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# The Effects of Using The Geometer's Sketchpad on Student Learning of Transformations in the Coordinate Plane

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# THE EFFECTS OF USING THE GEOMETER'S SKETCHPAD ON STUDENT LEARNING OF TRANSFORMATIONS IN THE COORDINATE PLANE

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## Abstract

The primary purpose of this study was to investigate how grade nine honors geometry students and grade ten regular geometry students learn and retain information about three basic rigid geometric transformations with the support of the software package *The Geometer's Sketchpad* in comparison to the traditional, non-software supported method of instruction.

The study was conducted in a high school with two tenth grade regular geometry classes and two ninth grade honors geometry classes. The researcher and another teacher each conducted a one-week instructional unit on the three basic, isometric transformations. Thirty-four students agreed to participate in the study. Nineteen students were in the computer group and fifteen students were in the non-computer group. All students worked through a series of explorations related to the three isometric transformations.

The analysis compared the honors geometry non-computer group to the honors geometry computer group and the regular geometry non-computer group to the regular geometry computer group. An overall comparison was made between the computer group and the non-computer group. Several statistical comparisons were used to compare groups of students. In all of these analyses, the mean score of the computer group was higher than the mean score of the non-computer group, but none of the results were statistically significant at the 0.05 level. This was perhaps due to the rather small sample sizes associated with the classes and the large variability in the post-test and differenced scores.

All students were given a pre- and post-attitudinal survey initially developed by Todd Moyer (2003) that addresses students' likes and dislikes using The Geometer's Sketchpad and could only be used for people using *The Geometer's Sketchpad*. The scores on the attitudinal survey did not show that students enjoyed using *The Geometer's Sketchpad*.

**Introduction**

The primary purpose of this study is to investigate how grade nine honors geometry students and grade ten regular geometry students learn and retain information about three basic rigid geometric transformations with the support of the software package *The Geometer's Sketchpad* in comparison to the traditional, non-software supported method of instruction. On pages 314-315 of its *Principles and Standards for School Mathematics* (2000), the National Council of Teachers of Mathematics (NCTM) makes the following statement.

> Middle-grades students should have had experience with such basic geometric transformations as translations, reflections, rotations, and dilations (including contractions). In high school, they will learn to represent these transformations with matrices, exploring the properties of the transformations using both graph paper and dynamic geometry tools. (NCTM, 2000, p. 314-315)

Not all teachers agree with the above recommendation about the use of dynamic geometry tools. As a result, many students are taught transformations in a traditional manner using only pencil and paper. In addition, some teachers use an abstract approach, relying strictly on the theory, to this topic before their students have sufficient experience with concrete examples to understand a generalized approached. Because transformations is a unifying topic that is used throughout mathematics and in applications of mathematics to other areas it is important that educators understand what teaching and learning environments are best for today's students.

When I first started teaching, I used *The Geometer's Sketchpad* to support my instruction about the three basic transformations, translations, reflections and rotations. However, I did not include the use of the coordinate plane. The students only applied the transformations by hand and by the use of *The Geometer's Sketchpad.* They did not study the change of specific coordinates when transforming the figures. During my second year of teaching, I developed a series of computer labs that incorporated the use of the coordinate plane while translating, reflecting or rotating figures. Because I perceived that, this was the best way for me to teach the topic, I now teach transformations almost entirely with *The Geometer's Sketchpad* and the coordinate plane. However, as mentioned before, some of my colleagues feel that the use of software is not beneficial. They contend that the computer is a passing fad and students have, for centuries, learned geometry without the use of computers. Although there are theses, dissertations, papers, articles and discussions in other formats that address the use of dynamic software in the geometry classroom, the conclusions of the studies are mixed. It is for this reason that I chose to research this topic myself.

## Transformations in School Mathematics

> In the 1800s, a mathematician named Felix Klein promoted the development of all of geometry through transformations. He defined geometry as the study of properties that remain unchanged under different sets of transformations. This view of geometry ties it closely to the study of algebra. (Dossey, 2002, p. 217-218)

Transformations were brought to the secondary educational community in 1971 with the publication of the book *Geometry: A Transformation Approach* by co-authors Arthur F. Coxford and Zalman P. Usiskin. Six chapters of their book are devoted strictly to the study of transformations. One of their main reasons for emphasizing transformations was "The use of transformations simplifies and unifies the mathematics presented" (Coxford & Usiskin, 1971, p. 5).

> Usiskin's doctoral dissertation involved the field-testing of this book. This book greatly influenced the way geometry is taught in many American high schools, according to the NCTM citation. In addition, Usiskin's pioneering work with applications of mathematics in high school algebra has influenced current trends in school mathematics. (Harms, 2001, p.1)

## Motivation for the Study

I became interested in this topic because of my experiences teaching transformations, both with and without the support of dynamic software such as *The Geometer's Sketchpad*, for the past eight years. During that time, I have observed that this is a very difficult topic for many students to master. I have also observed that most students appear to learn and retain information about transformations better when they are engaged with a dynamic software program such as *The Geometer's Sketchpad*. Thus, after the main purpose of investigating how students learn and retain information about the three basic rigid transformations with the support of *The Geometer's Sketchpad*, another purpose of this study is to investigate students' attitudes about studying geometric transformations with and without the support of interactive geometry software.

It has been my experience that combining the use of coordinate geometry and laboratory experiences with *The Geometer's Sketchpad* creates a supportive environment for students to learn about transformations. Through discussions with colleagues in my district, at graduate school and at professional conferences, I have learned that not everyone agrees with this point of view. After discussions with members of my faculty about this issue, I conducted an initial review of the literature and found a number of dissertations, theses and other studies that addressed the use of dynamic software to support the teaching and learning of geometry. Although the results of those research studies do not clearly support the use of dynamic software, some researchers expressed very strong opinions, such as the following, about the NCTM recommendation to include the use of dynamic software to support students' understanding of transformations.

> The way we teach mathematics-geometry in particular-is beginning to change, thanks to a few important developments in recent years. The deductive approach to teaching geometry is finally being seriously challenged and alternatives are available after more than a century of failing to reach a majority of students. (Bennett, 2002, p. viii)

The position of the Association of Mathematics Teacher Educators (AMTE) is "Mathematics teacher preparation programs must ensure that all mathematics teachers and teacher candidates have opportunities to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning" (AMTE, http://www.amte.net/). AMTE claims that the more technologically prepared students are the more efficient they will be at their work in the future.

Although much of the recent research supports the use of dynamic software that has evolved rapidly since the early 1990's, some researchers are not so sure that the use of software to support the teaching and learning of geometry is producing positive results (Koblitz, 1996; Day, 2006). While reviewing the literature, I found that the results varied as to whether or not *The Geometer's Sketchpad* is an effective tool. Some research such as the study by Todd Moyer (2003) found that the results did not support the use of *The Geometer's Sketchpad*. In those studies, there was no significant difference between test scores and the use of *The Geometer's Sketchpad*. Karen Flanagan's (2001) study showed that *The Geometer's Sketchpad* significantly helped the students' knowledge of translations. In another study by J. K. Dixon (1995), students' knowledge was significantly higher for those that used *The Geometer's Sketchpad* than for those who did not use the software. Dixon (1995) also found that there were no significant differences between the students who were limited English speakers versus native English speakers.

**Review of literature on the subject**

Numerous math practitioners, masters and doctoral students have investigated the use of dynamic geometry software in the classroom (Flannagan (2001), Baharvand (2001), Moyer (2003), Glass (2001), Dixon (1995), McDougall (1997), Molnar (1997), Dixon (2002), McGrath (1998), and Powers & Blubaugh (2005)). In this section, I highlight a few theses and dissertations that are pertinent to the study I conducted.

Karen Ann Flanagan's (2001) dissertation, *High School Students' Understandings of Geometric Transformations in the Context of a Technological Environment,* examines the effects of using *The Geometer's Sketchpad* on student learning. Her study sought to understand the nature of students' understanding of mathematical concepts within a technological context. The purpose of her study was to examine high school students' understanding of geometric transformations that included translations, rotations, reflections and dilations, when both *The Geometer's Sketchpad* and the TI-92 calculator were used. Flanagan (2001) researched whether a student understands geometric transformations better when using *The Geometer's Sketchpad* and how the computer helps the student learn transformational geometry. She conducted a seven-week instructional unit on geometric transformations with two high school honors geometry classes. She chose six students in these classes to serve as participants in the study. Four of the six students were selected for case studies because these students "...verbalized their thinking most often and the data on these four students was most complete" (Flanagan, 2001, p. 267). Each of these four students was the brightest in the class and scored very high on the van Hiele test. The van Hiele levels are five levels of geometric reasoning that students sequentially move through when learning geometry. The student's verbatim transcripts of the three in-depth clinical interviews, small group and whole group discussions, and their written work were analyzed. The first analysis focused on students' understanding of geometric transformations and the second analysis focused on the ways in which students made use of and interpreted the computer and calculator displays and other feedback. The researcher found there were key understandings that seemed to reflect deeper understandings of transformations. The ways in which the computer software helped students' understanding of transformations seemed related to both their understanding of the tool and their understanding of mathematics. While the computer environment may have encouraged a concrete approach, students' reasoning appeared to move along a continuum between the concrete and the theoretical, and often included an interaction between their theoretical and concrete understanding (Flanagan, 2001, p. iii).

The objective of the 2001 pre-test-post-test experimental study by M. Baharvand, *A Comparison of the Effectiveness of Computer-Assisted Instruction versus Traditional Approach to Teaching Geometry*, was to determine the effects of the computer software instruction with *The Geometer's Sketchpad.* This was compared to instruction by teacher-lecture and pencil-and-paper activities on three independent variables including: students' assessment performance, students' retention level and students' attitude toward learning geometric concepts. Twenty-six students in seventh grade made up a control group that received instruction by teacher lecture and another seventh grade class with twenty-four students made up an experimental group that learned the same concepts using *The Geometer's Sketchpad.* The data was analyzed using a t-test and the results indicated that students taught with Sketchpad scored significantly higher ($p < .05$) on the researcher-constructed post-test than the control group. Furthermore, the results revealed significant differences in students' positive attitude toward math/geometry in favor of the experimental group. The evidence of this study indicates that use of *The Geometer's Sketchpad* for learning and teaching geometry at the middle school level is an effective tool.

The purpose of T. Moyer's 2003 study, *An Investigation of the Geometer's Sketchpad and van Hiele Levels*, was to see if students' achievement levels increased with the use of the software package *The Geometer's Sketchpad.* The study took place in four geometry classes with two different teachers. Each teacher was instructed to use *The Geometer's Sketchpad* in one class and not in the other. Students in all classes were given van Hiele pre-tests and post-tests written by Dr. Zalman Usiskin, and the Purdue Spatial Visualization test. Moyer (2003) designed a content-based pre-test and content-based post-test. He also measured the attitudes and beliefs of students who used *The Geometer's Sketchpad*. Moyer (2003) did not find that *The Geometer's Sketchpad* had a significant impact on student learning. With Dr. Moyer's permission, I used his attitudinal survey in my study.

Brad Glass (2001) from the University of Iowa, investigated the use of two dynamic geometry software programs, *Cabri Geometry* from Texas Instruments and *The Geometer's Sketchpad* from Key Curriculum Press, to see if the use of the software enhanced students' learning and retention. In his 2001 study, *Students' Reification of Geometric Transformations in the Presence of Multiple Dynamically Linked Representation,* he used the three basic transformations: translations, reflections and rotations. Eighth grade students were given a pre-image and image of some figures and had to identify what transformation was being used. The study revealed that students understood translations best, followed by reflections and then rotations. Although the researcher did not come to the conclusion that he expected, he did determine that the use of dynamic software for the use of translations had a significantly positive effect on student learning and retention of information about translations. He did not have sufficient evidence that the dynamic software had the same effect on the retention of information about reflections and rotations.

In 1995, J. K. Dixon conducted a very interesting study, *English Language Proficiency and Spatial Visualization in Middle School Students' Construction of the Concept of Reflection and Rotation Using The Geometer's Sketchpad.* She studied eighth grade students' understanding of reflections and rotations while using *The Geometer's Sketchpad* and took into account the students' English language proficiency and visualization levels. She designed computer labs for *The Geometer's Sketchpad* where students were "discovering" the properties of both reflections and rotations. She determined that students who used *The Geometer's Sketchpad* significantly outperformed the students who used the traditional pencil and paper method of practice. Interestingly, she found that there were no significant differences between the students who were limited English speakers versus their classmates who were native English speakers.

In 1997, D. E. McDougall did a very interesting study on *Mathematics Teachers' Needs in Dynamic Geometric Computer Environments: In Search of Control.* In this study, McDougall (1997) analyzed the data from four teachers who agreed to participate in his study. McDougall (1997) found the following:

> …the four teachers participating in the study experienced an initial loss of control due to the new environment, in three categories: (1) Management control (they believed the new environment impaired their ability to maintain discipline), (2) Personal control (they were unable to determine their own expectations of the students and to assess students' achievement), and (3) Professional control (they felt they no longer had all the answers). (McDougall, 1997, p. ii)

Initially, the teachers were very anxious about using the computer and most particularly *The Geometer's Sketchpad*, but as the study progressed, they became more confident in "their ability to teach effectively with the new methods, and were even moved to reflect upon their previous teaching practices" (McDougall, 1997, ii). The teachers were wary about the lack of discipline and structure in the classroom and that the students would not behave or remain on task in the computer lab. McDougall (1997) found that the teachers were more comfortable as a "facilitator" of the lesson by the end of the study than they were at the beginning of the study. "Teachers play an important role in mathematics education. Use of dynamic geometric software programs can help teachers to develop or redevelop an enthusiasm for investigating geometric relationships" (McDougall, 1997, p. 39). McDougall (1997) found that many factors influenced students' success in mathematics. He found the newness of the materials influences the instructional time and requires teachers to plan and prepare more. He also recommends that teachers explore mathematical concepts and ideas on their own and reflect on their teaching style. A teacher also needs to become less of a provider of information and more of a facilitator. If teachers are confident in their mathematical and computer abilities they will be more open to allowing students to work independently and in cooperative groups. This requires teachers to use inquiry-based instruction.

In a 1997 article, *Computers in Education: A Brief History*, Andrew Molnar contends that there is a demand for students to be technologically savvy in the future. He refers to many articles and books on technology, math and education. "...we are preparing children for a new type of society that did not yet exist" (Molnar, 1997, p. 2). He references Seymour Papert saying that "...we should not teach mathematics, but should teach children to be mathematicians" (Molnar, 1997, p. 3). Molnar states that computers facilitate student visualization of a problem.

> It is said that computer visualization not only changes how we see phenomena, but also how we think about them. It is believed that it restructures a problem and shifts more work to our perceptual systems thus freeing the brain for high levels of analysis and synthesis and thus approaches the speed of thought. (Molnar, 1997, p. 7)

In summary, if technology is appropriately used, it can provide an effective means for learning (Molnar, 1997).

*ON-MATH* is an online periodical, published by the National Council of Teachers of Mathematics, that focuses on technology-based activities for mathematics. There are numerous articles written by teachers and researchers on the many different ways to infuse technology in the curriculum. Juli Dixon (2002), whose dissertation is cited earlier in this literature review, created one particular set of activities for transformations. In addition to creating lessons to use on the computer with *The Geometer's Sketchpad*, Dixon (2002) also created Java Applets on reflections and rotations. This resource is beneficial to teachers in districts without a license for *The Geometer's Sketchpad* because it allows them to do these activities with their students.

In a 1998 article, *Partners in Learning: Twelve Ways Technology Changes The Teacher-Student Relationship* in T.H.E. Journal, Beth McGrath cited twelve ways that technology changes classroom dynamic-for the better.

> Over the last decade, we have observed astounding effects of technology integration on teachers and students: veteran teachers who undergo a dramatic

transformation and find a sense of enthusiasm for their craft which they felt they'd lost; tech-savvy teachers who create wondrous classroom experiences and lessons that engage their students in real-world problem-solving; disaffected students suddenly curious about new areas of inquiry with the help of technology tools; and reticent students who've blossomed into eager, motivated members of a group investigating a common problem. (McGrath, 1998, p. 1)

She maintains that technology can break past the barriers of socio-economics, grade-level, subject, and teacher experience and found twelve common themes among all of the teachers. The twelve themes that McGrath (1998) observed are:

*1.* Technology increases student motivation, and motivated students are more receptive, more engaged, and more likely to learn.

*2.* Technology promotes cooperation and collaboration among students and good teachers can capitalize on these opportunities. Cooperative learning approaches with technology give students with different talents a chance to excel.

3. In classrooms with computers, conversations between teachers and students and among students themselves become deeper and more probing.

4. Technology use encourages teacher-as-facilitator approaches.

5. Technology promotes a "balance of power" between the teacher and his or her students.

6. With technological tools, students show more persistence in solving problems.

7. Technology encourages varied methods of assessment.

*8.* Despite all the challenges of a one-computer/one Internet-connection classroom, even this classroom environment enables good teachers to work effectively with diverse students.

9. Technology fosters increased and improved oral and written communication.

10. Technology enables opportunities for more depth of understanding, but the breadth of the curriculum is still problematic.

11. Technology provides increased opportunities for thematic, interdisciplinary explorations; teachers can use these interdisciplinary connections to further engage and excite students.

12. Technology makes classroom activities "feel" more real-world and relevant, and students often take these activities more seriously.

(McGrath, 1998, p. 2-7).

This 'magic' does not happen overnight. McGrath (1998) insists that teachers need time, patience and the support of the administration to make this work. When these needs are met students benefit. "'Technological resources, such as *The Geometer's Sketchpad*, have freed students from the tedium of calculations and their problem-solving skills become more advanced,' observes Doris Buxbaum" ((qtd. in) McGrath, 1998, p.6). In her seventh grade mathematics class, she used Sketchpad to explore the sum of the measure of the interior angle measures of different polygons, without even suggesting to the students that the sum is consistent for a particular number of sides.

> 'The students came to it in seconds! That wasn't even the point of this particular lesson, but in the process of learning one tool on the program, they learned a lot about polygons. So, when I get to that lesson, they'll zoom ahead.' (McGrath, 1998, p. 6)

*Contemporary Issues in Technology and Teacher Education (CITE Journal)* is an online publication with an extensive database of articles. The article that I chose from this site to review discusses the need for pre-service teachers to be adept at and learn to incorporate technology in their lessons. Robert Powers and William Blubaugh (2005) of the University of Northern Colorado wrote an article about the incorporation of technology in a math methods course. "The prep of preservice teachers to use technology is one of the most critical issues facing teacher education programs" (Powers and Blubaugh, 2005). In response to this, numerous teacher education programs have incorporated technology methods courses for their pre-service teachers. The researchers found that the mathematics majors did have experience using technology in their mathematics courses such as Mathematica, SAS and Excel, but did not have the opportunity to learn how to teach with it. They found by including technology in the math methods courses, pre-service teachers are more comfortable and familiar with technology.

> …their semester-long student-teaching experience, host teachers and university faculty members evaluate student teachers on their ability to integrate technology in the classroom. Upon graduation, these future teachers should not only be knowledgeable as to which mathematics concepts are best learned

through technology, but also will have had many successful experiences in developing and carrying out lesson plans that involve a variety of different technologies. (Powers and Blubaugh, 2005, p. 7)

The high quality of their pre-service teachers is evident, not only from the pre-service teacher point of view, but also from their mentor teachers. "As university supervisors, we often hear from the host teachers that our graduates are highly knowledgeable in dealing with technological instructional tools. Many host teachers admit to learning valuable teaching strategies using technology from individuals in our program" (Powers and Blubaugh, 2005).

In summary, my review of relevant literature revealed largely mixed results. One observation that I made through my reading was that use of dynamic geometry software does not impede student learning. Students are still learning the material, but the question still exists, does technology increase learning and retention?

**Research Setting**

The study took place in a suburban high school in northeast, New Jersey. The town occupies an area of approximately ninety-square miles, two-thirds of which is state protected forest and watershed land. The town is a very economically diverse community. Homes range from converted summer bungalows to larger estate-type homes on two or more acres. According to the United States Census Bureau, the median annual income for residents in 2000 was approximately $45,000.

The school district consists of six elementary schools, one middle school, and one high school. The high school enrolls approximately 1,500 students, ninety-eight percent of which speak English at home. Ninety-five percent of all students graduate and, of that ninety-five percent, seventy-four percent plan to attend a two-year or four-year college. The student to teacher ratio is 12.7 to 1. (This does include special education teachers that have approximately 5-10 students in their classes).

The study involved two freshman, grade nine, honors geometry classes and two sophomore, grade ten, regular geometry classes. The researcher was the instructor for the two freshman honors geometry classes. A colleague of the researcher was the instructor for the two sophomore regular geometry classes. The sophomore regular geometry classes had an elementary introduction to translations, reflections and rotations that overlapped minimally with the material used in this study. One section of each of the honors geometry and regular geometry classes was taught with the support of *The Geometer's Sketchpad*, while the remaining sections were taught without the support of *The Geometer's Sketchpad.* All students in all four classes had some previous experience with the software but had not experienced an entire unit of instruction with it as a teaching and learning tool.

## Data Collection

Data analysis for this study was conducted after permission had been secured from the parents and students. All students in the four classes were given a pre- and post-attitudinal survey developed by Todd Moyer (2003) that addresses students' likes and dislikes about studying geometry with or without the computer. Dr. Moyer's (2003) attitudinal survey was used because it was a tested instrument and used in a similar study. In addition, before and after the unit of instruction, all students were given a pre-test and post-test to measure their knowledge of translations, reflections and rotations. Data analysis only took place with data from students who agreed to participate in the study.

Because this study involved human participants, an application that included all testing materials, was prepared and submitted to Montclair State University's Institutional Review Board (IRB). Approval to conduct the study was granted.

Prior to unit instruction, a large envelope with each student's name on the outside was prepared. A teacher, not affiliated with the classes, had assigned code numbers to each student and placed these code numbers on the four envelopes for each student. In the large envelope there were four smaller envelopes labeled with the student's code number that neither I nor the other instructor had knowledge about. On any given day, as students entered the class, they picked up or were given the large envelope with their name on it. The instructors distributed the instruments, or in-class assignments. At the end of the period, the student removed one of the coded envelopes from his or her large envelope and placed that envelope in a large "ballot box" along with the envelope with his or her name on it in a different "ballot box." Both boxes were locked in a secure cabinet in the classroom. The envelopes with the completed instruments were picked up by a non-instructor (research assistant) and stored in a secure location, the math supervisor's office. After the unit was completed, a teacher, other than the instructors, removed one envelope at a time, placed the students' code on each of the documents it contained and returned these documents to the envelope. The same teacher also opened the returned assent/consent forms and separated the coded envelopes into two groups, those from students who were in the study and those who were not. Only the data from those in the study was analyzed. This procedure limited 'injury' to the students because the instructors did not know who was in the study and who was not in the study, while they were teaching it.

In the next section, I discuss the evolution of geometric software followed by an overview of transformations including the use of *The Geometer's Sketchpad.*

<u>**Geometric Software**</u>

The evolution of software to aid in the teaching and learning of mathematics was not a slow process, but getting teachers and districts to be willing to consider using them has taken time. However, with the help of the National Council of Teachers of Mathematics, the push to use such software has been made (NCTM, 2000). The software for the teaching and learning of geometry that first captured the attention of a number of mathematics educators was *The Geometric Supposer.*

> First, in 1985, Judah Schwartz and Michal Yerushalmy of the Education Development Center developed a landmark piece of instructional software that enabled teachers and students to use computers as teaching and learning tools rather than just as drillmasters. *The Geometric Supposer*, for Apple II computers, encouraged students to invent their own mathematics by making it easy to create simple geometric figures and make conjectures about their properties. Learning geometry could become a series of open-ended explorations of relationships in geometric figures rather than a rehashing of proofs of theorems that students tend to take for granted anyway. (Bennett, 1993, p. 1)

A short time later, in 1989, NCTM published *Curriculum and Evaluation Standards for School Mathematics* that called for significant changes in the way mathematics was taught. In particular, for the teaching of geometry, the standards call for decreased emphasis on the presentation of geometry as a complete deductive system and on two-column proofs. Instead, the standards called for an increase in open exploration and conjecturing and to topics in transformational geometry. In that call for change, the standards recognized the impact that technological tools, including *The Geometric Supposer*, would have on the way math was taught through their capacity to free students from time-consuming, mundane tasks while providing a means to see and explore interesting relationships (Bennett, 2002, p. viii; NCTM, 1989, p. 158-159).

Although *The Geometric Supposer* was the first software of its kind, soon after its release other software packages, building on its basic ideas, were created. For example, *The Geometer's Sketchpad* developed through the Visual Geometry Project, which was funded by the National Science Foundation, under the direction of Dr. Doris Schattschneider of Moravian College and Dr. Eugene Klotz of Swathmore College. Nicholas Jackiw began programming the beta version for the Macintosh computer in 1987, just two years after the release of *The Geometric Supposer*.

> The openness with which *Sketchpad* was developed generated incredible enthusiasm for the program. By the time of its release in the spring of 1991, it had been used by hundreds of teachers, students and other geometry lovers and was already the most talked about and awaited piece of software in recent memory. (Bennett, 1993, p. 2)

In 1993, *The Geometer's Sketchpad* was developed for Windows.

> *The Geometer's Sketchpad* was designed primarily for use in high school geometry classes. Testing has shown though, that its ease of use makes it possible for younger students to use *Sketchpad* successfully and the power of its features

has made it attractive to teachers of college level mathematics and teacher education courses. (Bennett, 1993, p. 3)

*The Geometer's Sketchpad* is discussed in more detail later in this thesis. Key Curriculum Press is expanding the use of *The Geometer's Sketchpad* from strictly geometry to algebra, trigonometry and calculus. This is even evidenced by the evolution of their logo. The word '*Sketchpad'* is getting bigger, as the word '*Geometer's'* is getting smaller. Will Key Curriculum Press eventually drop '*Geometer's'* all together to order to make *The Geometer's Sketchpad* marketable to all mathematicians, not only geometers?

*The Geometer's Sketchpad* was not the only educational software that expanded the functionality and usefulness of *The Geometric Supposer*. Two other well-known software programs are *Logo* and *Cabri Jr*., and are discussed next.

*Logo* was also developed during the 1980's. It is often used to introduce students, especially young students, to programming and is said to "develop [their] problem-solving and logical thinking skills" (Harper, 1989, p. v). By observing the motion of *Logo's* on screen "turtle," students can immediately observe the results of the commands as they enter and execute them. See http://www.softronix.com/logoex.html for screen shots, which link to samples that include some screen shots of logo programs and a site where readers can try a few commands. Unlike *The Geometer's Sketchpad, Logo* does not have ready-made applications for users to try. There are some Java Applets on the Internet for users to try, but these are not compatible with Logo.

Transformations can be taught using *Logo*, but this can only be done with some difficulty. I have written code to transform figures, and it is very challenging. Teachers without programming experience would most likely not be able to write such code. *Logo* is a staple in worldwide education, except for the United States.

> By the early 1990's some educators in the United States began to see *Logo* as old and out of date. The lack of innovation in *LogoWriter* and the sluggish pace of upgrading of the classic *Logo* was in sharp contrast to the rapid development of modern, flashy educational software that took advantage of the Macintosh / Windows graphical user interface. There were some *Logo* drop outs and *Logo* did not attract its share of interest among the many new computer-using educators in the United States and Canada. (Logo Foundation, 2000, Innovation section)

This is not the case in the rest of the world. Costa Rica pledged to provide *Logo* to over 50% of teachers and students in both elementary and secondary schools.

> The Costa Rican projects have provided extensive teacher education and support with a strong emphasis on Logo's constructionist educational approach. They (constructionist educational approaches) have been taken as models for similar endeavors in a dozen other Latin American countries. (Logo Foundation, 2000, Innovation section)

Japan has accepted and is now using *Logo* in their schools. "In England, Logo is a mandated part of the national curriculum. This guarantees that *Logo* is widely, if not necessarily well used" (Logo Foundation, 2000, Innovation section). I have used *Logo* with my students, but my students seem to have more success with *The Geometer's*

*Sketchpad*. The students find the programming in Logo too difficult and, therefore, give up easily.

Texas Instruments (TI) has created a number of geometry applications for its graphing calculators. *Cabri Geometry*, which was first released in 1988, is a software program where (as with *The Geometer's Sketchpad*) geometric figures can be constructed and then dynamically moved on the computer screen, while retaining the constructed properties of the figures. In 1992, *Cabri Geometry II* was developed for the TI-92 graphing calculator. This was the first time that handheld, interactive geometry software was available. The TI-92 graphing calculator has a QWERTY Keyboard. It is not very popular in the classroom, because calculators with such keyboards are not approved for use with high stake tests such as the Advanced Placement exams, SAT and the New Jersey HSPA. "Since this introduction to handhelds, the program has been modified as a flash application to run on the TI-89, TI-92 Plus and the Voyage 200 handhelds" (Vonder Embase, 2004, p. IV).

> *Cabri Jr.* is the next logical step in the family tree, opening the door to interactive geometry on the TI-84 Plus and TI-83 Plus families of graphing handhelds. Because of the limitation of the TI-84 Plus and TI-83 Plus families of graphing handhelds, *Cabri Jr.* does not have all of the functionality of other versions of the program, especially the computer versions. (Vonder Embase, 2004, p. iv)

Texas Instruments contends that *Cabri Jr.* is the perfect solution for teachers and students in the high school classroom. Although it is claimed that *Cabri* applications are easy to use on the calculators, I have found, from discussions with several college faculty, some individuals feel that the screen is too small and thus hard to read and use while constructing figures.

Transformations can be drawn, with the use of the graphing calculator application *Cabri Jr.*, but in a limited way. For example, students can easily reflect a pre-constructed menu-item such as a triangle, but if they construct their own polygon such as an octagon, it cannot be reflected because it is constructed with segments, not a predetermined polygon such as a triangle. The polygons in the construct menu are limited to triangles and quadrilaterals. Even though the portability and price of *Cabri Jr.* is attractive to cash-strapped school districts, I believe that using a calculator with *Cabri Jr.* is not the best environment in which to learn geometry.

There are also many multimedia and interactive applications on the Internet. Many of these websites are created to stimulate a student to "play" while learning. An application that demonstrates transformations is available at the Math Cove website at http://www.utc.edu/Faculty/Christopher-Mawata/transformations/translations/ . This website allows students to "drag" points to see how they are affected by the chosen transformation: translation, reflection, or rotation.

Another interactive application from the Internet is the National Library of Virtual Manipulatives (NLVM), which was developed at the Utah State University, with funding from the National Science Foundation. This website, http://nlvm.usu.edu/en/nav/topic_t_3.html, allows students to test each of the three basic transformations. The transformations can be used with elementary through high school level students. At the high school level, students can also investigate compositions of transformations. The NLVM not only has transformation applets, but a rich library that supports the learning of geometric concepts.

Another class of programs that can be used to study transformations are known as Computer Algebra Systems (CAS). An example is *Mathematica* created in 1988 by Wolfram Research. *Mathematica* and other CAS software such as *Derive* and *Maple* are used more widely in college settings than in high schools. Using computer algebra systems, transformations can be created by constructing and applying specific matrices to perform linear transformations.

I find that my students respond best to *The Geometer's Sketchpad*, which is why I chose to conduct my study with that software. Another reason for choosing *The Geometer's Sketchpad* was not only for its ease of use but because it was readily available to the students.

## **Transformations in the Plane**

### Overview

This section discusses transformations in general and explains related technical terms. It also provides specific examples of each type of transformation used in the study. One of the reasons transformations are taught is because they are a unifying topic with applications to other areas of secondary and collegiate mathematics and with applications to areas such as carpentry, design, and the movie industry. Their importance is confirmed by the inclusion of questions about transformations on the high stakes tests such as the HSPA, SAT, ACT and the PRAXIS exam for teachers.

Although students may view transformations as relevant only to geometry, there are many other applications and methods for use at higher levels. For instance, algebra 2 students learn how to use matrices to transform the vertices of a polygon. In an undergraduate abstract algebra course, most students study the non-abelian group with six elements that represent rotations and reflections of an equilateral triangle. Two and three-dimensional figures can be transformed using matrices. As an undergraduate student, I studied the groups of symmetries of several two-dimensional figures under rigid transformations. I used matrices with the help of *Mathematica* to obtain each transformation and then determined if it belonged to a group or not.

More recently, I took the graduate-level computer graphics course in the computer science department. When transforming three-dimensional figures, transformation matrices were applied. Before any transformation is applied, the figure needs to be moved to the origin, and then the transformation is applied and the figure is moved back to its original spot in the three-dimensional plane. This is done because it is much easier to manipulate a figure around the origin and then translate it back to its original space.

"Intuitively, a transformation transforms a geometric figure, shifting it around, flipping it over, rotating it, stretching it, or deforming it" (Serra, 2003, p. 357A). Technically, a transformation acts on the entire plane. This study will be limited to the effects of transformations on two-dimensional, that is, planar, shapes. Figure 1 shows an example of a 90 degree, counter-clockwise rotation of a 2-dimensional figure. The figure shows the original figure being rotated about a point P called the "center point" or center of rotation." In this example and for the remainder of the study, the center of rotation is the origin (0, 0) of the coordinate plane.

**Counter-Clockwise Rotation around Point P**

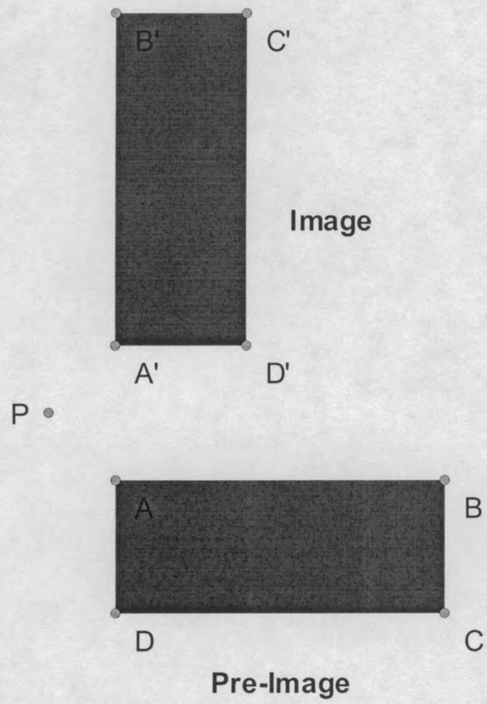


Figure 1

A rigid transformation or isometry is the movement of a figure in the plane so that the pre-image (original figure) and image (transformed figure) remain congruent (same size and same shape). Barron's *Dictionary of Mathematics Terms* (1995) defines an isometry as a way of transforming a figure that does not change the distances between any two points on the figure (Downing, 1995, p. 177). A transformation that produces an image that is not congruent to the pre-image is considered a non-rigid or non-isometric transformation. For example, a dilation that describes the scaling of a figure is not an isometry. Translations, reflections and rotations are isometries.

Although transformations can be investigated without the use of the coordinate plane, the use of coordinates helps students understand the actual movement and orientation of the figure. The use of the coordinate plane also prepares students to understand transformations studied in algebra and statistics, such as those used in the study of quadratic equations and those used when normalizing data. In the next section, I will formally define and provide examples of the three basic rigid transformations in the coordinate plane: translations, reflections and rotations.

**Rigid Transformations**

A basic translation is a slide to the left, right, up or down, parallel to one of the coordinate axes. Other slides can be obtained through a combination of those four basic movements. In Figure 2, the image is produced by translating the pre-image eight units to the right and three units down.

**Translation along vector <-8, 3>**

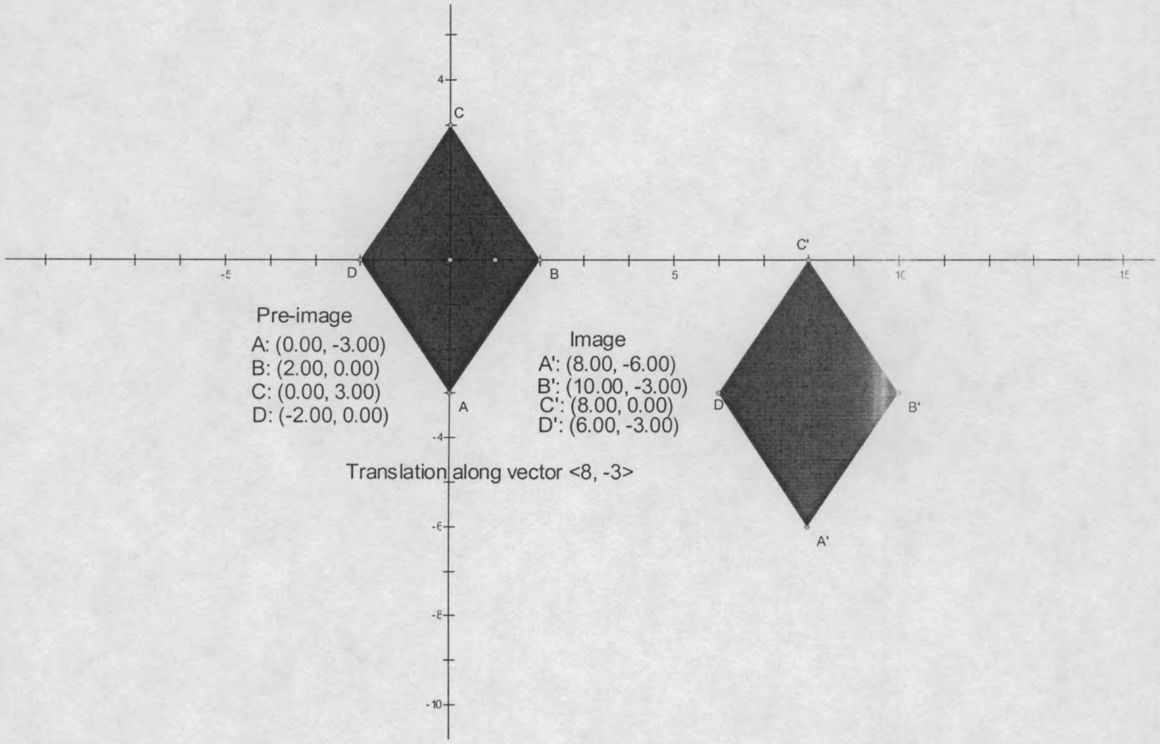


Figure 2

During the course of this study, translations will be obtained through the use of translation vectors. A vector is a quantity that has both magnitude and direction. In this study, a vector will describe the motion of the figure (or each point of the figure) by specifying the distance and direction of that motion. In Figure 3, the translation along vector <-2, 1> can move a point A (1, 1) to the location A′ (-1, 2). The coordinates of the image A′ of the point A with coordinates (1, 1) are obtained by adding the coordinates of the translation vector <-2, 1> to the coordinates of A. This is often written symbolically as

$(1, 1) \rightarrow (1 + (-2), 1 + 1) = (-1, 2).$

**Translation along vector <-2, 1>**

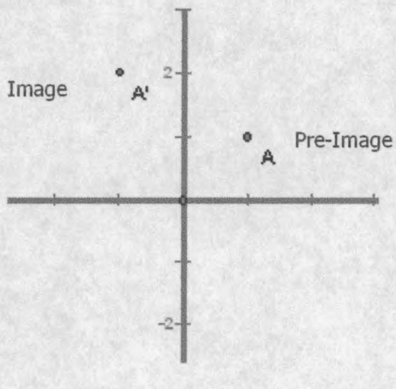


Figure 3

A reflection is another example of an isometry. According to Serra (2003) in *Discovering Geometry*, a reflection is a type of isometry that produces the "mirror image" of a figure. If you draw a figure on a piece of paper, place the edge of a mirror perpendicular to your paper and look at the figure in the mirror, you will see the reflected image of the figure (p. 360). The line where the mirror is placed is called the line of reflection, as shown in Figure 4 and Figure 5. Figure 4 has one line of reflection and line of symmetry. Figure 5 has two lines of reflection and two lines of symmetry. Larson defines a line of symmetry as a line through which a figure that can be mapped onto itself by a reflection (Larson, 1995, p. 750).

**One-Line Symmetry**

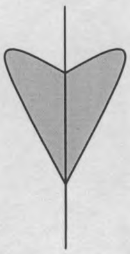

Figure 4

**Two-Line Symmetry**

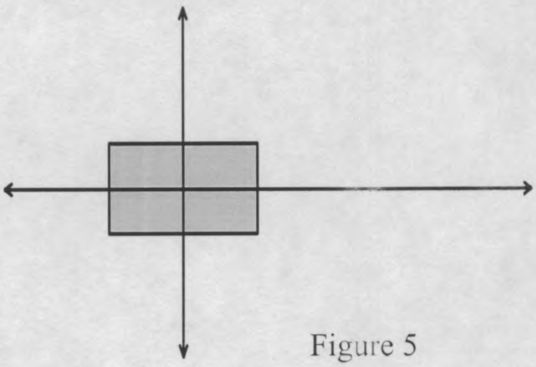

Figure 5

Note that the translation shown in Figure 6 can be obtained by performing two consecutive reflections over parallel lines. The diagram below shows two consecutive reflections. This is a non-basic translation, where the lines of reflection are not either of the axes.

**Consecutive Reflections over Parallel Lines**

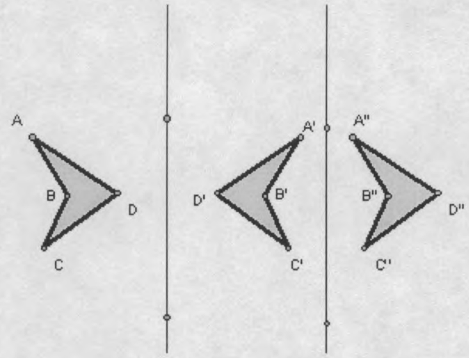


Figure 6

The pre-image and image remain congruent. Under a sequence of isometric transformations, a figure remains congruent to its image throughout all of the transformations. During the course of this study, all figures will be reflected in either the x-axis or y-axis of the coordinate plane as shown in Figure 7.

**Reflection over the y-axis and x-axis**

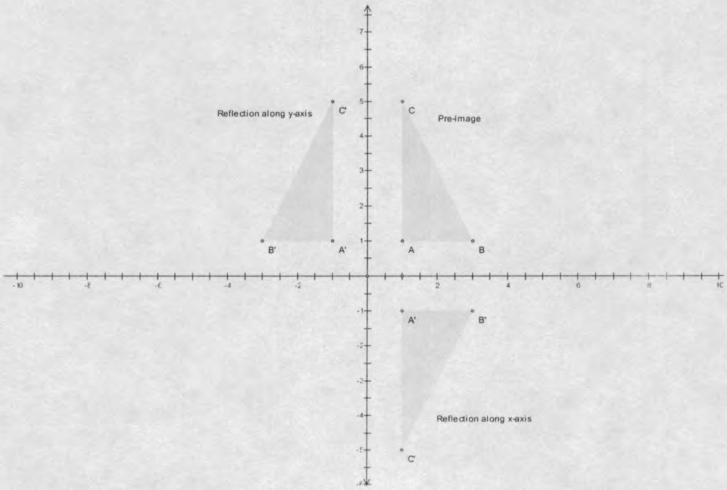


Figure 7

Rotations, such as shown in Figure 8, are the third type of isometry used in this study. Serra defines a rotation as an isometry where all points in the original figure rotate, or turn, an identical number of degrees about a fixed center point. A rotation is defined by its center point, the angle of rotation and whether it is a clockwise or counter-clockwise rotation. In this study, counter-clockwise rotations are used. If no direction is given, we assume the direction of a rotation is counter clock-wise (Serra, 2003, p. 359). During the course of the study, all figures were rotated 90, 180 or 270 degrees, counter-clockwise around the origin. Three examples of these rotations can be seen in Figure 8.

**\90, 180 and 270 Degree Rotation around the Origin**

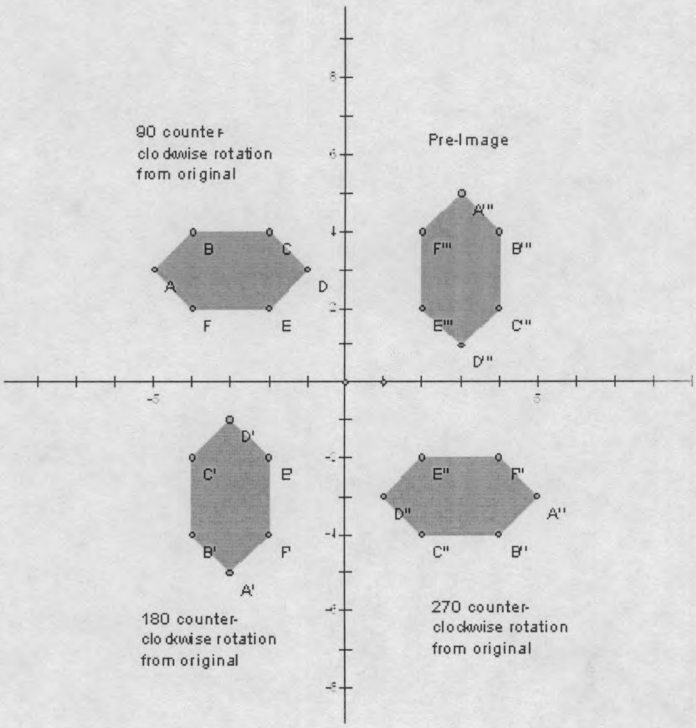


Figure 8

**Applications of Rigid Transformations**

Students should have had previous experience with transformations in their elementary and middle school mathematics classes. Both the National Council of Teachers of Mathematics (NCTM) and the New Jersey Core Curriculum Content Standards (NJCCCS) address this topic at those grade levels.

> In the middle grades, students should learn to understand what it means for a transformation to preserve distance, as translations, rotations and reflections do. High school students should learn multiple ways of expressing transformations, including using matrices to show how figures are transformed on the coordinate plane as well as by function-notation. They should also begin to understand the effects of compositions of translations. At all grade levels, appropriate consideration of symmetry provides insights into mathematics and into art and aesthetics. (NCTM, 2000. p. 43)

It is likely that most students have had some experience with transformations. For example, *Tetris* is a computer game that most children and adults have played. *Tetris* is strictly a game of translations, reflections and rotations. The object of the game is to transform your tetromino to a position that will enable it to fit into a line of other tetrominos. Transformations, or applications of transformations, can also be found in daily life (such as patterns in floor tiles, company logos and building foundations). Other examples include activities such as creating computer-generated images in animated films, weaving patterns into cloth, and basketry.

Native Americans have much to contribute to the study of geometry, specifically, transformations. As a fellow in the American Indian department at the Smithsonian Institution's Natural History Museum, I gained a greater appreciation for Native Americans and their artwork. Native Americans continue to create beautiful baskets and ceremonial costumes using patterns based on transformations. Frieze patterns are the key component to creating these beautiful designs.

> A frieze pattern is a pattern that extends infinitely to the left and right in such a way that the pattern can be mapped onto itself by a horizontal translation. Some frieze patterns can be mapped onto themselves by other transformations such as: a 180-degree rotation, a horizontal line reflection, a vertical line reflection or a horizontal glide reflection. (Larson, 1998, p. 358)

> In the past, baskets were made primarily for utility and versatility. A few baskets were made for ceremonial purposes. Now, because of modern technology and the existence of better storage solutions, the primary reason for the making of baskets is symbolism or ceremony -- with a substantial portion being made for commercial ventures. (Jolles, 1995)

Over centuries, the Hopi basket artists have developed many different designs for their baskets. Traditionally, women weave the baskets with many different colors.

> They are also innovative artists, developing new methods and designs from traditional ones. Red, yellow, and black are the usual colors skillfully arranged to produce *katsina* (sic) (dolls), animal, blanket, and geometric designs.

The natural colors of plant materials used to construct the baskets serve as a background for the designs, contrasting with the vivid colors of commercial dyes. The symbolism and tradition in Hopi basketry designs link each unique handmade basket to other parts of Hopi life, past and present. In particular, basketry designs reflect aspects of Hopi religion and agriculture. For the Hopi, just as the basket's fibers are woven together, so are all the pieces of Hopi culture: none is unrelated to another. (Hopi Cultural Preservation Office, 2007)

The most common designs on Hopi basketry are geometric, animal or Kachina related. A Kachina is an ancestral spirit that acts as a medium between humans and gods. The following figures of baskets demonstrate both beauty and mathematics, each demonstrating a different aspect of transformational geometry. Stewart Koyiyumptewa, Tribal Archivist, of the Hopi Cultural Preservation Office (HCPO) has granted me permission to use the following photographs, (Figures 9-14). The baskets can be viewed at the Hopi Cultural Preservation Office website http://www.nau.edu/~hcpo-p/arts/bas1.htm#table1 by clicking on the link 'photographs'.

The basket in Figure 9 has 90-degree rotational symmetry as well as point symmetry. This basket can be rotated 90 degrees, about its center point and not look any different from its original position. The basket can also be rotated 180 and 270 degrees with the same results. Point symmetry, which is equivalent to a 180-degree rotational symmetry, is also found in this basket.

**Basket with Rotational and Point Symmetry**

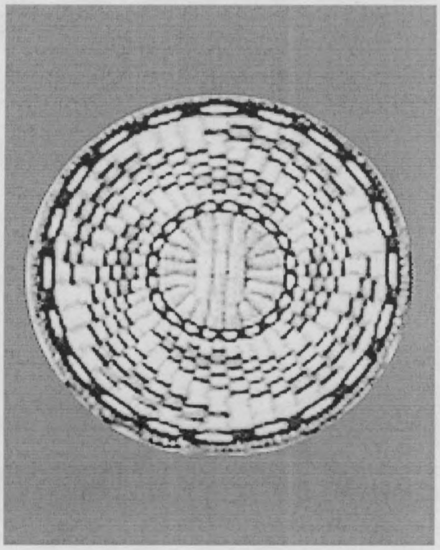

Figure 9

The basket in Figure 10 has one-line symmetry, as demonstrated in Figure 11. Line symmetry is present when you can fold a figure along the line of reflection and have two congruent figures. Figure 11 includes the line of reflection for the artwork shown in Figure 10.

**Basket with One-Line Symmetry** **Line of Symmetry Drawn**

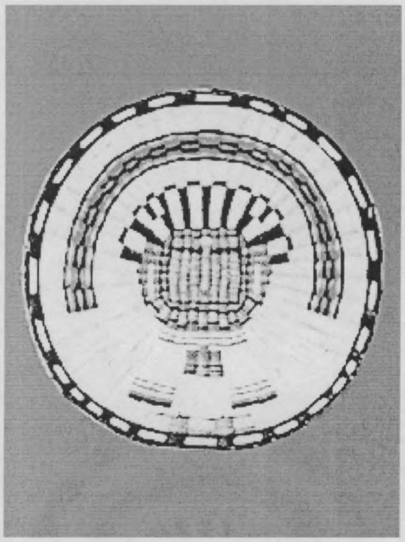

Figure 10

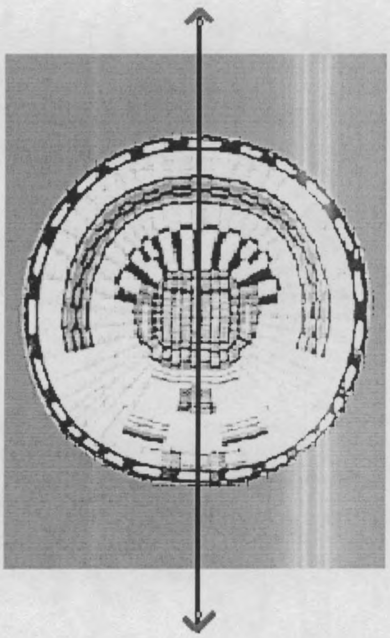

Figure 11

The basket in Figure 12 has line symmetry, rotational symmetry and point symmetry. Because of the use of different colors, there are only two lines of symmetry. If the colors on the sides were either both red or both green, the basket would have four lines of symmetry. The basket has 180-degree rotational symmetry as well as point symmetry.

**Basket with Rotational Symmetry and Point Symmetry**

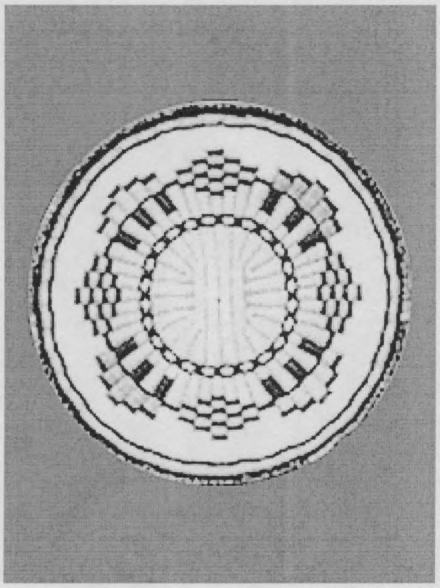

Figure 12

"Triangles and concentric circles round out the remaining majority of baskets, making their own statements with bright colors and intricate designs" (Jolles, 1995). This is shown in the previous four figures or below in Figure 13 and Figure 14.

**Basket with Concentric Circles**

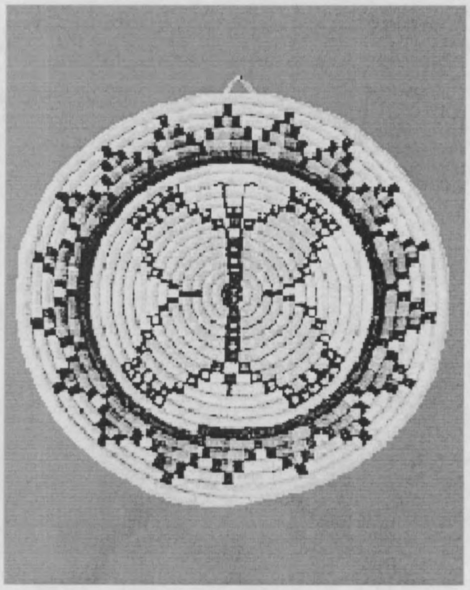

Figure 13

**Basket with Concentric Circles**

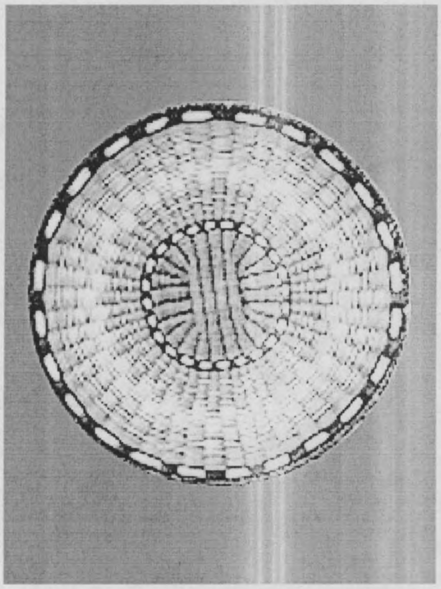

Figure 14

In a recent lecture (March 2006), Dr. Robert Devaney of Boston University demonstrated fractal image compression and explained how transformations are a key component of the process used to create backgrounds for movies. A part of a figure is shrunk, then rotated, reflected or translated and then reapplied to the picture. Using this process, the image of a forest or a galaxy in outer space can be created. An example that Dr. Devaney (2006) spoke of was the "dragon fractals" shown in Figure 15. This was drawn using *The Geometer's Sketchpad*.

**Dragon Fractal created on *The Geometer's Sketchpad***

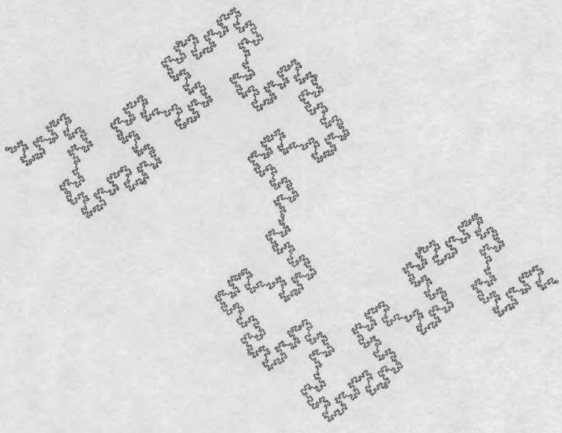

Figure 15

## Tiling and Tessellating

Most students have also been exposed to tiling and tessellating in the mathematics classroom. A tiling is defined as a complete covering of an entire plane without gaps or overlaps. "For shapes to fill the plane without gaps or overlaps, their angles, when arranged around a point, must have measures that add up to exactly 360 degrees. If the sum is less, there will be a gap. If the sum is greater, the shapes will overlap" (Serra, 2003, p. 379). Two basic types of tilings are shown below. A monohedral tiling is a tiling of one shape as in Figure 16. A checkerboard and a simple brick wall are all examples of monohedral tiling. A tiling with two tiles (Figure 17 shows an octagon and a square) is called a dihedral tiling.

**Monohedral Tiling**

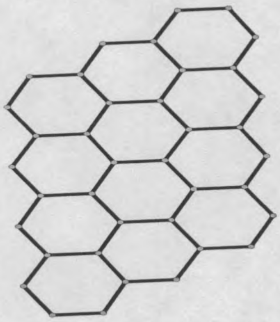

Figure 16

**Dihedral Tiling**

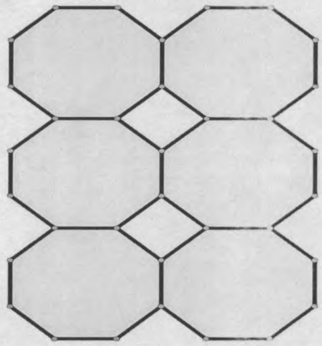

Figure 17

A tessellation is slightly different from a tiling. For a tessellation, the original figure is altered before it is used for tiling. The most important thing to remember is that what ever is done to one side of a figure must be done to the opposite side. To tessellate, start with a regular polygon such as a square, then take out one part of a side, and translate it to the opposite side. As stated above, the most important thing to remember is that what is done to one side, must be done to the opposite side. A demonstration of how to start a tessellation is shown in Figure 18.

**Starting a Tessellation**

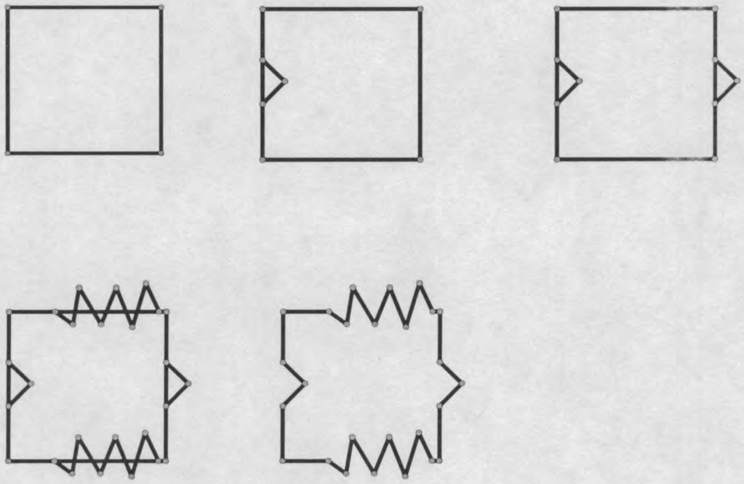

Figure 18

When finished with creating the figure you will now tile, for fun, you can color in and draw a face on the figure as shown in Figure 19.

**Face**

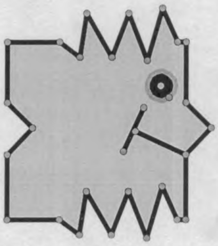

Figure 19

Once finished, the shape in Figure 19 can be used to tile the plane. As shown in Figure 20, the shape is translated to the right three times. That entire line of four shapes in then translated down three times.

**Tessellation**

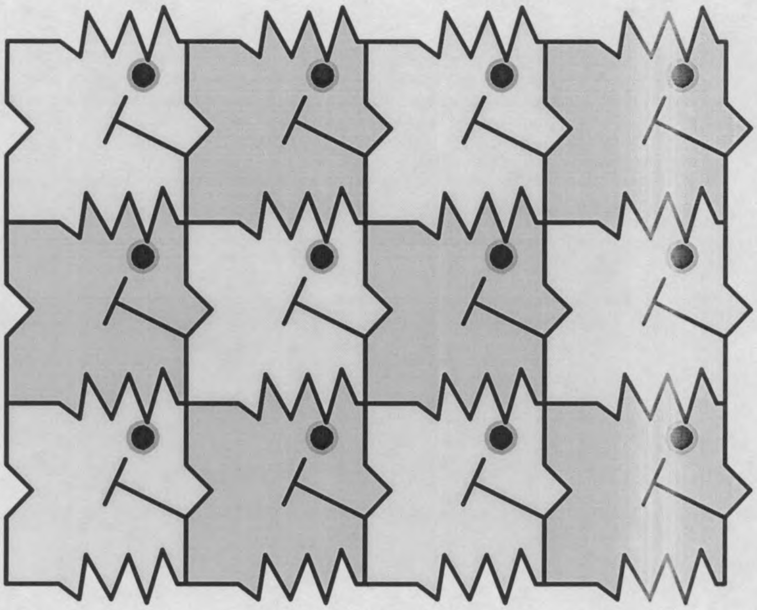

Figure 20

Other uses for transformations can be found in the area of computer graphics. As stated previously, the game *Tetris* is a game that is based solely on transformations. Most other computer games use the principles of geometric transformations. Transformations are also used with computer animation to create movies. Computer animation in movies such as *Toy Story, Finding Nemo* and *The Polar Express* could not have been developed without the use of geometric transformations. M. C. Escher has contributed much to transformations and particularly tessellations.

> Among his greatest admirers were mathematicians, who recognized in his work an extraordinary visualization of mathematical principles. This was the more remarkable in that Escher had no formal mathematics training beyond secondary school. As his work developed, he drew great inspiration from the

mathematical ideas he read about, often working directly from structures in plane and projective geometry, and eventually capturing the essence of non-Euclidean geometries, as we will see below. He was also fascinated with paradox and "impossible" figures, and used an idea of Roger Penrose's to develop many intriguing works of art. Thus, for the student of mathematics, Escher's work encompasses two broad areas: the geometry of space, and what we may call the *logic* of space. (Math Academy, 2007)

**Transformations in the Mathematics Curriculum**

It is not surprising that students are asked about transformations on all of the standardized tests, when one considers statements in the National Council of Teachers of Mathematics Principles and Standards (pages 42 and 43), the New Jersey Core Content Curriculum Standards (Standard 4.2 Section B) and other state standards such as those in the Massachusetts Mathematics Curriculum Framework. Elementary school students are asked to explain how a figure was moved to be in a different orientation than that of the original figure and high school students are asked for the new vertex coordinates of a triangle after it is rotated 90 degrees counter-clockwise. It is clear that transformations are important in the study of mathematics. According to The National Council of Teachers of Mathematics, students

> …in grades 9-12 should be able to do the following:
>
> - use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations

- understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- use various representations to help understand the effects of simple transformations and their compositions

(NCTM, http://standards.nctm.org/document/chapter7/geom.htm)

The New Jersey Department of Education Standard 4.2 (Geometry and Measurement) states

> …all students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.
>
> Transforming Shapes. Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.
>
> Coordinate Geometry. Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

(New Jersey Department of Education, http://www.state.nj.us/njded/cccs/02/s4_math.htm).

*The Geometer's Sketchpad* can be used to help students see how transformations move and affect the new orientation of a figure. This allows students, especially those who are visual learners, to see how a transformation affects the coordinates of an object. Students get immediate feedback that helps them draw conclusions about a transformation. Students who are tactile learners also learn through this hands-on approach to learning. These students are able to actively participate and explore the affects of transformations. Auditory learners are also helped because, in computer labs, students are encouraged to compare the results they have and to discuss the hypotheses they develop.

**Data Collection Activities**

As noted earlier, (see Research Settings, page 15), the study took place over a period of five days. Copies of all study instruments are included in the appendices. On the first day of the study, all students in the four classes completed the pre-attitudinal survey and the pre-content knowledge assessment. The classes using the computer lab engaged in the following activities on the first day of instruction.

- Introduced to the idea of a vector.
- Given the basic definition of a translation, went to the lab and used *The Geometer's Sketchpad* to construct figures.
- Completed a series of computer labs using *The Geometer's Sketchpad.*

Students in the two classes in the control groups were doing the same activities and had the same instruction as the students in the two treatment classes, except they were not using computers. Members of the control group were learning about transformations by sketching them on graph paper and engaging in other activities such as paper folding. In class, the control group was first introduced to the idea of a vector and then given the basic definition of a translation. Next, they were given graph paper on which to diagram their representations of translations.

On the second day, all classes reviewed and discussed the previous day's lab. Students were introduced to the definition of a reflection after which students in the experimental group went to the lab to use *The Geometer's Sketchpad* and complete a reflection lab. Students in the control group constructed representations of reflections on graph paper.

On each day, after the first day, students went over the previous day's lab and discussed it. On day three, they were introduced to the definition of a rotation, the center or point of rotation (in this study the point of rotation was the origin) and rotational symmetry. After the basic definitions were discussed, the students moved to the computer lab. They used *The Geometer's Sketchpad* to construct the rotated figures. The control group was introduced to the definition of rotations and point symmetry. They were given graph paper to use to construct rotations. On the fifth and last day, all students in the four classes completed the post-attitudinal survey and the post-content knowledge assessment.

## Analysis of Results

The objective of the study was to determine if students learn and retain information better when using the dynamic geometry program *The Geometer's Sketchpad*. The study was open to ninety-two ninth grade and tenth grade honors and regular geometry students. Only 34 students (37%) returned the Institutional Review Board (IRB) consent forms, thereby agreeing to participate in the study. The research assistant, not the instructors, explained the study to all ninety-two students. The breakdown of the study participants is as follows:

- 19 students were in the computer *(The Geometer's Sketchpad)* group.
  - Period 2: 6 students out of 27 students in class participated
  - Period 4: 13 students out of 20 students in class participated
- 15 students were in the non-computer group.
  - Period 5: 7 students out of 29 students in class participated
  - Period 7: 8 students out of 16 students in class participated

Students' retention and understanding of the three basic isometric transformations (translations, reflections, and rotations), was studied in students using and not using *The Geometer's Sketchpad*. Before and after the unit of instruction all students were given a pre-test and post-test to measure their knowledge. The pre-test and post-test questions were the same except for questions seven, nine, ten, eleven and twelve. However, the content of the questions was not different. The students were asked to do the same exact calculation, but with different numbers (Appendices 1 & 2). The scores of the pre- and post-tests are as follows for the computer group and non-computer group respectively: (Tables 1 and 2).

**Computer Group**

| Class | Pre | Post | Diff | Class | Pre | Post | Diff |
|---|---|---|---|---|---|---|---|
| Regular | 38 | 50 | 12 | Honors | 29 | 80 | 51 |
| Regular | 57 | 100 | 43 | Honors | 24 | 70 | 46 |
| Regular | 42 | 75 | 33 | Honors | 86 | 75 | -11 |
| Regular | 57 | 85 | 28 | Honors | 5 | 65 | 60 |
| Regular | 57 | 75 | 18 | Honors | 10 | 40 | 30 |
| | | | | Honors | 23 | 70 | 47 |
| | | | | Honors | 5 | 70 | 65 |
| | | | | Honors | 19 | 90 | 71 |
| | | | | Honors | 33 | 75 | 42 |
| | | | | Honors | 24 | 75 | 51 |
| | | | | Honors | 10 | 75 | 65 |
| | | | | Honors | 10 | 35 | 25 |
| | | | | Honors | 0 | 90 | 90 |
| Total | 251 | 385 | 134 | Total | 278 | 910 | 632 |
| Mean | 50.2 | 77 | 26.8 | Mean | 21.4 | 70 | 48.6 |

Table 1

**Non-Computer Group**

| Class | Pre | Post | Diff | Class | Pre | Post | Diff |
|---|---|---|---|---|---|---|---|
| Regular | 19 | 30 | 11 | Honors | 24 | 75 | 51 |
| Regular | 100 | 90 | -10 | Honors | 76 | 77 | 1 |
| Regular | 19 | 65 | 46 | Honors | 14 | 65 | 51 |
| Regular | 14 | 75 | 61 | Honors | 24 | 80 | 56 |
| Regular | 10 | 30 | 20 | Honors | 19 | 90 | 71 |
| Regular | 38 | 60 | 22 | Honors | 48 | 60 | 12 |
| Regular | 19 | 45 | 26 | Honors | 43 | 70 | 27 |
| Regular | 57 | 90 | 33 | | | | |
| Total | 276 | 485 | 209 | Total | 248 | 517 | 269 |
| Mean | 34.5 | 60.6 | 26.1 | Mean | 35.4 | 73.9 | 38.4 |

Table 2

The mean score of the post-test for the computer group was 71.9 (Table 4) and the mean score for the non-computer group was 66.8 (Table 6). The mean score for the computer group (both honors geometry and regular geometry) was 5.1 points higher than the non-computer group. The regular geometry computer groups mean score was a 77 whereas the honors geometry computer groups mean was a 70 (Table 3). The post-test mean for the honors geometry non-computer group was a 73.9 and the mean for the regular geometry non-computer group was 60.6 (Table 5).

**Computer Group Mean Score on Pre- and Post-test by Class**

| Regular | | Honors | |
|---|---|---|---|
| Pre | Post | Pre | Post |
| 50.2 | 77 | 21.4 | 70 |

Table 3

**Computer Group Mean Score on Pre- and Post-test by Group**

| Mean Pre-Test | Mean Post-Test |
|---|---|
| 29.4 | 71.9 |

Table 4

**Non-Computer Group Mean Score on Pre- and Post-test by Class**

| Regular | | Honors | |
|---|---|---|---|
| Pre | Post | Pre | Post |
| 34.5 | 60.6 | 35.4 | 73.9 |

Table 5

**Non-Computer Group Mean Score on Pre- and Post-test by Group**

| Mean Pre-Test | Mean Post-Test |
|---|---|
| 34.9 | 66.8 |

Table 6

**One-Way Analysis of Pre-Test by Group**

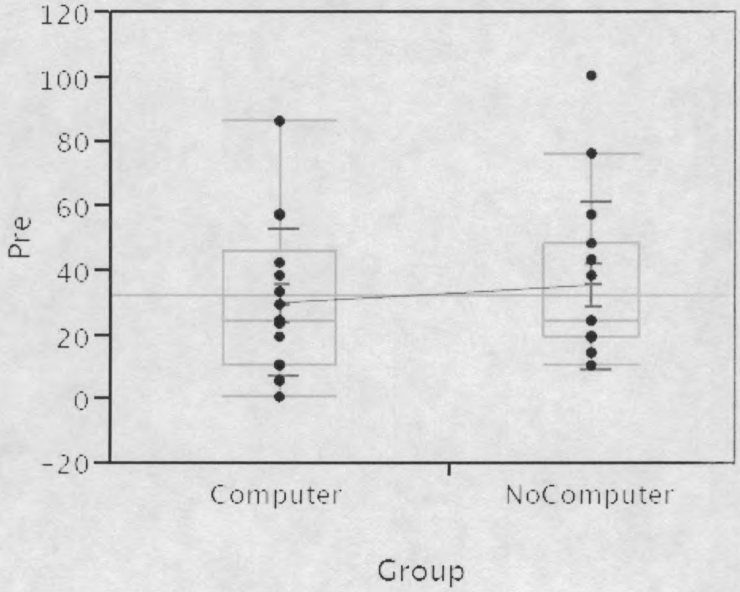


Figure 21

One-Way Analysis of Post-Test by Group

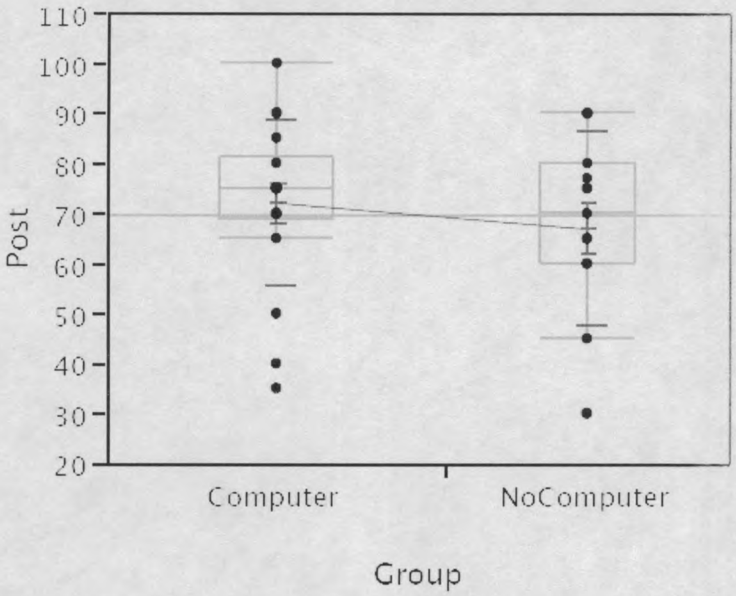


Figure 22

One-Way Analysis of Differences by Group

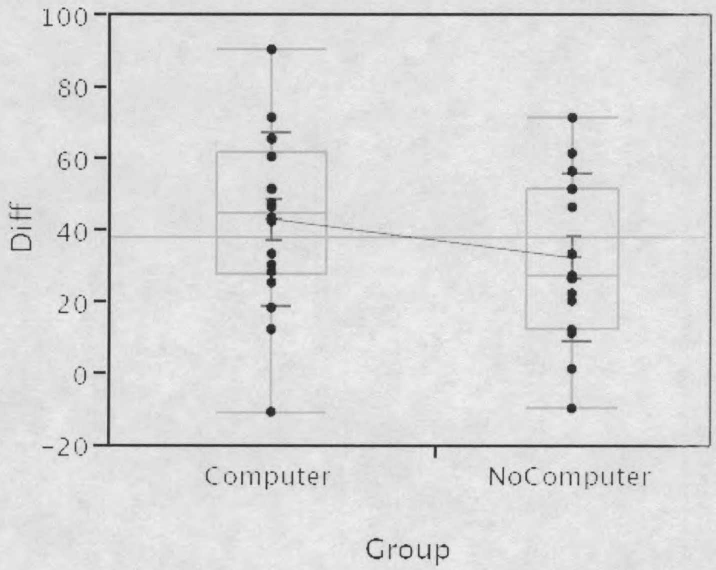


Figure 23

With the help of Montclair State University's Statistical Consulting Program, specifically Dr. Andrew J. McDougall, a variety of statistical methods were employed to analyze the data. Analysis of Variance (ANOVA) is a procedure, which is used to test the hypothesis that three or more samples were selected from a population with the same mean (Downing, 1995, p. 7). Specifically, a nested ANOVA model of the form, Y = Group + Group (Class), with Y separately representing the pre-test, post-test, and difference between the pre- and post-test scores was used. The category, Group, was the two-level factor representing the Computer versus Non-Computer groups, and Class is the two classes within each Group. There were no significant differences of the means between the groups.

The Tukey HSD (honestly significantly different) test was applied to determine if there is a difference among all pairs of groups and tests, and across all four classes. None of the means evaluated by the Tukey HSD test were significantly different.

The last analysis that was performed was Fisher's Exact test. Fisher's Exact test is a test that can be used for a two by two contingency table. Fisher's Exact Test was used to compare the results of the computer and non-computer group's, pre- and post-tests. The computer and non-computer group performed differently on the pre- and post-test at the 0.05 level of significance.

In addition to the pre- and post-test, all students completed a pre- and post-attitudinal survey (Appendix 3). The attitudinal survey, developed by Todd Moyer (2003), assessed the students' likes and dislikes about studying geometry with or without the computer. In discussion with my advisor, Dr. Kenneth Wolff, we decided not to use the attitudinal results for the non-computer group, since upon careful study of the questions we determined that the questions were only valid for the computer groups. The attitudinal survey data was compiled by calculating an overall score to each survey taken by students in the computer group. In this survey, scoring was based on a four-point Likert scale, where students received one point for '*strongly disagree*', two points for '*disagree*', three points for '*agree*' and four points for '*strongly agree*'. The sums of the points of the ten-question survey were scored according to a Likert Scale (see Table 7). For example, to obtain a score of 34 on the Likert scale a student would have had to choose at least eight '*strongly agree*' statements.

**Scale for Attitudinal Survey**

| | |
|---|---|
| **Key** | **10-17 Strongly Disagree** |
| | **18-25 Disagree** |
| | **26-33 Agree** |
| | **34-40 Strongly Agree** |

Table 7

The mean scores of the attitudinal survey for the pre- and post- survey were similar (Table 8).

**Computer Group**

| Regular | Pre-Survey | Post-Survey | | Honors | Pre-Survey | Post-Survey |
|---|---|---|---|---|---|---|
| Mean | 25.2 | 24.3 | | Mean | 26.0 | 26.5 |

Table 8

No student selected '*strongly agree*' or '*strongly disagree*' on the survey. In addition, the pre-test total scores had exactly nine '*disagree*' for the pre-survey and nine '*disagree*' responses for the post-survey. The pre-survey scores had exactly ten '*agree*' for the pre-survey and ten '*agree*' responses for the post-survey. Neither computer group (honors or regular) showed a significant difference in their attitude towards the use of *The Geometer's Sketchpad.*

**Discussion**

In the analysis of the mean score, on the post-test, the computer group (Table 4) was higher than the mean score of the non-computer group (Table 6), but none of the results were statistically significant at the 0.05 level. The rather small sample sizes associated with each class and the large variability in the post-test and differenced scores might have impacted the outcome. The small sample size might be explained by the fact that the research assistant did not encourage the students or get them excited about participating in the study. There was no incentive to participate in the study. After the consent forms were initially collected, the research assistant did not follow up with the students who did not turn in a consent form to help reduce the non-response bias. I did not pressure my students or the students in the other teacher's classes to be in the study, because I felt that pressuring could 'injure' (cause undo harm to the participants emotionally) them in the study, meaning I wanted the results to be unbiased. Had I been the one to 'pitch' the study to all of the classes, I predict the participation rate would have been greater than 37%.

After administering and scoring the pre- and post-tests, it became clear that some questions were inappropriate and unreliable. In the future, I would like to modify the following questions on the content assessment. For example, the first five questions (Appendices 1 & 2) ask students for a definition and an example. Asking for a definition and an example caused a problem because of the way in which these questions were worded. Most students provided a definition or an example, but not both. I should have separated the "definition" and "example" into two separate questions to ensure reliability of the questions. Question number six was incorrect (Appendices 1 & 2). Although this was a very good question, in both the pre- and post-test, the scale of the grid, in the diagram, was incorrect. The coordinates of the image were not at a "nice" decimal coordinate, (not even at the midpoint of the interval), which confused the students. The coordinate plane axes originally had increments of one, but when the graph was resized, the image was scaled incorrectly. As a result, it appears that students were trying to 'guess' the new coordinate. The reviewers and I missed these errors prior to the start of the study.

Finally, question 11B (Appendix 2) was a problem for many students. They were asked to calculate the degree of rotation in Figure 25. Almost all students correctly answered question 11A, Figure 24 (72 degree rotation), but a good percentage of students incorrectly answered question 11B. The students were confused with the 60-degree rotation in 11B, because they did not group the two reflections (Figure 25). Instead, they viewed it as another rotation and most answered 30 degrees instead of the correct answer, 60 degrees. Students looked at Figure 25 as having twelve "branches" when it actually has six "branches."

**Question 11A**

Figure 24

**Question 11B**

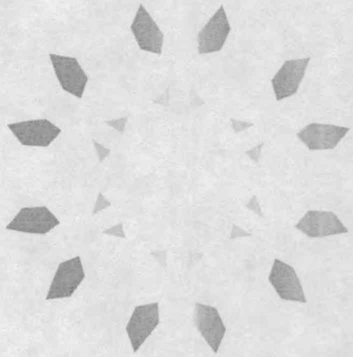

Figure 25

The computer and non-computer transformation labs (Appendices 4 and 5) may not have taught the students as well as I intended. Although a few questions on the post-test were similar to the labs, many were not. Students were not asked to think critically while working on the labs, but were asked to work more critically on the post-test. In the future, I would create computer labs that have questions that require critical thinking. I would also have a lab where students make their own kaleidoscopes so they have a better understanding of how they are created.

I think that if the above changes were made, the results might have been different and we might have seen that the use of *The Geometer's Sketchpad* would have improved students achievement, in a unit on transformations in the coordinate plane, at a statistically significant level. Although many of the problems that I encountered dealt with the pre- and post-test content, I also had problems with the attitudinal survey.

In retrospect, I should not have used Todd Moyer's (2003) attitudinal survey because it did not effectively differentiate the students' likes and dislikes using *The Geometer's Sketchpad* and could only be used for people using *The Geometer's Sketchpad*. Dr. Moyer (2003) and I had similar results on the attitudinal survey. I used his survey because it has been tested before in a similar study.

**Reflections**

Educational research can be very challenging. It took many letters and discussions to obtain permission to perform my study in the high school. I requested only one week when I would have preferred a longer time period. A pilot study would have been helpful because problematic questions on the pre- and post-test could have been addressed before the major study. Since computer labs equipped with *The Geometer's Sketchpad* were only available during certain periods during the day, it hindered the possibility of using other classes in my study and, as a result, decreased my sample size.

The students in the non-computer group told me that they felt their input to my study was not as important as those in the computer group. I explained to them that everyone's input was valuable. I would have liked the students to voice this concern to me earlier so I would have been able to explain the research to them and the importance of their participation in either the control or treatment group.

Students in both the computer group and non-computer group asked me if their results on the pre- and post-tests counted towards their grade. I told them it did not. This posed a problem because in both groups I got answers such as "peanut-butter," "I love math" and "chocolate." I wonder how a study can be valid when you get responses such as those stated above. How can you make the students try as hard on the pre- and post-tests in any study without its affecting their grade? I am sure I am not the first researcher to have this problem.

I have learned a great deal while conducting this research. First, I learned that before you administer a study instrument, have a statistician review it to check for reliability. The statistician can determine whether or not a question is fair or if it could be misconstrued in some way. Dr. McDougall's first comment to me was that I could have had better results had he looked over my pre- and post-test because some of the questions were not statistically fair.

Much of the above could have been worked out had I had conducted a pilot study. After the pilot study, I would have used a modified pre- and post-test, a completely different attitudinal survey and explained the study to all the students myself.

**Recommendations for Further Research**

If I had the opportunity to replicate this study, I would like to do so. I would use a different attitudinal survey, modify the content assessment and conduct the study on a larger scale, so that I would have a much larger sample size. Absurd answers such as "peanut-butter," "I love math," and "chocolate" would not affect a future study as they did in this study. Since the sample size was small to begin with, the above answers affected my data. I was fascinated with Juli Dixon's (1995) study of ESL learners and native speakers, because *The Geometer's Sketchpad* leveled the students' ability to learn geometry without English getting in the way. I would like to do more research in that area. From her results, *The Geometer's Sketchpad* helped students who were not native speakers succeed in geometry.

The research in this area of study is very mixed, some saying the use of dynamic geometry software significantly improves learning and retention of transformations, while other studies show there is no significant improvement. Technology, particularly *The Geometer's Sketchpad,* may not be the "miracle" we are all looking to find. No one has shown that the use of dynamic geometry technology impedes learning. In my opinion and in the opinions of my colleagues, it does seem to make learning about transformations more interesting so that students are more engaged. *The Geometer's Sketchpad* leads students into mathematical conversations that they would not normally have without the use of the software. I had hoped that I would put to rest the debate with my research, but I just added to its complexity.

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# Appendices

Appendix 1--Pre-Test

Appendix 2--Post-Test

Appendix 3--Attitudinal Survey

Appendix 4--Computer Group Labs

Appendix 5--Non-Computer Group Labs

ID# 000496

Appendix 1

## Pre-Transformation Opportunity



Define **and** give an example of the following:

1. Rotation

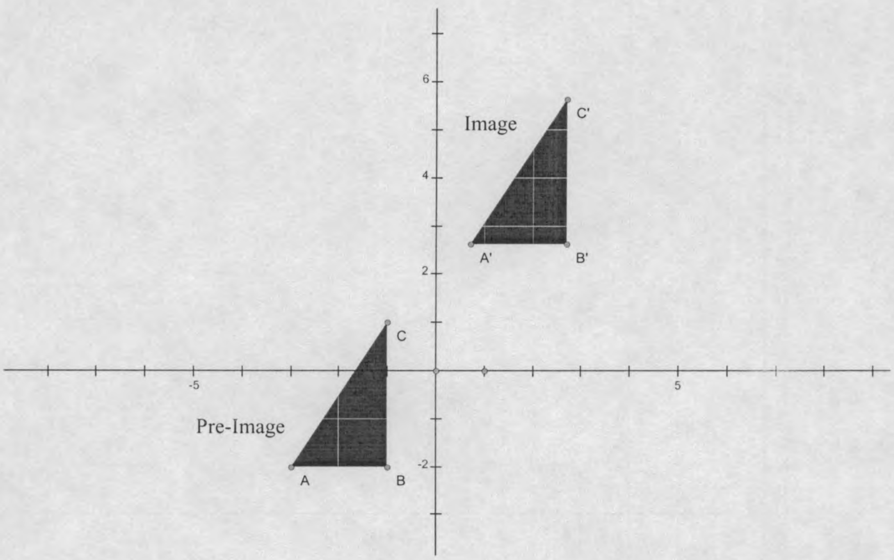
2. Translation

3. Reflection

4. Transformation

5. Isometry

6. Describe the vector that translated the ABC to A'B'C. What is the translation vector?




ID# 000496

7. Describe what angle the mirrors are at to produce this figure.

8. What is the result of two consecutive reflections over parallel lines?

For the following, fill out the chart.

9.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| Reflection x-axis | | | | | | |
| Reflection y-axis | | | | | | |

10.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| 90 Degree rotation | | | | | | |
| 180 degree rotation | | | | | | |
| 270 degree rotation | | | | | | |

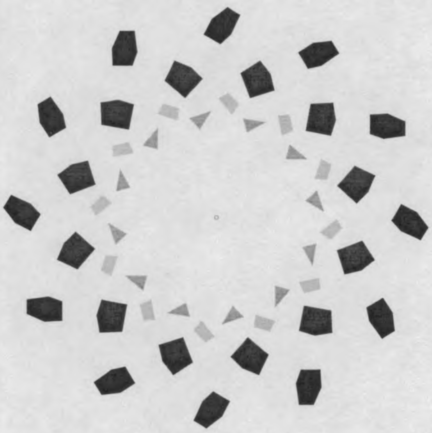
10. **A(-3,-2) B(-3,-5) C(-5,-5)**
Translate this set of points along vector **<4, 5>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| | | |

11. Describe any rotations (of less than 180 degrees) that will map the figure onto itself. Give each angle for the rotational symmetry. Also discuss if the figure has point symmetry.

a)

b)



12. Explain whether or not the composition of two or more rigid transformations is a rigid transformation.

ID# 00496

Appendix 2



## Post-Transformation Opportunity

Define **and** give an example of the following:

1. Rotation

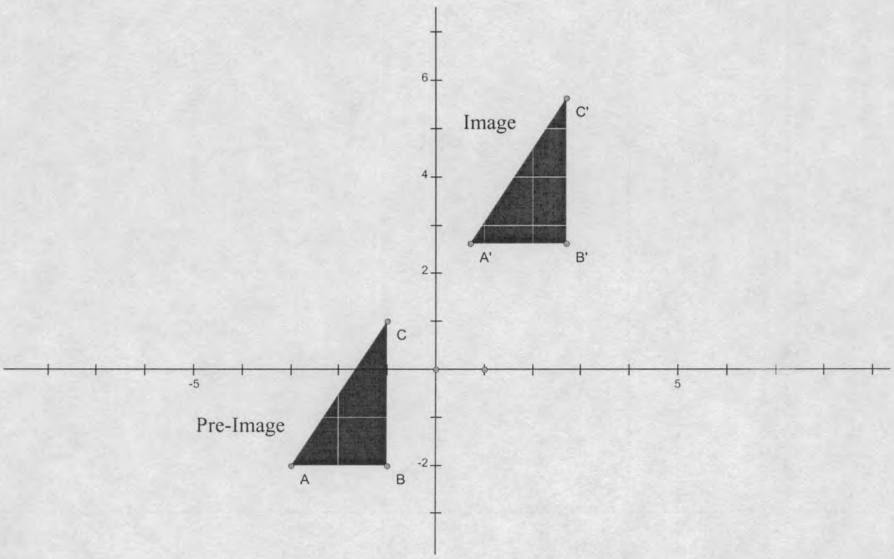
2. Translation

7. Reflection

8. Transformation

9. Isometry

10. Describe the vector that translated the ABC to A'B'C. What is the translation vector?




ID# 00496

7. Describe what angle the mirrors are at to produce this figure.

8. What is the result of two consecutive reflections over parallel lines?

For the following, fill out the chart.

9.

| **Original** | **A(-6,4)** | **B(-5,5)** | **C(-3,5)** | **D(-2,4)** | **E(-3,3)** | **F(-5,3)** |
|---|---|---|---|---|---|---|
| Reflection x-axis | | | | | | |
| Reflection y-axis | | | | | | |

10.

| **Original** | **A(-4,4)** | **B(-3,5)** | **C(-1,5)** | **D(0,4)** | **E(-1,3)** | **F(-3,3)** |
|---|---|---|---|---|---|---|
| 90 Degree rotation | | | | | | |
| 180 degree rotation | | | | | | |
| 270 degree rotation | | | | | | |

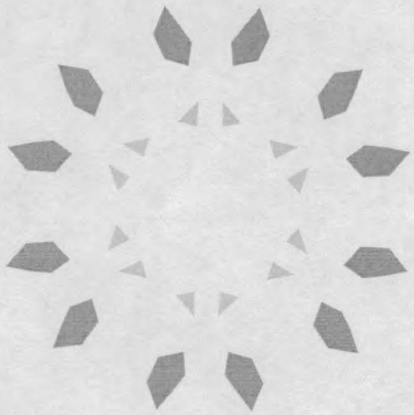
11. **A(1,-5) B(4,0) C(8,-3)**
Translate this set of points along vector **<-3, 2>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| | | |

11. Describe any rotations (of less than 180 degrees) that will map the figure onto itself. Give each angle for the rotational symmetry. Also discuss if the figure has point symmetry.

a)

b)



12. Explain whether or not the composition of two or more rigid transformations is a rigid transformation.

ID# 00496 Appendix 3

Survey on the use of The Geometer's Sketchpad

1. Class is more interesting when we use Sketchpad than when we do not use Sketchpad.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

2. I enjoy using Sketchpad as an instructional tool.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

3. I feel that I have learned more since we started using Sketchpad.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

4. Using Sketchpad has helped me raise my grade in this class.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

5. Using Sketchpad has helped me understand geometry better than before when we did not use Sketchpad.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

6. For transformations, I would rather use Sketchpad.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

7. I would rather do the constructions by hand than using Sketchpad.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

8. I am glad we did not use Sketchpad to investigate the properties of triangles.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

9. "Clicking and dragging" figures on the screen is more helpful to understand geometry than pictures on the whiteboard or overhead.

   1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

10. I prefer the teacher lecturing and demonstrating information instead of using Sketchpad to find that information.

    1: Strongly disagree 2: Disagree 3: Agree 4: Strongly agree

Appendix 4

# Student Worksheets and answer sheets

# With the computer

Name:

**Plotting Points Activity**

1. Go to **Graph->Define Coordinate System**
2. With the arrow click on the point (1,0) hold and drag:(this will make your window size larger or smaller)
3. Go to **Graph->Plot Points**
4. Enter in the points (Rectangular) then click done.
5. Label the points using the "A tool" OR
6. Highlight all of the points either by:
   a) Holding down the shift key and using your arrow or
   b) Go to **Edit->Select All (points)** (Highlight Point tool)

   Go to **Measure->Coordinates** (this will measure where your points are on the grid)
7. Highlight on point and drag it, what happens?
8. Highlight all three points in each case. Go to **Construct->Triangle (polygon) Interior**
9. If you go to **Display->Color** you can change the color of your polygon.

Plot each set of points separately.

a.) (1,3), (7, 0),(5,7)

b.) (-1,-5),(-4,-6),(-6,-8)

c.) (-3,5),(-5,3), (-1,0), (-1,3)

Name:

## Translations

Plot the following sets of points, separately. Translate each set of points separately along the indicated vector. After the points are plotted highlight them and then go to Translate. A window will pop up. In the first box (Translation Vector) mark -> Rectangular. The window will change and ask you to enter the horizontal coordinate (x) and the vertical coordinate (y).

1. **A(1,3) B(5,3) C(5,1) D(1,1)**
Translate this set of points along **vector <-6, -3>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| | | | |

2. **A(1,-4) B(3,-1) C(5,-4)**
Translate this set of points along vector **<0, -7>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| | | |

3. **A(-3,-2) B(-3,-5) C(-5,-5)**
Translate this set of points along vector **<4, 5>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| | | |

4. **A(-4,5) B(-3,3) C(-1,3) D(-1,1) E(-4,1)**
Translate this set of points along vector **<8, -3>**
Record the new set of points.

| | | | | |
|---|---|---|---|---|
| A' | B' | C' | D' | E' |
| | | | | |

5. **A(1,1) B(4,1) C(4,3) D(1,3)**
Translate this set of points along vector **<-6, 0>**
Record the new set of points.

| | | | |
|---|---|---|---|
| A' | B' | C' | D' |
| | | | |

6. **A(-2,0) B(0,3) C(2,0) D(0,-3)**
Translate this set of points along vector **<5, 7>**
Record the new set of points.

| | | | |
|---|---|---|---|
| A' | B' | C' | D' |
| | | | |

How do the coordinates change when you translate left or right?

How do the coordinates change when you translate up or down?

Say you had to plot the following:
A(-99,5) B(73, 80) C(0, 0)
And translate these points along vector <21, -31>
What would the new coordinates be?

Did you plot them or do this operation some other way?

Name:

## Reflections

**Plot the following sets of points, separately. Reflect each set (the original - pre-image) along the x-axis and then reflect along the y-axis. Record your results for each.**

1.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| Reflection x-axis | | | |
| Reflection y-axis | | | |

2.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| Reflection x-axis | | | | |
| Reflection y-axis | | | | |

3.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| Reflection x-axis | | | | | | |
| Reflection y-axis | | | | | | |

How do the coordinates change when you reflect in the x-axis?

How do the coordinates change when you reflect in the y-axis?

Name:

## Rotations

YOU ARE ROTATING AROUND THE ORIGIN, SO TO ROTATE FOLLOW THESE INSTRUCTIONS:

1- HIGHLIGHT THE ORIGIN (0,0)
2- GO TO TRANSFORM->MARK CENTER
3- HIGHLIGHT ENTIRE FIGURE
4- GO TO TRANSFORM->ROTATE
5- TYPE IN ROTATE 90 DEGREE COUNTER-CLOCKWISE THEN FROM ORIGINAL 180 THEN 270
6- DO THIS THREE TIMES FOR EACH. GO FROM THE ORIGINAL

**Plot the following sets of points, separately. Rotate each set of points separately. Plot your points and construct the polygon interior. You are rotating around the origin (counter clock-wise).**

1.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| 90 Degree rotation | | | |
| 180 degree rotation | | | |
| 270 degree rotation | | | |

2.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| 90 Degree rotation | | | | |
| 180 degree rotation | | | | |
| 270 degree rotation | | | | |

3.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| 90 Degree rotation | | | | | | |
| 180 degree rotation | | | | | | |
| 270 degree rotation | | | | | | |

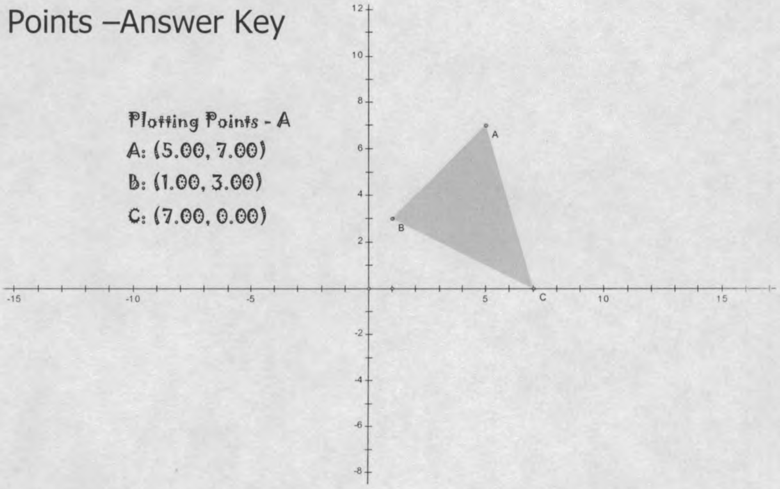
How do the coordinates change when you rotate 90 degrees around the origin?

How do the coordinates change when you rotate 180 degrees around the origin?

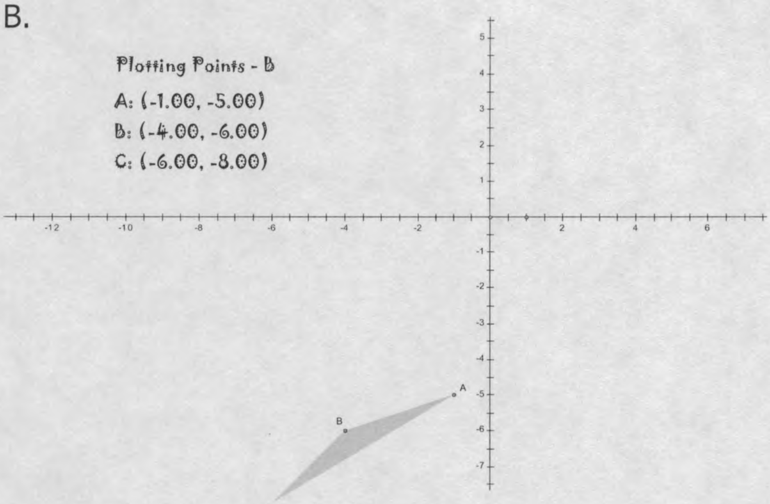
How do the coordinates change when you rotate 270 degrees around the origin?

Plotting Points –Answer Key

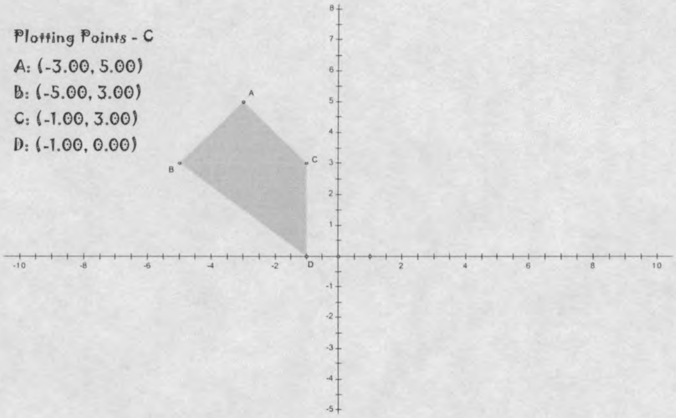
A.




B.




C.



Name:

## Translations—ANSWER KEY

**1. A(1,3) B(5,3) C(5,1) D(1,1)**
Translate this set of points along vector **<-6, -3>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| **-5, 0** | **-1, 0** | **-1,-2** | **-5,-2** |

**2. A(1,-4) B(3,-1) C(5,-4)**
Translate this set of points along vector **<0, -7>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| **1,-11** | **3,-8** | **5,-11** |

**3. A(-3,-2) B(-3,-5) C(-5,-5)**
Translate this set of points along vector **<4, 5>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| **1,3** | **1,0** | **-1,0** |

**4. A(-4,5) B(-3,3) C(-1,3) D(-1,1) E(-4,1)**
Translate this set of points along vector **<8, -3>**
Record the new set of points.

| A' | B' | C' | D' | E' |
|---|---|---|---|---|
| **4, 2** | **5, 0** | **7, 0** | **7, -2** | **4, -2** |

**5. A(1,1) B(4,1) C(4,3) D(1,3)**
Translate this set of points along vector **<-6, 0>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| **-5, 0** | **-2, 1** | **-2, 3** | **-5, 3** |

6. A(-2,0) B(0,3) C(2,0) D(0,-3)
Translate this set of points along vector <5, 7>
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| **3, 7** | **5, 10** | **7, 7`** | **5, 4** |

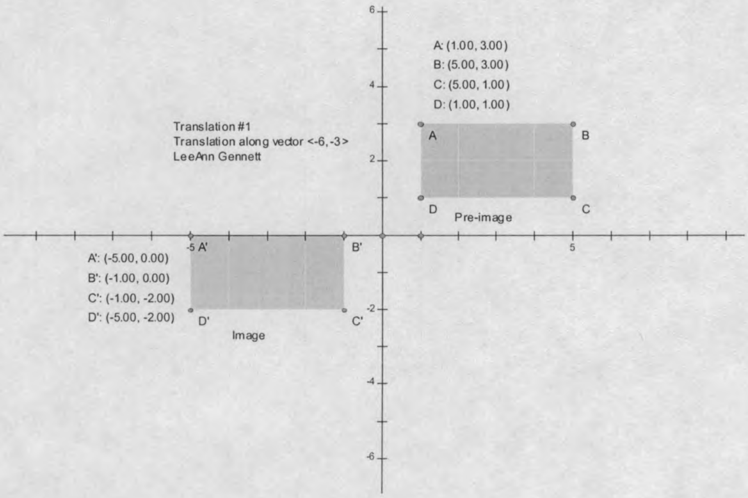
How do the coordinates change when you translate left or right?
**The y-coordinate stays the same, while the x-coordinate changes depending on if the translation is left or right.**

How do the coordinates change when you translate up or down?
**The x-coordinate stays the same, while the y-coordinate changes depending on if the translation is up or down.**

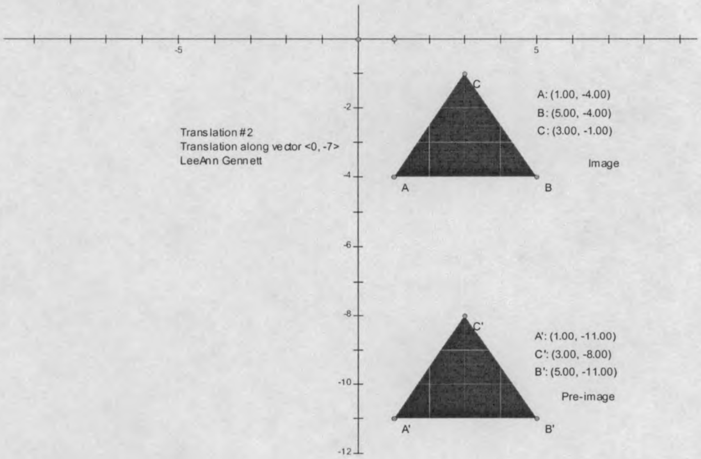
Say you had to plot the following: **A(-99,5) B(73, 80) C(0, 0)**
And translate these points along vector <21, -31>
What would the new coordinates be? **A′(-78, -26) B′(94, 49) C′(21, -31)**
Did you plot them or do this operation some other way?
**Add the x and y coordinates.**

# Translations- Answer Key

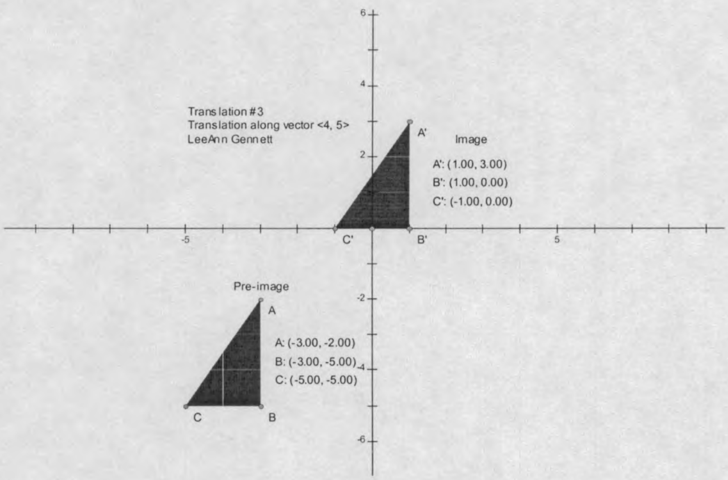
## Translation 1



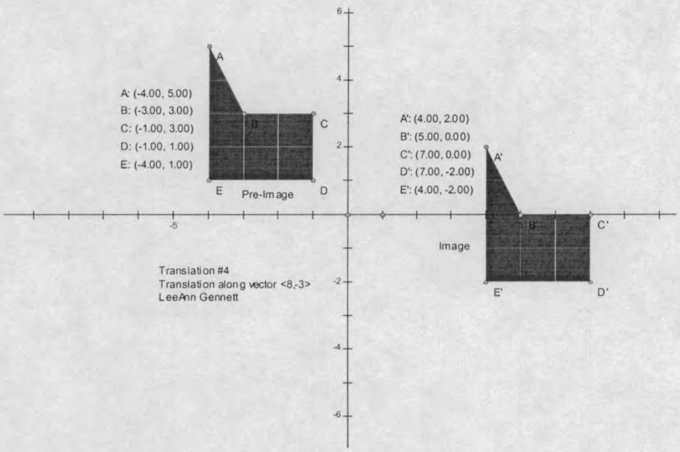

## Translation 2



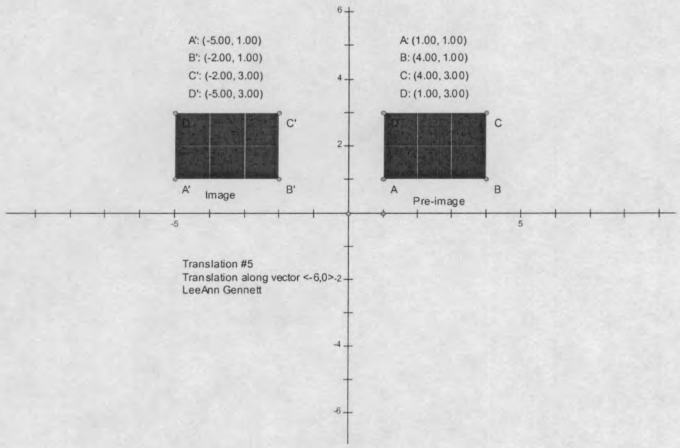

## Translation 3



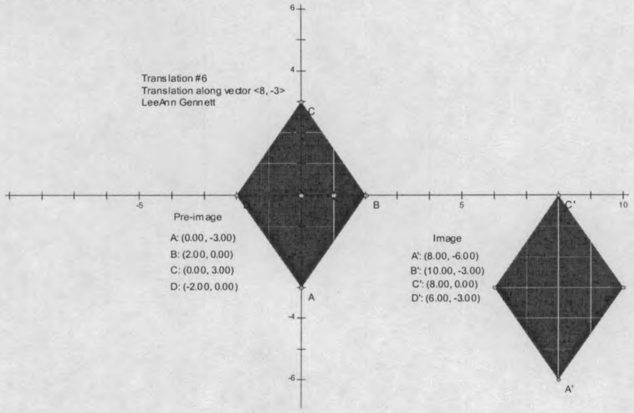
## Translation 4




## Translation 5




## Translation 6



Name:

## Reflections- ANSWER KEY

Plot the following sets of points, separately. Reflect each set
(the original - pre-image) along the x-axis and then reflect along the y-axis.
Record your results for each.
To reflect, plot each set of points and construct the polygon interior.
Measure and label the points.
Highlight the x-axis and go to **Transform->Mark Mirror.**
Then highlight the polygon with the points and go to **Transform->Reflect**.
Measure and label the points.
Record the results in the chart.
Repeat the following for the y-axis.

A.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| Reflection x-axis | **1,-1** | **3,-1** | **1,-5** |
| Reflection y-axis | **-1,1** | **-3,1** | **-1,5** |

B.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| Reflection x-axis | **1,1** | **6,1** | **6,3** | **1,3** |
| Reflection y-axis | **-1,-1** | **-6,-1** | **-6,-3** | **-1,-3** |

C.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| Reflection x-axis | **-5,-3** | **-4,-4** | **-2,-4** | **-1,-3** | **-2,-2** | **-4,-2** |
| Reflection y-axis | **5,3** | **4,4** | **2,4** | **1,3** | **2,2** | **4,2** |

How do the coordinates change when you reflect in the x-axis?

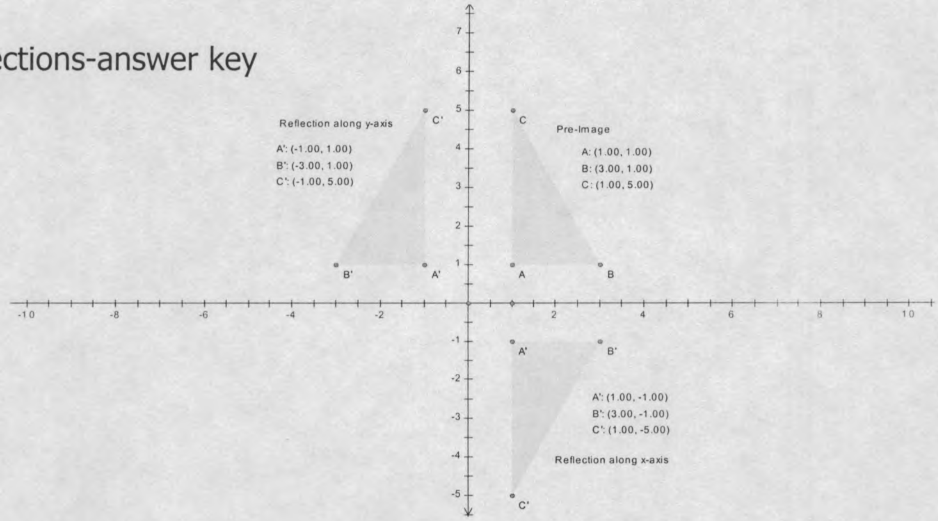
**The x-coordinates stays the same and the y-coordinate gets the opposite sign.**

How do the coordinates change when you reflect in the y-axis?

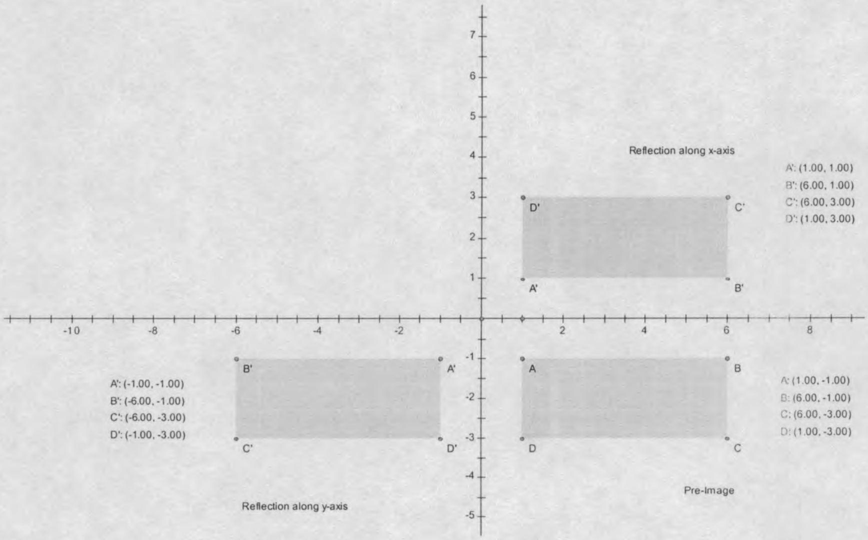
**The x-coordinates gets the opposite sign and the y-coordinate stays the same.**

# Reflections-answer key

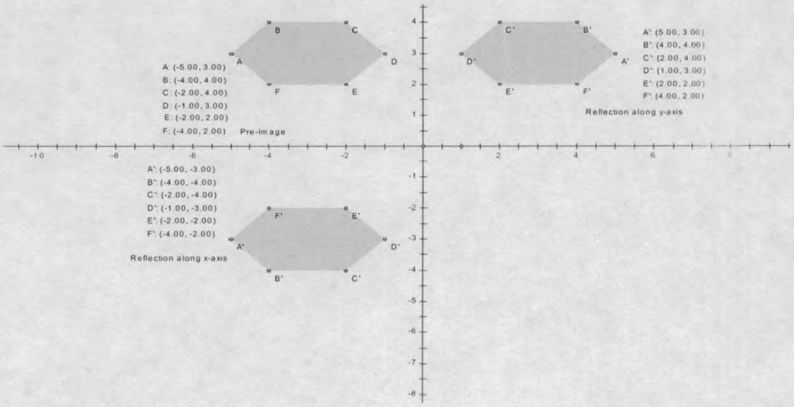
A.




B.




C.



Name:

## Rotations—ANSWER KEY

YOU ARE ROTATING AROUND THE ORIGIN, SO TO ROTATE FOLLOW THESE INSTRUCTIONS:

7- HIGHLIGHT THE ORIGIN (0,0)
8- GO TO TRANSFORM->MARK CENTER
9- HIGHLIGHT ENTIRE FIGURE
10- GO TO TRANSFORM->ROTATE
11- TYPE IN ROTATE 90 DEGREE COUNTER-CLOCKWISE THEN FROM ORIGINAL 180 THEN 270
12- DO THIS THREE TIMES FOR EACH. GO FROM THE ORIGINAL

**Plot the following sets of points, separately. Rotate each set of points separately. Plot your points and construct the polygon interior. You are rotating around the origin (counter clock-wise).**

1.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| 90 Degree rotation | **-1, 1** | **-1, 3** | **-5, 1** |
| 180 degree rotation | **-1, -1** | **-3, -1** | **-1, -5** |
| 270 degree rotation | **1, -1** | **1, -3** | **5, -1** |

2.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| 90 Degree rotation | **1, 1** | **1, 6** | **3, 6** | **3, 1** |
| 180 degree rotation | **-1, 1** | **-6, 1** | **-6, 3** | **-1, 3** |
| 270 degree rotation | **-1, -1** | **-1, -6** | **-3, -6** | **-3, -1** |

3.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| 90 Degree rotation | **-3, -5** | **-4, -4** | **-4, -2** | **-3, -1** | **-2, -2** | **-2, -4** |
| 180 degree rotation | **5, -3** | **4, -4** | **2, -4** | **1, 3** | **2, -2** | **4, -2** |
| 270 degree rotation | **3, 5** | **4, 4** | **4, 2** | **3, 1** | **2, 2** | **2, 4** |

How do the coordinates change when you rotate 90 degrees around the origin?

**X and y coordinates change places and the new x-coordinate gets an opposite sign**

How do the coordinates change when you rotate 180 degrees around the origin?

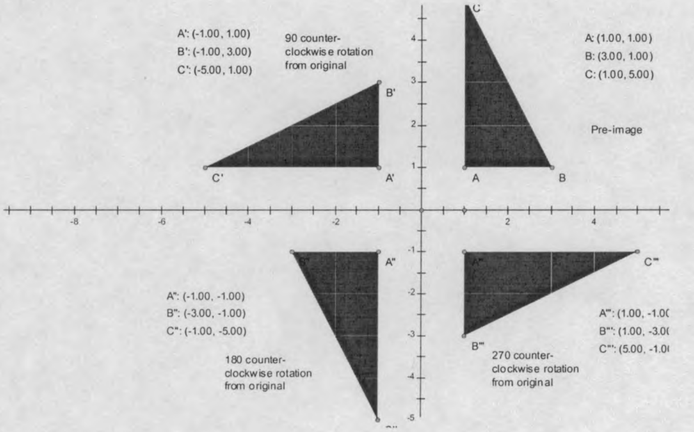
**X and y coordinates stay the same, but both the x and y coordinates get an opposite sign**

How do the coordinates change when you rotate 270 degrees around the origin?

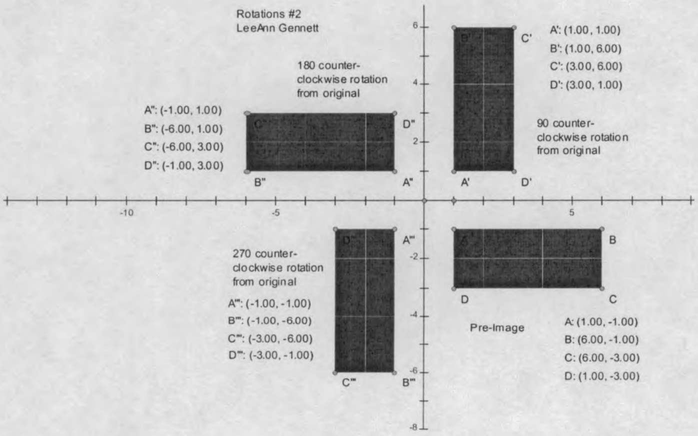
**X and y coordinates change places and the new y-coordinate gets an opposite sign**

# Rotations- Answer Key

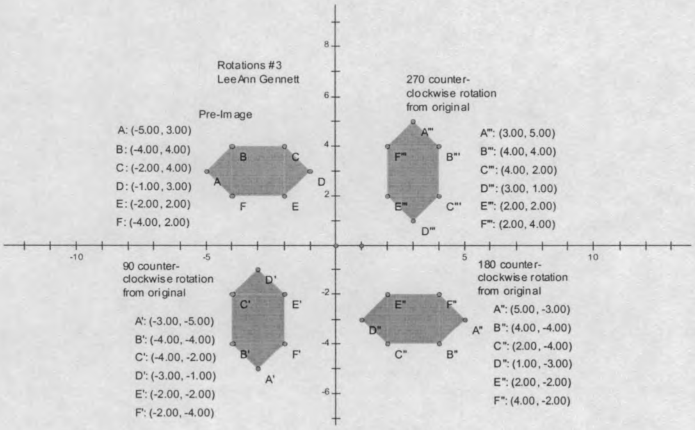
## Rotation #1




## Rotation #2




## Rotation #3



Appendix 5

# Student Worksheets and answer sheets

# Without the computer

Name:

**Plotting Points Activity**

Plot each set of points separately on graph paper.

a.) (1,3), (7, 0),(5,7)

b.) (-1,-5),(-4,-6),(-6,-8)

c.) (-3,5),(-5,3), (-1,0), (-1,3)

Name:

## Translations

Plot the following sets of points, separately. Translate each set of points separately along the indicated vector.

1. **A(1,3) B(5,3) C(5,1) D(1,1)**
Translate this set of points along **vector <-6, -3>**
Record the new set of points.

| | | | |
|---|---|---|---|
| A' | B' | C' | D' |
| | | | |

2. **A(1,-4) B(3,-1) C(5,-4)**
Translate this set of points along vector **<0, -7>**
Record the new set of points.

| | | |
|---|---|---|
| A' | B' | C' |
| | | |

3. **A(-3,-2) B(-3,-5) C(-5,-5)**
Translate this set of points along vector **<4, 5>**
Record the new set of points.

| | | |
|---|---|---|
| A' | B' | C' |
| | | |

4. **A(-4,5) B(-3,3) C(-1,3) D(-1,1) E(-4,1)**
Translate this set of points along vector **<8, -3>**
Record the new set of points.

| A' | B' | C' | D' | E' |
|---|---|---|---|---|
| | | | | |

5. **A(1,1) B(4,1) C(4,3) D(1,3)**
Translate this set of points along vector **<-6, 0>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| | | | |

6. **A(-2,0) B(0,3) C(2,0) D(0,-3)**
Translate this set of points along vector **<5, 7>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| | | | |

How do the coordinates change when you translate left or right?

How do the coordinates change when you translate up or down?

Name:

## Reflections

**Plot the following sets of points, separately. Reflect each set (the original - pre-image) along the x-axis and then reflect along the y-axis. Record your results for each.**

1.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| Reflection x-axis | | | |
| Reflection y-axis | | | |

2.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| Reflection x-axis | | | | |
| Reflection y-axis | | | | |

3.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| Reflection x-axis | | | | | | |
| Reflection y-axis | | | | | | |

How do the coordinates change when you reflect in the x-axis?

How do the coordinates change when you reflect in the y-axis?

Name:

## Rotations

**Plot the following sets of points, separately. Rotate each set of points separately. You are rotating around the origin (counter clock-wise).**

1.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| 90 Degree rotation | | | |
| 180 degree rotation | | | |
| 270 degree rotation | | | |

2.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| 90 Degree rotation | | | | |
| 180 degree rotation | | | | |
| 270 degree rotation | | | | |

3.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| 90 Degree rotation | | | | | | |
| 180 degree rotation | | | | | | |
| 270 degree rotation | | | | | | |

How do the coordinates change when you rotate 90 degrees around the origin?

How do the coordinates change when you rotate 180 degrees around the origin?

How do the coordinates change when you rotate 270 degrees around the origin?

# Plotting Points –Answer Key

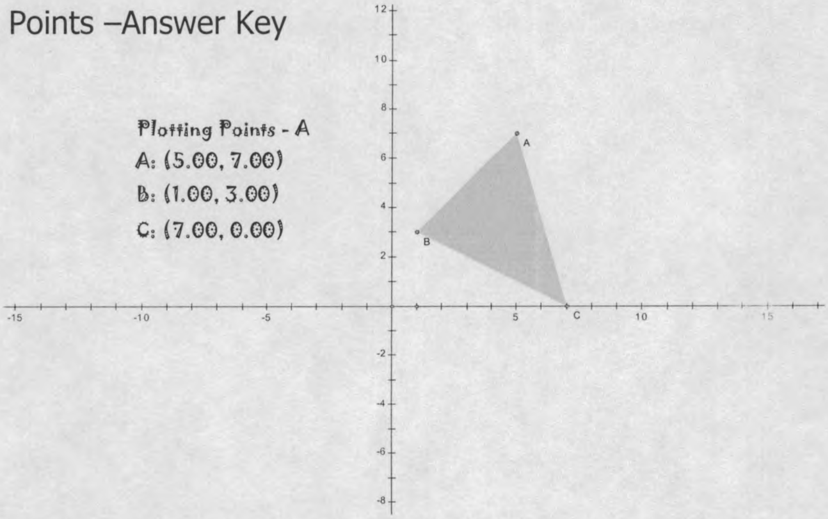
## A.

**Plotting Points - A**

A: (5.00, 7.00)

B: (1.00, 3.00)

C: (7.00, 0.00)



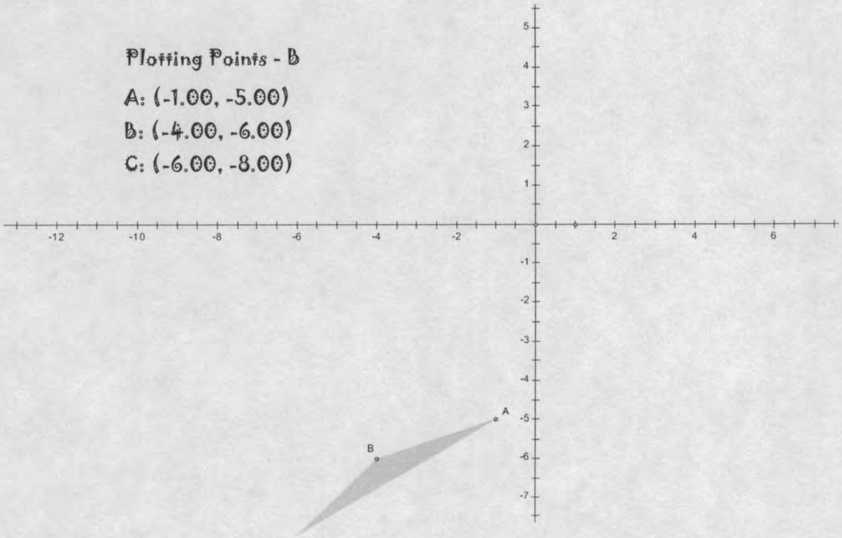
## B.

**Plotting Points - B**

A: (-1.00, -5.00)

B: (-4.00, -6.00)

C: (-6.00, -8.00)



## C.

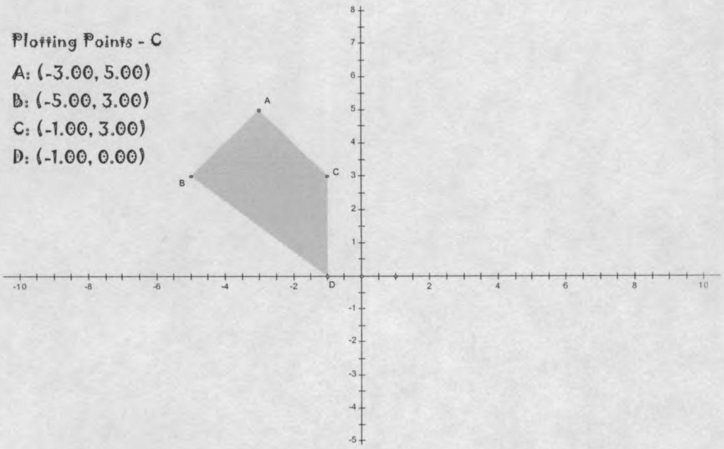
**Plotting Points - C**

A: (-3.00, 5.00)

B: (-5.00, 3.00)

C: (-1.00, 3.00)

D: (-1.00, 0.00)



Name:

## Translations—ANSWER KEY

**1. A(1,3) B(5,3) C(5,1) D(1,1)**
Translate this set of points along vector **<-6, -3>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| **-5, 0** | **-1, 0** | **-1,-2** | **-5,-2** |

**2. A(1,-4) B(3,-1) C(5,-4)**
Translate this set of points along vector **<0, -7>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| **1,-11** | **3,-8** | **5,-11** |

**3. A(-3,-2) B(-3,-5) C(-5,-5)**
Translate this set of points along vector **<4, 5>**
Record the new set of points.

| A' | B' | C' |
|---|---|---|
| **1,3** | **1,0** | **-1,0** |

**4. A(-4,5) B(-3,3) C(-1,3) D(-1,1) E(-4,1)**
Translate this set of points along vector **<8, -3>**
Record the new set of points.

| A' | B' | C' | D' | E' |
|---|---|---|---|---|
| **4, 2** | **5, 0** | **7, 0** | **7, -2** | **4, -2** |

**5. A(1,1) B(4,1) C(4,3) D(1,3)**
Translate this set of points along vector **<-6, 0>**
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| **-5, 0** | **-2, 1** | **-2, 3** | **-5, 3** |

6. A(-2,0) B(0,3) C(2,0) D(0,-3)
Translate this set of points along vector <5, 7>
Record the new set of points.

| A' | B' | C' | D' |
|---|---|---|---|
| **3, 7** | **5, 10** | **7, 7`** | **5, 4** |

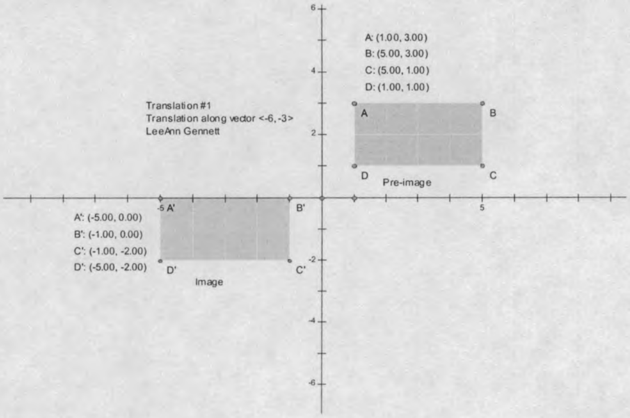
How do the coordinates change when you translate left or right?
**The y-coordinate stays the same, while the x-coordinate changes depending on if the translation is left or right.**

How do the coordinates change when you translate up or down?
**The x-coordinate stays the same, while the y-coordinate changes depending on if the translation is up or down.**

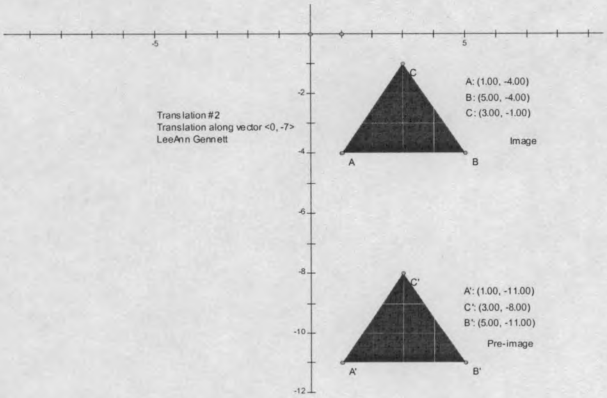
Say you had to plot the following: **A(-99,5) B(73, 80) C(0, 0)**
And translate these points along vector <21, -31>
What would the new coordinates be? **A'(-78, -26) B'(94, 49) C'(21, -31)**
Did you plot them or do this operation some other way?
**Add the x and y coordinates.**

# Translations- Answer Key

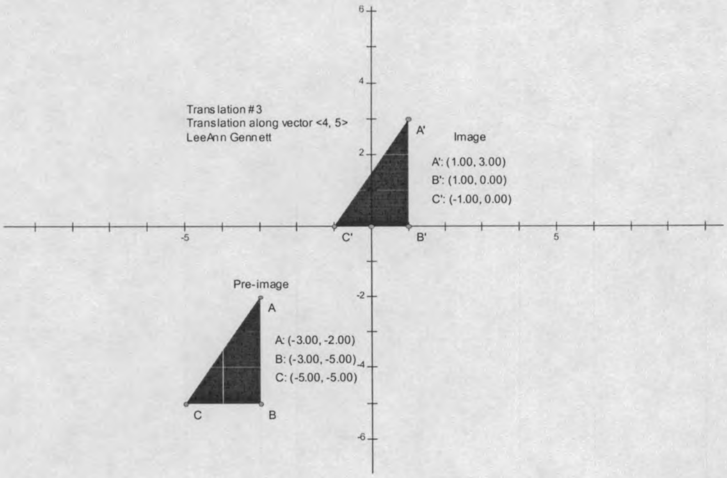
## Translation 1



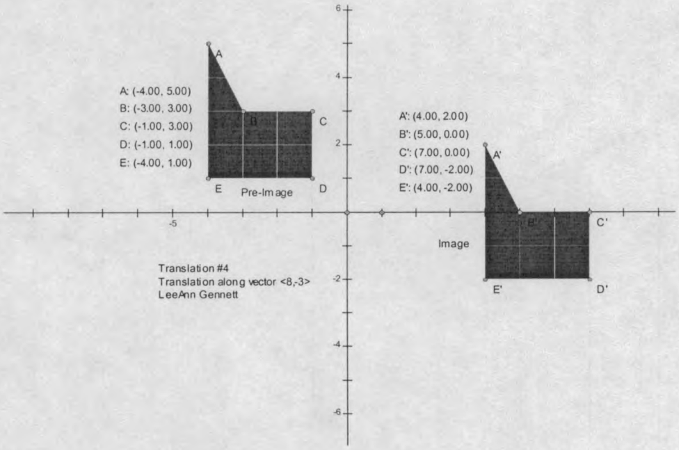

## Translation 2



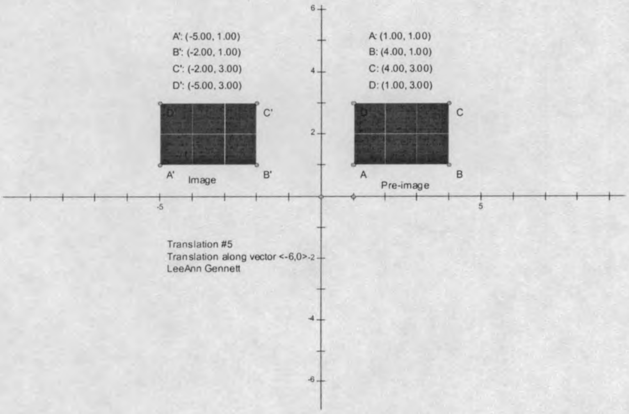

## Translation 3



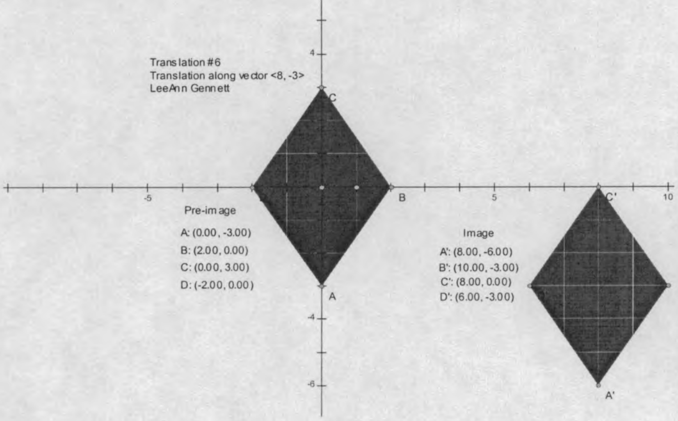
# Translation 4




# Translation 5




# Translation 6



Name:

## Reflections- ANSWER KEY

Plot the following sets of points, separately. Reflect each set (the original - pre-image) along the x-axis and then reflect along the y-axis. Record your results for each.
To reflect, plot each set of points and construct the polygon interior.
Measure and label the points.
Highlight the x-axis and go to **Transform->Mark Mirror.**
Then highlight the polygon with the points and go to **Transform->Reflect**.
Measure and label the points.
Record the results in the chart.
Repeat the following for the y-axis.

A.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| Reflection x-axis | **1,-1** | **3,-1** | **1,-5** |
| Reflection y-axis | **-1,1** | **-3,1** | **-1,5** |

B.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| Reflection x-axis | **1,1** | **6,1** | **6,3** | **1,3** |
| Reflection y-axis | **-1,-1** | **-6,-1** | **-6,-3** | **-1,-3** |

C.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| Reflection x-axis | **-5,-3** | **-4,-4** | **-2,-4** | **-1,-3** | **-2,-2** | **-4,-2** |
| Reflection y-axis | **5,3** | **4,4** | **2,4** | **1,3** | **2,2** | **4,2** |

How do the coordinates change when you reflect in the x-axis?

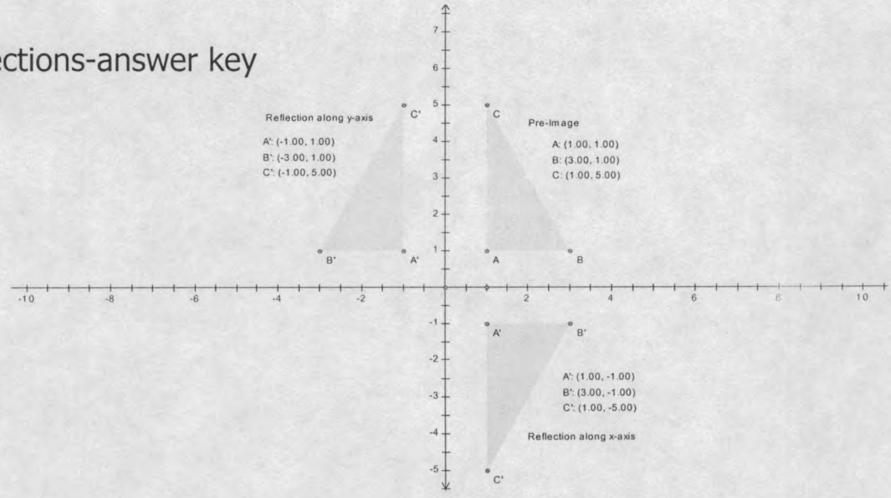
**The x-coordinates stays the same and the y-coordinate gets the opposite sign.**

How do the coordinates change when you reflect in the y-axis?

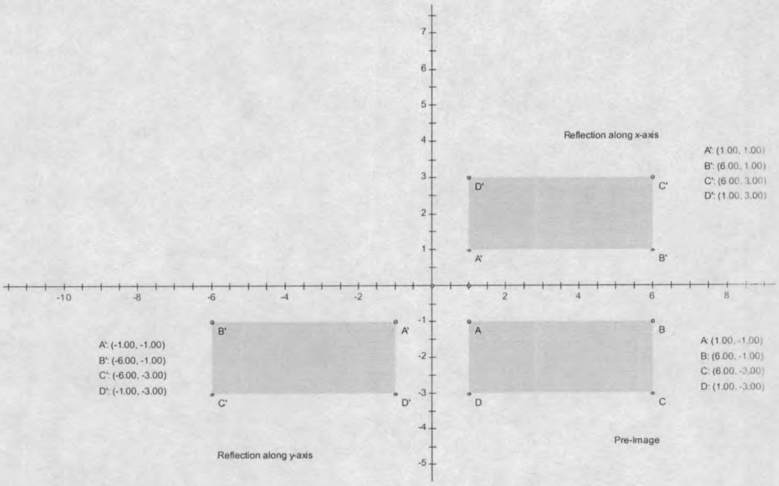
**The x-coordinates gets the opposite sign and the y-coordinate stays the same.**

# Reflections-answer key

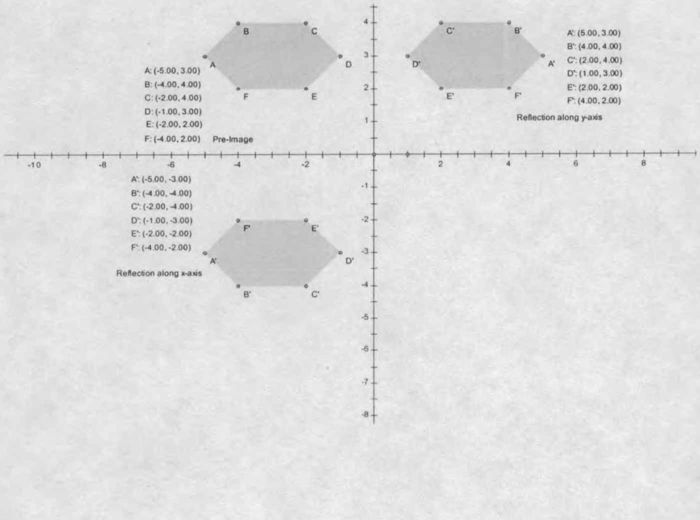
A.




B.




C.



Name:

**Rotations—ANSWER KEY**

YOU ARE ROTATING AROUND THE ORIGIN, SO TO ROTATE FOLLOW THESE INSTRUCTIONS:

13- HIGHLIGHT THE ORIGIN (0,0)
14- GO TO TRANSFORM->MARK CENTER
15- HIGHLIGHT ENTIRE FIGURE
16- GO TO TRANSFORM->ROTATE
17- TYPE IN ROTATE 90 DEGREE COUNTER-CLOCKWISE THEN FROM ORIGINAL 180 THEN 270
18- DO THIS THREE TIMES FOR EACH. GO FROM THE ORIGINAL

**Plot the following sets of points, separately. Rotate each set of points separately. Plot your points and construct the polygon interior. You are rotating around the origin (counter clock-wise).**

1.

| **Original** | **A(1,1)** | **B(3,1)** | **C(1,5)** |
|---|---|---|---|
| 90 Degree rotation | **-1, 1** | **-1, 3** | **-5, 1** |
| 180 degree rotation | **-1, -1** | **-3, -1** | **-1, -5** |
| 270 degree rotation | **1, -1** | **1, -3** | **5, -1** |

2.

| **Original** | **A(1,-1)** | **B(6,-1)** | **C(6,-3)** | **D(1,-3)** |
|---|---|---|---|---|
| 90 Degree rotation | **1, 1** | **1, 6** | **3, 6** | **3, 1** |
| 180 degree rotation | **-1, 1** | **-6, 1** | **-6, 3** | **-1, 3** |
| 270 degree rotation | **-1, -1** | **-1, -6** | **-3, -6** | **-3, -1** |

3.

| **Original** | **A(-5,3)** | **B(-4,4)** | **C(-2,4)** | **D(-1,3)** | **E(-2,2)** | **F(-4,2)** |
|---|---|---|---|---|---|---|
| 90 Degree rotation | **-3, -5** | **-4, -4** | **-4, -2** | **-3, -1** | **-2, -2** | **-2, -4** |
| 180 degree rotation | **5, -3** | **4, -4** | **2, -4** | **1, 3** | **2, -2** | **4, -2** |
| 270 degree rotation | **3, 5** | **4, 4** | **4, 2** | **3, 1** | **2, 2** | **2, 4** |

How do the coordinates change when you rotate 90 degrees around the origin?

**X and y coordinates change places and the new x-coordinate gets an opposite sign**

How do the coordinates change when you rotate 180 degrees around the origin?

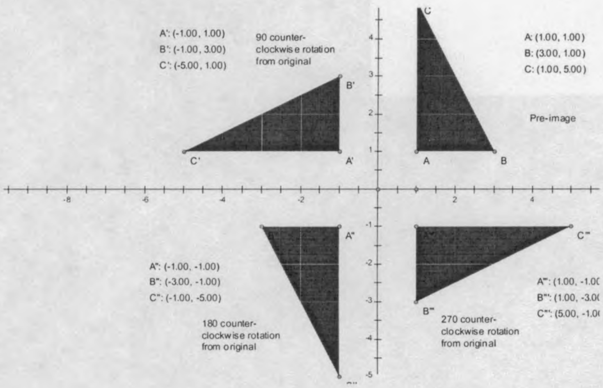
**X and y coordinates stay the same, but both the x and y coordinates get an opposite sign**

How do the coordinates change when you rotate 270 degrees around the origin?

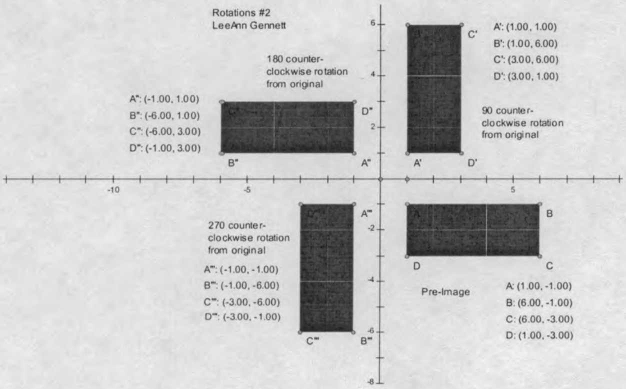
**X and y coordinates change places and the new y-coordinate gets an opposite sign**

# Rotations- Answer Key

## Rotation #1




## Rotation #2




## Rotation #3