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A Network-Based Analysis of Student Learning Trajectories and Andragogical Design

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Abstract

The core goal of this thesis project is to formalize the complex system that exists naturally in a formal classroom environment. The three factors that are considered in this study are the roles of the student, the roles of the teacher, and the respective environments from which these members arise and how these act as determinants of curriculum development and design. At the curricular scale, educational practices should be treated as a complex system composed of various inherently connected concepts and exchanges of ideas and ways of knowing. This synthesis of previous work and ongoing research efforts employs a network theory mediated analysis to investigate the affordances of curriculum and, in particular, its alignment with student learning processes.

Via the utilization of various lenses of network theory, connected curriculum design, and modern learning curve theory, this body of research generates the postulate that education is inherently a complex system at various scales and stages of the learning process. Therefore, the task of a proper educator lies in elucidating these connections and helping students make their own connections. A network theoretic perspective of the precalculus curriculum proves to be helpful in identifying and motivating key features of the subject as they appear in course texts. Of these, hubs and time-series developments of relevant computed metrics have been particularly useful in mapping the alignment between the preset goals of the precalculus course, as identified in previous literature and execution of taxonomic principles.

At the same time, this analysis has also been valuable in identifying the trajectory of the textbook curricula to adequately prepare precalculus students for success in calculus and beyond. Highly successful texts retain inherent commonalities, including the display of a power law with respect to frequency distribution of connected topics ($\alpha = 0.05$). The most well-connected topics (hubs) are regarded as necessary markers of classroom discourse, and the extent to which they are considered in taxonomical goals is measured with respect to the empirical distributions of both the intended curriculum and enacted curriculum.

Moreover, the implications of this work seek to assist in the optimization of the complex system synthesizing various feedback-based designs within educator roles, student response, and the interactions between the two as they pertain to various stages of the curriculum development process. This allows both students and educators alike to personalize learning through standardized exhaustive procedure and provide an equity-based environment conducive towards 'meaning making' or assigning new meaning from the foundation of old ideas. The results of this study aim to not only provide a deeper understanding of how intended and enacted curriculums interact with each other, but also considers to what extent the components of an andragogical system can be refined via the magnitude of their presence in both course materials and feedback provided directly by active participants in the learning environment.

Keywords: network theory, curriculum, creativity, complexity science, connectivity, equity

MONTCLAIR STATE UNIVERSITY

A NETWORK-BASED ANALYSIS OF STUDENT LEARNING TRAJECTORIES AND ANDRAGOGICAL DESIGN

by

JOHN KERRY O'MEARA

A Master's Thesis Submitted to the Faculty of

Montclair State University

In Partial Fulfillment of the Requirements

For the Degree of

Master of Science

May 2022

College of Science and Mathematics

Department of Mathematics

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Acknowledgements

Nothing in my life, scholastic or otherwise, would be possible without the love, faith, and support of my parents. Thank you, mom and dad, for instilling in me the virtues I have carried through my life that have made me the man I am today. Your constant reassurance, all of the delicious dinners the last 26 years, affirmation of my Catholic faith, fierce Irish pride, and unwavering love will forever plough the path before which I set for myself. Thank you to my brother and sister for being my best friends in every facet of the word, for putting up with my mathematical tangents (pun fully intended), persistently loud music at all hours of the day, and for always being the two most crucial members of my lifelong support system.

I would also like to acknowledge my thesis committee for the support system they have upheld since my first semester at Montclair State University. To Dr. Ashuwin Vaidya, thank you for always believing in me and giving me the opportunity to share my passion for teaching, mathematics as a vehicle for creativity and unification, and for being the pillar of a role model, mentor, collaborator, and a lifelong friend to you and your entire family. To Dr. Eileen Fernández, thank you for inspiring optimism, spatial reasoning, collaborative learning, and creativity through geometry in our MATH575 course: my very first graduate math course. You are a beacon of radiant happiness and inspiration, and I so look forward to continuing to learn from you in my doctoral studies. To Dr. Mika Munakata, thank you for allowing me to explore the creative and unexpected components of mathematical thinking, and becoming comfortable with the vast unknown as a driving force for investigation. Both our MATH572 and MATH574 courses have permeated directly into the ways in which I conduct my classrooms, and I will forever be grateful for the opportunity to both showcase my art through the medium of mathematics, and the lifelong acquired skill of juggling that will have a permanent residency on my curriculum vitae. I would also like to thank the entire family of Montclair State University for being so supportive and inclusive, and making the choice to stay here for my doctoral studies one of the easiest decisions I've ever had to make. I can finally confess in writing I only applied here, and am so eager to officially begin my next journey in the fall.

This work would also not be possible without the insights and efforts of Stephen Essien, Gabriella Vidal, and Vlad Nita. Thank you for all your past, current, and future contributions to this body of knowledge, and I look forward to all that is to come.

Thank you to Sam Harris for your constant love, support, and inspiring me to be a better mathematician and educator through your own teaching practices. I know in my heart that you will be an excellent teacher and mentor to all of your future students in the classroom, and I look forward to being with you every step of the way as we work to continue to spread our joy of learning, and optimize mathematics education as a whole.

Additionally, I would like to extend particular gratitude to every single member of the MATH350/575 Fall 2020 course instructed by Dr. Eileen Fernández. You are all so special to me, and I am so thankful for the memories we will continue to make together, and the paths we shall walk together.

This body of work is furthermore dedicated to every single person with whom I have ever crossed paths - listing out your names and the impact you've had on me would extend significantly longer than the confines of this thesis, and would fail to encapsulate the magnitude by which your existence has influenced me. You have sculpted who I am today, and who I am destined to become, and I thank you for all that you are.

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Chapter 1: Introduction

1.1 Background and Motivation

In the United States, mathematical performance is a long-standing issue that plagues the entire nation, as well as the world at large. The Program for International Student Assessment (PISA), a study of 15-year-old students' performance in reading, mathematics, and science literacy conducted every three years by the National Center for Education Statistics (NCES), aims to provide the world with a global view of students' performance compared to their peers in nearly eighty countries and education systems. Performance can be analyzed via many lenses, including through student scaffolding and concept mapping via network theory (Govorova, Benítez, & Muñiz, 2020). In the most recent administration of this assessment tool in 2018, the results of the United States students were compared to thirty-six other countries that serve as cooperating members of the Organization of Economic Co-operation and Development (OECD), a multinational collective that strives for global prosperity and aims to stimulate transnational progression of all involved nations. Upon comparison, out of a total possible score of 1000 with a performance range of 325 to 591, the United States' average score (478) proved to be markedly lower than the OECD average score (489). Compared to the thirty-six other OECD members, the U.S. average in mathematics literacy was lower than the average in twenty-four education systems, higher than in six, and not measurably different than in six (U.S. Dept. of Education, NCES, 2018). The United States ranked 30/77 across all countries involved in the study, with no statistically significant progress since the year 2000. In fact, only seven out of the seventy-seven included countries have had any statistically significant improvement since 2000 (OECD, 2007).

According to a study conducted by the National Science Foundation in 2016 concerning students of a similar educationally formative age group, among ninth graders who began high school in 2009 and completed high school in 2013, the majority (eighty-nine percent) completed algebra 2 or higher (U.S. Dept. of Education, Office for Civil Rights, 2016). Among these courses which receive a large proportion of students is precalculus, a standard stepping stone for many students' future studies in a variety of disciplines (Greene & Shorter, 2012). Precalculus in particular is a subject in which connections between concepts are absolutely paramount to go beyond rote memorization and instead motivate understanding and assimilation of concepts (Katz, 1991). Concerning the results of the PISA, it can be observed that the majority of involved nations have not improved their relative standing despite nearly two decades over which technology and teaching methods have developed immensely (Rose, 2013). This performance inequality can reasonably be thought of as a symptom of deeply inconsistent educational practices, lack of equity and agency in the learning environment, or consistently poor educational practices. A reasonable question to consider in this research effort is: how can we meaningfully and concretely change this?

1.2 Orientation Towards the Classroom Dynamic

Without any loss of generality, one of the many overarching desirable outcomes of education is to generate focus on exploring consensus of human thought throughout time, and utilize various practical methods to verify whether or not these principles hold true or ought to continue to be accepted as valid. By bringing forth the question of purpose in the classroom, the classroom dynamic generates a productive shift towards establishing agency and autonomy amongst all members of the complex system that is a well-balanced and successful learning environment. It is via this implementation of such a social system that we, as educators and pupils alike, become cohesive autonomous thinkers who are both well-represented by our educational environment and can make the transition to ultimately serve to represent the standard for future curricular design and considerations of our teaching surroundings and practices.

The archetypes of the roles of student and teacher play off one another in an incredibly dynamic system, one in which the ultimate goal is a mutual respect of both the material, each other, and the pursuit of understanding and enhanced knowledge (Biesta, 2009). As such, the role of an educator is to provide both a safe, comfortable environment that fosters creativity and thought, as well as serve as a role model to whom the students look up. Due to vast student diversity in every aspect, the best policy from the perspective of the teacher is to always aim to uphold representation, neutrality, and equity amongst their core values. To strive for individual and collective truth and accuracy through conveying the material at hand sits amongst these premier virtues of education. Such practices are highly conducive to developing passionate and deep thinkers that are well-represented and who wish to both solve problems rigorously as well as explore the significance of math in the real world: a topic that is imperative in the continuation of mathematics education throughout the country.

Lessons and discussions alike concerning any given topic of mathematics ought to be presented from two distinct perspectives; the deeper connections behind why the material holds true amongst itself (the ‘intra-connectivity’), and the applications of the material to the extenuating academic and personal lives of students and all others that might be influenced in a tangible manner (the ‘inter-connectivity’). Through this, there are two significant accomplishments. First of all, students move away from the tendency to present volumes of seemingly disconnected information that feels more like an exercise in memorization than anything else: there is now a root of true knowledge and context that will allow the material to seemingly branch out and connect itself to each other topic in many different ways, all of which are highly comparative and contrastive amongst each other. Second, applications to real-world scenarios answer the question that has echoed through the proverbial school hallway throughout human history: ‘When will I ever use this in the ‘real world’?’ Despite mathematics having the ability to retain ample significance in all walks of life, there certainly exist a variety of moments in the educational journey that lack meaningful emphasis on the ways in which one might need to use this information in an extensive manner throughout life. For example, to leverage a concept taught in calculus courses all over the world: an individual may compute the rate of change of a function describing volume in order to determine how to build a box with maximum volume given constraints on the amount of materials on hand. Through this, the classroom culture balances itself; lectures are taken both seriously and synthesized with great intrigue as the constant stream of variables and numbers now seemingly have a secure place in society and our local environments. This drives forth the understanding that maintenance of overarching learning and curriculum standards are not only imperative to bring to the classroom, but to ensure there is a strong alignment between intention and enaction (Carlson & Diefenderfer, 2015; Fernández & Jones, 2006).

Both in practice and in theory, modern mathematics education is effectively structured upon articulate, well-aligned communication between student and teacher (Doll, 1993; Fisher & Rickards, 1998). Within a modern dynamic educational environment, learning and instruction ought to influence each other in a feedback-based system (see Figure 1) in which intended curriculum aligns closely with enacted curriculum despite inevitable minor logistical constraints and/or obstacles within the classroom, which may be ascribed to random error in the highly dynamical complex system of educational interaction. In such a system, the dynamics of instructional practices may meaningfully be influenced and refined by student understanding. Such a reformation of teaching practices to better accommodate the student's needs ought to strive for a balance between ensuring the success and encapsulation of individual student's learning goals, as well as the goals of the course and classroom body at large (Morcom, 2016). How can we better inform periodic check-ins and implement evaluation tactics that ensure fairness, inclusion, and optimization of future quality of life in the pursuit of educational equity? Courses that have implemented hybrids of formative and summative 'check-in' and pacing assessments have been demonstrated to provide positive implications for the academic and career successes of students in such an equity-focused program (Jurdak, 2010; OECD, 2007).

1.3 Establishing an Andragogical Environment

While there exists a vast lexicon that encompasses educational disciplines, this work focuses on studying learning practices through the lens of *andragogy*. Andragogy refers to methods and principles utilized in the education of adult student groups. It should be noted that while the terms 'pedagogy' and 'andragogy' both refer to education as a means of communicating bodies of knowledge, this work considers college students to comprise an andragogical learning system versus a pedagogical system. This perspective is well-supported not only in the context of university academics, but specifically in the context of mathematics education at the college level (Rodrigues, 2012). The maxims and theory of andragogy are articulated heavily throughout the analysis of literature review and methodology design to structure a meaningful basis as to how educators ought to orient themselves properly and professionally to their student body. Moreover, this work emphasizes this basis in early collegiate education to promote equitable, professional, and engaging discourse in the everlasting pursuit of higher mathematics.

Chapter 2: Literature Review

2.1 Concept Mapping, Culture, and Creativity as Ways of Knowing

While this thesis aims to optimize learning processes in favor of student experiences, the role of highly intentional classroom design can not be understated, particularly with regard to connectivity and the human experience's role in education and educational understanding. This research project draws largely from current literature on concept mapping, learning curve theory, network modeling, and relational learning, particularly through the lens of constructivism. In terms of concept mapping, This novel approach to curriculum design merits particular interest in the work established by Jon Simon's "Curriculum Changes Using Concept Maps", in which the author details to what extent curricular concept maps can be applied in order to enhance module cohesion, integration with chosen texts, and the sequencing and/or pacing of topics (Simon, 2010). Implementing concept maps in mathematics education has a storied history of success in cohesion of student understanding, and also reflects promising study and review practices that provide meaningful progression towards enhancing performance as a whole (Ozdemir, 2005; Novak, 1990). This philosophical approach to establishing the trajectory of delivered educational content is reinforced by Dr. William Doll's observations of the connection between complexity and education in (Doll, 2008), in which Doll recounts that, "emergence of creativity from a complex flow of knowledge … results in emergent structures, i.e. creativity in the context of education should be thought of as a unique way to arrange information so as to make new meaning out of old ideas." (p. 196). Via this lens of synthesizing creativity, semantic-based learning, and complexity science, it is evident and self-emerging that learning processes amongst a collective body in the form of a classroom environment ought to retain some natural underlying architecture that is conducive to optimizing both retention and orientation with respect to *ethnomodeling*. Ethnomodeling is defined as “the study of mathematical phenomena within a culture because it is a social construct and culturally bound” (Milton & Orey, 2013). In conjunction with the study of the path mathematical creativity constructs to lead to robust learning and dynamic understanding outlined in (Monahan, Munakata, and Vaidya, 2019), these resources synthesize to verify the notion that education exists as a complex system, comprising various interwoven topics and relations that ought to be both inherently and intentionally connected. This revelation lends itself to approaching mathematics education analysis via the employment of tools commonly applied to complexity science, including but not limited to: network/graph theory, stochastic processes, probability and statistical theory, and nonlinear systems (Benham-Hutchins & Clancy, 2010).

2.2 Complexity Science and Other Quantitative Bases of Learning Analysis

The related quantitative research methods include building mathematical models that effectively outline the learning process from the student’s perspective, all of which work in synthesis with (Thurstone, 1919), which lays the groundwork for theorizing that a logistic growth model exemplifies the natural processes by which effective learning progresses (Thurstone, 1919). Additionally, relational learning and asymptotic learning analysis as conducted by human learning mechanisms can be meaningfully modeled in neural networks (Xiang & Neville, 2011), uncovering the revelation that a 'working alliance' between teacher and student operates best under conditions in which teachers act both as mentors assisting in a student’s personal growth, and as 'orienteers' maintaining the trajectory of the intended course of study (Spigler, Geiger, & Wyart, 2020). This requires teachers to monitor the quality of their relationships with students, and make adjustments that affirm students’ needs, interests, and position within the classroom. This serves to bridge the gap between qualitative observation and examination of classroom practices and student response, but also the quantitative and computational trending and forecasting ramifications of these interactions between educators and learners alike. The basis of such an approach to education is further reinforced in the study outlined in (O'Meara & Vaidya, 2021). The work conducted in this entire thesis project is based on the groundwork outlined in this paper, and hopes to push far beyond these axiomatic principles.

Any chosen framework for curriculum performs best not through breadth nor depth of topics alone, but through a balance of both, enacted through a large proportion of novel topic connections and a significant yet lesser relative proportion of deeper topic connections (O'Meara & Vaidya, 2021). Such a balance indicates the presence of a power law distribution, which is highly reinforced within natural and social systems (Clauset, Shalizi, & Newman, 2009; Newman, 2005). With the choice of textbook acting as a model for intended curricular content, one can gain insight into the efficiency and alignment of the enacted curriculum by conducting an analysis of student-provided data that serves as real-time feedback concerning how effectively this transfer of knowledge remains in line with standard classroom practices. This enables lessons to become more malleable in nature, and allows for near-immediate response from the teacher to analyze what portion of the content students are lacking. The corresponding questions to answer are: What are the roles that learning feedback models, creativity, and connectivity play in mathematics education? In the context of network theory, mathematical modeling, and probability/statistics theory, how can empirical data influence educational practices and improve curriculum design to ensure taxonomic principles and onboarding with future course material are met with little impedance to student learning goals?

One overarching means of understanding student learning is via the process of action and engagement in the classroom. With this, it can be hypothesized that finding methods of integrating tactile experiences that require active 'hands-on' processes ought to be highly engaging for students of all walks of life. Such considerations of serving students in the classroom brings about discourse surrounding affordances our environments might offer. Educators therefore posit that students make meaning by the way they interpret stimuli via classroom experiences and respond to it accordingly (Gibson, 1977). This information can then be applied as a sort of feedback model that might influence how we react to consequent events and continue to develop this ascribed meaning.

While there is a compelling case for curriculum as a complex system in the relevant literature (Doll, 2008; Doll, 1993; Barnett, 2000; Davis, 2018; Mason, 2008; Ovens & Butler, 2016; Wood & Butt, 2014), it is imperative to consider that the term 'complexity' itself is vaguely defined and depends on the field to which it is being applied. The primary defining feature of any complex system is such that it contains multiple ever-interacting parts, often in a non-linear or unpredictable pattern. Any novel venture into the area of complexity theory therefore requires clear articulation of its structure and evidence that this fundamental feature is met. Davis describes three areas of education where complexity theory has made its greatest mark thus far (Davis, 2018): (i.) "…contents of curriculum, complexity as a disciplinary discourse…", (ii.) "…beliefs on learning, complexity as a theoretical discourse…" and (iii.) "…pedagogical strategies, complexity as a pragmatic discourse…" (p. 75-88).

Chapter 3: Methodology and Results

Through the use of modern quantitative and qualitative methods alike, this research effort aims to improve and standardize the means by which we evaluate the success of student performance in a precalculus course through the role of a connectivity-based curriculum. This investigation ultimately aims to move throughout the content of precalculus and both motivate and develop the necessary information overlay that will serve students in future academic coursework, while considering how student ‘check-ins’ can deepen the reach of assessment tools as they center around the needs of both the collective and individual learner. Such future work includes traversing the calculus sequence for such collective observations, as well as any other relevant courses and/or areas of study specific to the student’s (singular or otherwise) intended academic trajectories. Through the scope of this project, the overarching hope is to employ educational data in order to uncover empirical patterns in successful models of intended curriculum to influence how educators highlight connections in the classroom. This goal will be accomplished in three phases:

The first phase will center on previously conducted research that acts as the basis for identifying strong curricular content. This prior work serves to deepen the analysis of a precalculus course with respect to established taxonomies and pedagogical/andragogical goals. Such previous work includes establishing the efficacy of a power law relationship among effectively executed learning systems (Newman, 2005; O’Meara & Vaidya, 2021), and the role of network theory in the pursuit of outlining meaningful curricular connections. This will be further supported by student-generated data that includes both qualitative responses to prompts regarding personal and academic orientation to central topics taught in class, as well as a quantitative concept map that displays how they view the connections between different topics throughout the entire course. The ability to meaningfully implement concept maps in the classroom demonstrates student ability to orient themselves in the context of the curriculum at large (Novak, 1990), shifting focus towards the penultimate steps in their educational paths established by this stepping-stone course. This first phase is the central focus of this thesis, with considerations from phase two and the outlining of the preliminary steps of phase three to be completed in an ongoing study that will culminate ideally in both a publication specifying the ramifications of such an analysis, as well as acting as the genesis for upcoming doctoral research by the author.

The second phase will consider the construction and efficacy of a section-wise time-series development of the relevant computed metrics in (O'Meara & Vaidya, 2021) such as average path length (APL), average local clustering coefficient (CC), and number of hubs (Hubs) based on the network mapping of the textbook used in Montclair State University's MATH111 precalculus course (See Figure 4), whose various sections over several semesters act as the focus of this study. The literature review conducted for this work, as well as preliminary statistical analysis on empirical data, has revealed an inherent logistic pattern which is well-aligned with student learning processes (Leibowitz, Baum, Enden, & Karniel, 2010). This will yield natural next steps in future work, for which similar curves will be constructed for the university's MATH122 calculus course to be qualitatively analyzed for repetition of topics introduced from precalculus in order to build meaningful concept maps and establish critical periods of overlap between courses. These moments of intersection will ultimately aid in the identification of student onboarding, and serve to allow the instructor the opportunity to ensure enacted student learning aligns properly with future coursework.

The construction of these logistic curves will serve as the springboard towards the third and final phase of this study, which will utilize data derived from the encoding of student responses in order to build composite concept maps. This process will then be encoded as networks as outlined in (O'Meara & Vaidya, 2021) to analyze how closely aligned both the aggregate mean of the relevant metrics and the individual students' obtained values are to the baseline intended curriculum of the corresponding course's text. Analyzing the evolution of this data will serve to demonstrate where students lie in the development of the course throughout the semester, whereas end-of-semester evaluations of this nature will ultimately provide a meaningful tool for administration at all levels to uncover how successfully students have achieved the goals of the precalculus course itself. This information will allow for meaningful individualization of learning processes and ensure that academic trajectories are well-aligned both in intention and in execution.

3.1 Understanding Curriculum Design and Choice through Network Analysis

The primary lens of understanding the exchange and discourse of knowledge in the classroom is understanding the set of tools the teacher brings forth in order to generate cohesive lesson planning and execution. Therefore, a reasonable place to start is in the choice of textbook, and to what extent this might serve as a strong template for productive and effective mathematics education. Factors such as repetition, connectivity, and support for identification of personal representation are explored through consideration of precalculus curricula, and how choices for textbook content can influence and improve standard of learning and improved instruction[1]. It is with the theories, assumptions, methodologies, and findings outlined in this publication that the

[1] This chapter uses excerpts from (O'Meara & Vaidya, 2021), which is an open access article distributed under the terms and conditions of the Creative Commons (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

following two phases are built upon in order to further develop both the bridge between teacher and student roles in curriculum, and the efficacy of research-based assessment and intervention opportunities.

Educators recognize the fact that teaching and learning cannot be spoken of independently; they proceed through feedback between each other (see Figure 1). We can think of the curricular aspect of teaching as an interaction between an intended curriculum (IC) and enacted curriculum (EC) (In the Figure 1, IC and EC compartments are mediated by their corresponding network models.). The intended curriculum can be reflected through a textbook which an instructor typically would utilize to design her own lesson plans which is enacted in the classroom, and referred to as the enacted curriculum. The differences between the IC and EC can vary drastically with instructor, students, topic, level of course etc. In this paper, each text chosen is mapped to a unique graph (described in detail below). In mathematical terms, teaching as defined in our study, is the mapping

$$T = T_C \cup T_E \cup T_S \ldots$$

where T_C represents the curricular or content aspect of teaching, T_E refers to the teaching environment and T_S to the skill level of the teacher, among others. We specifically focus on T_C which can be represented as the transformation:

$$T_C : IC - Model \rightarrow EC - Model$$

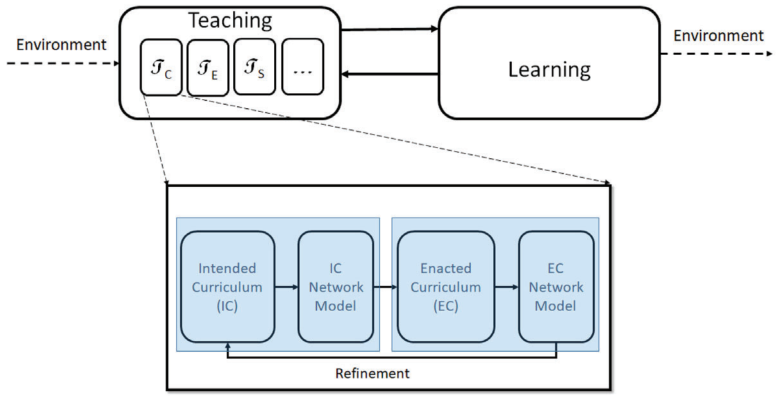

Figure 1: Schematic showing the complex nature of education and curricular design. While teaching and learning are recognized to proceed along an a-causal feedback loop, teaching at its intrinsic scale is feedback between the intended and enacted curriculum, each of which gets refined over time.

Based on Figure 1, the map T_C can itself be decomposed into several individual components that outline the feedback model generated between IC and EC, and how these steps in the development of curricular content might influence one another. While T is represented as a one-way mapping, in the hands of an experienced instructor, the EC can result in a transformation of the IC as well. It is also not unreasonable to assume, especially in the case of inexperienced instructors, that $T_C = I$ (identity map), i.e., the IC is the same as the EC.

It is important to recognize that teaching and learning are embedded within a specific environment, which they are shaped by and influence. Environmental factors directly impact IC (and directly or indirectly, EC), which takes the form of various editions/updates of a book, new books and revisions to course content etc. This interaction between education and its various components with the environment maintains education in an out-of-equilibrium state, much like a dissipative system (Doll, 1993; Kondepudi, 2008). If we persist with the thermodynamic language, we can expect this mutual interaction to result in emergent self-organized states, under fixed conditions, which translates to an optimal (meta)stable curriculum (or network pattern) which would change with time and environmental factors. While this argument relies primarily on analogies, it is evident that the educational system is a complex combination of interacting self-similar components, each of which represents a complex system at its own level (Mason, 2008; Ovens, 2016; Wood & Butt, 2014).

A complexity-based approach to education was perhaps best articulated by William Doll in his numerous works on the subject (see, for instance, (Doll, 1993)). In his article (Doll, 2008) states: “Order emerges from interactions having just the ‘right amount’ of tension or difference or imbalance among the elements interacting”. In the context of education, one could argue that this emergence of order is nothing but creativity. (Hiebert and Carpenter, 2006) associate the notion of ‘understanding’ to that of a complex network. They state: “Understanding increases as networks grow and as relationships become strengthened with reinforcing experiences and tighter network structuring” (p. 69). It has also been argued that learning through identification of similarities and differences between alternate representations of the same information can stimulate the construction of useful connections (Monahan, Munakata, & Vaidya, 2019). Therefore it is quite reasonable to think of education at all levels as an “adaptive and self-organizing complex system”.

This thesis is based on the premise that a textbook, which represents a particular pedagogy of the subject being treated, is a collection of multiple interacting parts. As students progress in a course, they navigate from one topic to another, united in the endeavor of comprehending, connecting, and unifying concepts. It is the potential to form their own bridge across different topics that is of significance here, since it relates to creativity: an oft spoken about but understudied phenomenon in higher education. Previous studies on creativity, in mathematics education in particular, have pointed to the importance of making connections as a necessary condition for creativity in the classroom (Monahan, Munakata, & Vaidya, 2019; Monahan, 2020; Munakata & Vaidya, 2013). The ‘lego model’ of creativity discussed in

(Monahan, Munakata, & Vaidya, 2019) argues for personalization of knowledge and connection between different ideas which can result in stability and longevity of understanding. The ‘connected curriculum’ program based in (UCL, 2021) espouses the importance of connections in higher education, highlighting the ‘real value’ of education, which lies in preparing students for the ‘real world’ where students must solve complex problems which do not appear under a disciplinary guise. The rationale for such a curriculum is well articulated in (Fung, 2017; Wraga, 2009) and other related works, pointing to the fact that “social problems and issues transcend disciplinary boundaries” (Wraga, 2009; Davies & Fung, 2018). Our approach in this paper is based on a similar philosophy of learning and clarified using a simpler example, which lends itself to very novel, rigorous and interesting mathematical analysis. We contend that such a hybrid modeling approach which combines qualitative and quantitative aspects of mathematics is extremely appropriate in the context of education, especially mathematics education.

The approach to introducing and evaluating this particular philosophy took the following form:

- Step 1: Mapping precalculus curricular content for quantitative and qualitative analysis
- Step 2: Applying relevant metrics to study the encoded curriculum and extract trends and patterns
- Step 3: Aggregate results and identify partitions in the data between well-revered and poorly received textbooks
- Step 4: Consider the basis of these results with respect to a ‘union network’ in which all possible connections between sampled texts might be made, followed by a stochastic analysis of the sensitivity of computed metrics

- Step 5: Discuss the interpretations of the network-based analysis in the context of curricular design and education, and reflect on the prospects and importance of such an analysis in other problems related to education.

Naturally, elements of this procedure are somewhat subjective; some instructors may highlight connections others might find unimportant or overlook. However, in the pursuit of developing a more significant understanding of the properties that make a text suitable for a given course, instructor, or student body, it is imperative that causal connections be strictly observed, both in proximity and conceptual basis, revealing an inherently complex nature amongst the association between any given curricular themes.

In our efforts to generate uniform procedure, we conducted an initial trial in which the same text was encoded by two separate researchers, and then results were compared in order to verify that results were not only as identical to one another as possible, but also to ensure that all included nodes and edges were strictly obtained from the textbook at hand, and were not the result of any personal bias or opinion as to which topics might warrant connectivity from the perspective of the researcher; the goal of this encoding process was to establish routine procedure and strict adherence to intentional language dictated by the text, while also taking advantage of the ability for human-based encoding to pick up on nuanced connections similar to the examples outlined in the sample text above.

By regarding each topic as a node and each link as an edge, we are able to meaningfully translate each text into a graph or network of ideas and their corresponding connections. A uniform set of principles were developed to help maintain consistency in mapping the various books. These include:

1. To start with, we make a list of all the topics in the chosen text under each chapter, section, and subsection. These topics are listed in a Table A1 in Appendix A. Each topic is given a numeric code. In the language of sets, $X = \{k_i \mid 1 < i < N\}$ pertains to the set of $N \in \mathbb{N}$ topics covered in the textbook, where k_i refers to the *i*-th assigned code for each topics
2. Based on the topics listed in first two columns of the Table A1, we then create a third column as shown in Appendix A, where the elements constitute the set $Y = \{Z_{ij} \mid Z_{ij} \in X, 1 < i < N, 1 < j < M, M \leq N - 1\}$: That is, topic k_i may contain up to *M* direct connections outlined in the corresponding text being mapped, $M \in \mathbb{N}$. The elements of set *Y* therefore represent distinct topics in *X* which are related to k_i .

Once the entire table is created, it can be represented as a network of nodes (topics) and edges (connections). Figure 2 gives examples of such generated network representations of sampled textbooks in this study.

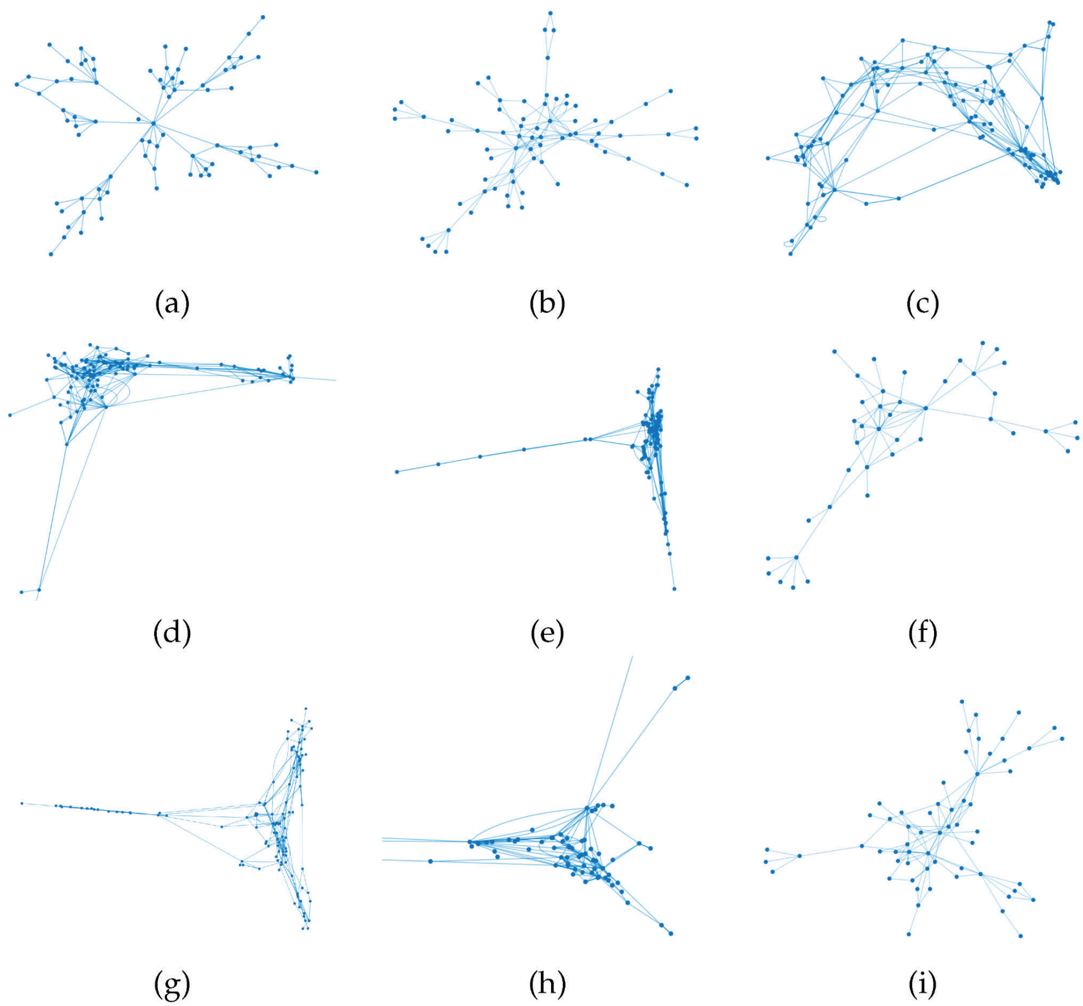

Figure 2: Network structure of the Pathways Combined curriculum. The various panels represent different books, namely: (a) Abramson (b) Blitzer, (c) CME, (d) COMAP, (e) Faires, (f) Larson, (g) Pathways (combined), (h) Rockswold and (i) Stewart. In cases (d,h), we provide a zoomed image to showcase the details of the connectivity.

Once represented as a graph, we can estimate several properties of such a graph which give us a glimpse of the underlying structure of the course and its potential to allow free and easy flow of ideas and foster new emergent understanding and creativity. By examining the network structure of various textbooks and pedagogical practices, we can help identify the kind of curricular plan that is likely to be most effective and creative. Specifically, the following metrics (Albert & Barabási, 2002; Newman, 2003) are examined:

1. The **Degree Distribution** (DD) helps us ascertain that the 'textbook network' does indeed display a power law profile and hence the metrics typically associated

with the analyses of such networks are meaningful in this context. The power law nature of such a network reveals that there is a specific structure to curriculum which is not random. The degree distribution of the network is given by the probability function

$$P(x) = cx^{-\alpha}$$

where c is a constant, x denotes the degree of the node and α is a scaling parameter that provides insight into the mass of the distribution's tail.

2. **Clustering Coefficient** (CC) tells us about the average number of connections for each node, giving us a glimpse into the variety of ways a particular topic in precalculus can be understood. A fundamental assumption of the constructivist model of mathematics is the potential to make meaning. Therefore the greater the clustering coefficient, the more diverse the ways in which a concept can be comprehended depending on the particular background and proclivity of the student. The local clustering coefficient, denoted C_i, is commonly given by the expression:

$$C_i = \frac{3(number\ of\ triangles)}{number\ of\ connected\ triples}$$

resulting in the average clustering coefficient for the network, $CC = \frac{1}{N}\sum_{1}^{N} C_i$.

3. **Average Path Length** (APL) tells us the average number of steps that must be taken to traverse between any two nodes. In the context of this study, the APL tells us about how efficiently one can move from one idea to another. It is particularly useful to strategize about how to resolve mathematical problems. A network possessing a low APL is preferable, since it makes explicit the links between concepts and provides a road-map to travel efficiently from one point to another. This, coupled with a high CC, makes for easy navigation between ideas and also increases the likelihood of exploring many possible ways to navigate between these ideas. The APL is given by the equation

$$APL = \frac{1}{N(N-1)}\sum_{i \neq j} d(V_i, V_j)$$

where $d(V_i, V_j)$ represents the path length from node i to node j.

4. **Hubs** (H) are nodes which have a large number of edges. The threshold number of edges to qualify to be a hub, in general, is determined by the nature of the problem itself. We use the minimum number of chapters from all the texts examined to decide a threshold to qualify for a hub. This number turns out to be 6 based on the book by Faires [30].

In addition, other general characteristics of the books are also considered such as the ratio of nodes to edges, and the specific topics that qualify as hubs across the different books.

When tasked with the penultimate goal of meaningfully analyzing the emergent quantitative patterns exhibited throughout network graph models of precalculus course textbooks, it became evident both empirically and statistically via the genesis of analyzing the degree distribution of each node (course topic) with respect to frequency that there exists a natural power law throughout these curricular connections. Moreover, one particularly remarkable result that emerged is that, through stochastic analysis of the generated union graph, the underlying chosen metrics yielded low sensitivity when edges were randomly removed, indicating that customization and individual choice of emphasis by the instructor can still yield desirable learning outcomes with respect to the intention of the authors of the chosen textbook and/or curricular map. Such an outcome is promising, as this sense of unique influence on learning practices places all classroom participants on the frontier of autonomy and agency as they navigate throughout precalculus and beyond.

Book	APL	CC	% H	# H	Edges/Nodes	Mean Edges per Node	Power-Law Exponent	*p*-Value
Abramson	19.0047	0.2662	11.54	9	116/78	1.487	−5.851	0.14
Blitzer	3.7652	0.0481	12.33	9	109/73	1.493	−16.881	0.16
Carlson	3.9001	0.2725	21.83	31	268/142	1.887	−16.6774	0.07
Stewart	3.361	0.0417	15.00	9	101/60	1.683	−28.357	0.06
Faires	3.2133	0.3414	35.25	43	327/122	2.680	−38.478	0.84
CME	3.4179	0.4152	32.03	41	314/128	2.453	−29.716	0.06
Larson	3.6915	0.156	14.63	6	66/41	1.610	−68.315	0.051
COMAP	3.6908	0.3397	21.19	32	302/151	2	−22.996	0.22
Rockswold	3.4128	0.2815	11.76	14	192/119	1.613	−55.960	0.07

Table 1: The results of the network analysis for all texts are summarized in this table. Quantities computed include the average path length, clustering coefficient, number of hubs, percentage of nodes that are hubs, number of edges and nodes for each text.

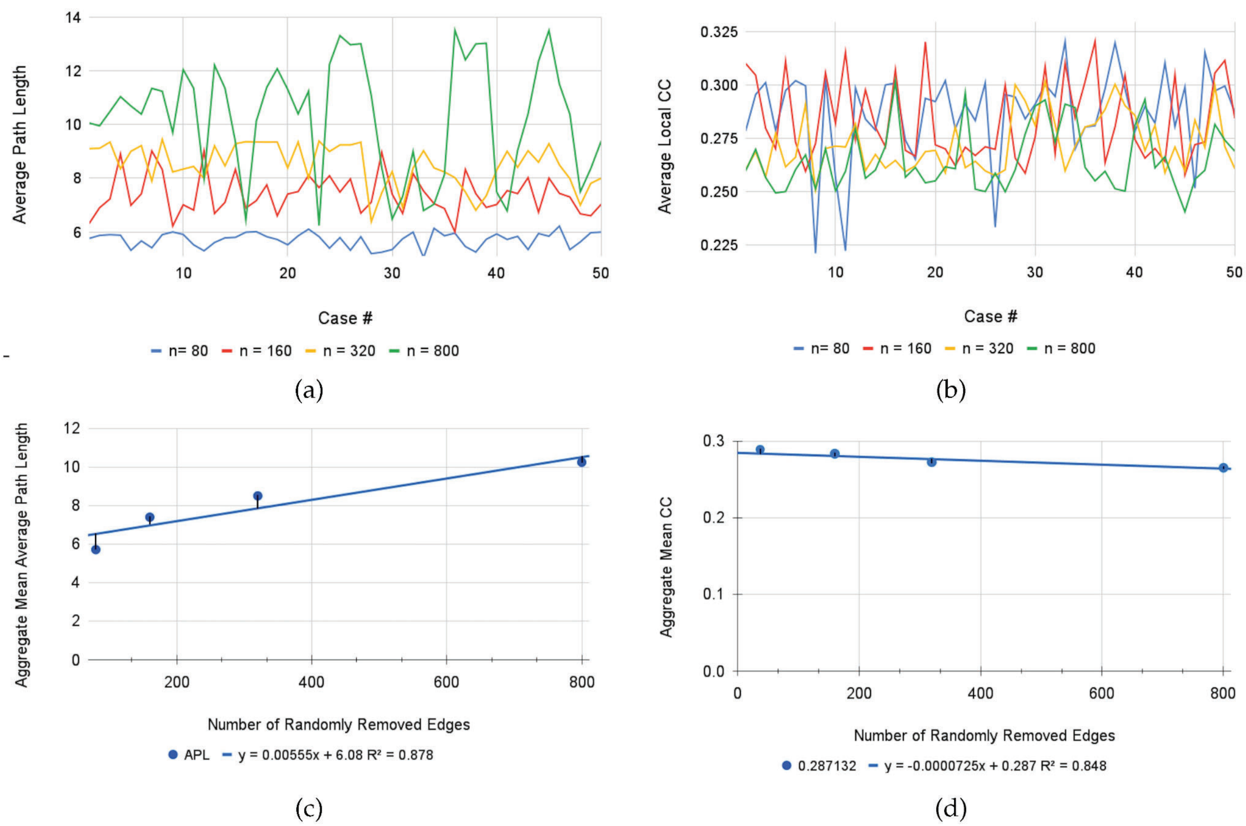

Figure 3: A visual display of both the variability of all fifty iterations for each value of n and the APL and CC metrics in (a,b), as well as a regression for the mean APL and CC across all iterations in (c,d).

No. of Removed Nodes, n	n = 5	n = 80	n = 160	n = 320	n = 800
Percent Equivalent of Total Edges	0.3097	4.9566	9.9132	19.8265	49.5662
Absolute Percent Error in APL	0.0666	11.7704	44.7580	66.4542	100.6488
Absolute Percent Error in CC	0.8559	9.3076	10.4080	13.9279	16.2400
Mean APL	5.1142	5.7123	7.3982	9.9291	12.9921
Standard Deviation of APL	0.0029	0.2820	0.7283	0.7835	2.1169
Mean CC	0.3138	0.2871	0.2836	0.2725	0.2651
Standard Deviation of CC	0.0028	0.0210	0.0190	0.0132	0.0146

Table 2: The results of the stochastic network simulation analysis for the union of all texts are summarized in this table. Quantities computed include the average path length, clustering coefficient, and error metrics relative to the original union graph.

3.2 Supporting the Learning Curve through Time-Series Concept Development

The concept of understanding learning curves as a logistical model permeates throughout various disciplines. This includes educational processes (MacLellan, Liu, & Koedinger, 2015) machine learning (Perlich, Provost, & Simonoff, 2003) and artificial intelligence (Schein & Ungar, 2007), modeling system factors of loan approval and financial modeling (Vaidya, 2017) as well as developing and motivating the dynamic behavior of ecological and social systems (Hartz, Ben-Shahar, & Tyler, 2001). All of these pillars of dynamic learning and social systems are of particular interest and relevance in the wake of the COVID-19 pandemic and an ever-accelerating need to improve technological, social, and andragogical/pedagogical practices in a vastly broader array of learning environments than ever before (Wang, Zheng, Li, & Zhu, 2020). As this work endeavors to uncover meaningful relationships within the initial framework of course design, this opens up a much larger question worthy of discussion: now that we have begun an exploration into the teacher's role, how can we continue to meaningfully understand the student's role in this complex system?

This current study aims to break down student comprehension and retention by examining both incremental changes in the metrics computed in the first stage of this research project as outlined in 1.1, and how the demonstration of logistic behavior can act as a meaningful model for ideal student performance along the trajectory of a semester-long course. By considering each student's perception of course trajectory as an individual network of observed topics and connections, not only can this be compared and contrasted with the idealized model of the course's textbook/curriculum, but this can also be used to seek correlations with past student experiences that might better inform what the source of unobserved meaning-making in class might be, and how one can correct that in a meaningful and andragogically productive way. Such patterns can be observed in Figure 4 below, along with theoretical growth phases in the development of a course in Figure 5.

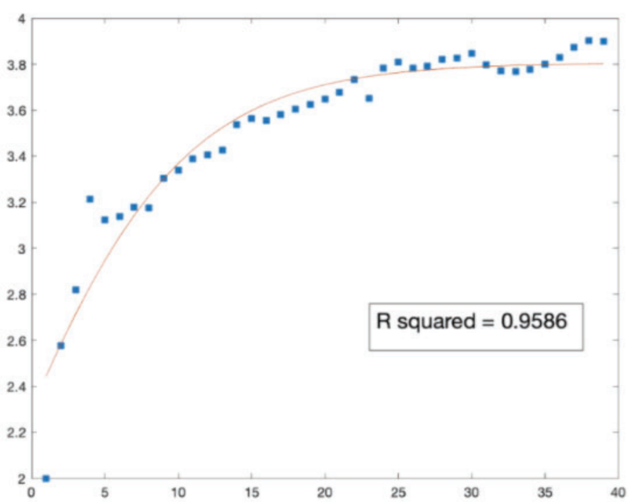

Figure 4: The evolution of the average path length (APL) in the Pathways precalculus text as a function of the section-wise chapters (or time). The APL is fit to a logistic curve with the provided correlation coefficient R^2.

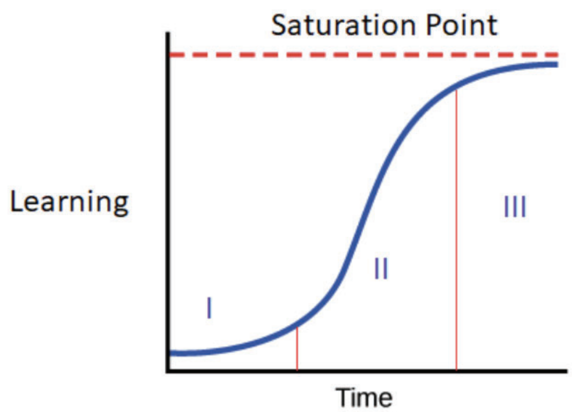

Figure 5: A depiction of theorized phases of learning development along a logistic curve, in which Phase I represents slow growth until critical knowledge is reached, Phase II outlines a rapid learning phase in which information is learned at an efficient pace, and Phase III depicts the approach of a saturation point.
We can define a couple of critical points (a) accumulation of critical mass to jump-start the learning process, and (b) a critical saturation point beyond which learning slows down considerably.

Via an initial case study conducted with the textbook and student handbook for Montclair State University's MATH111 precalculus course, it can be observed that these materials offer a promising exemplar of a logistic profile with respect to their APL. This parallels the general student observation that a course that is designed to bridge two distinct bodies of mathematical knowledge together (algebra and higher-order mathematics, typically through the lens of calculus or calculus-dependent studies) will offer a 'ramp-up' phase towards critical knowledge acquisition, ultimately culminating in an asymptotic 'saturation point' at which all theoretical knowledge is obtained by the students as stipulated in the course goals. The intent behind constructing such a model is that, with an outline for how knowledge and connections ought to be formed throughout the semester, we now have a yardstick by which we can assess the connections students are making across all topics, and ensure that they are well-aligned with the desired pacing of the course, as influenced by individual teaching practices and andragogical choices along the way to cater to the needs of the classroom at large. With respect to how such a student analysis might be conducted, we now move into the third phase of this body of work that uses student data to assess learning trajectories and provide opportunities for intervention and alignment.

3.3 Student Data & Classroom Support via Ethnomodeling and Educational Equity

In order to collect, process, and analyze student data, IRB-FY21-22-2488 - "An Analysis of Complexity and Connected Curriculum Practices in Upholding Course Goals in the Classroom" was submitted to and approved by the Institutional Review Board at Montclair State University. This collection of data is comprised of two parts:

- An assignment titled "Semester in Review" (See Appendix D) was administered to students for a portion of their homework grade, in which students were asked to write qualitative personal statements that relate the most well-connected topics in the course to each other, to other topics in the course of their own personal interest, as well as their academic and personal lives in any capacity observed.
- An assignment titled "Connectivity Survey" (See Appendix E) was also administered to students for a portion of their homework grade, in which students were provided with the list of every topic discussed throughout the entire semester ($n=127$), and asked to draw connections between topics as they saw fit.

All responses were anonymized by assigning random numbers in place of names, and removing all identifying characteristics from submissions. However, both assignments were given the same identification number, along with their final grade in the course (on a decimal scale from zero to one hundred) so that, while students cannot be identified by the author or any external observer, the respective collection of submissions and scholastic performance can be aggregated to draw connections and trends between the body of data.

In order to implement aspects of agency, authority, and equity as commanders of their educational paths, both prompts were provided with minimal requirements with respect to content and formatting. “Semester in Review” allowed students to submit in any format they desired, as long as the written contents accumulated to approximately two pages in length. While many students opted to write traditional essays, there were many instances of outwardly displayed creativity in the assignment. For example, some students opted to write poems, others short stories, and a handful of students even designed and coded the front and back end of their own websites with the assistance of HTML5 and CSS3 (see Figure 6 below). This component was important, as it allowed students to experience first-hand how their identities are intimately intertwined with their academic achievements and personal lives. For many, this was an opportunity for the educator to orient themselves deeply to the interests of the student which then motivated future lessons - an outcome that is invaluable to the adaptive and malleable nature of curriculum design and developing mathematical thinkers.

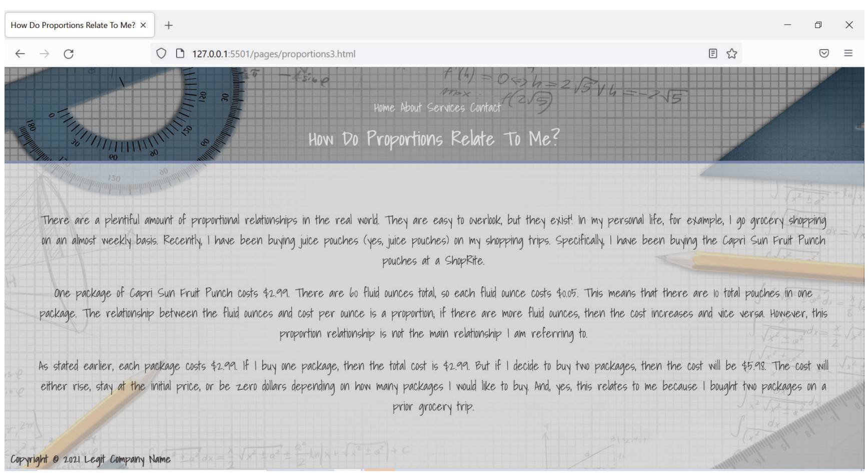

Figure 6: A submission for the assignment stipulated in Appendix D wherein a given student designed a website from scratch to draw connections between precalculus topics and implement elements of their personal and academic interests in the context of their studies. (ID#026)

Additionally, for the “Connectivity Survey”, students were provided without any minimum number of connections to draw, as well as the instruction to form connections without the necessity of going back to their notes. The purpose of these parameters was to allow students to express their true observations and intuitive relations amongst their studies without fear of repercussion or false inflation of their true observations. A preliminary analysis shows that many submissions have high degrees of variation and interpretation, as is reflected by both the quantity and quality of connections.

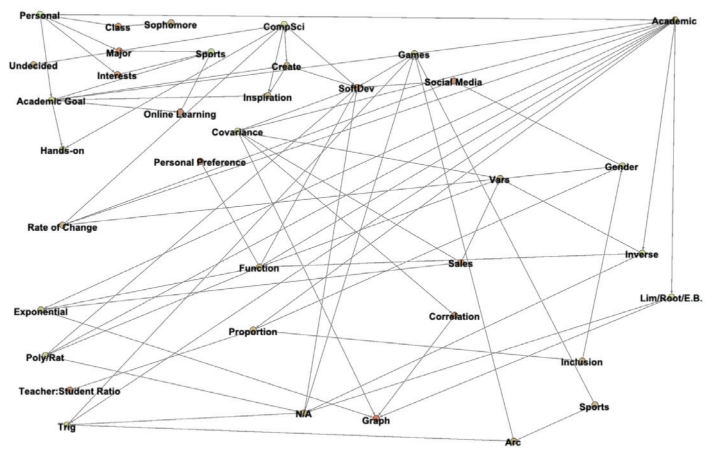

Figure 7: A network representation of a randomly selected student's submission for the assignment stipulated in Appendix D. (ID#044)

Although this third phase is still in progress, initial findings demonstrate a high correlation between observed connections, depth of connectivity between personal traits and academics, and performance in the course. Across an initial sample of ten randomly selected students, submissions were reviewed and encoded with respect to word count (*W*), number of distinct topics/concepts introduced (*T*), and performance in the course (on a numerical decimal scale from zero to one hundred). The values of *W* and *T* were compiled to form a root-mean standard error (RMSE) to provide an aggregate metric against which performance can be weighed in order to assess whether or not a correlation exists between initial measures of effort, insight, understanding of topics and their relevance to personal academic interests, and final grades in precalculus. This data is provided in Table 3 and Figure 8 below.

Student ID	Word Count (W)	# of Topics/Concepts Introduced (T)	RMSE (Relative Weight)	RMSE (Uniform Weight)	Performance	Sum of Max - RMSE (Uniform)
44	1118	37	0.01489580133	5.656854249	95	1157.343146
27	530	22	576.1634746	416.0967436	88	746.9032564
30	610	23	498.6855404	359.5469371	87	803.4530629
31	550	24	555.9986478	401.9110598	83	761.0889402
25	947	45	167.0764596	120.9152596	100	1042.08474
37	614	27	493.2711417	356.6090296	88	806.3909704
36	537	22	569.4523275	411.1508239	72	751.8491761
47	576	21	532.3820004	383.6274234	99	779.3725766
45	589	31	515.6054219	374.1904595	83	788.8095405
40	662	23	448.2791634	322.8157369	77	840.1842631

Table 3: Quantitative data extracted from a random sample of ten students in which the breadth and depth of 'Semester in Review' submissions are correlated with course performance. A standard error analysis is conducted with respect to the maximums of variables 'Word Count' (W) and '# of Topics/Concepts Introduced' (T).

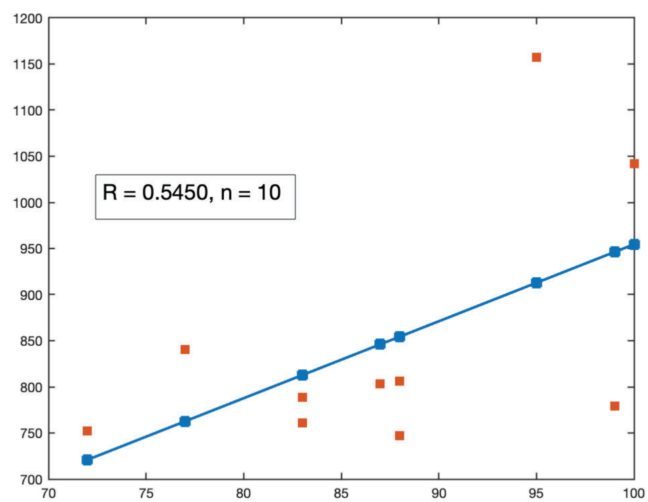

Figure 8: A graphical representation of the data in Table 3, coupled with an OLS linear regression to capture initial trending between course performance (x-axis) and adjusted RMSE (y-axis).. Sample size (*n*) and correlation coefficient (*R*) are displayed on the graph.

A much deeper analysis will follow these preliminary results, including the utilization of relevant metrics established in the first phase in order to evaluate where any given student's perception of connectivity between topics lies on the time-series progression of course metrics stipulated in Phase Two in order to allow for equitable attention and individualized assessment of knowledge formation, concept mapping, and understanding of ideal course connections and concepts as the weeks of the semester progress. While this study is performed retroactively, this work hopes to serve as the basis for informing alternative assessment opportunities and meaningful check-ins for precalculus educators in order to better serve students, both as the collective body of classroom thinkers and as autonomous agents of equitable access to optimal learning opportunities and trajectories.

Chapter 4: Discussion

The aim of this study is to standardize the way we as educators synthesize taxonomic goals in our lesson plans with enacted feedback from the student body, and how standardization of intended curricular design can act as a basis for the empirical analysis of student alignment at various stages of the learning process. The author seeks to answer this question by synthesizing network theory and learning curve theory as a real-time analytic device in the precalculus classroom to chart ideal learning outcomes, as well as actual student progress, and how this informs educator response in a meaningful and objective manner which can ultimately inform methods the teacher may use to supplement the provided curriculum.

4.1 Andragogical Outcomes of Connectivity in Curriculum

What is the significance of providing rigorous mathematical models to present a standard model of educational practice? Through highly focused exploration of examining intentional connection of meaning between various topics explored across all sampled texts, the emergence of fairly predictable models indicates that neither random distributions of navigating the depths of curricula, nor a uniform presentation in which all topics maintain the same relative connectivity indices, are proper formats of effective and successful learning platforms. Instead, the data generated thus far is highly indicative of finding a meaningful balance between high frequency of low connectivity, providing temporary satellites which yield fruitful conclusions on a particular pedagogical path with respect to the curriculum as a whole, as well as low frequency of high connectivity, by which the instructor and student alike are able to extend meaning-making into many novel branches of thought that yield a unique individual interpretation within the context of the global pursuit of holistic and cohesive course understanding. It is therefore highly relevant that these well-regarded outlines of classroom instruction follow similar mathematical models, as this can allow future educators and textbook writers to establish a deeper analysis of the efficacy of intelligent design in conjunction with the student experience at the forefront of pedagogical innovation.

Aside from the analysis of the generated degree distributions, the networks themselves yield incredibly valuable insights into not only the extent to which global connectivity is achieved, but also how local behavior provides a direct lens into higher-order navigation of scaffolding ideas and collective classroom procedure. Two of the most valuable metrics in this discussion that emerged, despite the consideration of many others, are average path length (APL) and average local clustering coefficient (CC). Nearly every text sampled retained an APL between 3.0 and 4.0, indicating that efficient navigation of precalculus discourse should keep local topics well-connected, requiring no more than a few select topics to serve as the roadmap between any given ideas. Surely, with our minds focused on the educational ramifications, this feels to be an appropriate conclusion, as the goal in such a foundational mathematics course is to consistently relay the notion that the depth of the curriculum is circumvented by the relevance of all chosen topics with respect to one another. A successful classroom experience is not one of isolation; it is one of intention, retention, and ever furthering the endeavor to highlight the reasons behind studying and analyzing functions, relations, and all that which makes up a proper course. Similarly, most CC values tend to lie between 0.3 and 0.4, indicating that any given topic should be connected to between 30% and 40% of its direct neighbors, again easing the path each student takes in their endurance to tether their interpretation with all previously learned and future material.

The minimum requirement of six directly connected nodes to qualify as a hub was chosen through the observation that the minimum number of chapters provided across all texts was itself six, providing the initial conception that we are indicating a topic that has the potential to reach across the minimum span of any given network. After implementing this criterion and generating each respective network, it was discovered that there are only two common hubs across every single textbook sampled: 'Functions and Relations' (F/R) and 'Polynomial Functions' (PF). (see Appendix B and Appendix C for a comprehensive list of all hubs across all the texts analyzed and a book by book breakdown) This deepens our pursuit of answering a fundamental question that permeates the mathematical community as a whole: What is the purpose of precalculus as its own course? Does it simply serve as a precursor to calculus, or is there deeper intention behind the syllabus? This pattern properly provides a reasonable response in that precalculus is the study of functions and their individual and collective properties. In everyday life, relations bring about our human desire to establish meaning behind call-and-response, input and output relationships. We study mathematics because we desire the understanding of the world around us: Similarly, students and instructors study precalculus because it refines our rigor and analytical ability to effectively communicate the similarities and differences in classes of relationships and experiences through the natural world, and perhaps the unnatural that we simply have yet to understand. There serves no better bridge between the world of entry-level and high-level mathematics than elucidating the meaning of functions and polynomials, as these serve as the backbone of analysis, the construction of meaning-making, and the ever-driving wandering through the question, 'How does this connect to our understanding, and therefore connect understanding itself'?

4.2 Future Work

I also wish to investigate the development and relevance of a multi-sigmoidal model that meaningfully explains how course content from a prerequisite can be optimized to give a proper introduction to the content that follows, and how these might build a narrative for a student's entire mathematics curriculum throughout the course of their studies. While this initial project focuses largely on precalculus, this work opens up the possibility to introduce algebra as a necessary previous component of mathematical knowledge bases, and how this may further inform the development of the calculus sequence, as well as establishing critical intersections of corresponding courses to ensure alignment of mathematical learning paradigms to structure both future mathematics courses, and the courses that follow for all students, regardless of their intended academic path. Additionally, to further strengthen the development of our comprehensive dynamic learning model, both quantitative and qualitative student responses ought to be incorporated to channel their perceived connectivity not only within the confines of the classroom, but how their mathematics education influences both their own life and informs their intended career path and/or area of study. This leads to the natural discussion of how the format and layout of the classroom might impact both teaching and learning practices, of which there is ample evidence of success within various modalities of classroom design that are conducive towards student problem solving (Fernandez, Kazimir, Vandemeulebroeke, & Burgos, 2002). Preliminary data showcased in this thesis shows that the introduction and development of such 'check-in' tools also allows for natural, meaningful discourse between student and teacher throughout the semester. Such communication is strengthened both through frequent administration of these tools, as well as ensuring outlines of materials provided to students remain focused and aligned with what the educator hopes the student body obtains from the course content. It is the intention of this ongoing study to forge forth in the pursuit of learning and developing grounding for the ultimate purpose of accessible research-based learning goals.

Chapter 5: Conclusion

This project aims to utilize the lenses of network theory, curriculum design, and modern learning curve theory amongst various mathematical methods to understand the inherent underlying connectivity of the learning process, and optimize the complex system comprised of various roles rooted in feedback loops within educator roles, student response, and the interactions between the two as they pertain to various stages of the curriculum development process. The results of this study hope to contribute to the role learning analytics may hold in mathematics education, and how these quantitative methods can better inform educational standards that allow for the improved analysis of qualitative outcomes.

A major limitation of the nature of this work is the reliance on human interpretation to take on the role of objectifying the role of course materials. While significant steps have been taken to reduce the level of error committed in this stage and establish checks and balances along the way, the bias of human involvement in such work is unavoidable. However, the perspective a math educator adds to this work naturally highlights connections between various topics whose vitality gathers large consensus amongst other teachers and educational specialists. This further reinforces the position that frameworks of curriculum design are just that; they are intended to be interpreted, tested, and adapted to serve the needs of the environment in which they are provided.

The overarching principle one can derive from such a balance of subjectivity in the pursuit of uniform andragogical standards and practices is that the execution of such a learning framework inherently adapts and continues to remain fluid over time. This will allow optimal results to naturally emerge, and continue to allow for the dynamic feedback between learners and educators as the roles between the two are ever-exchanging. It is with this in mind that the author hopes that we all continue to forge this path with the pursuit of continually expanding our individual and collective knowledge bases.

This line of research will contribute to our understanding of how students learn mathematics, how this learning can be individualized and understood in a deeper manner, and how these observations may be aggregated such that curriculum design ensures academic equity and ensures student successes in all future academic endeavors. The beauty of mathematics we ought to convey to those who are in the primary stages of advancing their education is that problem-solving is an abstract process where we consider a variety of tools best suited for the task at hand. The decision to choose one tool versus another lies uniquely in both the discretion of the student, and that which speaks loudest as a guide towards progress and understanding.

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Appendix

Appendix A: Network Construction

The table below shows the coding for the topics identified in each text and also the connections between topics (column 3). We demonstrate the text to network mapping for the Stewart text as an example. A similar procedure is used to encode and map the other texts.

Table A1. This table provides the codes and connected topics in the text by Stewart [33].

Topic Name	Code	Directly Connected Topics
Polynomial Functions	PF	RATZEROTHM, RF, REMAINALG, DIVALG, FACTTHM, GRAPHTECH, UPLOWBOUND, QUAD, LINEARFUNC, NONLIN, INEQ, F/R, DESCARTES,
Rational Zero Theorem	RATZEROTHM	PF
Rational Functions	RF	PF, GRAPH, MOD, PARTFRAC, LINEARFUNC, NONLIN, INEQ, F/R
Transformations	TRANS	GRAPH
Graphs	GRAPH	RF, TRANS, ASYM, MOD, EXP, LOG, EQN, PERIODICFUNC, LINEARFUNC, SYSLIN, LINPROG
Asymptotes	ASYM	GRAPH
Modeling	MOD	RF, GRAPH, EXP, LOG, EQN, PERIODICFUNC, LAWS, TRIGRAT, MATRIX, SYSLIN, HYPERBOLA,

		ELLIPSE, PARABOLA, SEQUENCES, BIN, RECURS
Partial Fractions	PARTFRAC	RF
Remainder Algorithm	REMAINALG	PF, SYNTH
Synthetic Division	SYNTH	REMAINALG, DIVALG
Division Algorithm	DIVALG	PF, SYNTH, LONGDIV
Long Division	LONGDIV	DIVALG
Factor Theorem	FACTTHM	PF
Intermediate Value Theorem	IVT	GRAPHTECH
Graphing with Technology	GRAPHTECH	PF, IVT
Descartes' Rule	DESCARTES	PF
Upper and Lower Bounds	UPLOWBOUND	PF
Quadratic Functions	QUAD	PF, QUADGRAPHING, OPTIM
Graphing Quadratics	QUADGRAPHING	QUAD
Optimization	OPTIM	QUAD
Exponential Functions	EXP	GRAPH, MOD, LOG, EQN, NAT, F/R
Logarithms	LOG	GRAPH, MOD, EXP, EQN, NAT, COM, LIQU, F/R
Equation Representations	EQN	GRAPH, MOD, EXP, LOG, TRIG, IDENT, HYPERBOLA, ELLIPSE, PARABOLA, POLAR
Natural Log	NAT	LOG, EXP
Common Log	COM	LOG
Application: Liquids	LIQU	LOG
Trigonometry	TRIG	EXP, LOG
Right Triangle Trig	RTTRI	LOG
Unit Circle	UNITCIRCLE	LOG
Trig Identities	IDENT	EQN, UNITCIRCLE

Periodic Functions	PERIODICFUNC	MOD, RTTRI
Inverses	INVERSE	RTTRI
Trig Laws	LAWS	MOD, RTTRI
Trig Ratios	TRIGRAT	MOD, RTTRI
Matrices	MATRIX	MOD, METHODS, SYSOFEQNS
Methods of Evaluating Systems	METHODS	MATRICES, DET, CRAMER, GAUSSJORD
Systems of Equations	SYSOFEQNS	MATRIX, LINEARFUNC
Determinants	DET	METHODS
Cramer's Rule	CRAMER	METHODS
Gauss-Jordan	GAUSSJORD	METHODS
Linear Functions	LINEARFUNC	PF, RF, GRAPH, SYSOFEQNS, F/R
Non-Linear Functions	NONLIN	PF, RF, SYSLIN, F/R
Systems of Non-Linear Eqns	SYSNLIN	GRAPH, MOD, NONLIN, INEQ
Inequalities	INEQ	PF, RF, SYSLIN, F/R
Linear Programming	LINPROG	GRAPH
Conic	CONIC	HYPERBOLA, ELLIPSE, PARABOLA, POLAR, F/R
Hyperbolas	HYPERBOLA	MOD, EQN, CONIC
Ellipses	ELLIPSE	MOD, EQN, CONIC
Parabolas	PARABOLA	MOD, EQN, CONIC
Polar	POLAR	EQN, CONIC
Sequences	SEQUENCES	MOD, INDUCTION, BIN, RECURS, GEO, ARITH, SERIES, F/R
Proof by Induction	INDUCTION	PROOF, SEQUENCES
Binomial Theorem	BIN	MOD, SEQUENCES
Recursion	RECURS	MOD, SEQUENCES

Geometric Sequences	GEO	SEQUENCES, PARTIALSUMS
Arithmetic Sequences	ARITH	SEQUENCES, PARTIALSUMS
Series	SERIES	SEQUENCES, PARTIALSUMS
Partial Sums	PARTIALSUMS	GEO, ARITH, SERIES
Proofs	PROOF	INDUCTION

Appendix B: Code for Hubs in All Books

Table A2. This table provides the codes used for the hubs identified in all the books.

Hub Name	Code
Functions and Relations	F/R
Linear Functions	LF
Polynomial Functions	PF
Rational Functions	RF
Trig	TRIG
Conics	CONICS
Sequences	SEQUENCES
Limits	LIMITS
Series	SERIES
Graphing	GRAPH
Modeling	MOD
Inequalities	INEQ
Equations	EQN
Rate of Change	ROC
Average Speed	AS
Constant Rate of Change	CROC
Quadratic	QUAD
Composition	COMPOSITION
Function Notation	FUNCTNOT
Inverses	INV
Domain and Range	D/R
Exponential Function	EF
Growth and Decay	GROW/DEC
Transformations	TRANSF
Roots and End Behavior	ROOTS/EB

Circular Motion	CIRCMOT
Angle Measure	ANGMES
Cosine	COS
Sine	SIN
Right Triangle	RTTRI
Non-Right Triangles	NRTTRI
Average Rate of Change	AROC
Tangent	TAN
Change in Quantity	DELQ
Covariation	COV
Proportions	PROP
Box Activity	BOX
Percent Change	%DEL
Logarithm	LOG
Technology	TECH
Real Number Line	REALLN
X-Y Plane	XYPLANE
Applications	APPS
Distance Formula	DISTF
Pythagorean Theorem	PYTHTHM
Parabola	PARABOLA
Symmetry	SYMM
Calculus	CALC
Complex Numbers	COMPLEX
Periodic	PERIODIC
Cotangent	COT
Secant	SEC
Cosecant	COSEC
Trigonometric Identities	IDENTITIES
Sum and Difference Formulas	SUMDIFF

Hyperbola	HYPERBOLA
Ellipse	ELLIPSE
Circle	CIRC
Reflection	REFLECT
Quadratic Formula	QUADF
Polar Form	POLAR
Radians	RAD
Reciprocal	RECIP
Roots of Unity	ROOTUNITY
DeMoivre's Theorem	DEMOIVRE
Secant Line	SECLINE
Slope	SLOPE
Euler's Constant	E
Factorial	FACTORIAL
Tangent Line	TANLN
Combinatorics	COMBINATORICS
Mathematical Modeling	MATHMODEL
Dependent Variable	DV
Independent Variable	IV
Applications: Free Falling Objects	FALLINGOBJECTS
Oblique Triangles	OBLIQUE
Vectors	VECTORS
Differential and Difference Equations	DIFFEQ
Set Representations	SETS
Function Representations	REP
Role of Numbers and Quantity	NUMBERS
Permutations and Combinations	PERMUTCOMB
Counting Principles	COUNTPRINC

Probability	PROB
Binomial Expansion	BINEXP
Recursion	RECURS
Difference Tables	DIFFTAB
Tables	TAB
Proof	PROOF
Geometry	GEOM
Analytic Geometry	ANALGEOM
Coordinate Plane	COORDPL
Exponent Value	EXPVAL
Events	EVENTS
Area Under Curve	AREAUNDCURVE
Division Algorithm	DIVALG
Operations	OPER

Appendix C: Hubs

The table below provides the hubs for each precalculus text studies in this paper.

Table A3. The hub topics for each textbook. Note that the threshold to qualify as a hub is based on the book by Faires which contains a minimum of six chapters.

Book	**Hub Names**
Abramson	RF, LF, PF, RF, TRIG, CONICS, SEQUENCES, LIMITS, SERIES
Blitzer	GRAPH, MOD, INEQ, RF, PF, EQN, CONICS, SERIES, F/R
Pathways	ROC, AS, CROC, QUAD, F/R, COMPOSITION, FUNCTNOT, INV, D/R, EF, GROW/DEC, PF, TRANSF, ROOTS/EB, RF, CIRCMOT, ANGMES, COS, SIN, RTTRI, NRTTRI
Stewart	PF, RF, GRAPH, MOD, LOG, EF, SEQUENCES, EQN R/F
Faires	R/F, LF, REALLN, XYPL, RF, APPLICATION, QUAD, D/R, GRAPH, TECH, PYTHTHM, TRANSF, PARABOLA, SYMM, CALC, INV, ROOTS, PF, COMPLEX, TRIG, SIN, COS, PERANG, ANG, TAN, COTAN, SEC, COSEC, IDENTITIES, SUMDIFF, RTTRI, EF, LOG, GROW/DEC, CONICS, ELLIPSE, HYPERBOLA, CIRC, REFLECT, QUADF, POLAR, DISTF, COMPOSITION
CME	TAN, SIN, COS, PYTHTHM, GRAPH, ANG, RADIANS, TRIG, CIRC, EQN, RECIPROCAL, COMPLEX, DEMOIVRE, PF, IDENTITIES, SUM, RF, SECLINE, SLOPE, EF, E, FACTORIAL, TANLN, COMBINATORICS,

	PERMUTCOMB, R/F, PROB, BINEXP, RECURS, DIFFTAB, TAB, PROOF, GEOM, ANALGEOM, COORDPL, EXPVAL, EVENTS, CALC, AREAUNDCURVE, COUNTPRINC, ROOTUNITY
Larson	DIVALG, PF, GRAPHS, MOD, EQNS, OPER
COMAP	TRANSF, LINEAR, GEOMETRY, PF, F/R, GRAPHS, MATHMODEL, TABLES, EQN, DV, IV, EXP, LOG, INVERSE, MODELING, FALLINGOBJECTS, TECHNOLOGY, COMPLEX, PERIODIC, COS, SIN, RADIAN, TAN, RTTRI, OBLIQUE, VECTORS, POLAR, MATRIX, ANALGEO, PARABOLA, COUNTINGPRINC, DIFFEQ
Rockswold	F/R, SETS, REP, GRAPHS, LINEAR, INEQ, NUMBERS, MODELS, ZERO, EQN, QUADRATIC, PF, DIVISION, RF

Appendix D: Semester in Review

Math 111

Semester In Review Assignment

11/29/21

A common theme throughout this course is not only the idea that we can view functions and relationships as covariation between two or more quantities, but also how we can dynamically adapt our understanding of one class of functions in order to meaningfully understand subsequent categories of functional relations. This approach to learning math hopes to dispel the rumors that mathematics is a discipline rooted in rote memorization, and instead aims to push forward the natural emergence and organization of patterns in the subject.

This course aims to emphasize the philosophy that mathematics is to be understood and interacted with instead of a rigid list of routine procedures. Many times throughout our classes and other academic obligations, it can be a tempting tendency to reject considering how these topics relate to our everyday lives. In this survey, we will be considering how some of the highly relevant concepts we have investigated throughout the semester retain roots in the everyday occurrences of our individual journeys through life, and how this may strengthen the lens by which we view mathematics as a relevant component engrained in humanity and society as a whole.

Consider the following themes we have discussed in class this semester:

1. Covariation
2. Rate of Change
3. Functions and Relations
4. Exponential Functions
5. Polynomial Functions
6. Rational Functions
7. Trigonometry

8. Limits, Roots and End Behavior
9. Proportions
10. Inverses

Your goal is to synthesize these topics through your own eyes. Write a personal statement that introduces these relevant concepts by addressing the following:

- What do these topics mean to you?
- How do these topics relate to any other topics discussed in the context of our classroom, including any of the themes listed above?
- How do these topics relate to your life in a personal way? Be sure to be as descriptive and specific as possible, outlining at least one highly detailed example for each theme discussed.

Your statement must have a minimum length of two pages, and must be written professionally and in such a manner that your thoughts are communicated in an effective way. Feel free to add any visuals you would like, and feel free to play around with the format of your statement! An excellent submission should reflect that nobody except you could have written it. As always, I am here for guidance and/or support to help you on your way. I look forward to reading your thoughts!

Appendix E: Connectivity Survey

In the context of precalculus, considering the connection between varying topics of discussion is a highly valuable summative review tool to strengthen the underlying themes of the course as a whole.

For this assignment, I ask that you simply consider how given topics we have discussed throughout the entire class are connected to each other. In the attached spreadsheet, please use the drop-down menus to fill in a selected initial topic under the column titled 'Name of Topic', and select all topics you consider to be connected in the columns labeled 'Name of Identified Connected Topic'. I have provided ten columns under this name to allow for up to ten perceived connected topics, but if you observe more than ten please feel free to add additional columns! While there are many topics that we have discussed in class, I do not expect you to spend a great deal of time on this: simply select your connections based on current understanding and memory.

Your submission should entail two items:

- A filled-in copy of the spreadsheet
- A text file that answers the following three questions with brevity:
    - Have you ever taken a college-level algebra course, or equivalent? Taking Algebra II in high school would count as an equivalent college algebra course.
    - What is your intended major?
    - How long has it been since your most recent math course, preceding our course? (please provide answers in units of either months or years.)

Note: Each cell contains a drop-down menu with every topic covered in the scope of the course (n=126). Students may also add more columns if they feel any given topic has more than twelve directly connected topics. The original table provided had 127 total rows.

Name of Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic	Name of Identified Connected Topic

Appendix F: Proposed Analytical Methods of Aggregating Student Data

In the initial steps of the construction and evaluation of self-reported student connectivity models, a natural issue that was encountered was the emergence of networks composed of disconnected components. In the context of network-theoretic language, student networks often consisted of individual trees, creating a forest structure. Such a structure makes it initially impossible to compute metrics such as APL and CC, as these metrics are undefined for disconnected forests. Therefore, the following methodology was employed in order to provide significant meaning to these networks through the utilization of aggregately computed metrics.

Consider a forest $\mathfrak{I}$ of k trees $(T_1, T_2, \ldots, T_k)$ and n total vertices $(a_1, a_2, \ldots, a_n)$, with k less than or equal to n. Note that an isolated node, by definition, is a tree. The APL will be defined under the following conditions: We will compute the APL of each individual subgraph. For isolated nodes, we let the APL of tree $T_i = n$, where $i = 1, 2, \ldots, n$. The reasoning behind this is that if a student finds a given topic a_i to lack any connection to any other topic in the course, it stands to reason that one must traverse the entire course in order to find any relation between any two a_i and a_j, where $j = 1, 2, \ldots, n,\ i \neq j$. This is meaningfully represented by letting each isolated node be weighted by the size of the entire list of course topics itself. For connected trees (non-isolated nodes), we simply compute its APL. We then assign relative frequency weights w_p $(p=1,2, \ldots, k)$ to each tree T_p in forest $\mathfrak{I}$ where $w_p = \frac{\#\ of\ nodes\ in\ T_p}{n}$. To find the aggregate APL of forest $\mathfrak{I}$, we compute this as

$$APL_{final} = \sum_{p=1}^{k} w_p * APL_p$$

to obtain a weighted average of the APL of each subgraph. The CC for forest $\mathfrak{I}$ will be computed via the same weighted-average method. The difference here is that the CC for isolated nodes will be assigned to be zero, which can be justified by reasoning that disconnected topics have no tendency to cluster amongst any other vertices. Additionally, we will compute the CC for each connected subgraph, and then assign relative weights to each CC in the same manner as described above to obtain an aggregate CC. While this is just one case of a set of obstacles that have cropped up in this analysis, the author anticipates many more along the way, and hopes to address these (as well as their prescribed solutions, along with the results of such an analysis) in future work.