2 and row 4 place 3, are the first four numbers in which sequence?

a. Powers of 2
b. Natural Numbers
c. Triangular Numbers
d. Fibonacci Numbers

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

17. What is the sum of each of the first 6 rows of Pascal's Triangle?

18. Write a mathematical expression which would allow you to easily calculate the sum of any row of Pascal's Triangle.

Post Test Cluster 3: Patterns and Algebra

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

1. What is the product of $(x+8)(x+2)$?
   a. $x^2+10x+10$
   b. $x^2+10x+16$
   c. $x^2+16x+16$
   d. $x^2+16x+10$

2. Which of the following represents the product of factors of x^2+6x+9?
   a. $(x+9)(x+1)$
   b. $(x+4)(x+1)$
   c. $(x+5)(x-4)$
   d. $(x+3)(x+3)$

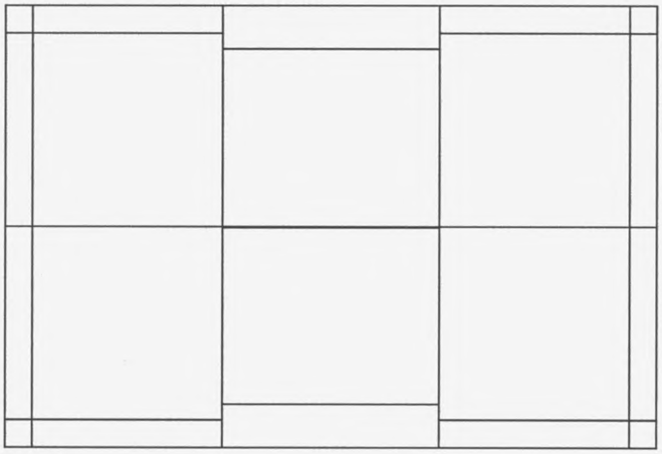
3. Which of the following represents the product of factors of $6x^2+14x+4$?
   a. $(8x+1)(-2x+3)$
   b. $(4x+4)(2x+1)$
   c. $(2x+4)(3x+1)$
   d. $(2x+1)(3x+3)$

4. Which of the following represents the product of factors of $18x^2+30x+8$?
   a. $(6x+2)(3x+4)$
   b. $(9x+6)(9x+2)$
   c. $(3x+2)(6x-6)$
   d. $(9x+2)(2x+6)$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

5. Your mom is building a garden in your backyard. She wants to have 6 sections in the middle and a brick border around. The lengths of the small squares in the corners are 1 inch. The long rectangles have a length of x and a height of 1 inch. Label the pieces that sum up to be the length and height of the garden.



6. Does the expression $6x^2+10x+4$ accurately represent the area of the quilt in question 5? If so, prove the factors of $6x^2+10x+4$ are equal to the side lengths of the quilt.

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

7. What is the slope of a line passing through points (2,3) and (8,6)?
   a. $\frac{1}{2}$
   b. $\frac{9}{10}$
   c. 2
   d. 3

8. What is the slope of a line passing through points (-4, 5) and (7,-5)?
   a. 0
   b. $-\frac{10}{11}$
   c. $-\frac{11}{10}$
   d. $-\frac{10}{3}$

9. What is the slope of a line passing through points (5,1) and (5,3)?
   a. $\frac{5}{2}$
   b. 0
   c. $\frac{2}{5}$
   d. *undefined*

10. What is the slope of a line passing through points (-5,3) and (3,3)?
    a. $-\frac{1}{3}$
    b. 0
    c. *undefined*
    d. $\frac{8}{7}$

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

11. John runs 3 miles for the Regional Valley School Track team everyday. Yesterday, he ran his first mile in 5 minutes. His second mile took him 6 minutes. He ran his third mile in 7 minutes. Draw a graph representing John's run in distance (miles) vs. time (minutes).

12. What was the average speed (slope) for each mile that John ran? What was his average speed (slope) for the entire 3 miles.

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

For questions 13 – 16, use Pascal's Triangle at the right.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

13. The sum of the 3rd row in Pascal's triangle is:

i. 4
j. 8
k. 16
l. 32

14. The 6th row of Pascal's Triangle looks like:

a. 1 6 15 20 15 6
b. 1 7 21 35 35 21 7 1
c. 1 5 10 10 5 1
d. 1 6 15 20 15 6 1

15. The binomial expansion (product) of $(x+1)^4 = 1x^4 + 4x^3 + 6x^2 + 4x + 1$. Which row of Pascal's Triangle matches with the coefficients of this expansion?

a. 2
b. 3
c. 4
d. 5

16. In Pascal's Triangle, the numbers in row 0 place 0, row 1 place 0, row 2 place 1, row 3 place 2 and row 5 place 4, are the first five numbers in which sequence?

a. Powers of 2
b. Natural Numbers
c. Triangular Numbers
d. Fibonacci Numbers

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

17. What is the sum of the 5^{th}, 6^{th} and 7^{th} rows of Pascal's Triangle?

18. Write a mathematical expression which would allow you to easily calculate the sum of any row of Pascal's Triangle.

Pre-Test Cluster 4: Data Analysis, Probability and Discrete Mathematics

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce your answers.

1. When rolling 1 die, what is the probability of rolling an even number?
   a. $\frac{1}{6}$
   b. 1
   c. $\frac{1}{18}$
   d. $\frac{1}{2}$

2. When rolling two dice, what is the probability of rolling doubles?
   a. $\frac{1}{6}$
   b. $\frac{1}{36}$
   c. $\frac{6}{12}$
   d. $\frac{1}{2}$

3. When rolling 1 die, what is the probability of rolling a prime number?
   a. $\frac{5}{6}$
   b. $\frac{1}{2}$
   c. $\frac{1}{6}$
   d. $\frac{1}{3}$

4. When rolling 2 dice, what is the probability of rolling a sum greater than or equal to 8?

   a. $\frac{5}{6}$

   b. $\frac{5}{12}$

   c. $\frac{1}{36}$

   d. $\frac{5}{36}$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

5. Complete the chart below filling in the combinations in the format (column, row). One has been done as an example.

Die 1: / Die 2:	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3				(4,3)		
4						
5						
6						

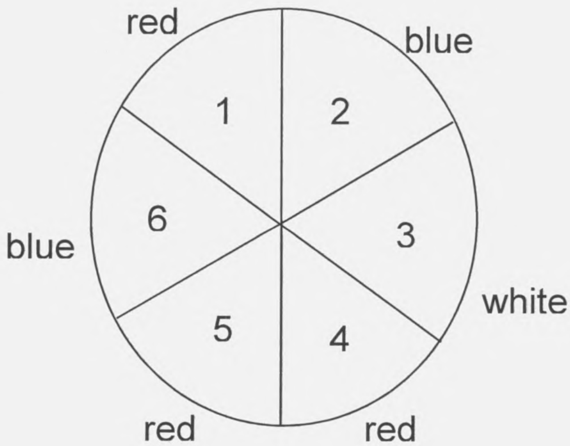
6. Using the chart from question 5:
   a. How many different outcomes are there when rolling a pair of dice?

      ____

   b. What is the most common sum of two dice? Explain how you found your answer. What is the probability of rolling that sum?

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Reduce all answers. For questions 7 – 10, use the spinner to the right.



7. What is the probability of spinning 4?
   a. $\frac{4}{6}$
   b. $\frac{1}{6}$
   c. $\frac{2}{3}$
   d. $\frac{1}{2}$

8. What is the probability of spinning an even number?
   a. $\frac{1}{6}$
   b. $\frac{1}{3}$
   c. $\frac{1}{2}$
   d. $\frac{1}{4}$

9. What is the probability of the spinner landing on a white space?

   a. $\frac{1}{4}$

   b. $\frac{1}{2}$

   c. $\frac{1}{3}$

   d. $\frac{1}{6}$

10. What is the probability of landing on a red odd?

    a. $\frac{1}{3}$

    b. $\frac{1}{6}$

    c. $\frac{1}{4}$

    d. $\frac{1}{2}$

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

11. You need to create a round spinner for a game you are playing. It will have eight slices numbered 1 through 8. Multiples of 3 are green. Multiples of 2 (excluding 6) are red. Any remaining slices are blue. Draw the spinner described.

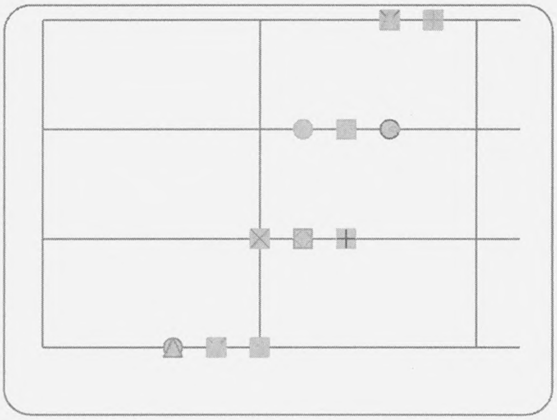
12. Using the spinner you created in question 11, what is:

    a. P(even and green):
    b. P(odd and blue):
    c. P(blue and green):
    d. P(prime numbers):

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

For questions 13 – 16, the scatter plot to the right shows the ages of 11 students in grades 9 – 12.



13. What is the range of the data?

a. 4
b. 6
c. 8
d. 10

14. What is the average age of a student in the 10th grade?

a. 14
b. 15
c. 16
d. 17

15. The data appears to have:

a. No correlation.
b. A negative correlation.
c. A positive correlation.
d. Even correlation.

16. The oldest student is ____ years of age.

a. 18
b. 18 ½
c. 19
d. 20

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

17. Below are the scores received by 15 students after studying for a certain period of time. Create a scatter plot showing the hours studied vs. exam grade earned. Label both axes and title your chart. Make sure to clearly label the values on you axes.

Hours Studied	2	1	1.5	2.5	.5	0	.5	1	0	.5	1	1.5	3	0	4
Grade	85	70	80	95	85	45	65	75	55	60	78	85	98	50	100

18. Does there seem to be a correlation in the data presented in question 17 above? If so, what type of correlation? Are the any outliers?

Post Test Cluster 4: Data Analysis, Probability and Discrete Mathematics

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Be sure to reduce you answers.

1. When rolling 1 die, what is the probability of rolling an odd number?
   a. $\frac{1}{6}$
   b. 1
   c. $\frac{1}{18}$
   d. $\frac{1}{2}$

2. When rolling two dice, what is the probability of rolling two consecutive numbers?
   a. $\frac{1}{6}$
   b. $\frac{5}{18}$
   c. $\frac{7}{12}$
   d. $\frac{1}{2}$

3. When rolling 1 die, what is the probability of rolling a number divisible by 3?
   a. $\frac{5}{6}$
   b. $\frac{1}{2}$
   c. $\frac{1}{6}$
   d. $\frac{1}{3}$

4. When rolling 2 dice, what is the probability of rolling a sum greater than or equal to 9?

    a. $\frac{5}{6}$

    b. $\frac{5}{12}$

    c. $\frac{1}{36}$

    d. $\frac{5}{18}$

Open – Ended

Read each question carefully before choosing a response. Be sure to reduce you answers. Be sure to show all work and to clearly mark your answers.

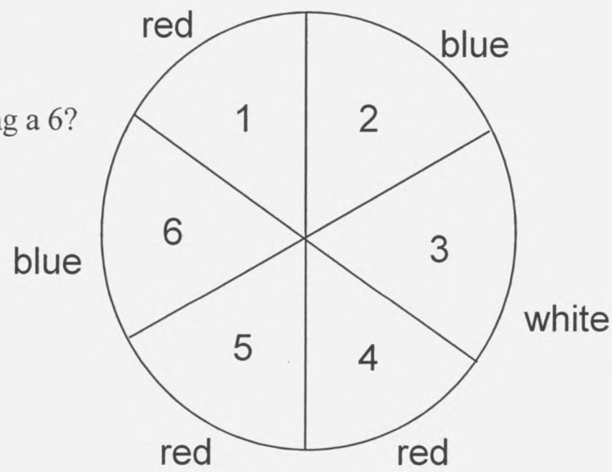
5. Complete the chart below filling in the sum of the roll in each box. One has been done as an example.

Die 1: Die 2:	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3				7		
4						
5						
6						

6. Using the chart from question 5:
    a. What is the probability of rolling a sum that is divisible by 2 and 3? Explain how you found your answer.

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided. Reduce all answers. For questions 7 – 10, use the spinner to the right.



7. What is the probability of spinning a 6?
    a. $\frac{4}{6}$
    b. $\frac{1}{6}$
    c. $\frac{2}{3}$
    d. $\frac{1}{2}$

8. What is the probability of spinning an odd number?
    a. $\frac{1}{6}$
    b. $\frac{1}{3}$
    c. $\frac{1}{2}$
    d. $\frac{1}{4}$

9. What is the probability of the spinner landing on a red space?
   a. $\frac{1}{4}$
   b. $\frac{1}{2}$
   c. $\frac{1}{3}$
   d. $\frac{1}{6}$

10. What is the probability of landing on a blue odd?
    a. $\frac{1}{3}$
    b. 0
    c. $\frac{1}{4}$
    d. $\frac{1}{2}$

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

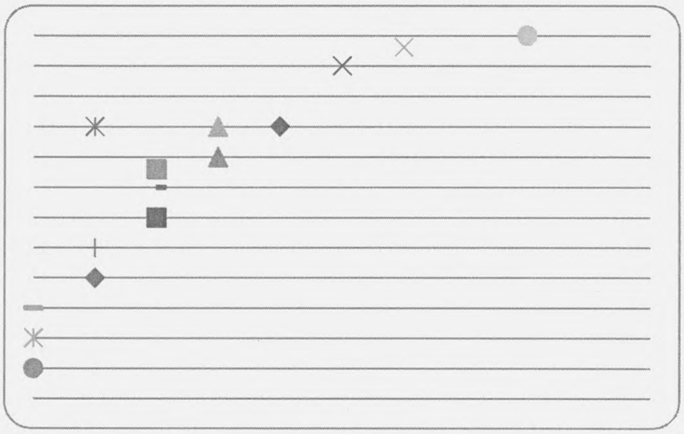
11. You need to create a round spinner for a game you are playing. It will have twelve slices numbered 1 through 12. Multiples of 4 are white. Multiples of 3 (excluding 12) are red. Any remaining slices are blue. Draw the spinner described.

12. Using the spinner you created in question 11, what is:
    a. P(even and white):
    b. P(odd and blue):
    c. P(blue or green):
    d. P(divisible by 2 and 3):

Multiple Choice

Read each question carefully before choosing a response. Please record your response on the bubble sheet provided.

For questions 13 – 16, the scatter plot to the right shows the hours studied by students vs. the grade they received on a test for 15 students.



13. What is the range of the grades?

a. 45
b. 55
c. 65
d. 100

14. What is the approximate average grade for a student who studied for 1 hour?

a. 78
b. 70
c. 74
d. 50

15. The data appears to have:

a. No correlation.
b. A negative correlation.
c. A positive correlation.
d. Even correlation.

16. The longest amount of time studied for was ______ hours.

a. 4.25
b. 4
c. 3
d. 3.5

Open – Ended

Read each question carefully before choosing a response. Be sure to show all work and to clearly mark your answers.

17. Below are the heights of sunflowers after a certain time period of growth. Create a scatter plot showing the time(t) vs. height (in). Label both axes and title your chart. Make sure to clearly label the values on your axes.

Time	1	1	3	4	2	2	0	1	3	4	5	3	2	0	1
Height	15	10	30	45	20	25	0	75	35	60	65	40	30	0	15

18. Does there seem to be a correlation in the data presented in question 17 above? If so, what type of correlation? Are the any outliers?

Appendix D

Student Attitudinal Survey

Please check the response that best describes your practices					
1	I like learning math	Strongly Disagree	Disagree	Agree	Strongly Agree
2	I have trouble understanding math	Strongly Disagree	Disagree	Agree	Strongly Agree
3	I like solving open ended questions	Strongly Disagree	Disagree	Agree	Strongly Agree
4	I need help with open ended questions	Strongly Disagree	Disagree	Agree	Strongly Agree
5	I find it difficult to participate when we are doing open ended questions	Strongly Disagree	Disagree	Agree	Strongly Agree
6	I can read but I do not understand open ended questions	Strongly Disagree	Disagree	Agree	Strongly Agree
7	I like to see examples of open ended questions from outside	Strongly Disagree	Disagree	Agree	Strongly Agree

	the text-book, in the real world to see how we use math				
8	I don't know where and how to start an open ended question	Strongly Disagree	Disagree	Agree	Strongly Agree
9	I have trouble adding fractions	Strongly Disagree	Disagree	Agree	Strongly Agree
10	I have trouble finding factors of numbers	Strongly Disagree	Disagree	Agree	Strongly Agree
11	I have trouble making Venn diagrams	Strongly Disagree	Disagree	Agree	Strongly Agree
12	I have trouble reflecting (flipping) an image	Strongly Disagree	Disagree	Agree	Strongly Agree
13	I have trouble drawing the side perspective of connected cubes	Strongly Disagree	Disagree	Agree	Strongly Agree
14	I have trouble calculating surface area	Strongly Disagree	Disagree	Agree	Strongly Agree
15	I have trouble calculating volume	Strongly Disagree	Disagree	Agree	Strongly Agree
16	I have trouble factoring polynomials	Strongly Disagree	Disagree	Agree	Strongly Agree
17	I have trouble finding the slope of a line	Strongly Disagree	Disagree	Agree	Strongly Agree

18	I have trouble working with probability	Strongly Disagree	Disagree	Agree	Strongly Agree
19	I have trouble making scatter plots	Strongly Disagree	Disagree	Agree	Strongly Agree
20	I enjoy using manipulatives	Strongly Disagree	Disagree	Agree	Strongly Agree
21	I enjoy using concrete (physical) manipulatives	Strongly Disagree	Disagree	Agree	Strongly Agree
22	I enjoy using virtual (computer-based) manipulatives	Strongly Disagree	Disagree	Agree	Strongly Agree
23	I learn better with concrete (physical) manipulatives	Strongly Disagree	Disagree	Agree	Strongly Agree
24	I learn better with virtual (computer-based) manipulatives	Strongly Disagree	Disagree	Agree	Strongly Agree
25	I liked using manipulatives to learn about fractions	Strongly Disagree	Disagree	Agree	Strongly Agree
26	I liked using manipulatives to learn about finding the factors of a number	Strongly Disagree	Disagree	Agree	Strongly Agree
27	I liked using manipulatives to	Strongly	Disagree	Agree	Strongly

	learn about Venn diagrams	Disagree			Agree
28	I liked using the manipulatives to learn about factoring quadratic equations	Strongly Disagree	Disagree	Agree	Strongly Agree
29	I liked using manipulatives to learn about finding the slope of a line	Strongly Disagree	Disagree	Agree	Strongly Agree
30	I liked using manipulatives to learn about finding patterns	Strongly Disagree	Disagree	Agree	Strongly Agree
31	I liked using manipulatives to learn about finding surface area and volume of structures made with cubes	Strongly Disagree	Disagree	Agree	Strongly Agree
32	I liked using manipulatives to learn about translations and reflections	Strongly Disagree	Disagree	Agree	Strongly Agree
33	I liked using manipulatives to learn about triangle inequalities	Strongly Disagree	Disagree	Agree	Strongly Agree
34	I liked using manipulatives to learn about probability	Strongly Disagree	Disagree	Agree	Strongly Agree
35	I liked using manipulatives to	Strongly	Disagree	Agree	Strongly

	learn about scatterplots	Disagree			Agree